

Chapter 9: Areas of Parallelograms and Triangles

NCERT Exemplar · Class 9 · Maths

Exercise 9.1 · Textbook page 85

Question 1

The median of a triangle divides it into two

- (A) triangles of equal area
- (B) congruent triangles
- (C) right triangles
- (D) isosceles triangles

Solution

(A) triangles of equal area

Explanation:

The median of a triangle divides it into triangles of equal area.

Hence, option (A) is the correct answer.

Question 2

In which of the following figures (Fig. 9.3) do you find two polygons on the same base and between the same parallels?

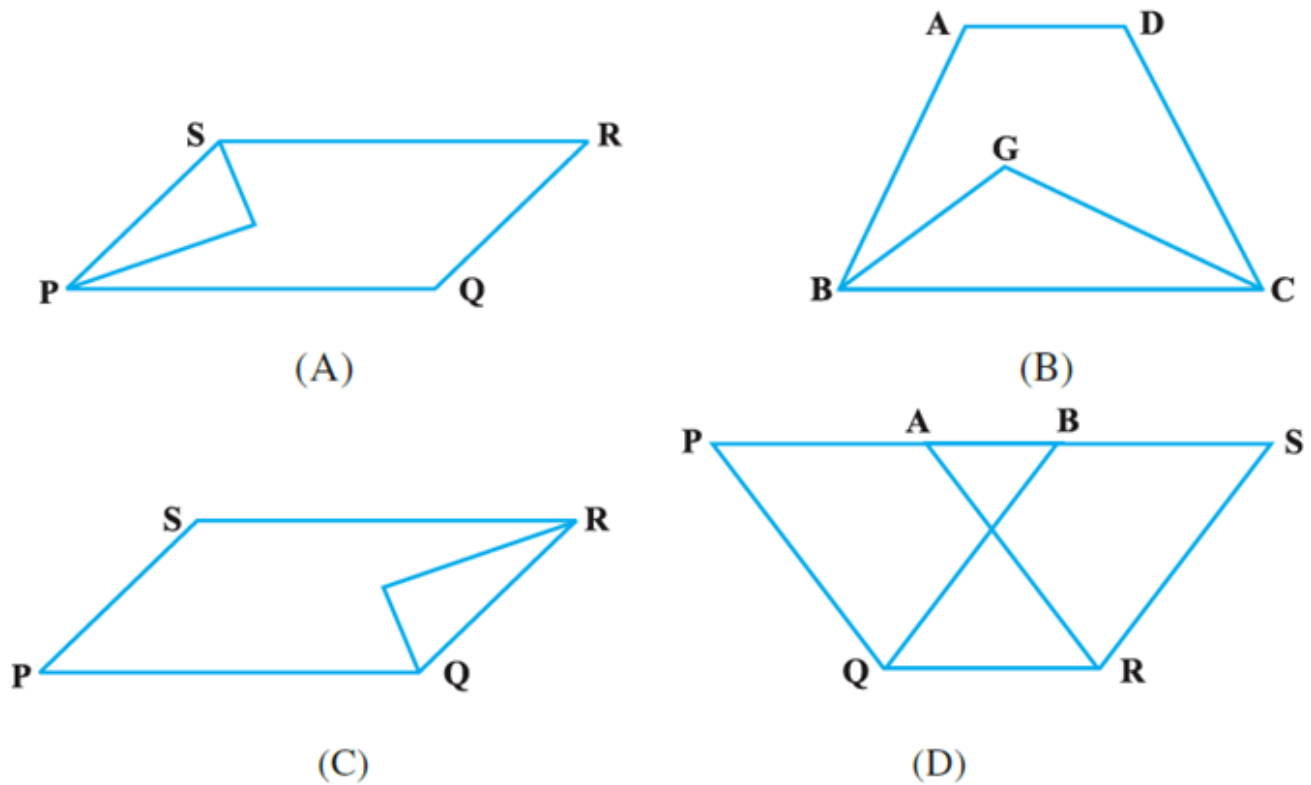


Fig. 9.3

Solution**(D)**

Explanation:

In figure (D), the parallelograms, PQRA and BQRS, are on the same base QR and between the same parallels QR and PS.

Hence, option (D) is the correct answer.

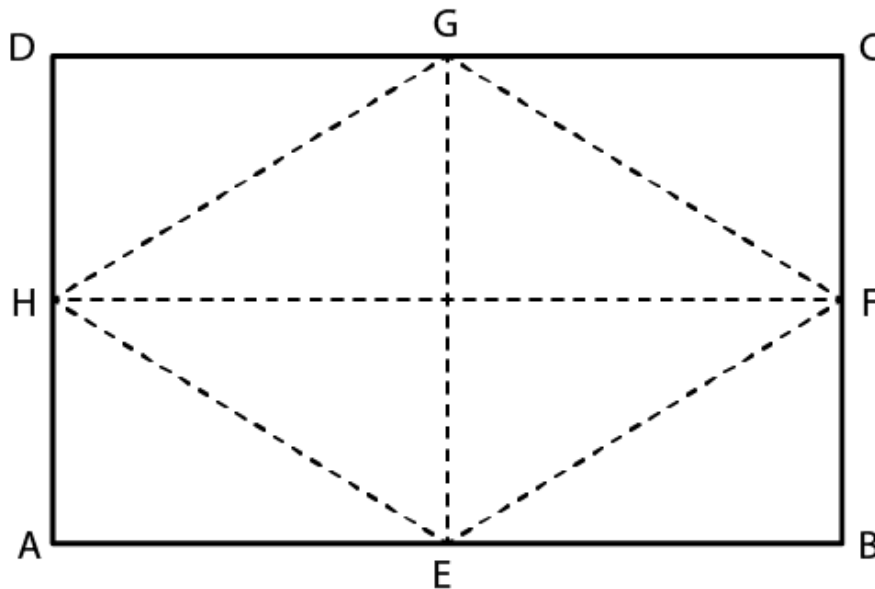
Question 3

The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is :

- (A) a rectangle of area 24 cm^2
- (B) a square of area 25 cm^2
- (C) a trapezium of area 24 cm^2
- (D) a rhombus of area 24 cm^2

Solution**(D) a rhombus of area 24 cm^2**

Explanation:



According to the question,

Let ABCD be the rectangle.

