

Chapter 2: Polynomials

NCERT Exemplar · Class 9 · Maths

Exercise 2.1

Question 1

Which of the following is a polynomial?

(A) $\frac{x^2}{2} - \frac{2}{x^2}$

(B) $\sqrt{2x} - 1$

(C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

(D) $\frac{x-1}{x+1}$

Solution

Ans: (C) is the correct option.

Polynomials are an algebraic expression that consist of variables and coefficients. Here the power of variables should be in whole number like 1,2,3 etc.

(A) $\frac{x^2}{2} - \frac{2}{x^2}$

Here the coefficient -2 is multiplied with x^{-2} . So here the power of x is not a whole number. Hence, it is not a Polynomial.

(B) $\sqrt{2x} - 1$

Here the coefficient $\sqrt{2}$ is multiplied with $x^{\frac{1}{2}}$. So here the power of x is not a whole number. Hence, it is not a Polynomial.

(C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

Here the coefficient 1 is multiplied with x^2 and the coefficient 3 is multiplied with

$$x \frac{3}{2} - \frac{1}{2} = x \frac{2}{2} = x$$

. So here the power of x is a whole number in the whole equation. Hence, it is a Polynomial.

(D) $\frac{x-1}{x+1}$

It is not a standard format of a Polynomial. Hence, it is not a Polynomial.

Question 2

$\sqrt{2}$ is a Polynomial of degree

(A) 2

(B) 0

(C) 1

(D) $\frac{1}{2}$

Solution

Ans: Option (B) is correct.

Here $\sqrt{2}$ is a constant Polynomial. We can also write it as $\sqrt{2}x^0$. Therefore, the degree of this Polynomial is 0.

Question 3

Degree of the Polynomial of $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(A) 4

(B) 5

(C) 3

(D) 7

Solution

Ans: Option (A) is correct.

The highest power of x in the Polynomial is called the degree of Polynomial. We have the term with the highest power of x is $4x^4$, which is 4 .

Therefore, the degree of Polynomial is 4 .

Question 4

Degree of the zero Polynomial

- (A) 0
- (B) 1
- (C) Any natural number
- (D) Not defined

Solution

Ans: Option (D) is the correct answer.

Like any constant value, the value 0 can be considered as a (constant) Polynomial, called the zero Polynomial. It has no nonzero terms, and so, strictly speaking, it has no degree either. Therefore, the degree of the zero Polynomial is not defined.

Question 5

If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

- (A) 0
- (B) 1
- (C) $4\sqrt{2}$
- (D) $8\sqrt{2} + 1$

Solution

Ans: The correct option is (B).

We have,

$$p(x) = x^2 - 2\sqrt{2}x + 1$$

Here, we have given the value of $x = 2\sqrt{2}$.

So, on putting the value in above equation, we get

$$p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1$$

$$p(2\sqrt{2}) = 4 \times 2 - 4 \times 2 + 1$$

$$p(2\sqrt{2}) = 8 - 8 + 1$$

$$p(2\sqrt{2}) = 1$$

Question 6

The value of the Polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(A) -6

(B) 6

(C) 2

(D) -2

Solution

Ans: The correct option is (A).

Here we have given a Polynomial $5x - 4x^2 + 3$.

In this we have to put $x = -1$, and then we have to find the value of Polynomial

Therefore,

$$p(x) = 5x - 4x^2 + 3$$

$$p(-1) = 5(-1) - 4(-1)^2 + 3$$

$$p(-1) = -5 - 4 + 3$$

$$p(-1) = -9 + 3$$

$$p(-1) = -6$$

Question 7

If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

- (A) 3
- (B) $2x$
- (C) 0
- (D) 6

Solution

Ans: The correct option is (D).

Here, we have $p(x) = x + 3$.

Now, on substituting x with $-x$, we get $p(-x) = -x + 3$

Now, on adding both the equations, we get

$$p(x) + p(-x) = x + 3 - x + 3$$

$$p(x) + p(-x) = 6$$

Question 8

Zero of a zero Polynomial is

- (A) 0
- (B) 1
- (C) Any real number
- (D) Not defined

Solution

Ans: (C) is the correct answer.

Zero Polynomial is a constant Polynomial whose all coefficients are equal to 0. Zero of a Polynomial or we can say that root of a polynomial is the value of a variable that is responsible to make the Polynomial equals to zero. Hence, zero of the zero Polynomial is any real number.

Question 9

Zero of the Polynomial $p(x) = 2x + 5$ is

(A) $-\frac{2}{5}$

(B) $-\frac{5}{2}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$

Solution

Ans: (B) is the correct option.

To get the zero of the Polynomial, we have to put the expression equals to zero.

$$p(x) = 0$$

$$2x + 5 = 0$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

Therefore, $-\frac{5}{2}$ is the zero of the Polynomial.

Question 10

One of the zeroes of given Polynomial $2x^2 + 7x - 4$ is

(A) 2

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) -2

Solution

Ans: (B) is the correct option.

To find the zeroes of a Polynomial, we have to put that expression equals to zero.

