

Chapter 6: Lines and Angles

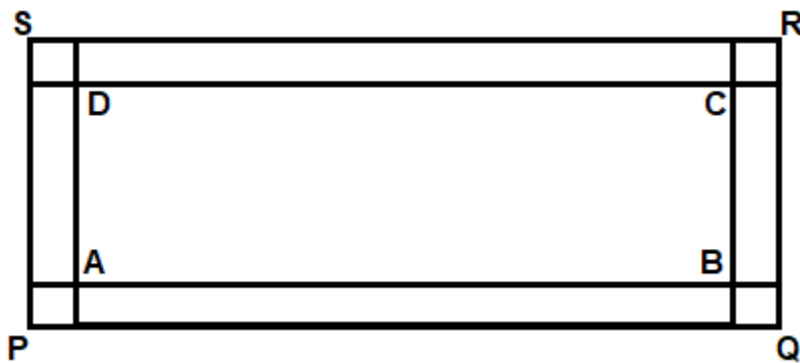
NCERT Exemplar · Class 9 · Maths

Exercise 6.1 · Textbook page 55

Question 1

In Fig. 6.1, if $AB \parallel CD \parallel EF$, $PQ \parallel RS$, $\angle RQD = 25^\circ$ and $\angle CQP = 60^\circ$, then $\angle QRS$ is equal to

- (A) 85°
- (B) 135°
- (C) 145°
- (D) 110°



Solution

(C) 145°

Explanation:

According to the given figure, we have

$AB \parallel CD \parallel EF$

$PQ \parallel RS$

$\angle RQD = 25^\circ$

$\angle CQP = 60^\circ$

$PQ \parallel RS$.

We know that,

If a transversal intersects two parallel lines, then each pair of alternate exterior angles is equal.

Now, since $PQ \parallel RS$

$$\Rightarrow \angle PQC = \angle BRS$$

We have $\angle PQC = 60^\circ$

$$\Rightarrow \angle BRS = 60^\circ \dots \text{eq.(i)}$$

We also know that,

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Now again, since $AB \parallel CD$

$$\Rightarrow \angle DQR = \angle QRA$$

We have $\angle DQR = 25^\circ$

$$\Rightarrow \angle QRA = 25^\circ \dots \text{eq.(ii)}$$

Using linear pair axiom,

We get,

$$\angle ARS + \angle BRS = 180^\circ$$

$$\Rightarrow \angle ARS = 180^\circ - \angle BRS$$

$$\Rightarrow \angle ARS = 180^\circ - 60^\circ \text{ (From (i), } \angle BRS = 60^\circ \text{)}$$

$$\Rightarrow \angle ARS = 120^\circ \dots \text{eq.(iii)}$$

$$\text{Now, } \angle QRS = \angle QRA + \angle ARS$$

From equations (ii) and (iii), we have,

$$\angle QRA = 25^\circ \text{ and } \angle ARS = 120^\circ$$

Hence, the above equation can be written as:

$$\angle QRS = 25^\circ + 120^\circ$$

$$\Rightarrow \angle QRS = 145^\circ$$

Therefore, option (C) is the correct answer.

Question 2

If one angle of a triangle is equal to the sum of the other two angles, then the triangle is

- (A) An isosceles triangle
- (B) An obtuse triangle
- (C) An equilateral triangle
- (D) A right triangle

Solution

(D) A right triangle

Explanation:

Let the angles of $\triangle ABC$ be $\angle A$, $\angle B$ and $\angle C$

Given that $\angle A = \angle B + \angle C \dots (\text{eq1})$

But, in any $\triangle ABC$,

Using the angle sum property, we have,

$$\angle A + \angle B + \angle C = 180^\circ \dots (\text{eq2})$$

From equations (eq1) and (eq2), we get

$$\angle A + \angle A = 180^\circ$$

$$\Rightarrow 2\angle A = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ / 2 = 90^\circ$$

$$\Rightarrow \angle A = 90^\circ$$

Hence, we get that the triangle is a right triangle

Therefore, option (D) is the correct answer.

