

Chapter 7: Triangles

NCERT Exemplar · Class 9 · Maths

Exercise 7.1 · Textbook page 64

Question 1

Which of the following is not a criterion for congruence of triangles?

A.

SAS

B.

ASA

C.

SSA

D.

SSS

Answer

Correct option: C

Solution

(C) SSA

Explanation:

We know that,

Two triangles are congruent, if the side(S) and angles (A) of one triangle is equal to another.

And criterion for congruence of triangle are SAS, ASA, SSS, and RHS.

SSA is not the criterion for congruency of a triangle.

Hence, option C is the correct answer.

Question 2

If $AB = QR$, $BC = PR$ and $CA = PQ$, then

- (A) $\triangle ABC \cong \triangle PQR$
- (B) $\triangle CBA \cong \triangle PRQ$
- (C) $\triangle BAC \cong \triangle RPQ$
- (D) $\triangle PQR \cong \triangle BCA$

Solution

(B) $\triangle CBA \cong \triangle PRQ$

Explanation:

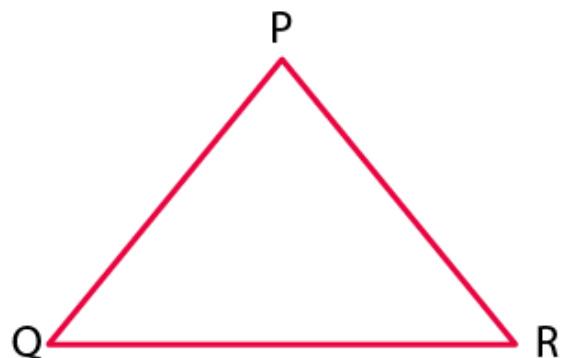
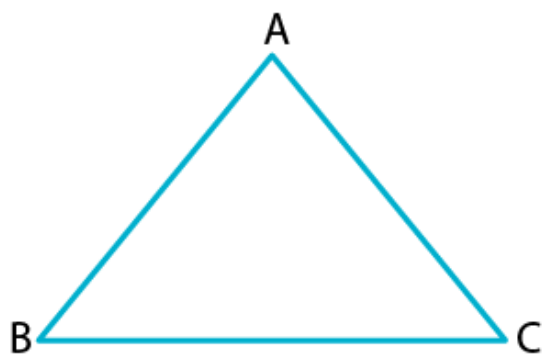
According to the question,

$AB = QR$, $BC = PR$ and $CA = PQ$

Since, $AB = QR$, $BC = PR$ and $CA = PQ$

We can say that,

A corresponds to Q, B corresponds to R, C corresponds to P.



Hence, (B) $\triangle CBA \cong \triangle PRQ$

Hence, option B is the correct answer.

Question 3

In $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$. Then $\angle C$ is equal to

(A) 40° (B) 50° (C) 80° (D) 130°

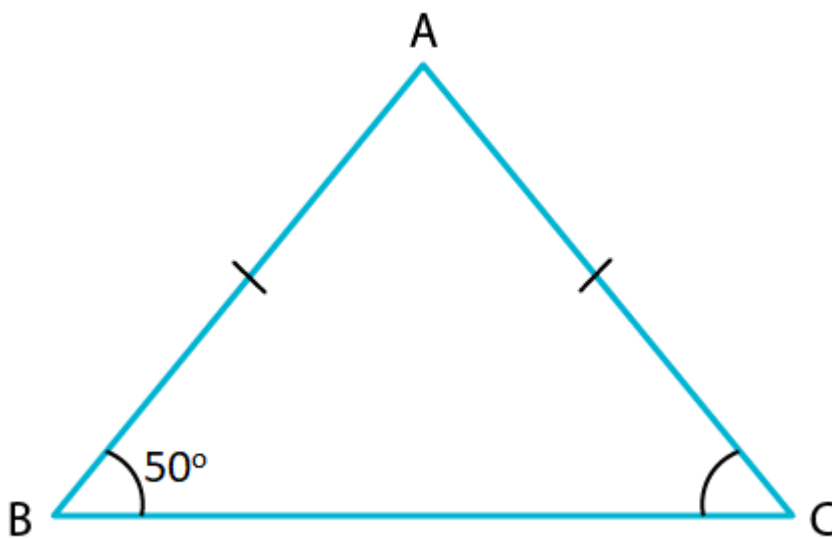
Solution

(B) 50°

Explanation:

According to the question,

$\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$.



Since, $AB = AC$

$\triangle ABC$ is an isosceles triangle.

Hence, $\angle B = \angle C$

$\angle B = 50^\circ$ (given)

$\Rightarrow \angle C = 50^\circ$

Hence, option B is the correct option.

Question 4

In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ is equal to

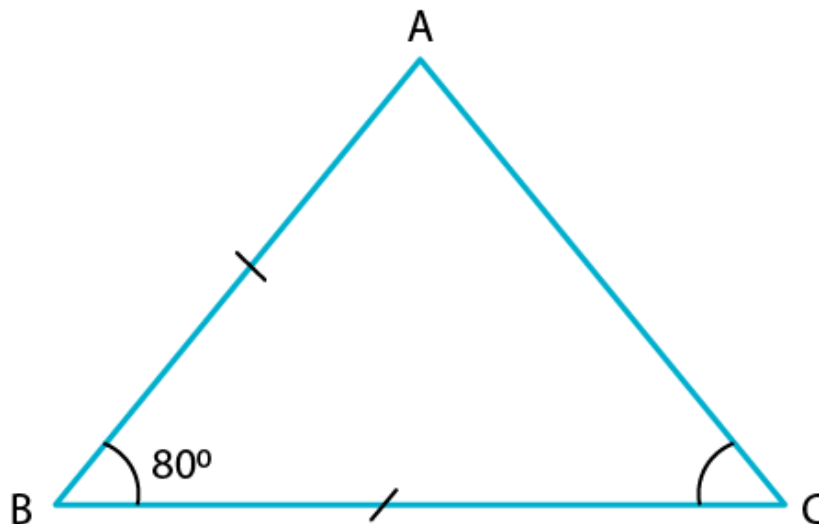
(A) 80° (B) 40° (C) 50° (D) 100°

Solution

(C) 50°

Explanation:

Given: $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$



Since, $BC = AB$

$\triangle ABC$ is an isosceles triangle.