Therefore,

$$2x^2 + 7x - 4 = 0$$

Using middle term splitting,

$$2x^2 + (8 - 1)x - 4 = 0$$

$$2x^2 + 8x - x - 4 = 0$$

$$2x(x + 4) - 1(x + 4) = 0$$

$$(2x - 1)(x + 4) = 0$$

So, there we have two Ansutions

$$2x - 1 = 0 \text{ and } x + 4 = 0$$

$$2x = 1 \text{ and } x = -4$$

$$x = \frac{1}{2} \text{ and } x = -4$$

Question 11

If $x^{51} + 51$ is divided by $x + 1$, the remainder is

- (A) 0
- (B) 1
- (C) 49
- (D) 50

Solution

Ans: Option (D) is the correct option.

Here we will use the remainder theorem.

If any Polynomial $f(x)$ is divided by $x - h$, then the remainder will be $f(h)$

Here given Polynomial is $x^{51} + 51$ and divided by $x + 1$

So, by using the remainder theorem we can say that

Remainder= $f(-1)$ since the divisor is $x + 1$

Therefore we can write it as

$$\begin{aligned}\Rightarrow f(x) &= x^{51} + 51 \\ \Rightarrow f(-1) &= (-1)^{51} + 51 \\ \Rightarrow f(-1) &= -1 + 51 \\ \therefore f(-1) &= 50\end{aligned}$$

Question 12

If $x + 1$, is a factor of the Polynomial $2x^2 + kx$, then the value of k is

- (A) -3
- (B) 4
- (C) 2
- (D) -2

Solution

Ans: Option (C) is correct.

Let $p(x) = 2x^2 + kx$

Since,

$$(x + 1)$$

is a factor of

$$p(x)$$

, then

Using, factor theorem

$$\begin{aligned}\Rightarrow p(-1) &= 0 \\ \Rightarrow 2(-1)^2 + k(-1) &= 0 \\ \Rightarrow 2 - k &= 0 \\ \Rightarrow k &= 2\end{aligned}$$

Therefore, the value of k is 2 .

Question 13

$x + 1$, is a factor of the Polynomial

- (A) $x^3 + x^2 - x + 1$
(B) $x^3 + x^2 + x + 1$
(C) $x^4 + x^3 + x^2 + 1$
(D) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution

Ans: (B) is the correct option.

If $x + 1$ is the factor of the Polynomial, then $x = -1$ is the root of the Polynomial.

(A) Let $p(x) = x^3 + x^2 - x + 1$

Now, put $x = -1$.

$$p(-1) = (-1)^3 + (-1)^2 - (-1) + 1$$

$$p(-1) = -1 + 1 + 1 + 1$$

$$p(-1) = 2$$

Therefore, $p(-1) \neq 0$.

So, $x + 1$ is not a factor of the $p(x)$.

(B) Let $p(x) = x^3 + x^2 + x + 1$

Now, put $x = -1$.

$$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

$$p(-1) = -1 + 1 - 1 + 1$$

$$p(-1) = 0$$

Therefore, $p(-1) = 0$.

So, $x + 1$ is a factor of the $p(x)$.

(C) Let $p(x) = x^4 + x^3 + x^2 + 1$

Now, put $x = -1$.

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + 1$$

$$p(-1) = 1 - 1 + 1 + 1$$

$$p(-1) = 2$$

Therefore, $p(-1) \neq 0$.

So, $x + 1$ is not a factor of the $p(x)$.

(D) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Now, put $x = -1$.

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$p(-1) = 1 - 3 + 3 - 1 + 1$$

$$p(-1) = 1$$

Therefore, $p(-1) \neq 0$.

So, $x + 1$ is not the factor of the $p(x)$.

Question 14

One of the factor of $(25x^2 - 1) + (1 + 5x)^2$ is

(A) $5 + x$

(B) $5 - x$

(C) $5x - 1$

(D) $10x$

Solution

Ans: (D) is the correct option.

$$\Rightarrow (25x^2 - 1) + (1 + 5x)^2$$

$$\Rightarrow ((5x)^2 - (1)^2) + (5x + 1)^2$$

$$\Rightarrow (5x + 1)(5x - 1) + (5x + 1)^2$$

Now, taking common $(5x + 1)$

$$\Rightarrow (5x + 1)((5x - 1) + (5x + 1))$$

$$\Rightarrow (5x + 1)(5x - 1 + 5x + 1)$$

$$\Rightarrow (5x + 1)(10x)$$

Therefore, one of the factor of $(25x^2 - 1) + (1 + 5x)^2$ is $10x$.

Question 15

The value of $249^2 - 248^2$ is

- (A) 1^2
- (B) 477
- (C) 487
- (D) 497

Solution

Ans: (D) is the correct option.

Here we will use the identity $a^2 - b^2 = (a + b)(a - b)$.

We have,

$$\begin{aligned} &\Rightarrow 249^2 - 248^2 \\ &\Rightarrow (249 - 248)(249 + 248) \\ &\Rightarrow (1)(497) \\ &\Rightarrow 497 \end{aligned}$$

Question 16

The factorization of $4x^2 + 8x + 3$ is

- (A) $(x + 1)(x + 3)$
- (B) $(2x + 1)(2x + 3)$
- (C) $(2x + 2)(2x + 5)$
- (D) $(2x - 1)(2x - 3)$

Solution

Ans: (B) is the correct option.