Question 3

An exterior angle of a triangle is 105° , and its two interior opposite angles are equal. Each of these equal angles is

(A) $37\frac{1}{2}^\circ$

(B) $52\frac{1}{2}^\circ$

(C) $72\frac{1}{2}^\circ$

(D) 75°

Solution

(B) $52\frac{1}{2}^\circ$

Explanation:

According to the question,

The exterior angle of triangle = 105°

Let the two interior opposite angles of the triangle = x

We know that,

The exterior angle of a triangle = sum of interior opposite angles

Then, we have the equation,

$$105^\circ = x + x$$

$$2x = 105^\circ$$

$$x = 52.5^\circ$$

$$x = 52\frac{1}{2}$$

Therefore, option (B) is the correct answer.

Question 4

The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is

(A) An acute angled triangle

(B) An obtuse-angled triangle

(C) A right triangle

(D) An isosceles triangle

Solution

(A) An acute angled triangle

Explanation:

According to the question,

The angles of a triangle are of the ratio 5 : 3 : 7

Let 5:3:7 be $5x$, $3x$ and $7x$

Using the angle sum property of a triangle,

$$5x + 3x + 7x = 180$$

$$15x = 180$$

$$x = 12$$

Substituting the value of x , $x = 12$, in $5x$, $3x$ and $7x$ we get,

$$5x = 5 \times 12 = 60^\circ$$

$$3x = 3 \times 12 = 36^\circ$$

$$7x = 7 \times 12 = 84^\circ$$

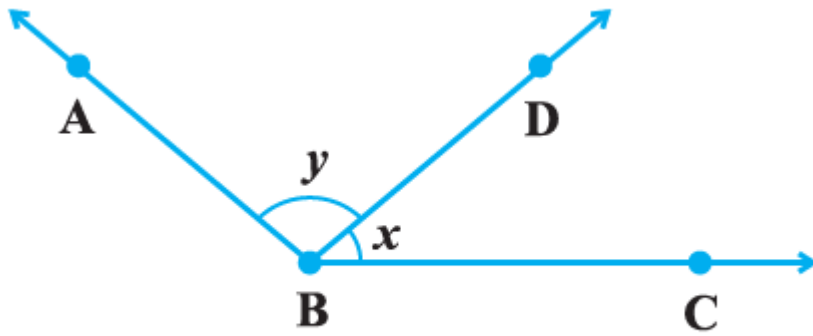
Since all the angles are less than 90° , the triangle is an acute-angled triangle.

Therefore, option (A) is the correct answer.

Exercise 6.2 · Textbook page 56

Question 1

For what value of $x + y$ in Fig. 6.4 will ABC be a line? Justify your answer.

**Fig. 6.4****Solution**

The value of $x + y$ should be 180° for ABC to be a line.

Justification:

From the figure, we can say that,

BD is a ray that intersects AB and BC at the point B, which results in

$$\angle ABD = y$$

$$\text{and, } \angle DBC = x$$

We know,

If a ray stands on a line, then the sum of two adjacent angles so formed is 180° .

$\Rightarrow$ If the sum of two adjacent angles is 180° , then a ray stands on a line.

Thus, for ABC to be a line,

The sum of $\angle ABD$ and $\angle DBC$ should be equal to 180° .

$$\Rightarrow \angle ABD + \angle DBC = 180^\circ$$

$$\Rightarrow x + y = 180^\circ$$

Therefore, the value of $x + y$ should be equal to 180° for ABC to be a line.

Question 2

Can a triangle have all angles less than 60° ? Give reasons for your answer.

Solution

No. A triangle cannot have all angles less than 60°

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be $= 180^\circ$.

Suppose, all the angles are 60° ,

Then we get, $60^\circ + 60^\circ + 60^\circ = 180^\circ$.

Now, considering angles less than 60° ,

Let us take 59° , which is the highest natural number less than 60° .

Then we have,

$$59^\circ + 59^\circ + 59^\circ = 177^\circ \neq 180^\circ$$

Hence, we can say that if all the angles are less than 60° , the measure of the angles won't satisfy the angle sum property.

Therefore, a triangle cannot have all angles less than 60° .

