

Chapter 8: Quadrilaterals

NCERT Exemplar · Class 9 · Maths

Exercise 8.1 · Textbook page 73

Question 1

Three angles of a quadrilateral are 75° , 90° and 75° . The fourth angle is

- (A) 90°
- (B) 95°
- (C) 105°
- (D) 120°

Solution

(D) 120°

Explanation:

According to the question,

Three angles of quadrilateral are 75° , 90° and 75°

Consider the fourth angle to be x .

We know that,

Sum of all angles of a quadrilateral = 360°

$$\Rightarrow 75^\circ + 90^\circ + 75^\circ + x = 360^\circ$$

$$\Rightarrow 240^\circ + x = 360^\circ$$

$$\Rightarrow x = 360^\circ - 240^\circ$$

$$\Rightarrow x = 120^\circ$$

Hence, the fourth angle is 120° .

Therefore, option (D) is the correct answer.

Question 2

A diagonal of a rectangle is inclined to one side of the rectangle at 25° . The acute angle between the diagonals is

- (A) 55°
- (B) 50°
- (C) 40°
- (D) 25°

Solution

(B) 50°

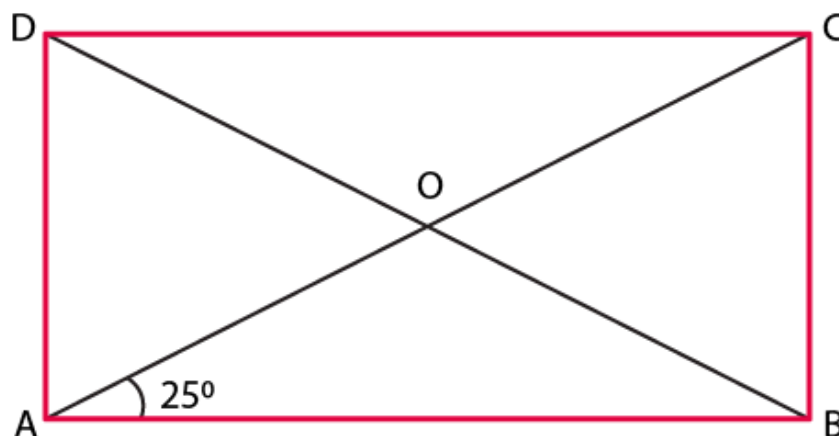
Explanation:

According to the question,

A diagonal of a rectangle is inclined to one side of the rectangle at 25°

i.e., Angle between a side of rectangle and its diagonal = 25°

Consider the acute angle between diagonals to be $= x$



We know that diagonals of a rectangle are equal in length i.e.,

$$AC = BD$$

Dividing RHS and LHS by 2,

$$\Rightarrow \frac{1}{2} AC = \frac{1}{2} BD$$

Since, O is mid-point of AC and BD

$$\Rightarrow OD = OC$$

Since, angles opposite to equal sides are equal

$$\Rightarrow \angle y = 25^\circ$$

We also know that,

Exterior angle is equal to the sum of two opposite interior angles.

$$\text{So, } \angle BOC = \angle ODC + \angle OCD$$

$$\Rightarrow \angle x = \angle y + 25^\circ$$

$$\Rightarrow \angle x = 25^\circ + 25^\circ$$

$$\Rightarrow \angle x = 50^\circ$$

Hence, the acute angle between diagonals is 50° .

Therefore, option (B) is the correct answer.

Question 3

ABCD is a rhombus such that $\angle ACB = 40^\circ$. Then $\angle ADB$ is

(A) 40°

(B) 45°

(C) 50°

(D) 60°

Solution

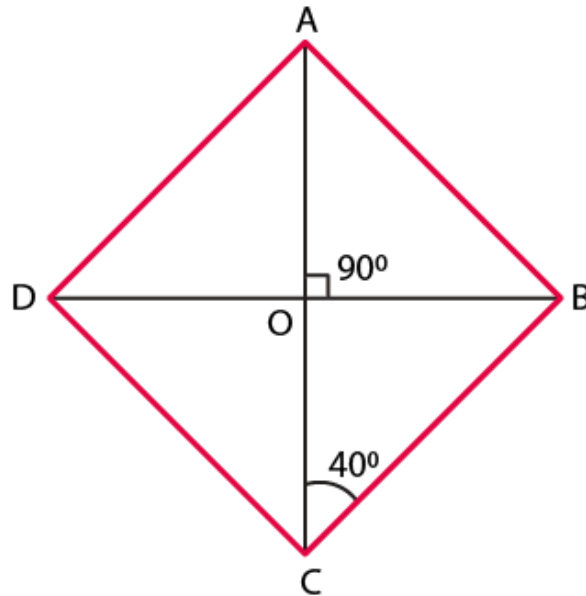
(C) 50°

Explanation:

According to the question,

ABCD is a rhombus

$$\angle ACB = 40^\circ$$



$$\because \angle ACB = 40^\circ$$

$$\Rightarrow \angle OCB = 40^\circ$$

$$\because AD \parallel BC$$

$$\Rightarrow \angle DAC = \angle BCA = 40^\circ \text{ [Alternate interior angles]}$$

$$\Rightarrow \angle DAO = 40^\circ$$

Since, diagonals of a rhombus are perpendicular to each other

We have,

$$\angle AOD = 90^\circ$$

We know that,

$$\text{Sum of all angles of a triangle} = 180^\circ$$

$$\Rightarrow \angle AOD + \angle ADO + \angle DAO = 180^\circ$$

$$\Rightarrow 90^\circ + \angle ADO + 40^\circ = 180^\circ$$

$$\Rightarrow 130^\circ + \angle ADO = 180^\circ$$

$$\Rightarrow \angle ADO = 180^\circ - 130^\circ$$

$$\Rightarrow \angle ADO = 50^\circ$$

$$\Rightarrow \angle ADB = 50^\circ$$

Hence, $\angle ADB = 50^\circ$

Therefore, option (C) is the correct answer.

Question 4

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral

PQRS, taken in order, is a rectangle, if

- (A) PQRS is a rectangle
- (B) PQRS is a parallelogram
- (C) diagonals of PQRS are perpendicular
- (D) diagonals of PQRS are equal.