In this question, we will use middle term splitting for factorization

$$\Rightarrow 4x^2 + 8x + 3$$

We can also write it as,

$$\Rightarrow 4x^2 + 6x + 2x + 3$$

$$\Rightarrow 2x(2x + 3) + 1(2x + 3)$$

$$\Rightarrow (2x + 1)(2x + 3)$$

Question 17

Which of the followings is factor of $(x + y)^3 - (x^3 + y^3)$?

(A) $x^2 + y^2 + 2xy$

(B) $x^2 + y^2 - xy$

(C) xy^2

(D) $3xy$

Solution

Ans: (D) is the correct option.

We have, $(x + y)^3 - (x^3 + y^3)$.

Now, in this question we will use the identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\Rightarrow x^3 + y^3 + 3xy(x + y) - (x^3 + y^3)$$

$$\Rightarrow 3xy(x + y)$$

So, we can say that $3xy$ is a factor of $(x + y)^3 - (x^3 + y^3)$.

Question 18

The coefficient of variable x in the expansion $(x + 3)^3$ is

(A) 1

(B) 9

(C) 18

(D) 27

Solution

Ans: (D) is the correct option.

Here in this question, we will use the identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Therefore,

$$(x + 3)^3 = x^3 + 3^3 + 3 \times x \times 3(x + 3)$$

$$(x + 3)^3 = x^3 + 27 + 9x^2 + 27x$$

Therefore, the coefficient of x is 27 .

Question 19

If $\frac{x}{y} + \frac{y}{x} = -1$ the value of $x^3 - y^3$ is

(A) 1

(B) -1

(C) 0

(D) $\frac{1}{2}$

Solution

Ans: The correct option is (C).

Here, it is given that $\frac{x}{y} + \frac{y}{x} = -1$

On taking LCM, we get

$$\frac{x^2 + y^2}{xy} = -1$$

On cross-multiplication, we get

$$x^2 + y^2 = -xy$$

Now, we know that

$$x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$$

On putting $x^2 + y^2 = -xy$ in above equation, we get

$$x^3 - y^3 = (x - y)(-xy + xy)$$

$$x^3 - y^3 = (x - y)(0)$$

$$x^3 - y^3 = 0$$

Question 20

If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is

(A) 0

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Solution

Ans: (C) is the correct option.

Here in this question, we will use the identity $a^2 - b^2 = (a + b)(a - b)$

$$\Rightarrow 49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$$

$$\Rightarrow 49x^2 - b = (7x)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow 49x^2 - b = 49x^2 - \left(\frac{1}{2}\right)^2$$

On comparing, we get

$$b = \left(\frac{1}{2}\right)^2$$

$$b = \frac{1}{4}$$

Question 21

If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is equal to

(A) 0

(B) abc

(C) $3abc$

(D) $2abc$

(i) $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ is a polynomial

(ii) $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}}$ is a polynomial, $x \neq 0$

Solution

Ans: (C) is the correct option.

Here we know an identity,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

It is given that, $a + b + c = 0$

Therefore, put it in the above equation

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = (0)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

Short Answer Questions with Reasoning

Sample Question 1: Write whether the following statements are True or False.

Justify your answer.

Ans: It is a false statement.

Here, the exponent of a variable is not a whole number. So, it is not a polynomial.

Ans: It is a true statement.

We can also write it as,

$$\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}} = \frac{\sqrt{x}(6 + x)}{\sqrt{x}} = 6 + x$$

Here, the exponent of variable is not a whole number. So, it is a polynomial.

Exercise 2.2

Question 1

Which of the following expressions are Polynomials? Justify your answer.

(i) 8

(ii) $\sqrt{3}x^2 - 2x$

(iii) $1 - \sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$

(v) $\frac{(x-2)(x-4)}{x}$

(vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

(viii) $\frac{1}{2x}$

Solution

Ans: Here we have given an expression 8 in which the degree of variable is zero. Therefore, it is a constant Polynomial.

Ans: $\sqrt{3}x^2 - 2x$

Here, in each term the power of x is a whole number. Therefore, this expression is a Polynomial.

Ans: $1 - \sqrt{5x}$

We can also write it as $1 - \sqrt{5}x^{\frac{1}{2}}$. Here the degree of variable is not a whole number. Therefore, this expression is not a Polynomial.

Ans: $\frac{1}{5x^{-2}} + 5x + 7$

We can also write it as $\frac{1x^2}{5} + 5x + 7$. Here in this expression each variable term has the power of x in a whole number. Therefore, this expression is a Polynomial.

Ans: $\frac{(x-2)(x-4)}{x}$

We can also write it as

$$\frac{x^2 - 6x + 8}{x} = x - 6 + \frac{8}{x} = x - 6 + 8x^{-1}$$

Here, the exponent of variable x in the third term $8x^{-1}$ is -1 , and this is not a whole number.

Therefore, this algebraic expression is not a Polynomial.