Let, $\angle C = \angle A = x$

$\angle B = 80^\circ$ (given)

We know that,

Using angle sum property,

Sum of interior angles of a triangle should be $= 180^\circ$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + 80^\circ + x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 80^\circ$$

$$\Rightarrow 2x = 100^\circ$$

$$\Rightarrow x = 50^\circ$$

Therefore, $\angle C = \angle A = 50^\circ$

Hence, option C is the correct answer.

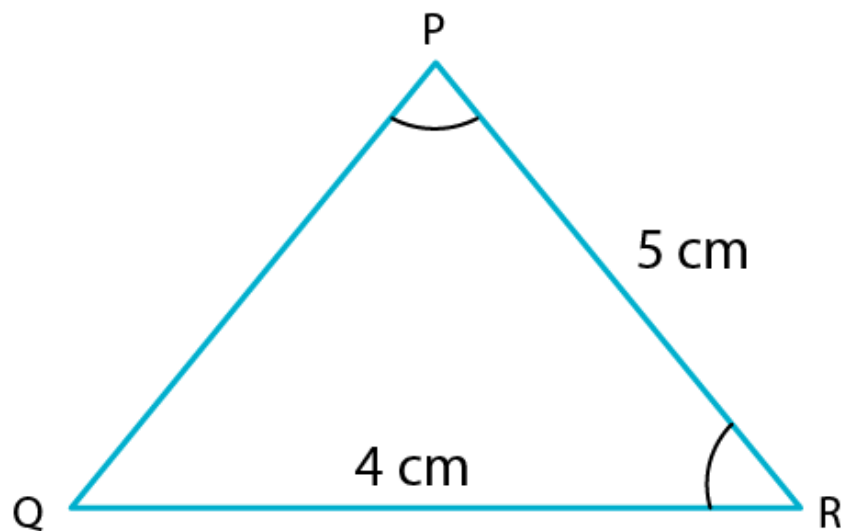
Question 5

In ΔPQR , $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm. Then the length of PQ is

(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm

Solution

Given: ΔPQR , $\angle R = \angle P$ and $QR = 4$ cm and $PR = 5$ cm



Since, $\angle R = \angle P$

ΔPQR is an isosceles triangle.

Hence, $PQ = QR$

$$\Rightarrow PQ = 4\text{cm}$$

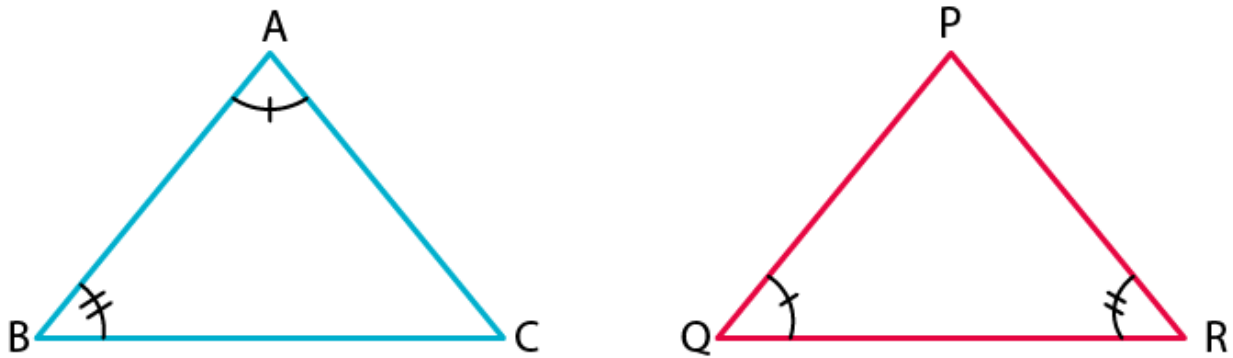
Hence, option A is the correct answer.

Exercise 7.2 · Textbook page 65

Question 1

In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reason for your answer.

Solution



In triangle ABC and PQR, we have

$$\angle A = \angle Q \text{ [Given]}$$

$$\angle B = \angle R \text{ [Given]}$$

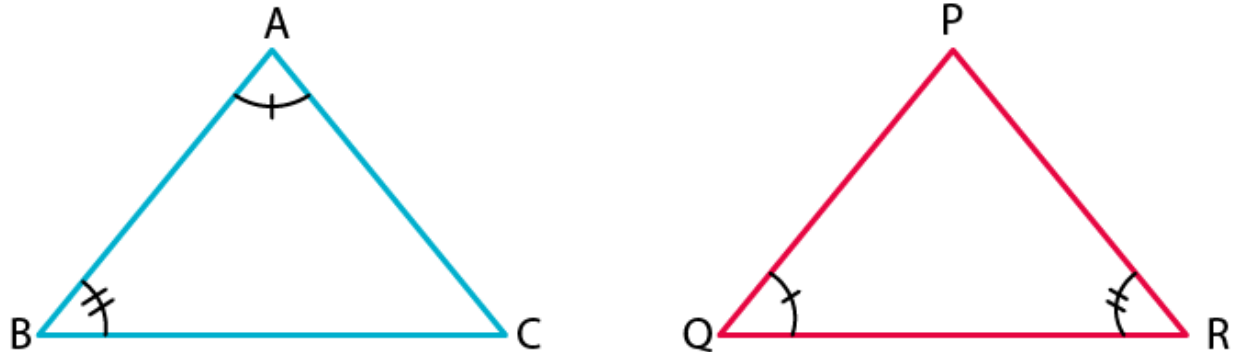
For the triangle to be congruent, AB should be equal to QR.