Question 3

Can a triangle have two obtuse angles? Give reasons for your answer.

Solution

No. A triangle cannot have two obtuse angles

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be $= 180^\circ$.

An obtuse angle is one whose value is greater than 90° but less than 180° .

Considering two angles to be equal to the lowest natural number greater than 90° , i.e., 91° .

According to the question,

If the triangle has two obtuse angles, then there are two angles which are at least 91° each.

On adding these two angles,

$$\text{Sum of the two angles} = 91^\circ + 91^\circ$$

$$\Rightarrow \text{Sum of the two angles} = 182^\circ$$

The sum of these two angles already exceeds the sum of three angles of the triangle, even without considering the third angle.

Therefore, a triangle cannot have two obtuse angles.

Question 4

How many triangles can be drawn having its angles as 45° , 64° and 72° ? Give reasons for your answer.

Solution

No triangle can be drawn having its angles 45° , 64° and 72° .

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be $= 180^\circ$.

But, according to the question,

We have the angles 45° , 64° and 72° .

$$\text{Sum of these angles} = 45^\circ + 64^\circ + 72^\circ$$

$$= 181^\circ, \text{ which is greater than } 180^\circ.$$

Hence, the angles do not satisfy the angle sum property of a triangle.

Therefore, no triangle can be drawn having its angles 45° , 64° and 72° .

Question 5

How many triangles can be drawn having its angles as 53° , 64° and 63° ? Give reasons for your answer.

Solution

Infinitely many triangles can be drawn having its angles as 53° , 64° and 63° .

Justification:

According to the angle sum property,

We know that the sum of all the interior angles of a triangle should be $= 180^\circ$.

According to the question,

We have the angles 53° , 64° , and 63° .

Sum of these angles $= 53^\circ + 64^\circ + 63^\circ$

$= 180^\circ$

Hence, the angles satisfy the angle sum property of a triangle.

Therefore, infinitely many triangles can be drawn having its angles as 53° , 64° and 63° .

Exercise 6.3 · Textbook page 58

Question 1

In Fig. 6.9, OD is the bisector of $\angle AOC$, OE is the bisector of $\angle BOC$ and $OD \perp OE$. Show that points A, O and B are collinear.

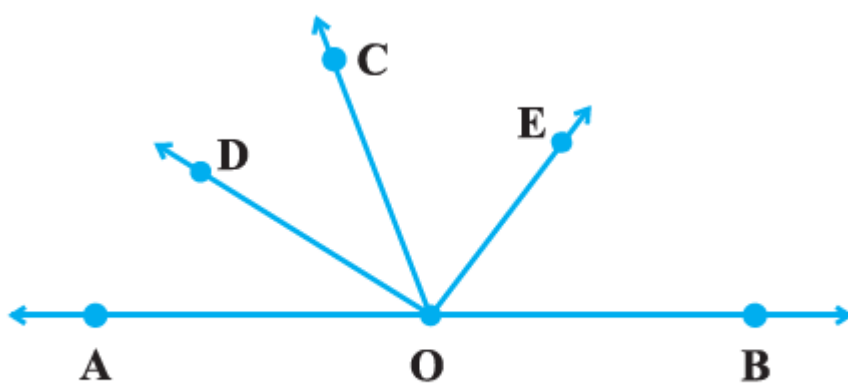


Fig. 6.9

Solution

According to the question,

In figure,

$OD \perp OE$,

OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$.

To prove: Points A, O and B are collinear

i.e., AOB is a straight line.

Proof:

Since OD and OE bisect angles $\angle AOC$ and $\angle BOC$, respectively.