Ans: $\frac{1}{x+1}$

We can also reduce it to $(x+1)^{-1}$, which cannot be reduced to an expression in which the exponent of the variable x had only whole numbers in each of its terms. Therefore, this algebraic expression is not a Polynomial.

Ans: $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

In this expression, the exponent of a in each term is a whole number, so this expression is a Polynomial.

Ans: $\frac{1}{2x}$

We can also write it as $\frac{1}{2}x^{-1}$. Here, the exponent of the variable x is -1 , which is not a whole number so, this algebraic expression is not a Polynomial.

Question 2

Write whether the following statements are True or False. Justify your answer.

(i) A binomial can have at most two terms

(ii) Every Polynomial is a binomial

(iii) A binomial may have degree 5

(iv) Zero of a Polynomial is always 0

(v) A Polynomial can't have more than one zeros

(vi) The power of the sum of two polynomials each of whose degree 5 is always 5 .

Solution

Ans: The given statement is False because binomials have exactly two terms.

Ans: Every Polynomial cannot be a binomial because a Polynomial had many terms but a binomial had only two terms in the expression. For example, $x^4 + x^3 + x^2 + x + 1$ is a Polynomial but not a binomial. Therefore, the given statement is false.

Ans: The given statement is True because a binomial is a Polynomial whose degree is a whole number and it is greater than or equals to one and contain two terms. For example, $x^5 - 1$ is a binomial of degree 5 .

Ans: The given statement is False, because zero of Polynomial can be any real number.

Ans: The given statement is False, because a Polynomial can have any number of zeroes which depends on the degree of the Polynomial.

Ans: The given statement is False. For example, consider the two Polynomials $-x^5 + 3x^2 + 4$ and $x^5 + x^4 + 2x^3 + 3$. The degree of each of these polynomials will be 5. Their sum is $x^4 + 2x^3 + 3x^2 + 7$. The degree of this Polynomial is 4 not 5 .

Exercise 2.3

Question 1

Classify the following Polynomial as Polynomials in one variable, two variables etc.

(i) $x^2 + x + 1$

(ii) $y^3 - 5y$

(iii) $xy + yz + zx$

(iv) $x^2 - 2xy + y^2 + 1$

Solution

Ans: $x^2 + x + 1$ is a Polynomial in one variable.

Ans: $y^3 - 5y$ is a Polynomial in one variable.

Ans: $xy + yz + zx$ is a Polynomial in three variables.

Ans: $x^2 - 2xy + y^2 + 1$ is a Polynomial in two variables.

Question 2

Determine the degree of each of the following Polynomials:

(i) $2x - 1$

(ii) -10

(iii) $x^3 - 9x + 3x^5$

(iv) $y^3(1 - y^4)$

Solution

Ans: Since the highest power of x is 1 , the degree of the Polynomial $2x - 1$ is 1 .

Ans: -10 is a non-zero constant. A constant term does not contain any variable and its degree is always 0 .

Ans: Since the highest power of x is 5 , the degree of the Polynomial $x^3 - 9x + 3x^5$ is 5 .

Ans: Here, we have $y^3(1 - y^4) = y^3 - y^7$. Since the highest power of y is 7 , the degree of the Polynomial is 7 .

Question 3

For the Polynomial $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write

(i) the degree of the Polynomial

(ii) the coefficient of x^3

(iii) the coefficient of x^6

(iv) the constant term.

Solution

Ans: $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$

We can also write it as,

$$\frac{x^3}{5} + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$

As we know that highest power of variable in a Polynomial is known as degree of a Polynomial. In given Polynomial, the term with highest of x is $-x^6$, and the exponent of x in this term is 6.

Ans: The coefficient of x^3 is $\frac{1}{5}$.

Ans: The coefficient of x^6 is -1 .

Ans: The constant term is $\frac{1}{5}$.

Question 4

Write the coefficient of x^2 in each of the following:

(i) $\frac{\pi}{6}x + x^2 - 1$

(ii) $3x - 5$

(iii) $(x - 1)(3x - 4)$

(iv) $(2x - 5)(2x^2 - 3x + 1)$

Solution

Ans: Coefficient of x^2 in the given Polynomial is 1.

Ans: The given Polynomial can also be written as $0x^2 + 3x - 5$. So, the coefficient of x^2 in the Polynomial is 0.

Ans: We can write the given polynomial as:

$$\Rightarrow (x - 1)(3x - 4)$$

$$\Rightarrow 3x^2 - 3x - 4x + 4$$

$$\Rightarrow 3x^2 - 7x + 4$$

Therefore, the coefficient of x^2 in the given Polynomial is 3.

Ans: We can write the given polynomial as:

$$\Rightarrow (2x - 5)(2x^2 - 3x + 1)$$

$$\Rightarrow 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$$

$$\Rightarrow 4x^3 - 16x^2 + 17x - 5$$

So, the coefficient of x^2 in the given Polynomial is -16 .

Question 5

Classify the following as a constant, linear quadratic and cubic Polynomials:

(i) $2 - x^2 - x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^3$

(ix) t^2

(x) $\sqrt{2}x - 1$

Solution

Ans: To do this question, we have to keep some points in mind:

(A) A Polynomial in which there is no variable term and there is only constant term, is known as a constant polynomial.