Hence, triangle ABC and PQR can be congruent by ASA congruence rule.

Question 2

In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side BC of ΔABC so that the two triangles are congruent? Give reason for your answer.

Solution



In triangle ABC and PQR, we have

$$\angle A = \angle Q \text{ and } \angle B = \angle R \text{ [Given]}$$

For the triangles to be congruent, we must have

$$BC = RP$$

Hence, triangle ABC and PQR will be congruent by AAS congruence rule.

Question 3

“If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent.” Is the statement true? Why?

Solution

No, the statement, “if two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent” is false.

Justification:

Because by the congruent rule,

The two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, i.e., SAS rule.

Question 4

“If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.” Is the statement true? Why?

Solution

The statement, “If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.” is true.

Justification:

The statement is true because, the triangles will be congruent either by ASA rule or AAS rule. This is because two angles and one side are enough to construct two congruent triangles.

Question 5

Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

Solution

No, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

Justification:

We know that,

Sum of any two sides of a triangle is always greater than the third side.

Here, the sum of two sides whose lengths are 4 cm and 3 cm = 4 cm + 3 cm = 7 cm,

Which is equal to the length of third side, i.e., 7 cm.

Hence, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

Question 6

It is given that $\triangle ABC \cong \triangle RPQ$. Is it true to say that $BC = QR$? Why?

Solution

It is False that $BC = QR$ because $BC = PQ$ as $\triangle ABC \cong \triangle RPQ$.

Exercise 7.3 · Textbook page 67

Question 1

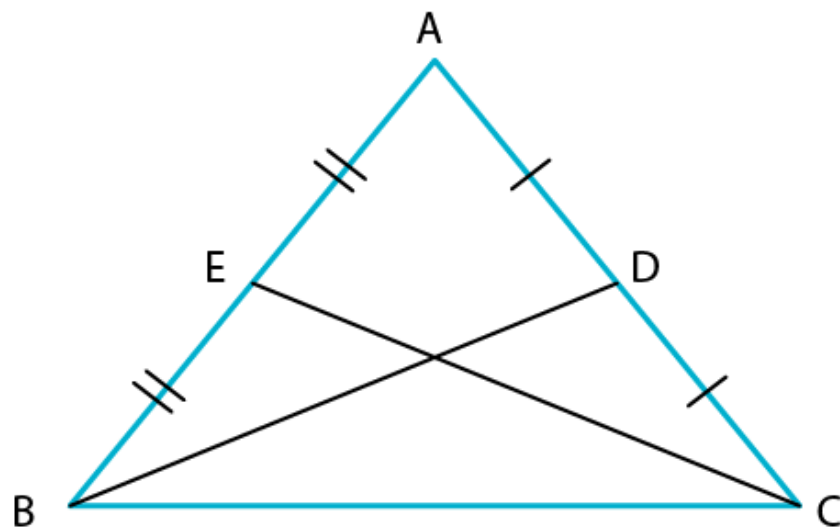
ABC is an isosceles triangle with $AB = AC$ and BD and CE are its two medians. Show that

$BD = CE$.

Solution

According to the question,

$\triangle ABC$ is an isosceles triangle and $AB = AC$, BD and CE are two medians



From $\triangle ABD$ and $\triangle ACE$,

$AB = AC$ (given)

$2 AE = 2 AD$ (as D and E are mid points)

So, $AE = AD$

$\angle A = \angle A$ (common)

Hence, $\triangle ABD \cong \triangle ACE$ (using SAS)

$\Rightarrow BD = CE$ (by CPCT)

Hence proved.

Question 2

In Fig.7.4, D and E are points on side BC of a ΔABC such that $BD = CE$ and $AD = AE$.

Show that $\Delta ABD \cong \Delta ACE$.

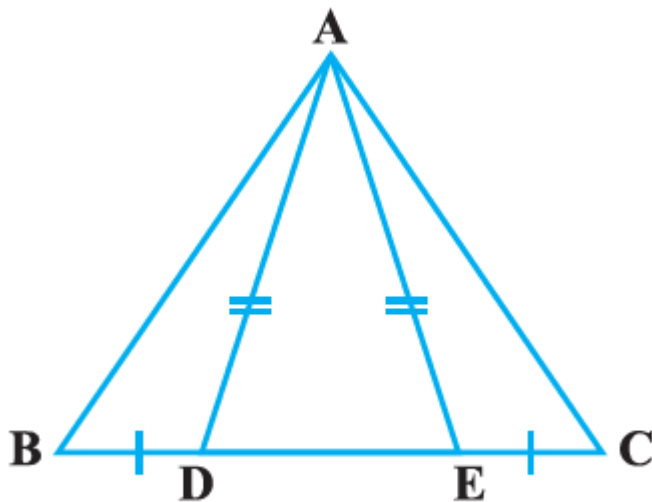


Fig. 7.4

Solution

According to the question,

In ΔABC ,

$BD = CE$ and $AD = AE$.