Solution

(C) diagonals of PQRS are perpendicular

Explanation:

Let the rectangle be ABCD,

We know that,

Diagonals of rectangle are equal

$$\therefore AC = BD$$

$$\Rightarrow PQ = QR$$

$\therefore$ PQRS is a rhombus

Diagonals of a rhombus are perpendicular.

Hence, diagonals of PQRS are perpendicular

Therefore, option (C) is the correct answer.

Question 5

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral

PQRS, taken in order, is a rhombus, if

- (A) PQRS is a rhombus
- (B) PQRS is a parallelogram
- (C) diagonals of PQRS are perpendicular
- (D) diagonals of PQRS are equal.

Solution

(D) diagonals of PQRS are equal.

Explanation:

Since, ABCD is a rhombus

We have,

$$AB = BC = CD = DA$$

Now,

Since, D and C are midpoints of PQ and PS

By midpoint theorem,

We have,

$$DC = \frac{1}{2} QS$$

Also,

Since, B and C are midpoints of SR and PS

By midpoint theorem

We have,

$$BC = \frac{1}{2} PR$$

Now, again, ABCD is a rhombus

$$\therefore BC = CD$$

$$\Rightarrow \frac{1}{2} QS = \frac{1}{2} PR$$

$$\Rightarrow QS = PR$$

Hence, diagonals of PQRS are equal

Therefore, option (D) is the correct answer.

Question 6

If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio

3:7:6:4, then ABCD is a

(A) rhombus

(B) parallelogram

(C) trapezium

(D) kite

Solution

(C) trapezium

Explanation:

As angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3: 7: 6: 4,

We have the angles A, B, C and D = 3x, 7x, 6x and 4x.

Now, sum of the angle of a quadrilateral = 360° .

$$3x + 7x + 6x + 4x = 360^\circ$$

$$\Rightarrow 20x = 360^\circ$$

$$\Rightarrow x = 360 \div 20 = 18^\circ$$

So, the angles A, B, C and D of quadrilateral ABCD are,

$$\angle A = 3 \times 18^\circ = 54^\circ,$$

$$\angle B = 7 \times 18^\circ = 126^\circ$$

$$\angle C = 6 \times 18^\circ = 108^\circ$$

$$\angle D = 4 \times 18^\circ = 72^\circ$$

AD and BC are two lines cut by a transversal CD

Now, sum of angles $\angle C$ and $\angle D$ on the same side of transversal,

$$\angle C + \angle D = 108^\circ + 72^\circ = 180^\circ$$

Hence, $AD \parallel BC$

So, ABCD is a quadrilateral in which one pair of opposite sides are parallel.

Hence, ABCD is a trapezium.

Therefore, option (C) is the correct answer.

Question 7

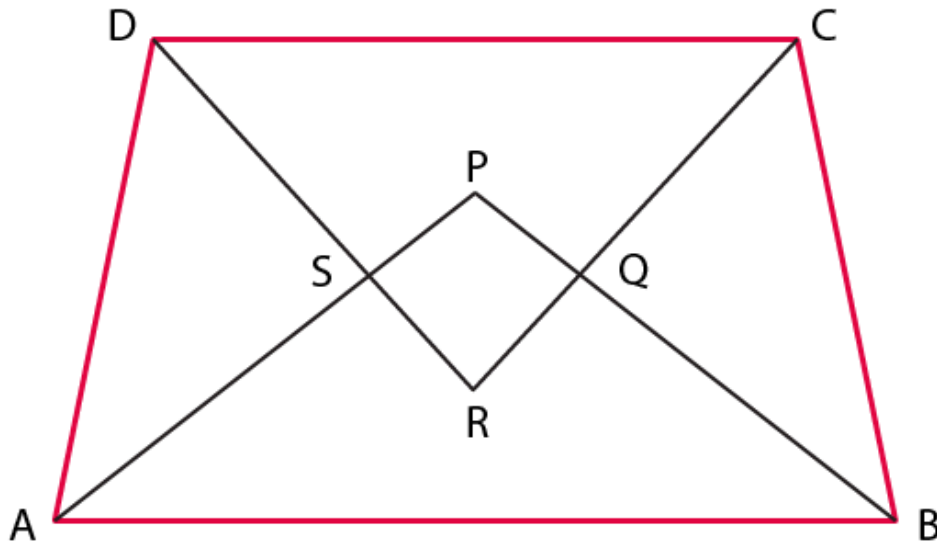
If bisectors of $\angle A$ and $\angle B$ of a quadrilateral ABCD intersect each other at P, of $\angle B$ and $\angle C$ at Q, of $\angle C$ and $\angle D$ at R and of $\angle D$ and $\angle A$ at S, then PQRS is a

- (A) rectangle
- (B) rhombus
- (C) parallelogram
- (D) quadrilateral whose opposite angles are supplementary

Solution

(D) quadrilateral whose opposite angles are supplementary

Explanation:



We know that,

Sum of all angles of a quadrilateral = 360°

$$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$$

Dividing LHS and RHS by 2,

$$\Rightarrow \frac{1}{2} (\angle A + \angle B + \angle C + \angle D) = \frac{1}{2} \times 360^\circ = 180^\circ$$

Since, AP, PB, RC and RD are bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$

$$\Rightarrow \angle PAB + \angle ABP + \angle RCD + \angle RDC = 180^\circ \dots (1)$$

We also know that,

Sum of all angles of a triangle = 180°

$$\angle PAB + \angle APB + \angle ABP = 180^\circ$$

$$\Rightarrow \angle PAB + \angle ABP = 180^\circ - \angle APB \dots (2)$$

Similarly,

$$\therefore \angle RDC + \angle RCD + \angle CRD = 180^\circ$$

$$\Rightarrow \angle RDC + \angle RCD = 180^\circ - \angle CRD \dots (3)$$

Substituting the value of equations (2) and (3) in equation (1),

$$180^\circ - \angle APB + 180^\circ - \angle CRD = 180^\circ$$

$$\Rightarrow 360^\circ - \angle APB - \angle CRD = 180^\circ$$

$$\Rightarrow \angle APB + \angle CRD = 360^\circ - 180^\circ$$

$$\Rightarrow \angle APB + \angle CRD = 180^\circ \dots(4)$$

Now,

$$\angle SPQ = \angle APB \text{ [vertically opposite angles]}$$

$$\angle SRQ = \angle DRC \text{ [vertically opposite angles]}$$

Substituting in equation (4),

$$\Rightarrow \angle SPQ + \angle SRQ = 180^\circ$$

Hence, PQRS is a quadrilateral whose opposite angles are supplementary.