$$\angle AOC = 2\angle DOC \dots(\text{eq.1})$$

$$\text{And } \angle COB = 2\angle COE \dots(\text{eq.2})$$

Adding (eq.1) and (eq.2), we get

$$\angle AOC = \angle COB = 2\angle DOC + 2\angle COE$$

$$\angle AOC + \angle COB = 2(\angle DOC + \angle COE)$$

$$\angle AOC + \angle COB = 2\angle DOE$$

Since, $OD \perp OE$

We get,

$$\angle AOC + \angle COB = 2 \times 90^\circ$$

$$\angle AOC + \angle COB = 180^\circ$$

$$\angle AOB = 180^\circ$$

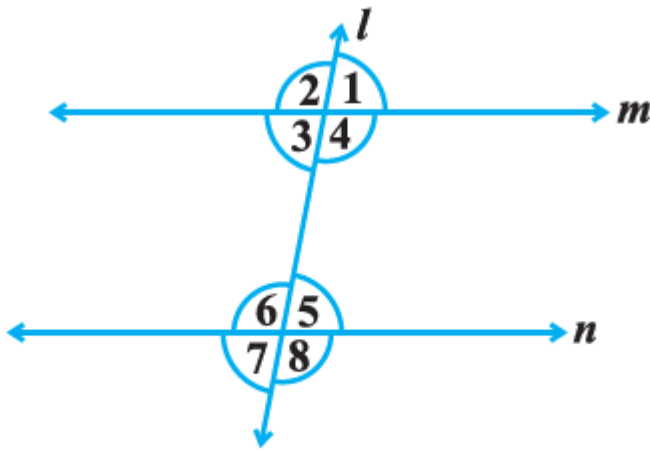
So, $\angle AOC + \angle COB$ form linear pair.

Therefore, AOB is a straight line.

Hence, points A, O and B are collinear.

Question 2

In Fig. 6.10, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$. Show that the lines m and n are parallel.

**Fig. 6.10****Solution**

According to the question,

We have from the figure $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$

Since, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$

Here, $\angle 1 = \angle 3$ [since they are vertically opposite angles]

$$\angle 3 = \angle 1 = 60^\circ$$

$$\text{Now, } \angle 3 + \angle 6 = 60^\circ + 120^\circ$$

$$\Rightarrow \angle 3 + \angle 6 = 180^\circ$$

We know that,

If the sum of two interior angles on the same side of l is 180° , then the lines are parallel.

Therefore, $m \parallel n$

Question 3

AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (Fig. 6.11). Show that $AP \parallel BQ$.

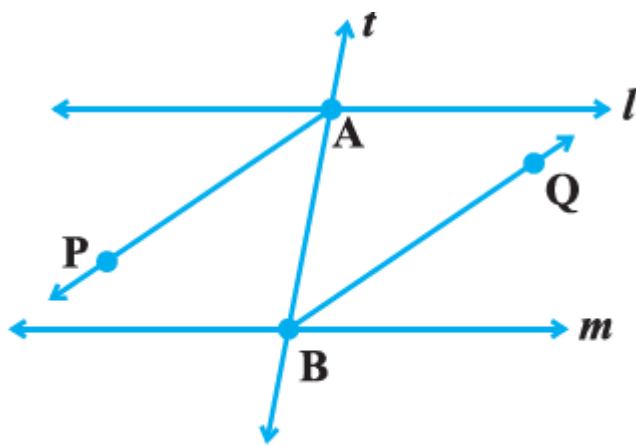
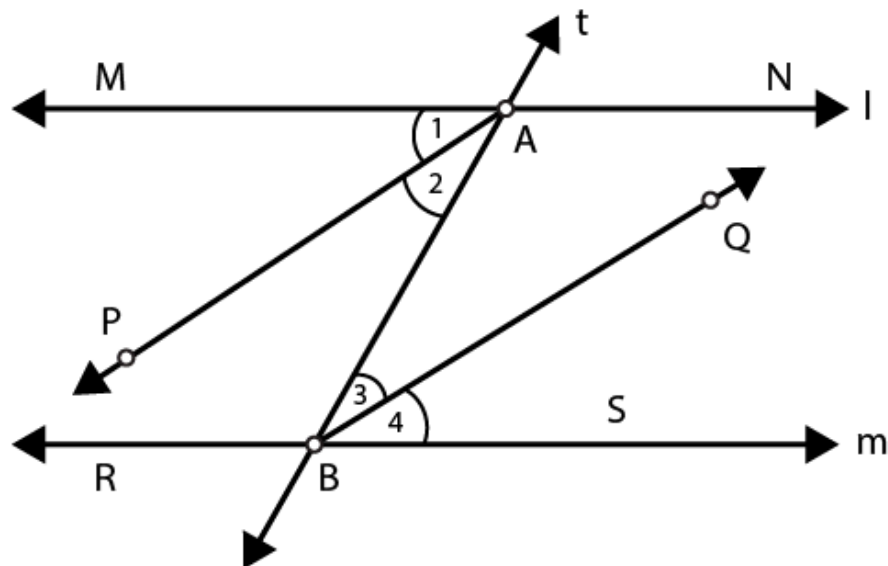