E is the mid-point of the side AB

F is the mid-point of the side BC

G is the mid-point of the side CD

H is the mid-point of the side DA

The figure obtained by joining the midpoints E, F, G and H is a rhombus.

The sides of the rectangle are given,

We know that,

Side of the rectangle AB = diagonal of the rhombus FH = 8cm

Similarly,

Side of the rectangle AD = diagonal of the rhombus EG = 6cm

Then,

$$\text{Area of the rhombus} = \frac{1}{2} \times EG \times FH$$

$$= \frac{1}{2} \times 6 \times 8$$

$$= 24 \text{ cm}^2$$

Hence, option (D) is the correct answer.

Question 4

In Fig. 9.4, the area of parallelogram

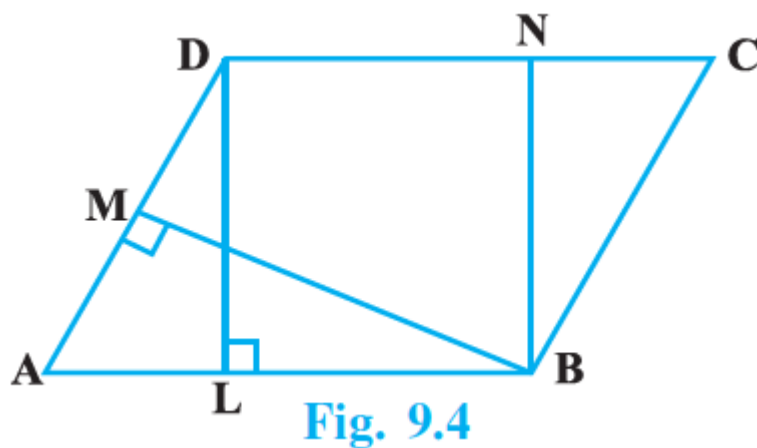
ABCD is:

(A) $AB \times BM$

(B) $BC \times BN$

(C) $DC \times DL$

(D) $AD \times DL$



Solution

(C) $DC \times DL$

Explanation:

$$\text{Area of parallelogram} = \text{Base} \times \text{Corresponding altitude}$$

$$= AB \times DL \dots (\text{eq 1})$$

Since opposite sides of a parallelogram are equal,

We get,

$$AB = DC$$

Substituting this in eq(1), we get,

$$\text{Area of parallelogram} = AB \times DL$$

$$= DC \times DL$$

Hence, option (C) is the correct answer.

Question 5

In Fig. 9.5, if parallelogram ABCD and rectangle ABEF are of equal area, then :

- (A) Perimeter of ABCD = Perimeter of ABEM
- (B) Perimeter of ABCD < Perimeter of ABEM
- (C) Perimeter of ABCD > Perimeter of ABEM
- (D) Perimeter of ABCD = $\frac{1}{2}$ (Perimeter of ABEM)

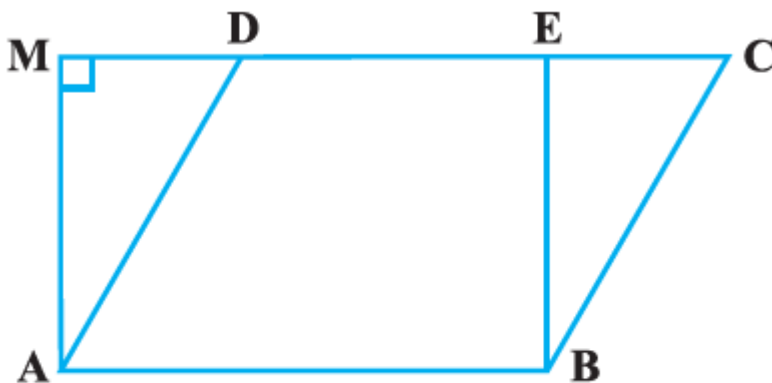


Fig. 9.5

Solution

- (C) Perimeter of ABCD > Perimeter of ABEM**

Explanation:

In rectangle ABEM,

$$AB = EM \dots (\text{eq.1}) \text{ [sides of rectangle]}$$

In parallelogram ABCD,

$$CD = AB \dots(\text{eq.2})$$

Adding equations (1) and (2),

We get

$$AB + CD = EM + AB \dots(\text{i})$$

We know that,

The perpendicular distance between two parallel sides of a parallelogram is always less than the length of the other parallel sides.

$$BE < BC \text{ and } AM < AD$$

On adding both above inequalities, we get

$$BE + AM < BC + AD \text{ or } BC + AD > BE + AM$$

On adding $AB + CD$ on both sides, we get

$$AB + CD + BC + AD > AB + CD + BE + AM$$

$$\Rightarrow AB + BC + CD + AD > AB + BE + EM + AM \quad [\because CD = AB = EM]$$

Hence,

We get,

The perimeter of parallelogram $ABCD >$ perimeter of rectangle $ABEM$

Hence, option (C) is the correct answer.