Therefore, option (D) is the correct answer.

Exercise 8.2 · Textbook page 75

Question 1

Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Solution

According to the question,

$$OA = 3 \text{ cm}$$

$$OD = 2 \text{ cm}$$

We know that,

Diagonals of parallelogram bisect each other.

Then,

$$AC = 2AO$$

$$AC = 2 \times 3 \text{ cm}$$

$$AC = 6 \text{ cm}$$

And,

$$BD = 2OD$$

$$BD = 2 \times 2 \text{ cm}$$

$$BD = 4 \text{ cm}$$

Hence, $AC = 6 \text{ cm}$ and $BD = 4 \text{ cm}$

Question 2

Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Solution

The statement “diagonals of a parallelogram are perpendicular to each other” is false.

Justification:

Diagonals of a parallelogram bisect each other but not at 90° .

So, they are not perpendicular to each other.

Hence, this statement is false.

Question 3

Can the angles 110° , 80° , 70° and 95° be the angles of a quadrilateral? Why or why not?

Solution

The angles 110° , 80° , 70° and 95° cannot be the angles of a quadrilateral.

Justification:

We know that,

$$\text{Sum of all angles of a quadrilateral} = 360^\circ$$

Sum of given angles,

$$110^\circ + 80^\circ + 70^\circ + 95^\circ = 355^\circ \neq 360^\circ$$

Hence, 110° , 80° , 70° and 95° cannot be the angles of a quadrilateral.

Question 4

In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$. What special name can be given to this quadrilateral?

Solution

According to the question,

In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$

We know that,

In a trapezium,

Sum of co-interior angles = 180°

Hence, the given quadrilateral is a trapezium.

Question 5

All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Solution

According to the question,

All the angles of a quadrilateral are equal.

Suppose all the angles of the quadrilateral = x

We know that,

Sum of all angles of a quadrilateral = 360°

$$\Rightarrow x + x + x + x = 360^\circ$$

$$\Rightarrow 4x = 360^\circ$$

$$\Rightarrow x = 360^\circ/4$$

$$\Rightarrow x = 90^\circ$$

Hence, the quadrilateral is a rectangle.

Question 6

Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

Solution

The statement “diagonals of a rectangle are equal and perpendicular” is false.

We know that,

Diagonals of a rectangle bisect each other.

Therefore, they are equal but they are not perpendicular.

Hence, the statement is not true.

Question 7

Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

Solution

All the four angles of a quadrilateral cannot be obtuse angles.

Justification:

We know that,

Sum of all angles of a quadrilateral = 360°

So, at least one angle should be acute angle.

Hence, all the four angles of a quadrilateral cannot be obtuse angles.

Exercise 8.3 · Textbook page 78**Question 1**

One angle of a quadrilateral is of 108° and the remaining three angles are equal. Find each of the three equal angles.

Solution

Let the remaining three equal angles be x .

We know,

Sum of all interior angles of a quadrilateral is $= 360^\circ$

$$108^\circ + x + x + x = 360^\circ$$

$$108^\circ + 3x = 360^\circ$$

$$3x = 360^\circ - 108^\circ$$

$$3x = 252^\circ$$

$$x = 252/3$$

$$x = 84^\circ$$

Each of three equal angles, $x = 84^\circ$.

Question 2

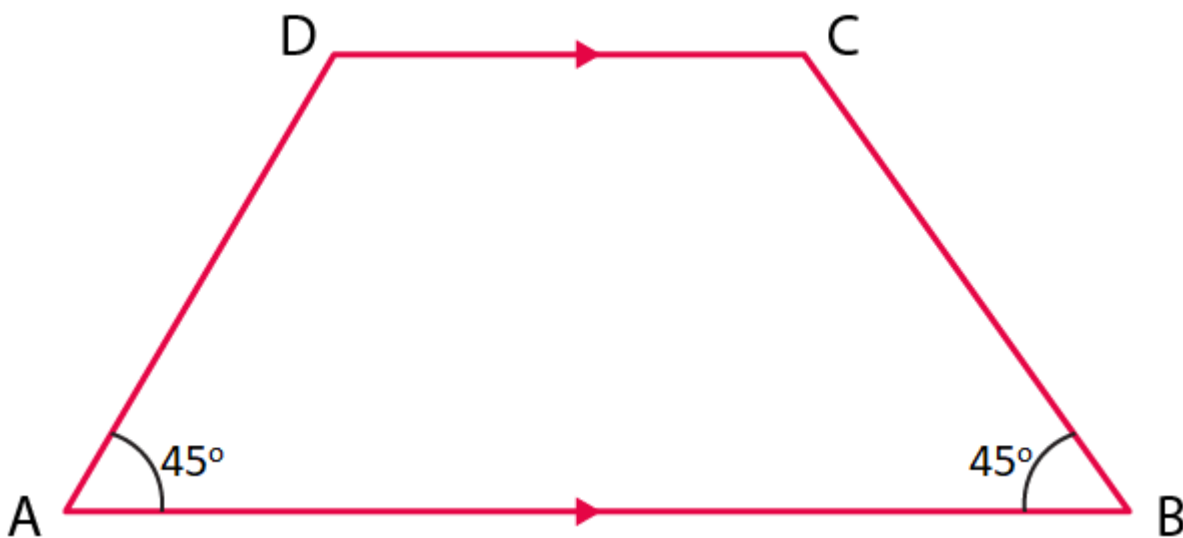
ABCD is a trapezium in which $AB \parallel DC$ and $\angle A = \angle B = 45^\circ$. Find angles C and D of the trapezium.

Solution

According to the question,

ABCD is a trapezium

$$\angle A = \angle B = 45^\circ$$



We know that,

Angles opposite to each other in quadrilateral are supplementary.