Fig. 6.11

Solution



$l \parallel m$ and t is the transversal

$\angle MAB = \angle SBA$ [alternate angles]

$\Rightarrow \frac{1}{2} \angle MAB = \frac{1}{2} \angle SBA$

$\Rightarrow \angle PAB = \angle QBA$

$\Rightarrow \angle 2 = \angle 3$

But, $\angle 2$ and $\angle 3$ are alternate angles.

Hence, $AP \parallel BQ$.

Question 4

If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that $l \parallel m$.

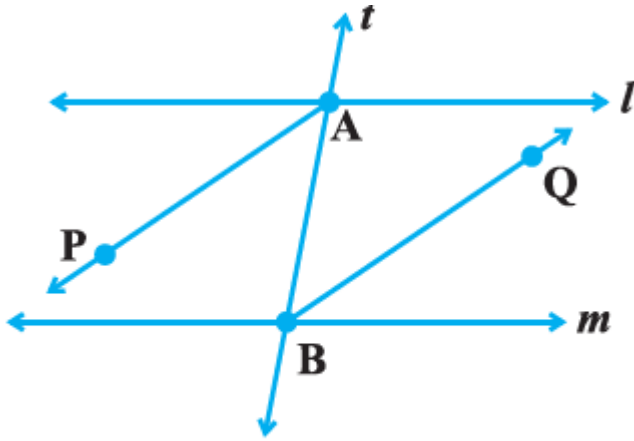


Fig. 6.11

Solution

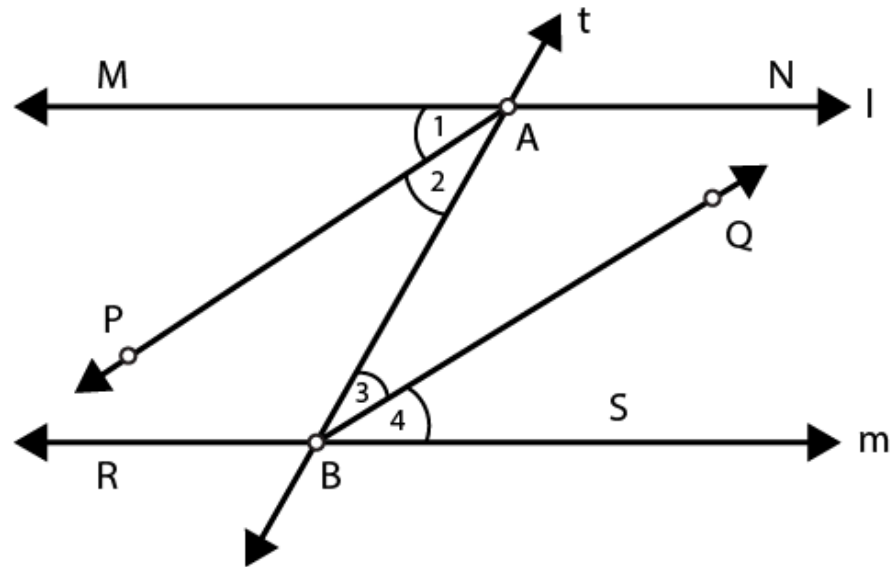
AP is the bisector of $\angle MAB$

BQ is the bisector of $\angle SBA$.

Given: $AP \parallel BQ$.

As $AP \parallel BQ$,

We have,



So $\angle 2 = \angle 3$ [Alternate angles]

$$2\angle 2 = 2\angle 3$$

$$\Rightarrow \angle 2 + \angle 2 = \angle 3 + \angle 3$$

From figure, we have $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$$

$$\Rightarrow \angle MAB = \angle SBA$$

But, we know that these are alternate angles.