In ΔADE ,

$AD = AE$

Since opposite angles to equal sides are equal,

We have,

$$\angle ADE = \angle AED \dots (1)$$

Now, $\angle ADE + \angle ADB = 180^\circ$ (linear pair)

$$\angle ADB = 180^\circ - \angle ADE \dots (2)$$

Also, $\angle AED + \angle AEC = 180^\circ$ (linear pair)

$$\angle AEC = 180^\circ - \angle AED$$

Since, $\angle ADE = \angle AED$

$$\angle AEC = 180^\circ - \angle ADE \dots (3)$$

From equation (2) and (3)

$$\angle ADB = \angle AEC \dots (4)$$

Now, In $\triangle ADB$ and $\triangle AEC$,

$AD = AE$ (given)

$BD = EC$ (given)

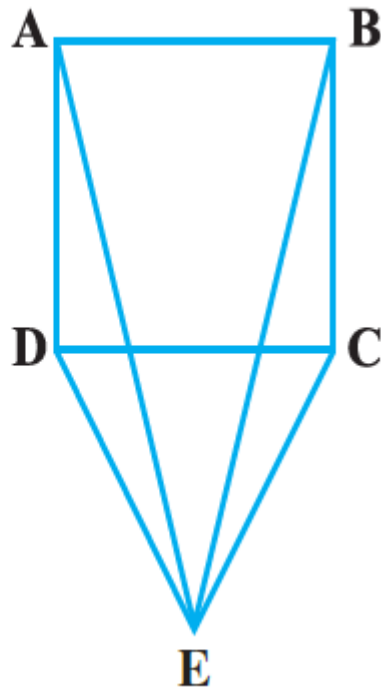
$\angle ADB = \angle AEC$ (from (4))

Hence, $\triangle ABD \cong \triangle ACE$ (by SAS)

Question 3

CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.7.5). Show that

$\triangle ADE \cong \triangle BCE$.

**Fig. 7.5****Solution**

According to the question,

CDE is an equilateral triangle formed on a side CD of a square ABCD.

In $\triangle ADE$ and $\triangle BCE$,

$DE = CE$ (sides of equilateral triangle)

Now,

$$\angle ADC = \angle BCD = 90^\circ$$

$$\text{And, } \angle EDC = \angle ECD = 60^\circ$$

$$\text{Hence, } \angle ADE = \angle ADC + \angle CDE = 90^\circ + 60^\circ = 150^\circ$$

$$\text{And } \angle BCE = \angle BCD + \angle ECD = 90^\circ + 60^\circ = 150^\circ$$

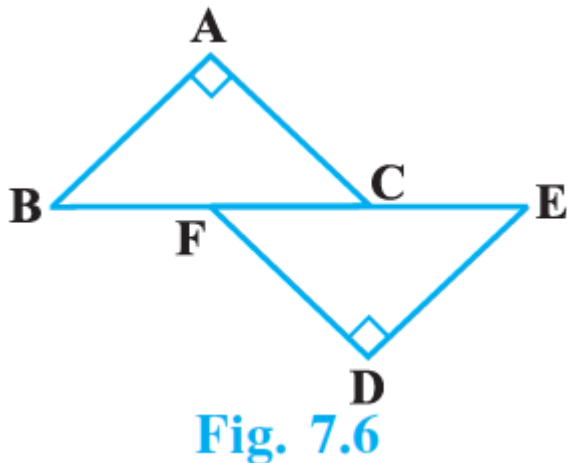
$$\Rightarrow \angle ADE = \angle BCE$$

$AD = BC$ (sides of square)

Hence, $\triangle ADE \cong \triangle BCE$ (by SAS)

Question 4

In Fig.7.6, $BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$. Show that $\triangle ABC \cong \triangle DEF$.

**Solution**

According to the question,

$BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$.

In $\triangle ABC$ and $\triangle DEF$

$BA = DE$ (given)

$BF = EC$ (given)

$\angle A = \angle D$ (both 90°)

$BC = BF + FC$

$EF = EC + FC = BF + FC$ ($\because EC = BF$)

$\Rightarrow EF = BC$

Hence, $\triangle ABC \cong \triangle DEF$ (by RHS)

Question 5

Q is a point on the side SR of a $\triangle PSR$ such that $PQ = PR$. Prove that $PS > PQ$.

Solution

Given: in $\triangle PSR$, Q is a point on the side SR such that $PQ = PR$.

In $\triangle PRQ$,

$PR = PQ$ (given)

$\Rightarrow \angle PRQ = \angle PQR$ (opposite angles to equal sides are equal)

But $\angle PQR > \angle PSR$ (exterior angle of a triangle is greater than each of opposite interior angle)

$\Rightarrow \angle PRQ > \angle PSR$

$\Rightarrow PS > PR$ (opposite sides to greater angle is greater)

$\Rightarrow PS > PQ$ (as $PR = PQ$)

Exercise 7.4 · Textbook page 69

Question 1

Find all the angles of an equilateral triangle.

Solution

In equilateral triangle,

All the sides are equal.

Therefore, all angles are also equal

Let the angles of an equilateral triangle = x

According to angle sum property,

We know that the sum of the interior angles is equal to 180° .

$$x+x+x=180^\circ$$

$$3x=180$$

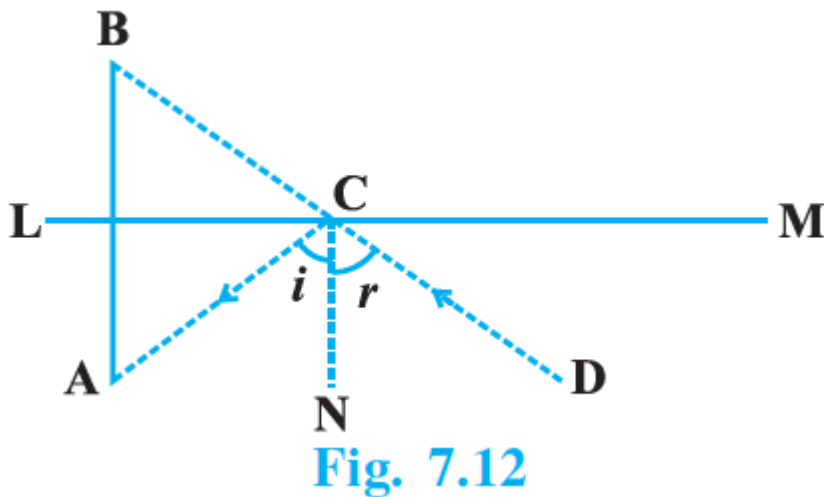
$$x=60^\circ$$

Therefore, all the angles of an equilateral triangle are 60°

Question 2

The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.