Then, we have,

$$\angle A + \angle C = 180^\circ$$

$$45^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 45^\circ$$

$$\angle C = 135^\circ$$

Similarly,

We have,

$$\angle B + \angle D = 180^\circ$$

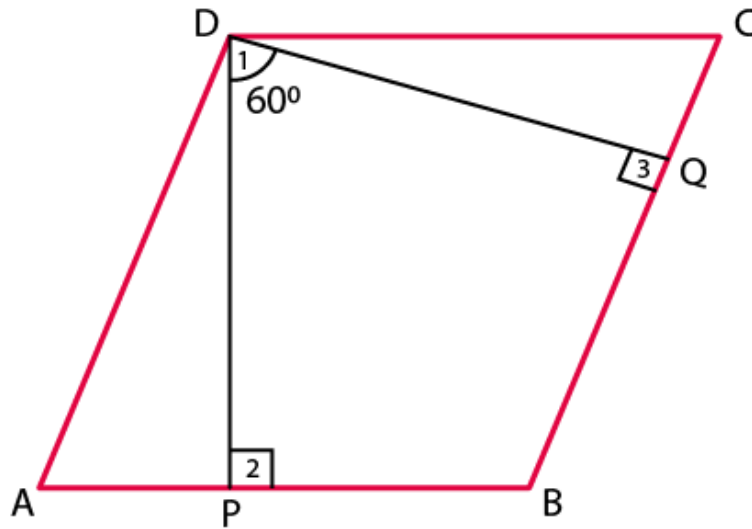
$$45^\circ + \angle D = 180^\circ$$

$$\angle D = 180^\circ - 45^\circ$$

$$\angle D = 135^\circ$$

Question 3

The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60° . Find the angles of the parallelogram.

Solution

According to the question,

ABCD is parallelogram,

$DP \perp AB$

$DQ \perp BC$.

$\angle PDQ = 60^\circ$

In quad. DPBQ,

Using angle sum property of a quadrilateral,

We have,

$$\angle PDQ + \angle Q + \angle P + \angle B = 360^\circ$$

$$60^\circ + 90^\circ + 90^\circ + \angle B = 360^\circ$$

$$240^\circ + \angle B = 360^\circ$$

$$\angle B = 360^\circ - 240^\circ$$

$$\angle B = 120^\circ$$

Since, opposite angles in parallelogram are equal,

We have,

$$\angle B = \angle D = 120^\circ$$

Since, opposite sides are parallel in parallelogram,

We have,

$$AB \parallel CD$$

Also, since sum of adjacent interior angles is 180° ,

We have,

$$\angle B + \angle C = 180^\circ$$

$$120^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 120^\circ$$

$$\angle C = 60^\circ$$

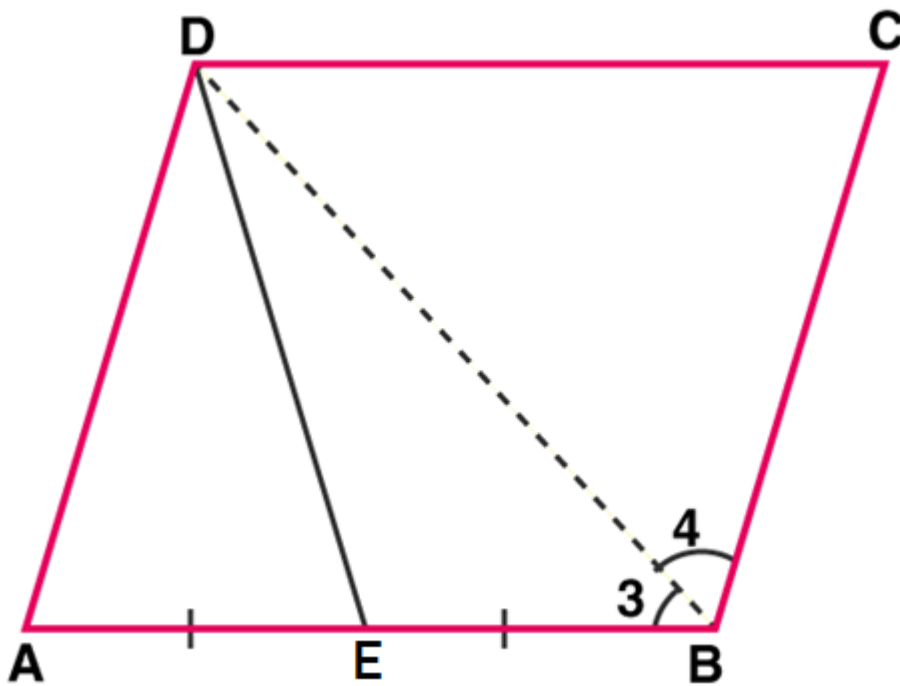
Since, opposite angles in parallelogram are equal,

We have,

$$\angle C = \angle A = 60^\circ$$

Question 4

ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

Solution

According to the question,

We have,

ABCD is a rhombus.

DE is the altitude on AB then $AE = EB$.

In $\triangle AED$ and $\triangle BED$,

We have,

$DE = DE$ (common line)

$\angle AED = \angle BED$ (right angle)

$AE = EB$ (DE is an altitude)

$\therefore \triangle AED \cong \triangle BED$ by SAS property.

$\therefore AD = BD$ (by C.P.C.T)

But $AD = AB$ (sides of rhombus are equal)

$\Rightarrow AD = AB = BD$

$\therefore$ ABD is an equilateral triangle.

$$\therefore \angle A = 60^\circ$$

Since, opposite angles of rhombus are equal, we get,

$$\Rightarrow \angle A = \angle C = 60^\circ$$

We also know that,

Sum of adjacent angles of a rhombus = supplementary.

So,

$$\angle ABC + \angle BCD = 180^\circ$$

$$\angle ABC + 60^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 60^\circ = 120^\circ$$

Since, opposite angles of rhombus are equal, we get,

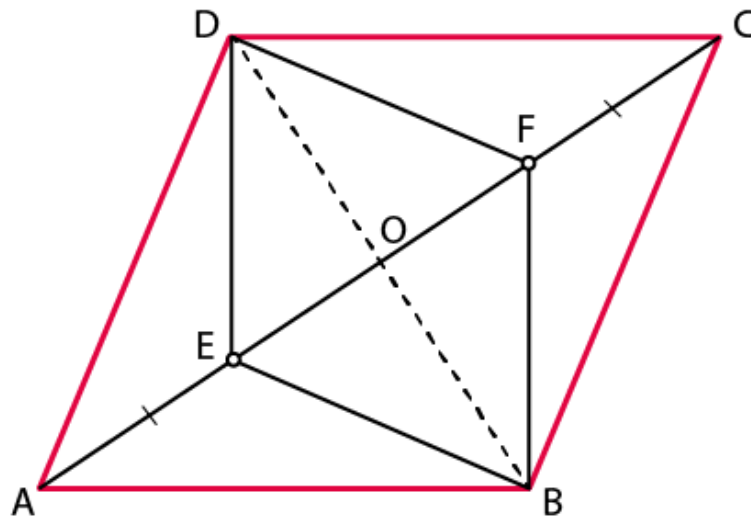
$$\angle ABC = \angle ADC = 120^\circ$$

Hence, Angles of rhombus are:

$$\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ, \angle D = 120^\circ$$

Question 5

E and F are points on diagonal AC of a parallelogram ABCD such that $AE = CF$. Show that BFDE is a parallelogram.