Exercise 9.2 · Textbook page 88

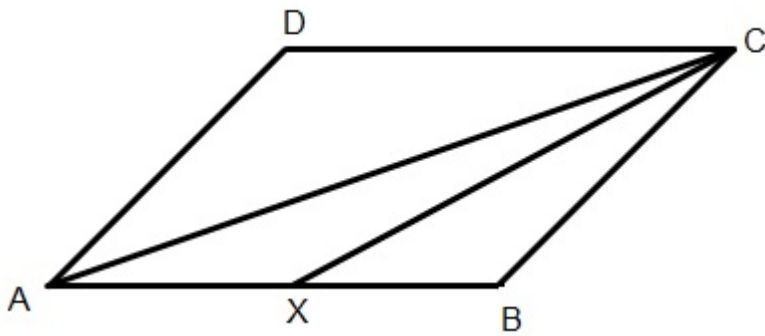
Question 1

$ABCD$ is a parallelogram and X is the mid-point of AB . If $\text{ar}(AXCD) = 24 \text{ cm}^2$, then $\text{ar}(ABC) = 24 \text{ cm}^2$.

Solution

False

Explanation:



In $\triangle ABC$, X is the midpoint of AB

Considering the given question,

$$\text{Hence } \text{ar}(\triangle AXC) = \text{ar}(\triangle BXC) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Therefore,

$$\text{ar}(\triangle AXC) = \text{ar}(\triangle BXC) = 12 \text{ cm}^2$$

$$\text{Area of } \parallel\text{gm } ABCD = 2 \times \text{ar}(\triangle ABC)$$

$$= 2 \times 24 = 48 \text{ cm}^2$$

But, according to the question,

$$\text{area of } \parallel\text{gm } ABCD = \text{ar}(\triangle AXC) + \text{ar}(\triangle BXC)$$

$$= 24 + 12 = 36 \text{ cm}^2$$

Hence, we find a contradiction here.

So, If $\text{ar}(\triangle AXC) = 24 \text{ cm}^2$, then $\text{ar}(\triangle ABC) \neq 24 \text{ cm}^2$

Question 2

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then $\text{ar}(\triangle PAS) = 30 \text{ cm}^2$.

Solution

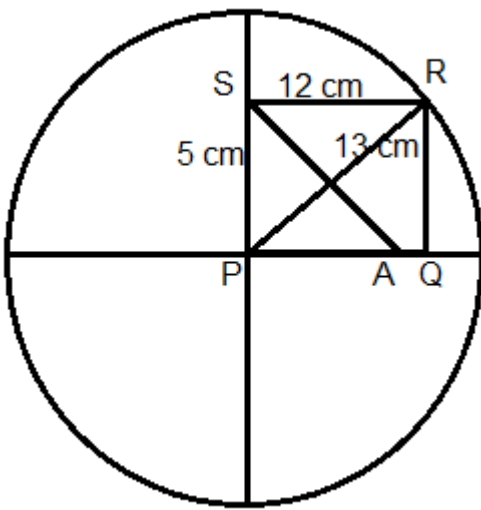
True. Since A is any point on PQ, then $\text{ar}(\triangle PAS) \neq 30 \text{ cm}^2$

But, the statement can be true if PA is equal to PS.

Justification:

According to the question,

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm.



Given,

A is any point on PQ

$PA < PQ$

$$\text{ar}(\triangle PQR) = \frac{1}{2} \times PQ \times QR$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

$$PS = 5 \text{ cm}$$

Suppose $PA < PQ$,

$$\text{ar}(\triangle PAS) < \text{ar}(\triangle PQR)$$

$$\text{ar}(\triangle PAS) < 30 \text{ cm}^2$$

Suppose $PA = PQ$,

$$\text{ar}(\triangle PAS) = \frac{1}{2} \times PQ \times PS$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Exercise 9.3 · Textbook page 89

Question 1

In Fig.9.11, PSDA is a parallelogram. Points Q and R are taken on PS such that $PQ = QR = RS$ and $PA \parallel QB \parallel RC$. Prove that $\text{ar}(\triangle PQE) = \text{ar}(\triangle CDF)$.

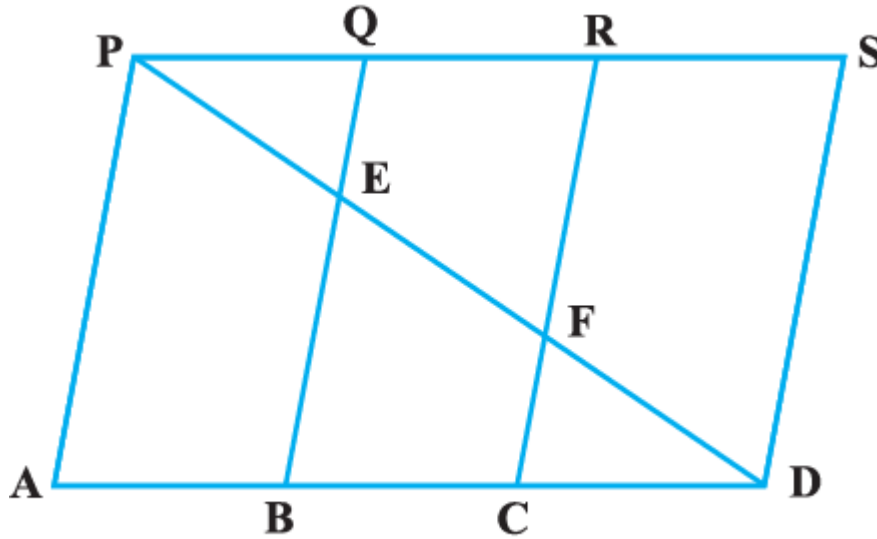


Fig. 9.11

Solution

According to the question,

$$PA \parallel QB \parallel RC \parallel SD \text{ \& } PQ=QR=RS$$

According to the Equal intercept theorem,

We know that if three or more parallel lines make equal intercepts on traversal, then they make equal intercepts on any other form of traversal.