[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].



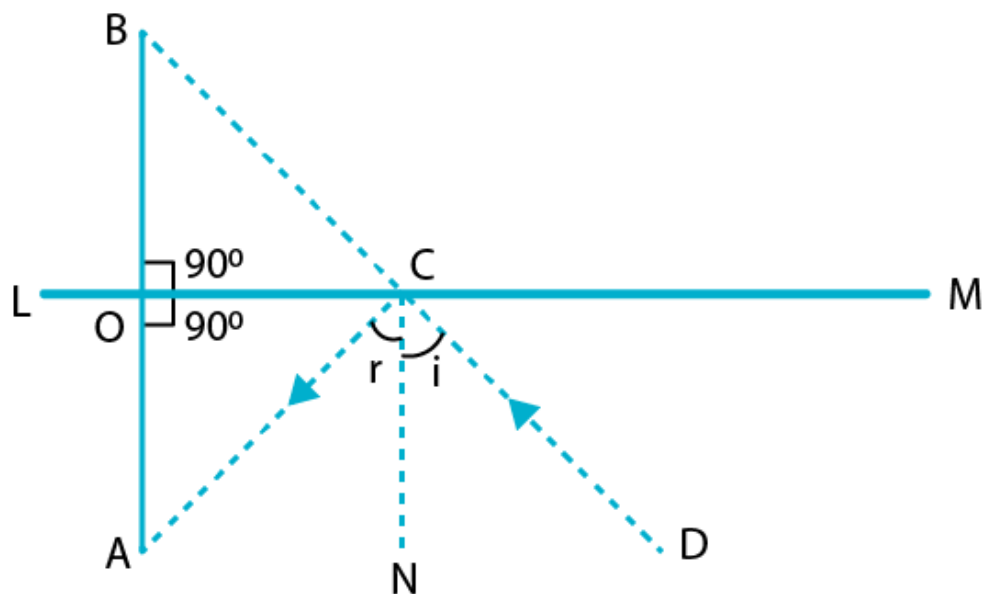
Solution

Let AB intersect LM at O. We have to prove that $AO = BO$.

Now, $\angle i = \angle r$... (1)

$\angle B = \angle i$ [Corresponding angles] ... (2)

And $\angle A = \angle r$ [Alternate interior angles] ... (3)



From (1), (2) and (3), we get

$$\angle B = \angle A$$

$$\Rightarrow \angle BCO = \angle ACO$$

In $\triangle BOC$ and $\triangle AOC$ we have

$$\angle 1 = \angle 2 \text{ [Each} = 90^\circ]$$

$$OC = OC \text{ [Common side]}$$

$$\text{And } \angle BCO = \angle ACO \text{ [Proved above]}$$

$$\triangle BOC \cong \triangle AOC \text{ [ASA congruence rule]}$$

$$\text{Hence, } AO = BO \text{ [CPCT]}$$

Question 3

ABC is an isosceles triangle with $AB = AC$ and D is a point on BC such that $AD \perp BC$ (Fig. 7.13). To prove that $\angle BAD = \angle CAD$, a student proceeded as follows:

In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \text{ (Given)}$$

$$\angle B = \angle C \text{ (because } AB = AC)$$

$$\text{And } \angle ADB = \angle ADC$$

$$\text{Therefore, } \triangle ABD \cong \triangle ACD \text{ (AAS)}$$

$$\text{So, } \angle BAD = \angle CAD \text{ (CPCT)}$$

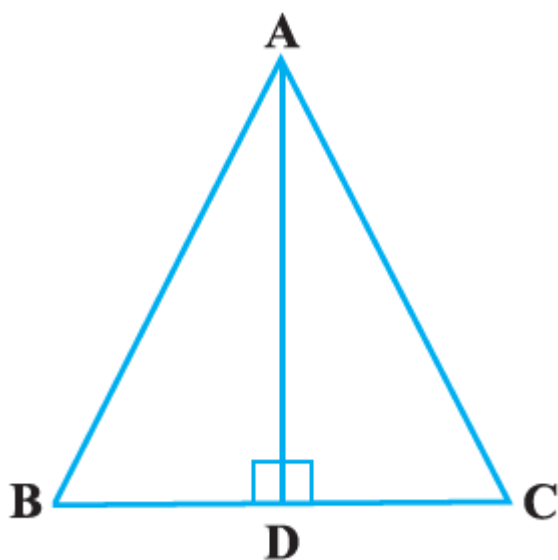
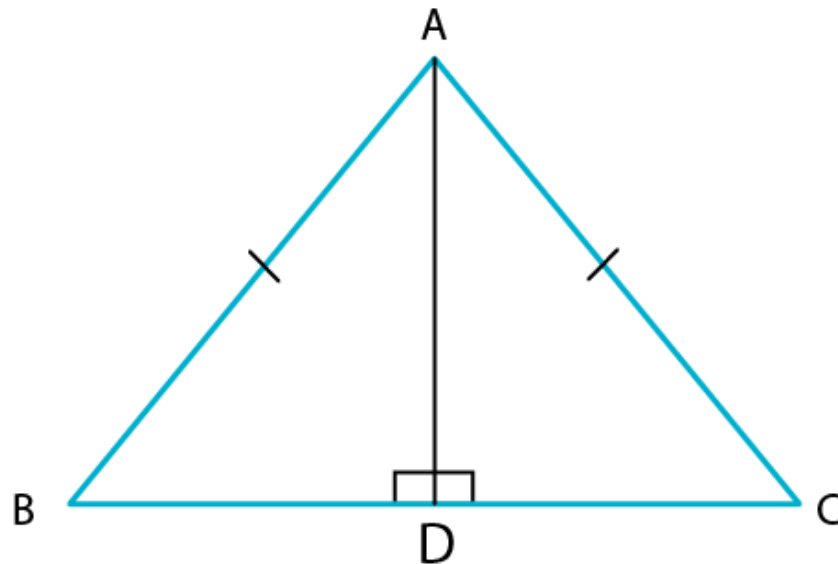


Fig. 7.13

What is the defect in the above arguments?