Solution

Construction:

Join BD, meeting AC at O.

According to the question,

Since diagonals of a parallelogram bisect each other,

We get,

$OA = OC$ and $OD = OB$.

And,

$OA = OC$ and $AE = CF$,

$OA - AE = OC - CF$

$OE = OF$

So, BFDE is a quadrilateral whose diagonals bisect each other.

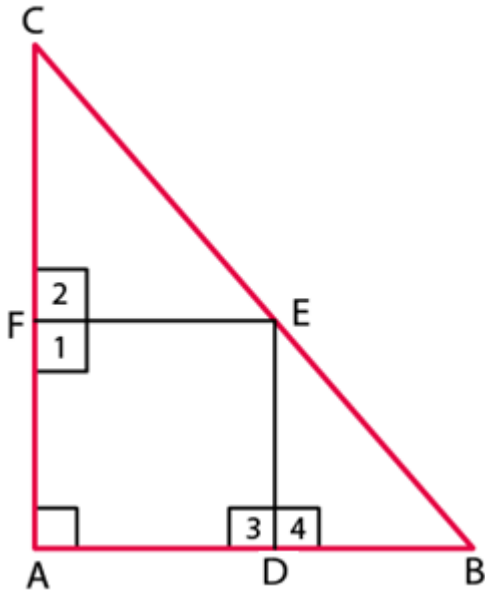
Hence, BFDE is a parallelogram.

Exercise 8.4 · Textbook page 82

Question 1

A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle in common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Solution



According to the question,

$\triangle ABC$ with $\angle A = 90^\circ$ and

Since, ABC is an isosceles triangle,

We get,

$$AB = AC \dots(i)$$

Let ADEF be the square inscribed in the isosceles triangle ABC.

Then, we have,

$$AD = AF = EF = AD \dots(ii)$$

Subtracting equation (ii) from (i),

$$AB - AD = AC - AF$$

$$BD = CF$$

Now,

Considering $\triangle CFE$ and $\triangle EDB$,

$$BD = CF$$

$$DE = EF$$

$$\angle CFE = \angle EDB = 90^\circ \text{ (Since, they are the side of a square)}$$

$$\triangle CEF \sim \triangle BED \text{ (By SAS criteria)}$$

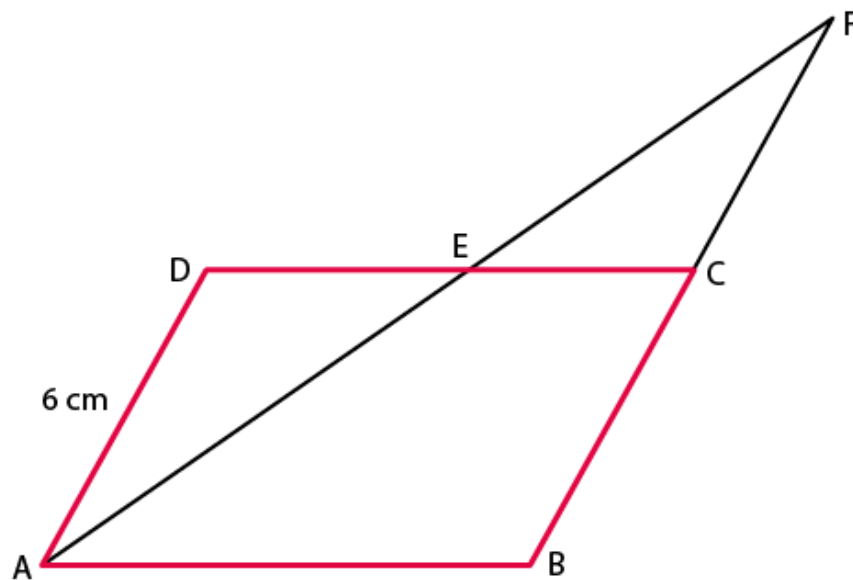
$$\text{Hence, } CE = BE$$

Therefore, vertex E of the square bisect the hypotenuse BC.

Question 2

In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of $\angle A$ meets DC in E. AE and BC produced meet at F. Find the length of CF.

Solution



According to the question,

We have,

ABCD is a parallelogram

$$AB = 10 \text{ cm}$$

$$AD = 6 \text{ cm.}$$

The bisector of $\angle A$ meets DC at E.

AE and BC produced meet at F.

Since, AF bisects $\angle A$,

We get,

$$\angle BAE = \angle EAD \dots (1)$$

$$\angle EAD = \angle EFB \dots (2) \text{ [Alternate angles]}$$

From equations (1) and (2),

We get,

$$\angle BAE = \angle EFB$$

Since sides opposite to equal angles are equal,

We get,

$$BF = AB$$

$$\text{Here, } AB = 10 \text{ cm}$$

$$\text{So, } BF = 10 \text{ cm}$$

$$\Rightarrow BC + CF = 10 \text{ cm}$$

$$6 \text{ cm} + CF = 10 \text{ cm} \text{ [BC = AD = 6 cm, opposite sides of a parallelogram]}$$