Hence, we get,

$$PE=EF=FD \text{ \& } AB=BC=CD$$

From $\triangle PQE$ & $\triangle DCF$,

We get,

$$\angle PEQ = \angle DFC$$

$$PE = DF$$

$$\angle QPE = \angle CDF$$

So,

$$\Delta PQE \cong \Delta DCF$$

Since Congruent figures have equal areas,

We get,

$$\text{ar } \Delta PQE = \text{ar } \Delta DCF$$

Hence proved.

Question 2

X and Y are points on the side LN of the triangle LMN such that $LX = XY = YN$. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig. 9.12). Prove that

$$\text{ar } (LZY) = \text{ar } (MZYX)$$

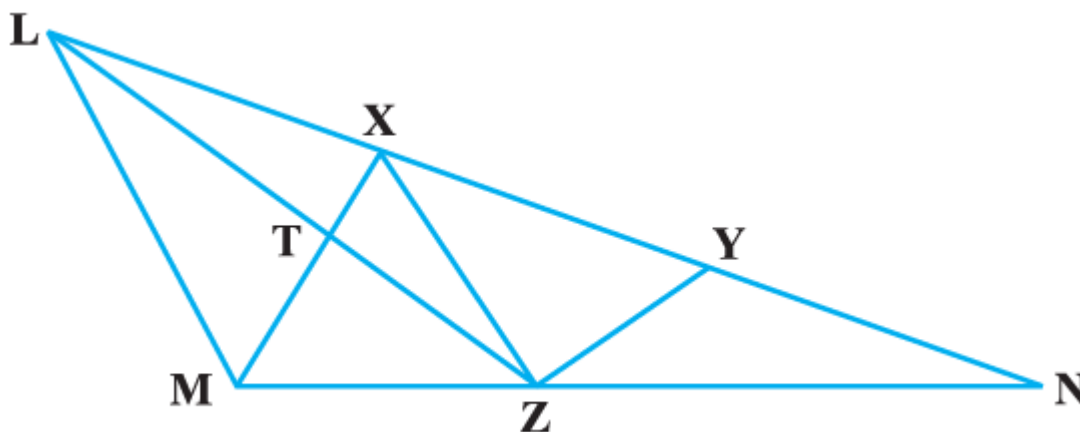


Fig. 9.12

Solution

According to the question,

$$LX = XY = YN$$

$$XZ \parallel LM$$

We have,

$$\text{ar}(LZX) + \text{ar}(XZY) = \text{ar}(LZY) \quad \text{--- (1)}$$

$$\text{ar}(MXZ) + \text{ar}(XZY) = \text{ar}(MZYX) \quad \text{--- (2)}$$

Both triangles LZX and MXZ are on the same base XZ and between the same parallels LM and XZ

$$\text{ar}(\text{LZX}) = \text{ar}(\text{MXZ})$$

Adding equations (1) and (2),

We get,

$$\text{ar}(\text{LZY}) = \text{ar}(\text{MZYX})$$

Hence proved

Question 3

The area of the parallelogram ABCD is 90 cm^2 (see Fig.9.13). Find

(i) $\text{ar}(\text{ABEF})$

(ii) $\text{ar}(\text{ABD})$

(iii) $\text{ar}(\text{BEF})$

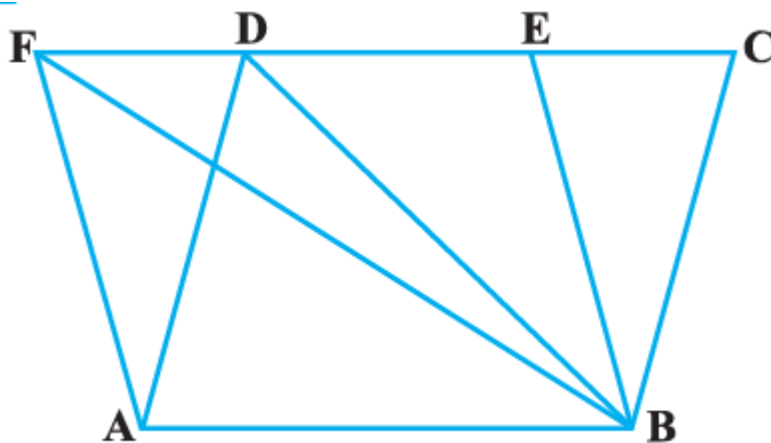


Fig. 9.13

Solution

According to the question,

$$\text{Area of parallelogram, ABCD} = 90 \text{ cm}^2$$

(i)

We know that,

Parallelograms on the same base and between the same parallel are equal in area.

Here,

The parallelograms ABCD and ABEF are on the same base AB and between the same parallels AB and CF.

$$\text{Therefore, ar (ABEF) = ar (ABCD) = 90 cm}^2$$

(ii)

We know that,

If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half of the area of the parallelogram.

Here,

$\triangle ABD$ and parallelogram ABCD are on the same base AB and between the same parallels AB and CD.

$$\text{Therefore, ar } (\triangle ABD) = \frac{1}{2} \text{ ar (ABCD)}$$

$$= \frac{1}{2} \times 90 = 45 \text{ cm}^2$$

(iii)

We know that,

If a triangle and a parallelogram are on the same base and between the same parallels, then the area of a triangle is equal to half of the area of the parallelogram.