[Hint: Recall how $\angle B = \angle C$ is proved when $AB = AC$].

Solution



In $\triangle ABD$ and $\triangle ADC$, we have

$$\angle ADB = \angle ADC$$

According to the question,

$$AB = AC$$

$$AD = AD \text{ [Common side]}$$

By RHS criterion of congruence,

We have,

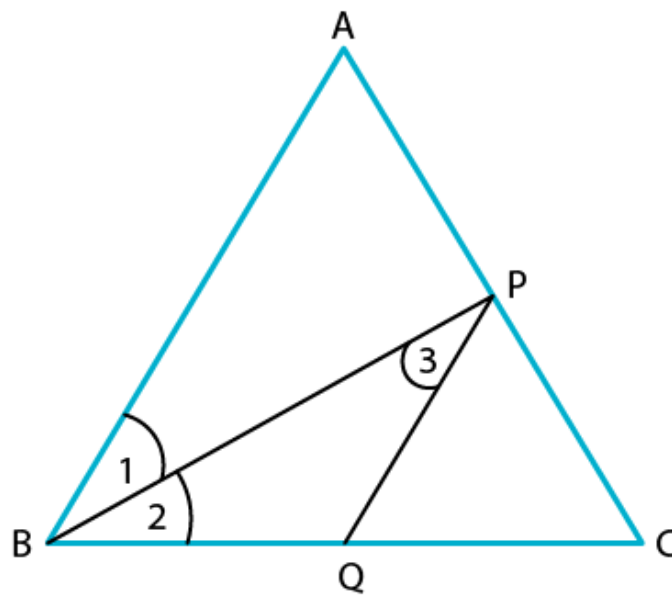
$$\triangle ABD \cong \triangle ACD$$

$$\angle BAD = \angle CAD \text{ [CPCT]}$$

Hence Proved.

Question 4

P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.

Solution

To prove: BPQ is an isosceles triangle.

According to the question,

Since, BP is the bisector of $\angle ABC$,

$$\angle 1 = \angle 2 \dots (1)$$

Now, PQ is parallel to BA and BP cuts them

$$\angle 1 = \angle 3 \text{ [Alternate angles]} \dots (2)$$

From equations, (1) and (2),

We get

$$\angle 2 = \angle 3$$

In $\triangle BPQ$,

We have

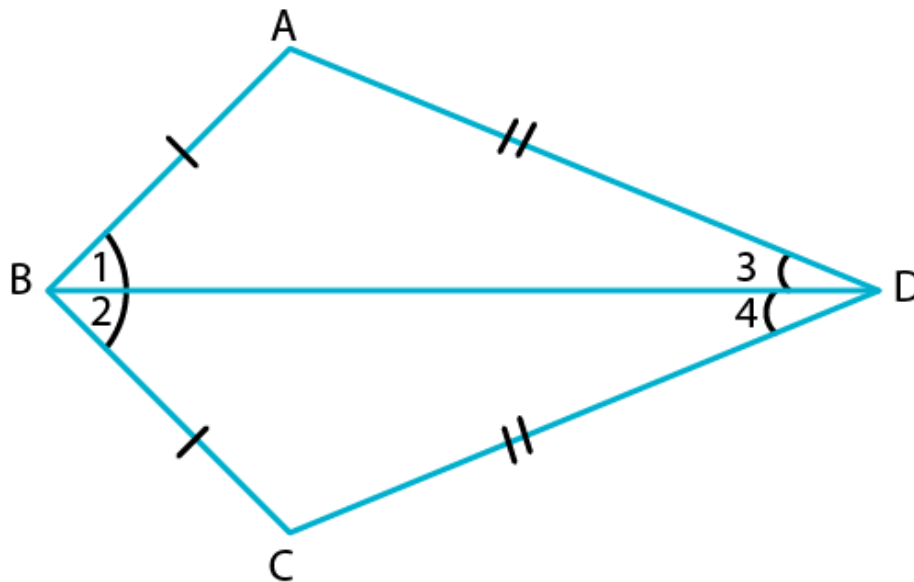
$$\angle 2 = \angle 3$$

$$PQ = BQ$$

Hence, BPQ is an isosceles triangle.

Question 5

ABCD is a quadrilateral in which $AB = BC$ and $AD = CD$. Show that BD bisects both the angles ABC and ADC.

Solution

According to the question,

In $\triangle ABC$ and $\triangle CBD$,

We have

$$AB = BC$$

$$AD = CD$$

$$BD = BD \text{ [Common side]}$$

$$\triangle ABC \cong \triangle CBD \text{ [By SSS congruence rule]}$$

$$\Rightarrow \angle 1 = \angle 2 \text{ [CPCT]}$$

$$\text{And } \angle 3 = \angle 4$$

Hence, BD bisects both $\angle ABC$ and $\angle ADC$.