(B) A Polynomial having degree 1 is known as a linear Polynomial.

(C) A Polynomial of having degree 2 is known as quadratic Polynomial.

(D) A Polynomial having degree 3 is known as cubic Polynomial.

Cubic Polynomial is the correct answer.

Ans: Cubic Polynomial is the correct answer.

Ans: Linear Polynomial is the correct answer.

Ans: Quadratic Polynomial is the correct answer.

Ans: Constant Polynomial is the correct answer.

Ans: Linear Polynomial is the correct answer.

Ans: Cubic Polynomial is the correct answer.

Ans: Cubic Polynomial is the correct answer.

Ans: Quadratic Polynomial is the correct answer.

Ans: Linear Polynomial is the correct answer.

Question 6

Give an example of a Polynomial, which is:

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

Solution

Ans: A Polynomial which contains only one term is called a monomial, a Polynomial having only two terms is called binomial, a Polynomial having only three terms is called a trinomial.

$5x$ is monomial of degree 1 .

Ans: $x^{20} + 5$ is a binomial of degree 20 .

Ans: $2x^2 + 2x + 1$ is a trinomial of degree 2 .

Question 7

Find the value of the Polynomial $3x^3 - 4x^2 + 7x + 5$, when $x = 3$ and also when $x = -3$.

Solution

Ans: Let $p(x)$ be the given Polynomial.

$$p(x) = 3x^3 - 4x^2 + 7x + 5$$

Now, put $x = 3$

$$p(3) = 3(3)^3 - 4(3)^2 + 7(3) + 5$$

$$p(3) = 3(27) - 4(9) + 7(3) + 5$$

$$p(3) = 61$$

Now,

Put $x = -3$

$$p(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$$

$$p(-3) = 3(-27) - 4(9) + 7(-3) - 5$$

$$p(-3) = -81 - 36 - 21 - 5$$

$$p(-3) = -143$$

Question 8

If $p(x) = x^2 - 4x + 3$, evaluate $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

Solution

Ans: We have $p(x) = x^2 - 4x + 3$

Now, put $x = 2$

$$p(2) = (2^2 - 4 \times 2 + 3) = (4 - 8 + 3) = (-4 + 3) = -1$$

Now, put $x = -1$

$$p(-1) = ((-1)^2 - 4 \times (-1) + 3) = (1 + 4 + 3) = 8$$

Now, put $x = \frac{1}{2}$

$$p\left(\frac{1}{2}\right) = \left(\left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 3\right) = \left(\frac{1}{4} - 2 + 3\right) = \left(1 + \frac{1}{4}\right) = \frac{5}{4}$$

Now,

$$p(2) - p(-1) + p\left(\frac{1}{2}\right) = -1 - 8 + \frac{5}{4}$$

$$p(2) - p(-1) + p\left(\frac{1}{2}\right) = -9 + \frac{5}{4} = \frac{-36 + 5}{4} = -\frac{31}{4}$$

Question 9

Find $p(0)$, $p(1)$, $p(-2)$ for the following Polynomials:

(i) $p(x) = 10x - 4x^2 - 3$

(ii) $p(y) = (y + 2)(y - 2)$

Solution

Ans: We have

$$p(x) = 10x - 4x^2 - 3$$

Put, $x = 0$

$$p(0) = 10(0) - 4(0)^2 - 3 = -3$$

Put, $x = 1$

$$p(1) = 10(1) - 4(1)^2 - 3 = 10 - 4 - 3 = 3$$

Put, $x = -2$

$$p(-2) = 10(-2) - 4(-2)^2 - 3 = -20 - 16 - 3 = -39$$

Ans: We have

$$p(y) = (y + 2)(y - 2)$$

Put, $y = 0$

$$p(0) = (0 + 2)(0 - 2) = -4$$

Put, $y = 1$

$$p(1) = (1 + 2)(1 - 2) = (3)(-1) = -3$$

Put, $y = -2$

$$p(-2) = (-2 + 2)(-2 - 2) = 0$$

Question 10

Verify whether the following are true or false.

(i) -3 is a zero of $x - 3$.

(ii) $-\frac{1}{3}$ is a zero of $3x + 1$.

(iii) $-\frac{4}{5}$ is a zero of $4 - 5y$.

(iv) 0 and 2 are the zeroes of $t^2 - 2t$

(v) -3 is a zero $y^2 + y - 6$

Solution

Ans: A number c can be a zero of a Polynomial $p(x)$ if a number c is such that

$$p(c) = 0$$

.

$$\text{Let } p(x) = x - 3$$

$$p(-3) = -3 - 3 = -6$$

$$p(-3) \neq 0$$

Hence, -3 is not a zero of $x - 3$.

So, the result is False.

Ans: A number c can be a zero of a Polynomial $p(x)$ if a number c is such that

$$p(c) = 0$$

.

$$\text{Let } p(x) = 3x + 1$$

$$\text{Put, } x = -\frac{1}{3}$$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1$$

$$p\left(-\frac{1}{3}\right) = -1 + 1 = 0$$

Hence, $-\frac{1}{3}$ is zero of $p(x) = 3x + 1$.