Here,

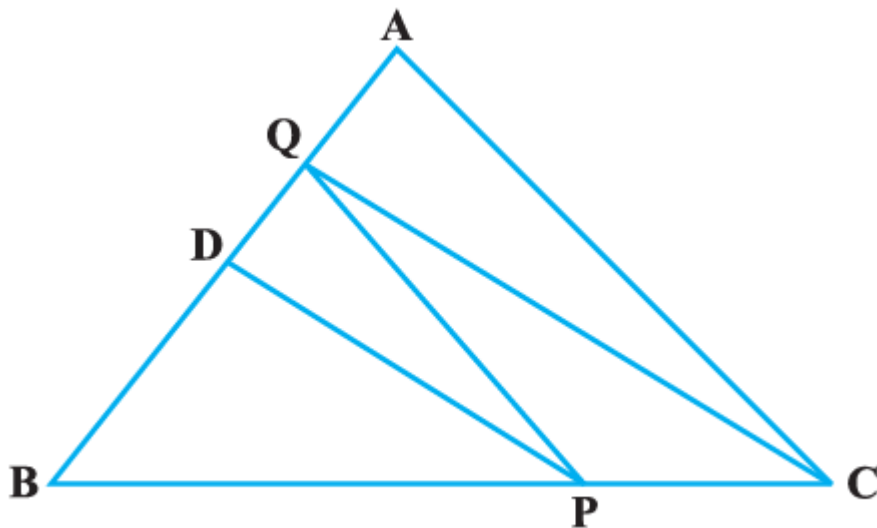
$\triangle BEF$ and parallelogram ABEF are on the same base EF and between the same parallels AB and EF.

$$\text{Therefore, ar } (\triangle BEF) = \frac{1}{2} \text{ ar (ABEF)}$$

$$= \frac{1}{2} \times 90 = 45 \text{ cm}^2$$

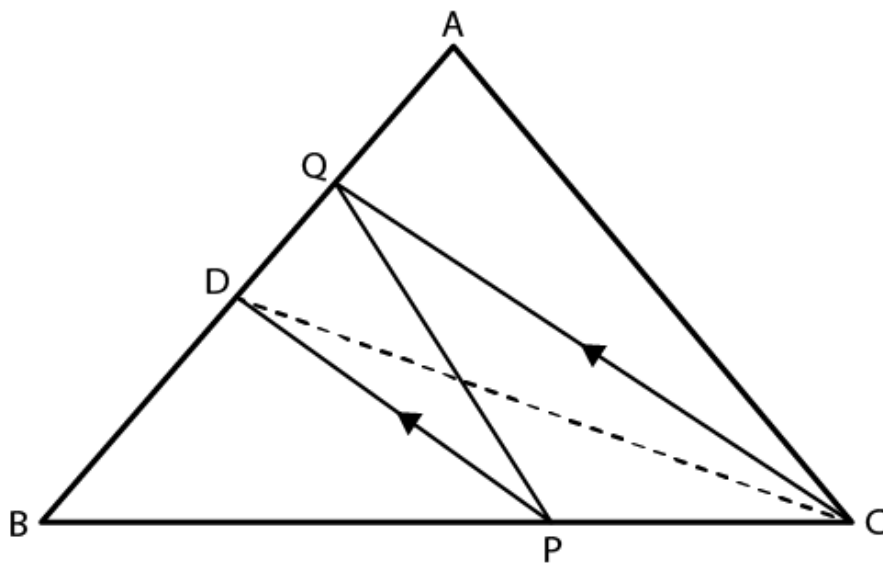
Question 4

In $\triangle ABC$, D is the mid-point of AB, and P is any point on BC. If $CQ \parallel PD$ meets AB in Q (Fig. 9.14), then prove that $\text{ar (BPQ)} = \frac{1}{2} \text{ ar (ABC)}$.

**Fig. 9.14****Solution**

According to the question,

We have the figure,



In $\triangle ABC$,

D is the mid-point of AB

P is any point on BC.

If $CQ \parallel DP$ meets AB at Q

To prove: $\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$

Construction: Join DC

Proof:- Since D is the mid-point of AB. so, in $\triangle ABC$, CD is the median.

$$\text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC) \dots\dots (1)$$

We know that,

$\triangle PDQ$ and $\triangle PDC$ are on the same base PD and between the same parallel lines PD QC.

$$\text{Therefore, } \text{ar}(\triangle PDQ) = \text{ar}(\triangle PDC) \dots\dots\dots (2)$$

From (1) and (2)

$$\text{ar}(\triangle BCD) = \frac{1}{2} \text{ar}(\triangle ABC)$$

$$\text{ar}(\triangle BCD) = \text{ar}(\triangle BPD) + \text{ar}(\triangle PDC)$$

$$= \frac{1}{2} \text{ar}(\triangle ABC)$$

$$\text{Area of } \triangle PDC = \triangle PDQ$$

So,

$$\text{ar}(\triangle BPQ) = \text{ar}(\triangle BPD) + \text{ar}(\triangle PDQ)$$

$$= \frac{1}{2} \text{ar}(\triangle ABC)$$

$$\text{Therefore, } \text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Hence Proved

Exercise 9.4 · Textbook page 94

Question 1

A point, E, is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F.