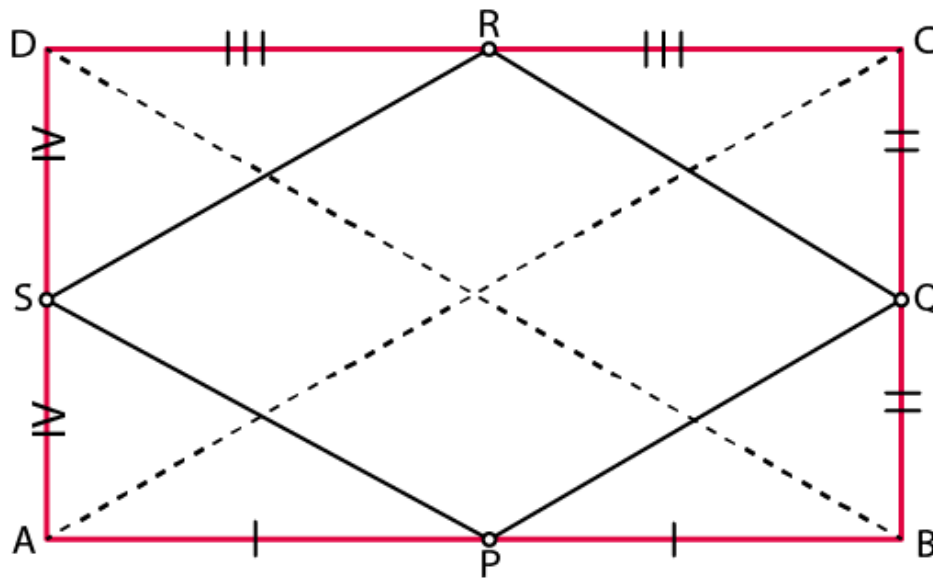
$$\Rightarrow CF = 10 - 6 \text{ cm} = 4 \text{ cm}$$

$$\Rightarrow CF = 4 \text{ cm}$$

Question 3

P, Q, R and S are, respectively, the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which $AC = BD$. Prove that PQRS is a rhombus.

Solution



According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also, we know that,

$$AC = BD.$$

In $\triangle ADC$, by mid-point theorem,

$$SR = \frac{1}{2} AC$$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$$PQ = \frac{1}{2} AC$$

And, $PQ \parallel AC$

$$\text{Hence, } SR = PQ = \frac{1}{2} AC$$

Similarly,

In $\triangle BCD$, by mid-point theorem,

$$RQ = \frac{1}{2} BD$$

And, $RQ \parallel BD$

In $\triangle BAD$, by mid-point theorem,

$$SP = \frac{1}{2} BD$$

And, $SP \parallel BD$

So, we get,

$$SP = RQ = \frac{1}{2} BD = \frac{1}{2} AC$$

Then,

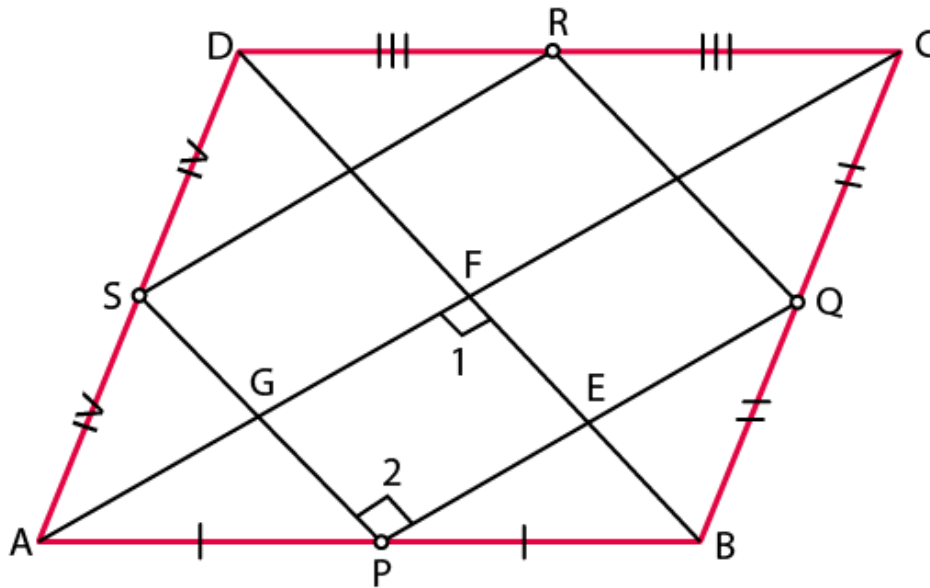
$$SR = PQ = SP = RQ$$

Hence, PQRS is a rhombus.

Question 4

P, Q, R and S are, respectively, the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that $AC \perp BD$. Prove that PQRS is a rectangle.

Solution



According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also,

$AC \perp BD$

$\angle COD = \angle AOD = \angle AOB = \angle COB = 90^\circ$

In $\triangle ADC$, by mid-point theorem,

$SR = \frac{1}{2} AC$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$PQ = \frac{1}{2} AC$

And, $PQ \parallel AC$

So, we have,

$$PQ \parallel SR \text{ and } SR = PQ = \frac{1}{2} AC$$

Similarly,

$$SP \parallel RQ \text{ and } SP = RQ = \frac{1}{2} BD$$

Now, in quadrilateral EOFR,

$$OE \parallel FR \text{ and } OF \parallel ER$$

So, we get,

$$\angle EOF = \angle ERF = 90^\circ$$

Hence, PQRS is a rectangle.

Question 5

P, Q, R and S are, respectively, the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which $AC = BD$ and $AC \perp BD$. Prove that PQRS is a square.

Solution

According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

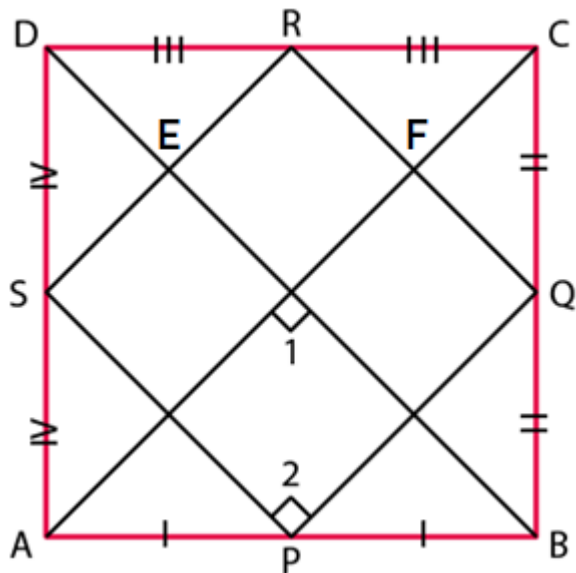
R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also,

$$AC \perp BD$$

$$\text{And } AC = BD$$



In $\triangle ADC$, by mid-point theorem,

$$SR = \frac{1}{2} AC$$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$$PQ = \frac{1}{2} AC$$

And, $PQ \parallel AC$

So, we have,

$$PQ \parallel SR \text{ and } PQ = SR = \frac{1}{2} AC$$

Now, in $\triangle ABD$, by mid-point theorem,

$$SP \parallel BD \text{ and } SP = \frac{1}{2} BD = \frac{1}{2} AC$$

In $\triangle BCD$, by mid-point theorem,

$$RQ \parallel BD \text{ and } RQ = \frac{1}{2} BD = \frac{1}{2} AC$$

$$SP = RQ = \frac{1}{2} AC$$

$$PQ = SR = SP = RQ$$

Thus, we get that,

All four sides are equal.