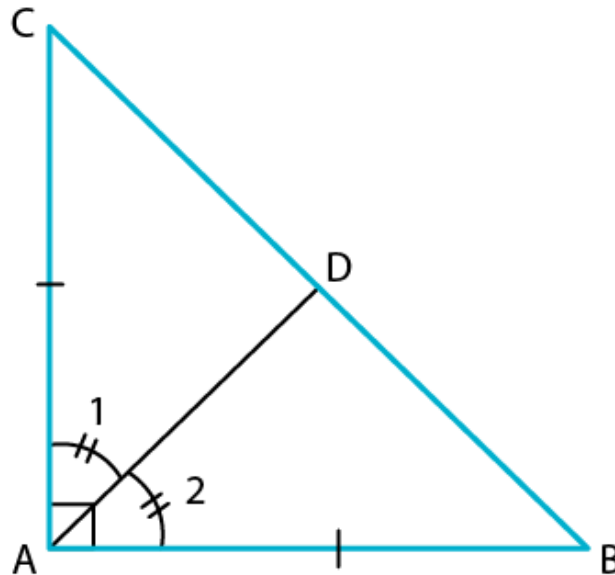
Question 6

ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D. Prove that $BC = 2 AD$.

Solution

Given: A right angles triangle with $AB = AC$ bisector of $\angle A$ meets BC at D .

To prove: $BC = 2AD$



Proof:

According to the question,

In right $\triangle ABC$,

$$AB = AC$$

Since, hypotenuse is the longest side,

BC is hypotenuse

$$\angle BAC = 90^\circ$$

Now,

In $\triangle CAD$ and $\triangle BAD$,

We have,

$$AC = AB$$

Since, AD is the bisector of $\angle A$,

$$\angle 1 = \angle 2$$

$AD = AD$ [Common side]

Now,

By SAS criterion of congruence,

We get,

$$\triangle CAD \cong \triangle BAD$$

$$CD = BD \text{ [CPCT]}$$

Since, Mid-point of hypotenuse of a right triangle is equidistant from the 3 vertices of a triangle.

$$AB = BD = CD \dots (1)$$

$$\text{Now, } BC = BD + CD$$

$$\Rightarrow BC = AD + AD \text{ [Using eq.(1)]}$$

$$\Rightarrow BC = 2AD$$

Hence, proved.

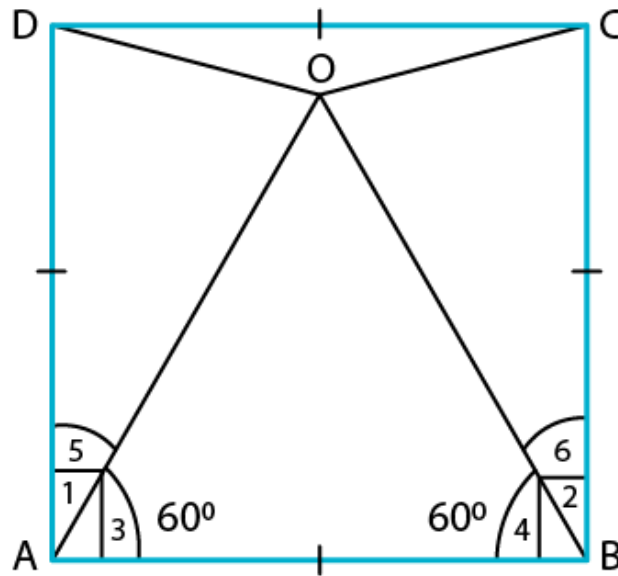
Question 7

O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that $\triangle OCD$ is an isosceles triangle.

Solution

According to the question,

It is given that: A square ABCD and $OA = OB = AB$.



To prove: $\triangle OCD$ is an isosceles triangle.

Proof:

In square ABCD,

Since $\angle 1$ and $\angle 2$ is equal to 90°

$$\angle 1 = \angle 2 \dots (1)$$

Now, in $\triangle OAB$, we have

Since $\angle 3$ and $\angle 4$ is equal to 60°

$$\angle 3 = \angle 4 \dots (2)$$

Subtracting equations (2) from (1),

We get

$$\angle 1 - \angle 3 = \angle 2 - \angle 4$$

$$\Rightarrow \angle 5 = \angle 6$$

Now,

In $\triangle DAO$ and $\triangle CBO$,

$AD = BC$ [Given]

$\angle 5 = \angle 6$ [Proved above]

$OA = OB$ [Given]

By SAS criterion of congruence,

We have

$$\triangle DAO \cong \triangle CBO$$

$$OD = OC$$

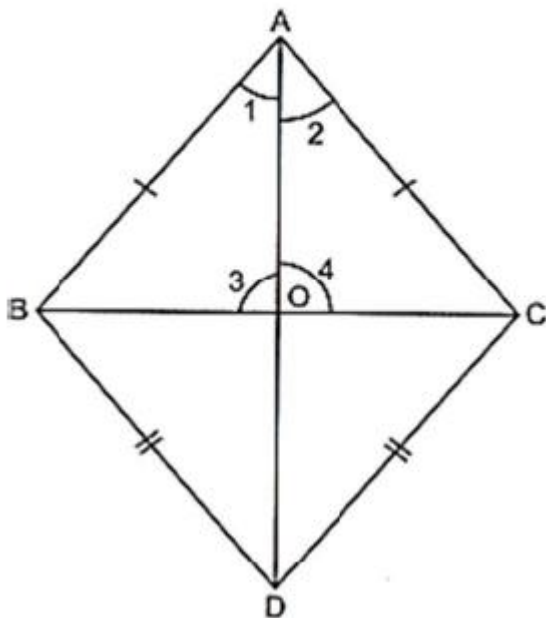
$\Rightarrow \triangle OCD$ is an isosceles triangle.

Hence, proved.

Question 8

ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC , $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC .

Solution



Given: $\triangle ABC$ and $\triangle DBC$ on the same base BC . Also, $AB = AC$ and $BD = DC$.