Prove that $\text{ar}(\triangle ADF) = \text{ar}(\triangle ABFC)$

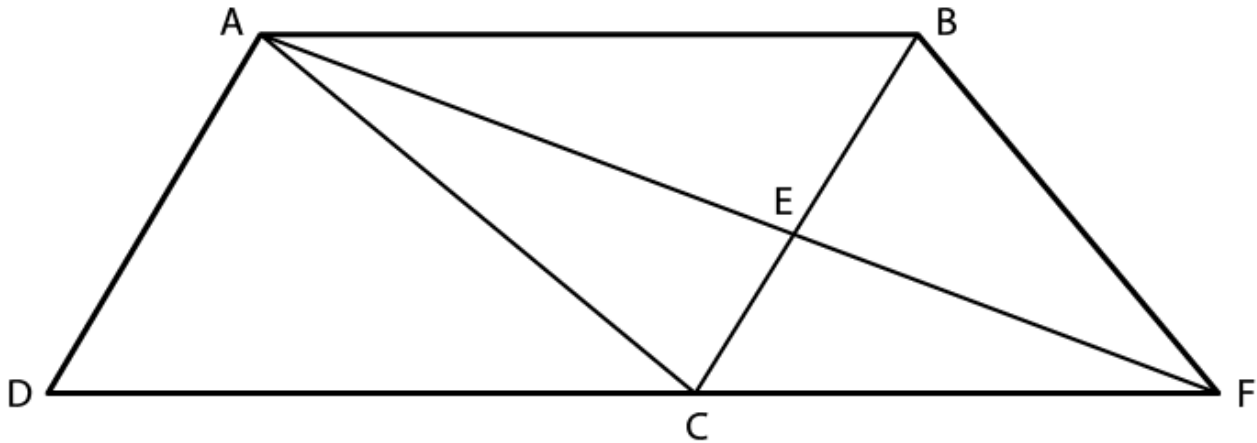
Solution

According to the question,

ABCD is a parallelogram.

E is a point on BC.

AE and DC are produced to meet at F.



To prove: $\text{area}(\triangle ADF) = \text{area}(\triangle ABFC)$.

Proof:

We know that,

Triangles on the same base and between the same parallels are equal in area.

Here,

We have,

$\triangle ABC$ and $\triangle ABF$ are on the same base AB and between the same parallels, $AB \parallel CF$.

$$\text{Area}(\triangle ABC) = \text{area}(\triangle ABF) \dots (1)$$

We also know that,

The diagonal of a Parallelogram divides it into two triangles of equal area

So,

$$\text{Area}(\triangle ABC) = \text{area}(\triangle ACD) \dots (2)$$

Now,

$$\text{Area}(\triangle ADF) = \text{area}(\triangle ACD) + \text{area}(\triangle ACF)$$

$\therefore \text{Area}(\triangle ADF) = \text{area}(\triangle ABC) + \text{area}(\triangle ACF) \dots$ (From equation (2))

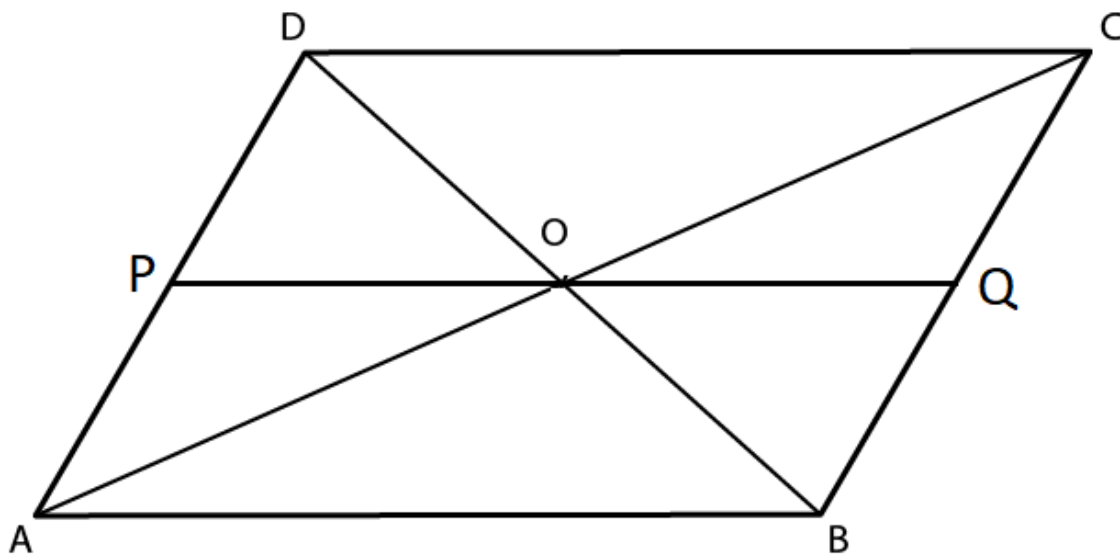
$\Rightarrow \text{Area}(\triangle ADF) = \text{area}(\triangle ABF) + \text{area}(\triangle ACF) \dots$ (From equation (1))

$\Rightarrow \text{Area}(\triangle ADF) = \text{area}(\triangle BFC)$

Question 2

The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

Solution



According to the question,

The diagonals of a parallelogram ABCD intersect at point O.

Through O, a line is drawn to intersect AD at P and BC at Q.

To Prove:

$\text{Ar}(\text{parallelogram PDCQ}) = \text{ar}(\text{parallelogram PQBA})$.