So, the result is True.

Ans: A number c can be a zero of a Polynomial $p(x)$ if a number c is such that

$$p(c) = 0$$

.

$$\text{Let } p(y) = 4 - 5(y)$$

Put, $y = -\frac{4}{5}$

$$p\left(-\frac{4}{5}\right) = 4 - 5\left(-\frac{4}{5}\right)$$

$$p\left(-\frac{4}{5}\right) = 4 + 4 = 8$$

Hence, $-\frac{4}{5}$ is zero of $p(y) = 4 - 5(y)$.

Ans: Let $p(t) = t^2 - 2t$

Put, $t = 0$

$$p(0) = (0)^2 - 2(0)$$

$$p(0) = 0$$

Put, $t = 2$

$$p(2) = (2)^2 - 2(2) = 4 - 4 = 0$$

Hence, 0 and 2 are zeroes of the Polynomial $t^2 - 2t$.

So, the result is True.

Ans: Let $p(y) = y^2 + y - 6$

Put, $y = -3$

$$p(-3) = (-3)^2 + (-3) - 6$$

$$p(-3) = 9 - 9 = 0$$

Hence, -3 is a zero of the Polynomial $y^2 + y - 6$.

So, the result is True.

Question 11

Find the zeroes of the Polynomial in each of the following:

(i) $p(x) = x - 4$

(ii) $g(x) = 3 - 6x$

(iii) $q(x) = 2x - 7$

(iv) $h(y) = 2y$

Solution

Ans: Here we have to Solve the equation $p(x) = 0$, we get

$$x - 4 = 0$$

$$x = 4$$

So, 4 is a zero of the Polynomial $x - 4$.

Ans: Here we have to Solve the equation $g(x) = 0$, we get

$$3 - 6x = 0$$

$$x = \frac{1}{2}$$

So, $\frac{1}{2}$ is a zero of the Polynomial $3 - 6x$.

Ans: Here we have to Solve the equation $q(x) = 0$, we get

$$2x - 7 = 0$$

$$x = \frac{7}{2}$$

So, $\frac{7}{2}$ is a zero of the Polynomial $2x - 7$.

Ans: Here we have to Solve the equation $h(y) = 0$, we get

$$2y = 0$$

$$y = 0$$

So, 0 is a zero of the Polynomial $2y$.

Question 12

Find the zeroes of the Polynomial $(x - 2)^2 - (x + 2)^2$.

Solution

Ans: Let $p(x) = (x - 2)^2 - (x + 2)^2$

To get the zeroes of $p(x)$, we have to Solve the equation

$$p(x) = 0$$

.

So,

$$p(x) = 0$$

$$(x - 2)^2 - (x + 2)^2 = 0$$

$$((x - 2) - (x + 2))((x - 2) + (x + 2)) = 0$$

$$(-4)(2x) = 0$$

$$x = 0$$

Hence, $x = 0$ is the only one zero of $p(x)$.

Question 13

By acute division, find the quotient and the remainder when the first Polynomial is divided by the second $x^4 + 1$; $x + 1$.

Solution

Ans: By acute division, we have

$$\begin{array}{r}
 x^3 + x^2 + x + 1 \\
 x-1 \overline{) \begin{array}{l} x^4 + 1 \\ x^4 - x^3 \\ \hline x^3 + 1 \\ x^3 - x^2 \\ \hline x^2 + 1 \\ x^2 - x \\ \hline x + 1 \\ x - 1 \\ \hline 2 \end{array}}
 \end{array}$$

Quotient= $x^3 + x^2 + x + 1$ **remainder =** 2

Question 14

By remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where

- (i) $p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$
(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$
(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$
(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$

Solution

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

We have, $g(x) = x + 1$

Now, put

$$g(x) = 0$$

Therefore,

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

$$\text{Remainder} = p(-1)$$

$$= (-1)^3 - 2(-1)^2 - 4(-1) = -1 - 2 + 4 - 1 = 0$$

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

We have, $g(x) = x - 3$

Therefore,

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

$$\text{Remainder} = p(3)$$

$$= (3)^3 - 3(3)^2 + 4(3) + 50 = 27 - 27 + 12 + 50 = 62$$

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

We have, $g(x) = 2x - 1$

Therefore,

$$\Rightarrow 2x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}$$

$$\text{Remainder} = p\left(\frac{1}{2}\right)$$

$$= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3 = 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 3 = \frac{1}{2} + 1 = \frac{3}{2}$$

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

$$\text{We have, } g(x) = 1 - \frac{3}{2}x$$

Therefore,

$$\Rightarrow 1 - \frac{3}{2}x = 0$$

$$\Rightarrow x = \frac{2}{3}$$

$$\text{Remainder} = p\left(\frac{2}{3}\right)$$

$$\begin{aligned} &= \left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) - 4 = \frac{8}{27} - 6 \times \frac{4}{9} + \frac{4}{3} - 4 \\ &= \frac{8}{27} - \frac{8}{3} + \frac{4}{3} - 4 = \frac{8 - 72 + 36 - 108}{27} = \frac{-136}{27} \end{aligned}$$