Hence, the lines l and m are parallel, i.e., $l \parallel m$.

Question 5

In Fig. 6.12, $BA \parallel ED$ and $BC \parallel EF$. Show that $\angle ABC = \angle DEF$ [Hint: Produce DE to intersect BC at P (say)].

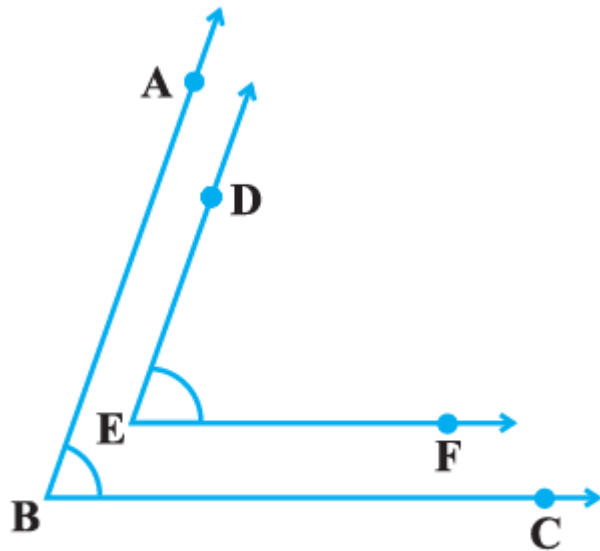


Fig. 6.12

Solution

Construction:

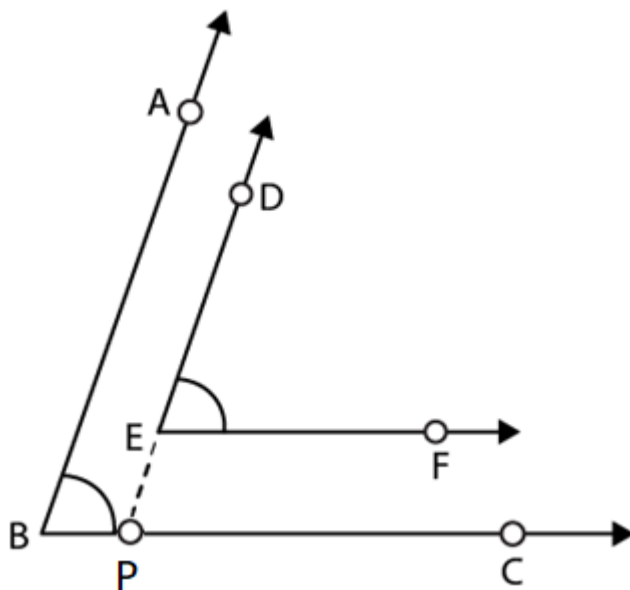
Extend DE to intersect BC at point, P.

Given, $EF \parallel BC$ and DP are the transversal,

$$\angle DEF = \angle DPC \dots (\text{eq.1}) \text{ [Corresponding angles]}$$

Also given, $AB \parallel DP$ and BC is the transversal,

$$\angle DPC = \angle ABC \dots (\text{eq.2}) \text{ [Corresponding angles]}$$



From (eq.1) and (eq.2), we get

$$\angle ABC = \angle DEF$$

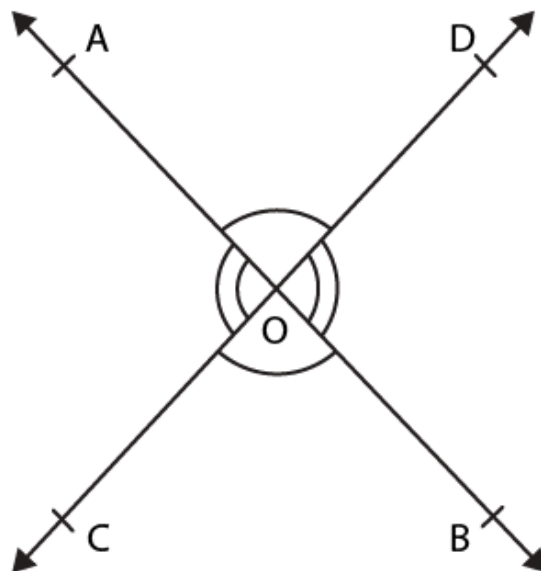
Hence, Proved.