Proof:

AC is a diagonal of || gm ABCD

$\therefore \text{ar}(\triangle ABC) = \text{ar}(\triangle ACD)$

$$= \frac{1}{2} \text{ar}(\parallel\text{gm ABCD}) \dots(1)$$

In $\triangle AOP$ and $\triangle COQ$,

$$AO = CO$$

Since, diagonals of a parallelogram bisect each other,

We get,

$$\angle AOP = \angle COQ$$

$$\angle OAP = \angle OCQ \text{ (Vertically opposite angles)}$$

$$\therefore \triangle AOP = \triangle COQ \text{ (Alternate interior angles)}$$

$$\therefore \text{ar}(\triangle AOP) = \text{ar}(\triangle COQ) \text{ (By ASA Congruence Rule)}$$

We know that,

Congruent figures have equal areas

So,

$$\text{ar}(\triangle AOP) + \text{ar}(\text{parallelogram OPDC}) = \text{ar}(\triangle COQ) + \text{ar}(\text{parallelogram OPDC})$$

$$\Rightarrow \text{ar}(\triangle ACD) = \text{ar}(\text{parallelogram PDCQ})$$

$$\Rightarrow \frac{1}{2} \text{ar}(\parallel \text{gm ABCD}) = \text{ar}(\text{parallelogram PDCQ})$$

From equation (1),

We get,

$$\text{ar}(\text{parallelogram PQBA}) = \text{ar}(\text{parallelogram PDCQ})$$

$$\Rightarrow \text{ar}(\text{parallelogram PDCQ}) = \text{ar}(\text{parallelogram PQBA}).$$

Hence Proved.

Question 3

The medians BE and CF of a triangle ABC intersect at G. Prove that the area of $\triangle GBC$ = area of the quadrilateral AFGE.

Solution

According to the question,

We have,

BE & CF are medians

E is the midpoint of AC

F is the midpoint of AB

$$\therefore \triangle BCE = \triangle BEA \dots (i)$$

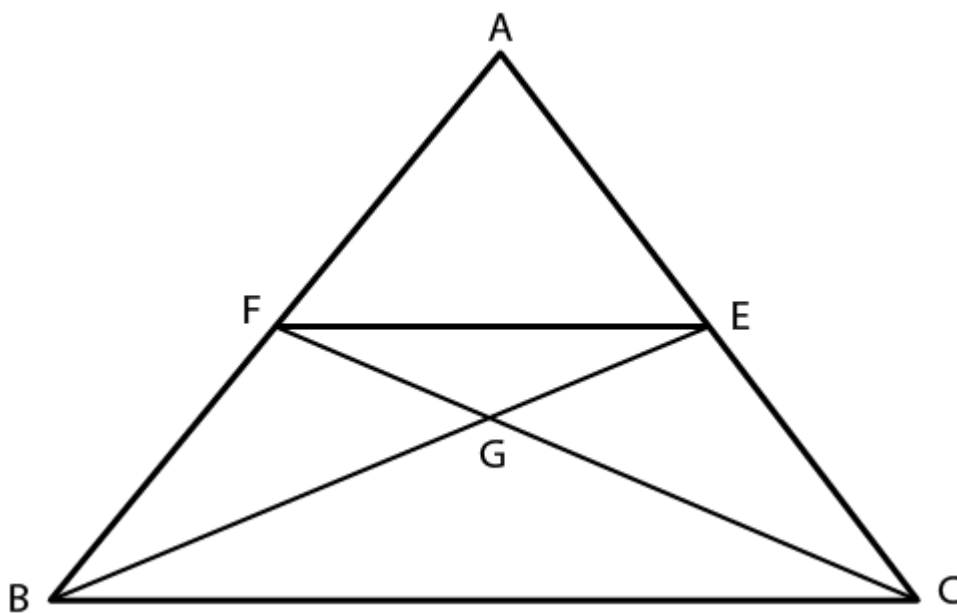
$$\triangle BCF = \triangle CAF$$

Construct:

Join EF,

By the midpoint theorem,

We get $FE \parallel BC$



We know that,

Δ on the same base and between the same parallels are equal in area

$$\therefore \triangle FBC = \triangle BCE$$

$$\triangle FBC - \triangle GBC = \triangle BCE - \triangle GBC$$

$$\Rightarrow \triangle FBG = \triangle CGE \text{ } (\triangle GBC \text{ is common})$$

$$\Rightarrow \triangle CGE = \triangle FBG \text{ } \dots (ii)$$

Subtracting equation (ii) from (i)

We get,

$$\triangle BCE - \triangle CGE = \triangle BEA - \triangle FBG$$

$$\therefore \triangle BGC = \text{quadrilateral AFGE}.$$

Question 4

In Fig. 9.24, $CD \parallel AE$ and $CY \parallel BA$. Prove that $\text{ar}(\triangle CBX) = \text{ar}(\triangle AXY)$.