Considering the quadrilateral EOFR,

$$OE \parallel FR, OF \parallel ER$$

$$\angle EOF = \angle ERF = 90^\circ \text{ (Opposite angles of parallelogram)}$$

$$\angle QRS = 90^\circ$$

Hence, PQRS is a square.

Question 6

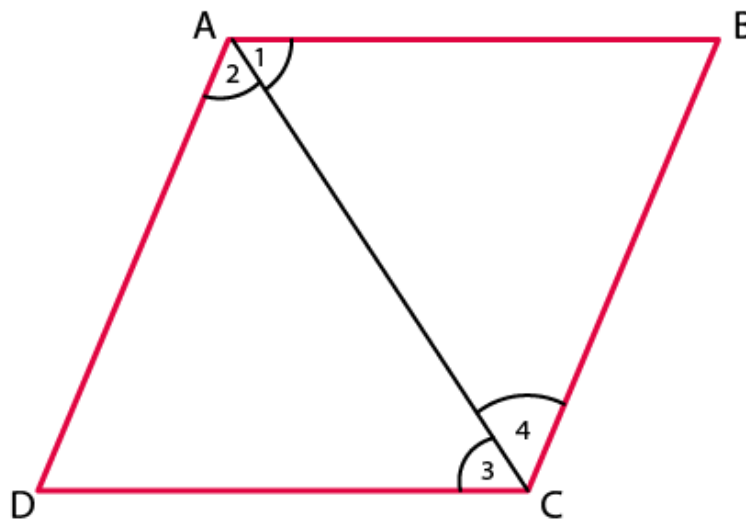
A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Solution

Let the parallelogram be = ABCD

Diagonal AC bisect $\angle A$.

$$\angle CAB = \angle CAD$$



Now,

$AB \parallel CD$ and AC is a transversal.

$$\angle CAB = \angle ACD$$

Again, $AD \parallel BC$ and AC is a transversal.

$$\angle DAC = \angle ACB$$

Now,

$$\angle A = \angle C$$

$$\frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\angle DAC = \angle DCA$$

$$AD = CD$$

But, $AB = CD$ and $AD = BC$ (Opposite sides of parallelograms)

$$AB = BC = CD = AD$$

Thus, ABCD is a rhombus.

Question 7

P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Solution

According to the question,

Q is the midpoint of AB

P is the midpoint of CD

Now,

$$AB \parallel CD,$$

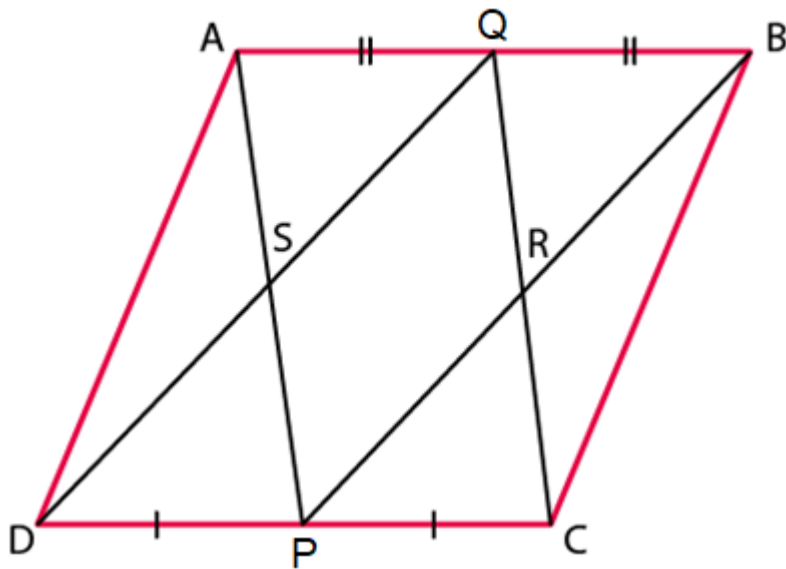
Also,

$$AP \parallel QC$$

$$\text{And, } AB = DC$$

$$\frac{1}{2} AB = \frac{1}{2} DC$$

$$AP = QC$$



Now,

$AP \parallel QC$ and $AP = QC$

$APCQ$ is a parallelogram.

$AQ \parallel PC$ or $SQ \parallel PR$

Again,

$AB \parallel DC$ means $\frac{1}{2} AB = \frac{1}{2} DC$

$BP = QD$

Now, $BP \parallel QD$ and $BP = QD$

$BPDQ$ is a parallelogram

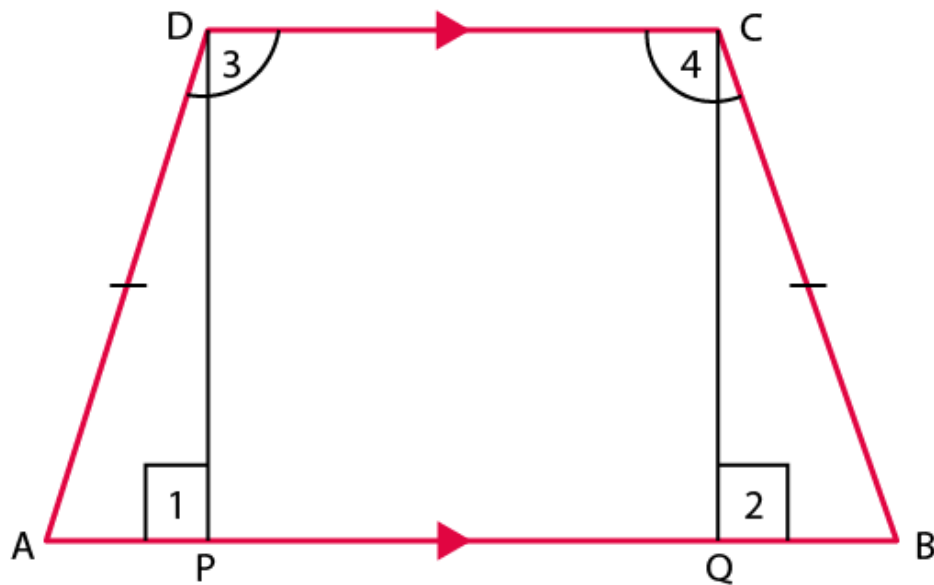
So, $PD \parallel BQ$ or $PS \parallel QR$

Thus, $SQ \parallel RP$ and $PS \parallel QR$

$PQRS$ is a parallelogram.