Exercise 6.4 · Textbook page 61

Question 1

If two lines intersect, prove that the vertically opposite angles are equal.

Solution



From the figure, we know that,

AB and CD intersect each other at point O.

Let the two pairs of vertically opposite angles be,

1st pair – $\angle AOC$ and $\angle BOD$

2nd pair – $\angle AOD$ and $\angle BOC$

To prove:

Vertically opposite angles are equal,

i.e., $\angle AOC = \angle BOD$, and $\angle AOD = \angle BOC$

From the figure,

The ray AO stands on the line CD.

We know that,

If a ray lies on a line, then the sum of the adjacent angles is equal to 180° .

$$\Rightarrow \angle AOC + \angle AOD = 180^\circ \text{ (By linear pair axiom) ... (i)}$$

Similarly, the ray DO lies on line AOB.

$$\Rightarrow \angle AOD + \angle BOD = 180^\circ \text{ (By linear pair axiom) ... (ii)}$$

From equations (i) and (ii),

We have,

$$\angle AOC + \angle AOD = \angle AOD + \angle BOD$$

$$\Rightarrow \angle AOC = \angle BOD \text{ --- (iii)}$$

Similarly, the ray BO lies on the line COD.

$$\Rightarrow \angle DOB + \angle COB = 180^\circ \text{ (By linear pair axiom) --- (iv)}$$

Also, the ray CO lies on line AOB.

$$\Rightarrow \angle COB + \angle AOC = 180^\circ \text{ (By linear pair axiom) --- (v)}$$

From equations (iv) and (v),

We have,

$$\angle DOB + \angle COB = \angle COB + \angle AOC$$

$$\Rightarrow \angle DOB = \angle AOC \text{ --- (vi)}$$

Thus, from equation (iii) and equation (vi),

We have,

$$\angle AOC = \angle BOD, \text{ and } \angle DOB = \angle AOC$$

Therefore, we get vertically opposite angles are equal.

Hence Proved.