Question 15

Check whether $p(x)$ is a multiple of $g(x)$ or not:

(i) $p(x) = x^3 - 5x^2 + 4x - 3$, $g(x) = x - 2$

(ii) $p(x) = 2x^3 - 11x^2 - 4x + 5$, $g(x) = 2x + 1$

Solution

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

$p(x)$ is a multiple of $g(x)$ if $g(x)$ divides $p(x)$

Now, $g(x) = x - 2$ gives $x = 2$ if we put $g(x) = 0$.

$$\begin{aligned} \text{Remainder} &= p(2) = (2)^3 - 5(2)^2 + 4(2) - 3 \\ &= 8 - 5(4) + 8 - 3 = 8 - 20 + 8 - 3 = -7 \end{aligned}$$

Hence, the remainder $\neq 0$

Therefore, function $p(x)$ is not a multiple of function $g(x)$.

Ans: According to the remainder theorem, if we divide a polynomial $p(x)$ by a factor

$$(x - a)$$

; then you will find a smaller polynomial with a remainder. This remainder which has been obtained is actually a value of $p(x)$ at $x = a$, specifically $p(a)$. So,

$$(x - a)$$

is the divisor of $p(x)$ if and only if

$$p(a) = 0$$

.

$p(x)$ will be a multiple of $g(x)$ if and only if $g(x)$ divides $p(x)$.

so,

$$g(x) = 2x + 1 \text{ give } x = -\frac{1}{2}$$

$$\text{remainder} = p\left(-\frac{1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 5$$

$$= 2\left(-\frac{1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5 = \frac{-1}{4} - \frac{11}{4} + 7$$

$$= \frac{-1 - 11 + 28}{4} = \frac{16}{4} = 4$$

Since the remainder $\neq 0$.

So, $p(x)$ will not be a multiple of $g(x)$.

Question 16

Show that:

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$

Solution

Ans: Let $p(x) = 69 + 11x - x^2 + x^3$, $g(x) = x + 3$

$$g(x) = x + 3 = 0$$

It will give, $x = -3$

$g(x)$ will be a factor of $p(x)$ if $p(-3) = 0$ (Factor theorem).

$$\begin{aligned}\text{Now, } p(-3) &= 69 + 11(-3) - (-3)^2 + (-3)^3 \\ &= 69 - 33 - 9 - 27 = 0\end{aligned}$$

Hence $p(-3) = 0$, $g(x)$ will be a factor of $p(x)$.

Ans: Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) = 2x - 3$

$$g(x) = 2x - 3$$

Now,

$$g(x) = 0$$

$$2x - 3 = 0$$

$$x = \frac{3}{2},$$

$g(x)$ will be factor of $p(x)$ if $p\left(\frac{3}{2}\right) = 0$ (Factor theorem)

$$\begin{aligned}\text{Now, } p\left(\frac{3}{2}\right) &= \frac{3}{2} + 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12 = \frac{3}{2} + 2\left(\frac{27}{8}\right) - 9\left(\frac{9}{4}\right) + 12 \\ &= \frac{3}{2} + \frac{27}{4} - \frac{81}{4} + 12 = \frac{6 + 27 - 81 + 48}{4} = \frac{0}{4} = 0\end{aligned}$$

Since, $p\left(\frac{3}{2}\right) = 0$, therefore, $g(x)$ is a factor of $p(x)$.

Question 17

Determine which of the following Polynomials has $x - 2$ the factor as:

(i) $3x^2 + 6x - 24$

(ii) $4x^2 + x - 2$

Solution

Ans: As we know that if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

Let $p(x) = 3x^2 + 6x - 24$

If $x - 2$ will be factor of $p(x) = 3x^2 + 6x - 24$, then $p(2)$ should be equal to 0.

$$\begin{aligned}\text{Now, } p(2) &= 3(2)^2 + 6(2) - 24 \\ &= 3(4) + 6(2) - 24 = 12 + 12 - 24 = 0\end{aligned}$$

Therefore, by factor theorem $x - 2$, is factor of $3x^2 + 6x - 24$.

Ans: As we know that if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

$$\text{Let } p(x) = 4x^2 + x - 2$$

If $x - 2$ will be factor of $p(x) = 4x^2 + x - 2$, then $p(2)$ should be equal to 0.

Now,

$$\begin{aligned}p(2) &= 4(2)^2 + 2 - 2 \\ &= 16 + 2 - 2 = 16\end{aligned}$$

Since, $16 \neq 0$

Therefore, $x - 2$ is not a factor of $4x^2 + x - 2$.

Question 18

Show that $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

Solution

Ans: $p - 1$ is a factor $p^{10} - 1$, then $(1)^{10} - 1$ should be equal to zero.

$$\text{Now, } (1)^{10} - 1 = 1 - 1 = 0$$

Therefore, $p - 1$ is a factor $p^{10} - 1$.

Again, if $p - 1$ is a factor of $p^{11} - 1$, then $(1)^{11} - 1$ should be equal to zero.

$$\text{Now, } (1)^{11} - 1 = 1 - 1 = 0$$

Therefore, $p - 1$ is a factor $p^{11} - 1$.