Question 8

$ABCD$ is a quadrilateral in which $AB \parallel DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

Solution

According to the question,

We have,

Quadrilateral ABCD

$AB \parallel CD$ and $AD = BC$.

To prove: $\angle A = \angle B$ and $\angle C = \angle D$.

Construction: Draw $DP \perp AB$ and $CQ \perp AB$.

Proof: In $\triangle APD$ and $\triangle BQC$,

Since $\angle 1$ and $\angle 2$ are equal to 90°

$$\angle 1 = \angle 2$$

Distance between parallel line,

$$AD = BC \text{ [Given]}$$

By RHS criterion of congruence,

We have

$$\triangle APD \cong \triangle BQC \text{ [CPCT]}$$

$$\angle A = \angle B$$

Now, $DC \parallel AB$

Since, sum of consecutive interior angles is 180°

$$\angle A + \angle 3 = 180 \dots (1)$$

And,

$$\angle B + \angle 4 = 180 \dots (2)$$

From equations (1) and (2),

We get

$$\angle A + \angle 3 = \angle B + \angle 4$$

Since, $\angle A = \angle B$,

We have,

$$\Rightarrow \angle 3 = \angle 4$$

$$\Rightarrow \angle C = \angle D$$

Hence, proved.

Question 9

In Fig. 8.11, $AB \parallel DE$, $AB = DE$, $AC \parallel DF$ and $AC = DF$. Prove that $BC \parallel EF$ and $BC = EF$.

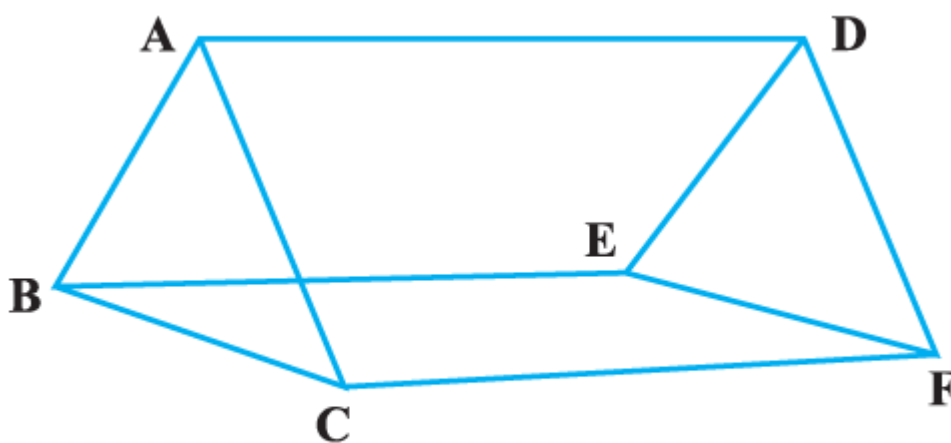
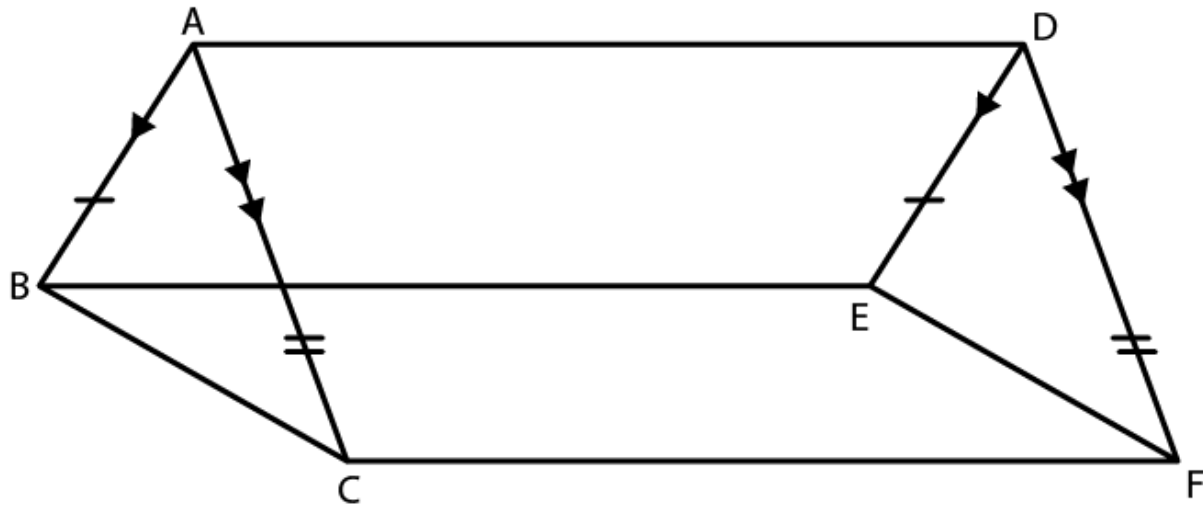


Fig. 8.11

Solution

According to the question,

In quadrilateral ABED,

We have,

$AB \parallel DE$ and $AB = DE$

ABED is a parallelogram.

$AD \parallel BE$ and $AD = BE$

In quadrilateral ACFD,

We have,

$AC \parallel FD$ and $AC = FD$

ACFD is a parallelogram.

$AD \parallel CF$ and $AD = CF$

$AD = BE = CF$ and $CF \parallel BE$

In quadrilateral BCFE,

$BE = CF$ and $BE \parallel CF$.

BCFE is a parallelogram.

$BC = EF$ and $BC \parallel EF$

Hence proved.