To prove: AD is the perpendicular bisector of BC i.e., $OB = OC$

Proof: In $\triangle BAD$ and $\triangle CAD$ we have

$$AB = AC \text{ [Given]}$$

$$BD = CD \text{ [Given]}$$

$$AD = AD \text{ [common side]}$$

So, by SSS criterion of congruence, we have

$$\triangle BAD \cong \triangle CAD$$

$$\angle 1 = \angle 2 \text{ [CPCT]}$$

Now, in $\triangle BAO$ and $\triangle CAO$, we have

$$AB = AC \text{ [Given]}$$

$$\angle 1 = \angle 2 \text{ [Proved above]}$$

$$AO = AO \text{ [Common side]}$$

So, by SAS criterion of congruence, we have

$$\triangle BAO \cong \triangle CAO$$

$$BO = CO \text{ [CPCT]}$$

$$\text{And, } \angle 3 = \angle 4 \text{ [CPCT]}$$

$$\text{But, } \angle 3 + \angle 4 = 180^\circ \text{ [Linear pair axiom]}$$

$$\Rightarrow \angle 3 + \angle 3 = 180$$

$$\Rightarrow 2\angle 3 = 180$$

$$\Rightarrow \angle 3 = 180/2$$

$$\Rightarrow \angle 3 = 90^\circ$$

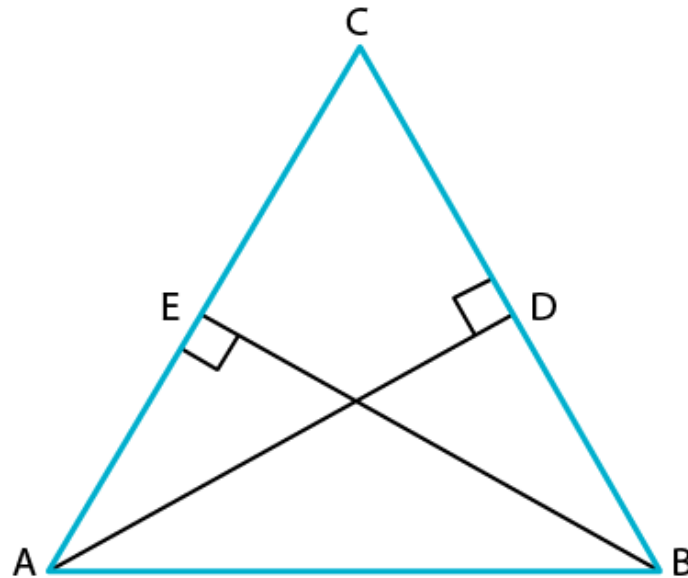
Since $BO = CO$ and $\angle 3 = 90^\circ$,

AD is perpendicular bisector of BC .

Hence, proved.

Question 9

ABC is an isosceles triangle in which $AC = BC$. AD and BE are respectively two altitudes to sides BC and AC. Prove that $AE = BD$.

Solution

According to the question,

In $\triangle ADC$ and $\triangle BEC$,

We have

$$AC = BC \text{ [Given] ... (1)}$$

Since $\angle ADC$ and $\angle BEC = 90^\circ$

$$\angle ADC = \angle BEC$$

$$\angle ACD = \angle BCE \text{ [Common angle]}$$

$$\triangle ADC \cong \triangle BEC \text{ [By ASA congruence rule]}$$

$$CE = CD \text{ ... (2) [CPCT]}$$

Subtracting equation (2) from (1),

We get

$$AC - CE = BC - CD$$

$$\Rightarrow AE = BD$$

Hence, proved.

Question 10

Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Solution

According to the question,

We have, ΔABC with median AD .

To prove:

$$AB + AC > 2AD$$

$$AB + BC > 2AD$$

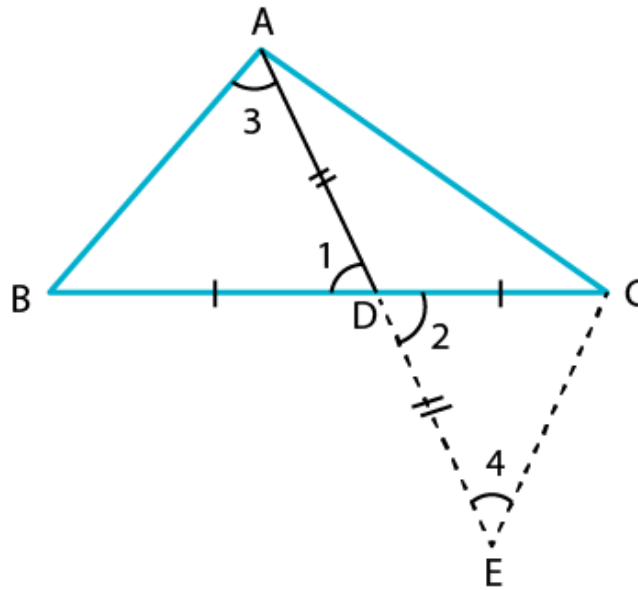
$$BC + AC > 2AD$$

Construction:

Extend AD to E such that $DE = AD$

Join EC .

Proof:



In $\triangle ADB$ and $\triangle EDC$,

$AD = ED$ [By construction]

$\angle 1 = \angle 2$ [Vertically opposite angles are equal]

$DB = DC$ [Given]

So, by SAS criterion of congruence, we have

$\triangle ADB \cong \triangle EDC$

$AB = EC$ [CPCT]

And $\angle 3 = \angle 4$ [CPCT]

Now, in $\triangle AEC$,

Since sum of the lengths of any two sides of a triangle must be greater than the third side,

We have

$AC + CE > AE$

$\Rightarrow AC + CE > AD + DE$

$\Rightarrow AC + CE > AD + AD$ [$AD = DE$]

$\Rightarrow AC + CE > 2AD$

$\Rightarrow AC + AB > 2AD$ [$\because AB = CE$]

Similarly,

We get,

$AB + BC > 2AD$ and $BC + AC > 2AD$.

Hence, proved.