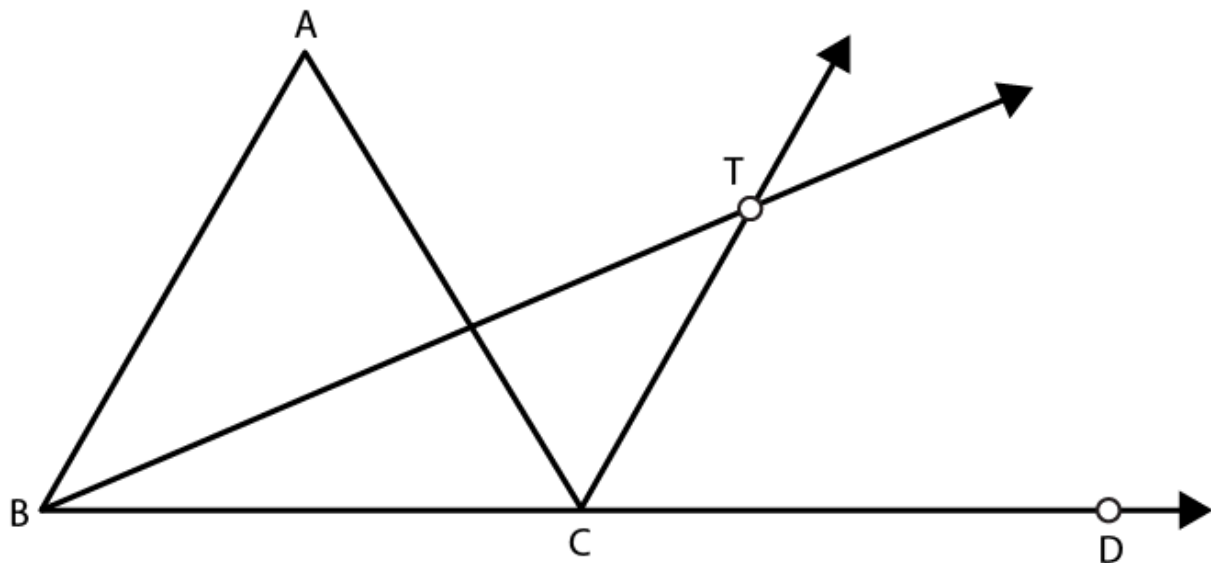
Question 2

Bisectors of interior $\angle B$ and exterior $\angle ACD$ of a $\triangle ABC$ intersect at point T.

Prove that $\angle BTC = \frac{1}{2} \angle BAC$.

Solution

Given: $\triangle ABC$, produce BC to D, and the bisectors of $\angle ABC$ and $\angle ACD$ meet at point T.



To prove:

$$\angle BTC = \frac{1}{2} \angle BAC$$

Proof:

In $\triangle ABC$, $\angle ACD$ is an exterior angle.

We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then,

$$\angle ACD = \angle ABC + \angle CAB$$

Dividing L.H.S and R.H.S by 2,

$$\Rightarrow \frac{1}{2} \angle ACD = \frac{1}{2} \angle CAB + \frac{1}{2} \angle ABC$$

$$\Rightarrow \angle TCD = \frac{1}{2} \angle CAB + \frac{1}{2} \angle ABC \dots(1)$$

We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then in $\triangle BTC$,

$$\angle TCD = \angle BTC + \angle CBT$$

$$\Rightarrow \angle TCD = \angle BTC + \frac{1}{2} \angle ABC \dots(2)$$

From equations (1) and (2),

We get,

$$\frac{1}{2} \angle CAB + \frac{1}{2} \angle ABC = \angle BTC + \frac{1}{2} \angle ABC$$

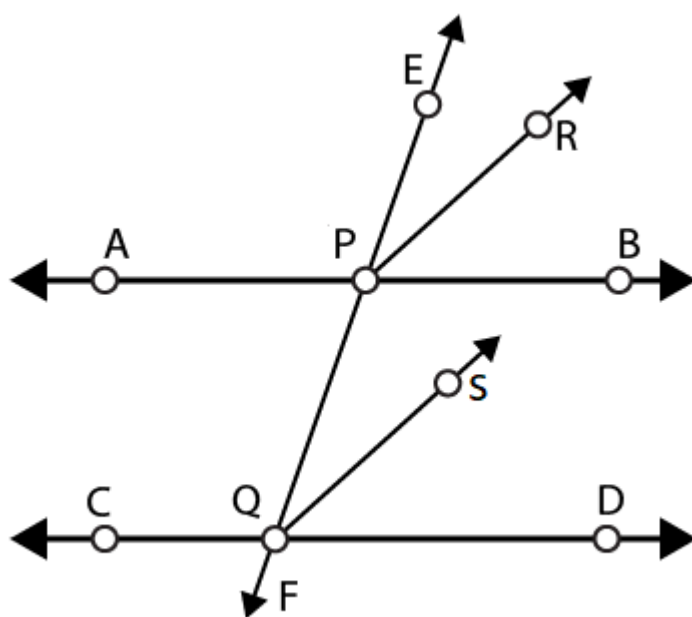
$$\Rightarrow \frac{1}{2} \angle CAB = \angle BTC \text{ or } \frac{1}{2} \angle BAC = \angle BTC$$

Hence, proved.

Question 3

A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Solution



Let,

$$AB \parallel CD$$

EF be the transversal passing through the two parallel lines at P and Q, respectively.

PR and QS are the bisectors of $\angle EPB$ and $\angle PQD$.

We know that the corresponding angles of parallel lines are equal,

$$\text{So, } \angle EPB = \angle PQD$$

$$\frac{1}{2} \angle EPB = \frac{1}{2} \angle PQD$$

$$\angle EPR = \angle PQS$$

But, we also know that they are corresponding angles of PR and QS

Since the corresponding angles are equal,

We have,

$$PR \parallel QS$$

Hence Proved.