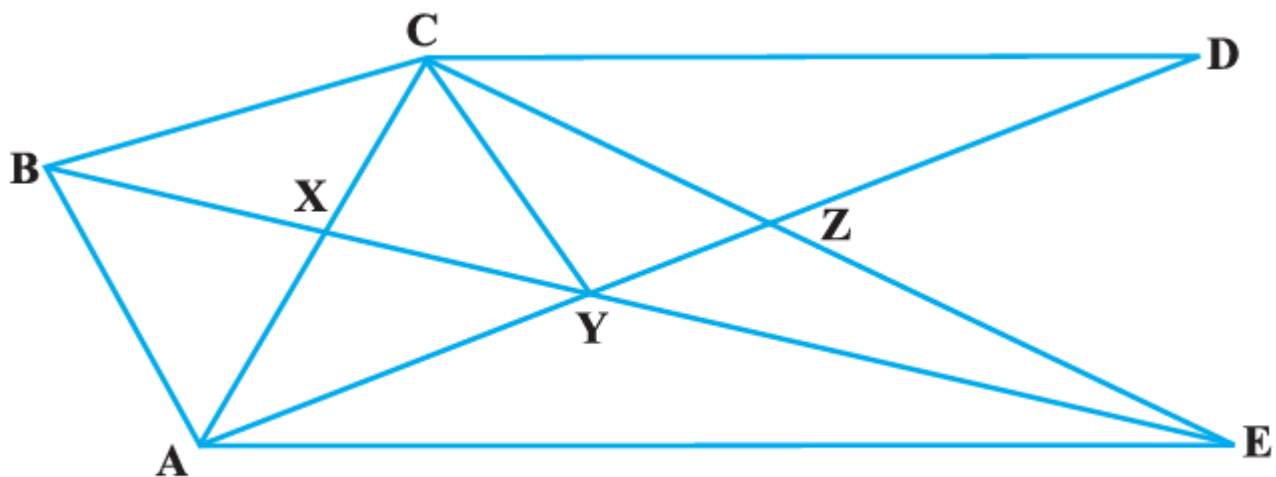


Fig. 9.24

Solution

According to the question,

From figure,

We get,

$$CD \parallel AE$$

$$\text{and } CY \parallel BA$$

To prove:

$$\text{ar}(\triangle CBX) = \text{ar}(\triangle AXY).$$

Proof:

We know that,

Triangles on the same base and between the same parallels are equal in area.

Here,

$\triangle ABY$ and $\triangle ABC$ both lie on the same base AB and between the same parallels CY and BA .

$$\text{ar}(\triangle ABY) = \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle ABX) + \text{ar}(\triangle AXY) = \text{ar}(\triangle ABX) + \text{ar}(\triangle CBX)$$

Solving and cancelling $\text{ar}(\triangle ABX)$,

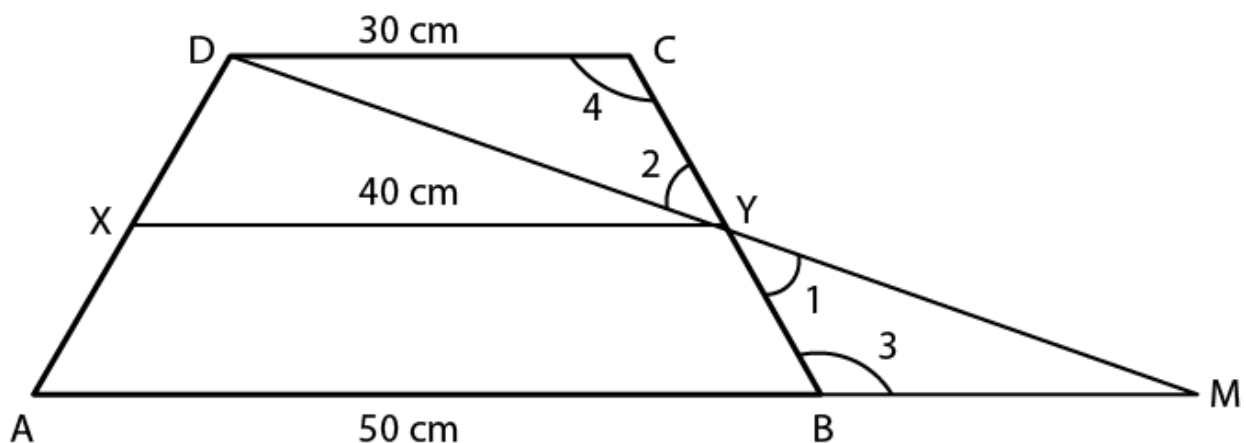
We get,

$$\Rightarrow \text{ar}(\triangle AXY) = \text{ar}(\triangle CBX)$$

Question 5

$ABCD$ is a trapezium in which $AB \parallel DC$, $DC = 30$ cm and $AB = 50$ cm. If X and Y are, respectively, the mid-points of AD and BC , prove that $\text{ar}(\triangle DCYX) = \frac{7}{9} \text{ar}(\triangle XYBA)$

Solution



According to the question,

We have,

ABCD is a trapezium with $AB \parallel DC$

Construction: Join DY and produce it to meet AB produced at P.

In $\triangle BYP$ and $\triangle CYD$

$\angle BYP = \angle CYD$ (vertically opposite angles)

Since alternate opposite angles of $DC \parallel AP$ and BC are the transversal

$\angle DCY = \angle PBY$

Since Y is the midpoint of BC,

$BY = CY$

Thus $\triangle BYP \cong \triangle CYD$ (by ASA congruence criterion)

So, $DY = YP$ and $DC = BP$

$\Rightarrow$ Y is the midpoint of AD

$\therefore XY \parallel AP$ and $XY = \frac{1}{2} AP$ (by midpoint theorem)

$\Rightarrow XY = \frac{1}{2} AP$

$= \frac{1}{2} (AB + BP)$

$= \frac{1}{2} (AB + DC)$

$= \frac{1}{2} (50 + 30)$

$= \frac{1}{2} \times 80 \text{ cm}$

$= 40 \text{ cm}$

Since X is the midpoint of AD

And Y is the midpoint of BC

Hence, trapezium DCYX and ABYX are of the same height, h , cm

Now

$$\frac{\text{area (DCYX)}}{\text{area (ABYX)}} = \frac{\frac{1}{2}(\text{DC} + \text{XY}) \times h}{\frac{1}{2}(\text{AB} + \text{XY}) h} = \frac{30+40}{50+40} = \frac{70}{90} = \frac{7}{9}$$

$$\Rightarrow 9 \text{ ar(DCXY)} = 7 \text{ ar(XYBA)}$$

$$\Rightarrow \text{ar(DCXY)} = 7/9 \text{ ar(XYBA)}$$

Hence Proved.