Hence, $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

Question 19

For what value of m is $x^3 - 2mx^2 + 16$ divisible by $x + 2$?

Solution

Ans: If $x^3 - 2mx^2 + 16$ is divisible by $x + 2$, then $x + 2$ is a factor of $x^3 - 2mx^2 + 16$.

Now, let $p(x) = x^3 - 2mx^2 + 16$

As, $x + 2 = x - (-2)$ is a factor of $x^3 - 2mx^2 + 16$.

So, $p(-2) = 0$

Now, $p(-2) = (-2)^3 - 2m(-2)^2 + 16$

$$= -8 - 8m + 16 = 8 - 8m$$

Now, $p(-2) = 0$

$$\Rightarrow 8 - 8m = 0$$

$$\Rightarrow 8m = 8$$

$$\Rightarrow m = 1$$

Hence, for $m = 1$, $x + 2$ is a factor of $x^3 - 2mx^2 + 16$ so that $x^3 - 2mx^2 + 16$ is completely divisible by $x + 2$.

Question 20

If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a .

Solution

Ans: Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$

If $x - (-2a)$ is a factor of $p(x)$, then $p(-2a) = 0$.

$$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3$$

$$= -32a^5 + 32a^5 - 4a + 2a + 3$$

$$= -2a + 3$$

Now, $p(-2a) = 0$

$$\Rightarrow -2a + 3 = 0$$

$$\Rightarrow a = \frac{3}{2}$$

Question 21

Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

Solution

Ans: Let $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$

As, $(2x - 1)$ is a factor of $p(x)$.

Then $p\left(\frac{1}{2}\right) = 0$ (using factor theorem)

$$\Rightarrow 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$$

$$\Rightarrow 8\left(\frac{1}{16}\right) + 4\left(\frac{1}{8}\right) - 16\left(\frac{1}{4}\right) + 5 + m = 0$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$$

$$\Rightarrow 1 + 1 + m = 0$$

$$\Rightarrow m = -2$$

Question 22

If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a .

Solution

Ans: Let $p(x) = ax^3 + x^2 - 2x + 4a - 9$

As $x + 1$ is a factor of $p(x)$

Then,

$$p(-1) = 0 \text{ (By factor theorem)}$$

$$\Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$$

$$\Rightarrow -a + 1 + 2 + 4a - 9 = 0$$

$$\Rightarrow 3a - 6 = 0$$

$$\Rightarrow 3a = 6$$

$$\Rightarrow a = 2$$

Question 23

Factorize:

(i) $x^2 + 9x + 18$

(ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$

(iv) $84 - 2r - 2r^2$

Solution

Ans: In order to factorize $x^2 + 9x + 18$, we have to split the middle term.

$$\Rightarrow x^2 + 9x + 18$$

$$\Rightarrow x^2 + (6 + 3)x + 18$$

$$\Rightarrow x^2 + 6x + 3x + 18$$

$$\Rightarrow x(x + 6) + 3(x + 6)$$

$$\Rightarrow (x + 3)(x + 6)$$

Ans: In order to factorize $6x^2 + 7x - 3$, we have to split the middle term.

$$\Rightarrow 6x^2 + 7x - 3$$

$$\Rightarrow 6x^2 + (9 - 2)x - 3$$

$$\Rightarrow 6x^2 + 9x - 2x - 3$$

$$\Rightarrow 3x(2x + 3) - 1(2x + 3)$$

$$\Rightarrow (3x - 1)(2x + 3)$$

Ans: If we want to factorise $2x^2 - 7x - 15$, we have to split the middle term.

$$\Rightarrow 2x^2 - 7x - 15$$

$$\Rightarrow 2x^2 - (10 - 3)x - 15$$

$$\Rightarrow 2x^2 - 10x + 3x - 15$$

$$\Rightarrow 2x(x - 5) + 3(x - 5)$$

$$\Rightarrow (2x + 3)(x - 5)$$

Ans: If we want to factorise $84 - 2r - 2r^2$, we have to split the middle term.

$$\Rightarrow -(2r^2 + 2r - 84)$$

$$\Rightarrow -2(r^2 + r - 42)$$

$$\Rightarrow -2(r^2 + (7 - 6)r - 42)$$

$$\Rightarrow -2(r^2 + 7r - 6r - 42)$$

$$\Rightarrow -2(r(r + 7) - 6(r + 7))$$

$$\Rightarrow 2(6 - r)(r + 7)$$

Question 24

Factorise:

(i) $2x^3 - 3x^2 - 17x + 30$

(ii) $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x + 4$

(iv) $3x^3 - x^2 - 3x + 1$

Solution

Ans: Let $f(x) = 2x^3 - 3x^2 - 17x + 30$ be the given Polynomial. The factors of the constant term $+30$ are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$. The factor of coefficient of x^3 is 2 .

Therefore, possible rational roots of $f(x)$ are:

$$\Rightarrow \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$$

We $f(2) = 2(2)^3 - 3(2)^2 - 17(2) + 30$

$$= 2(8) - 3(4) - 17(2) + 30$$

$$= 16 - 12 - 34 + 30 = 0$$

And $f(-3) = 2(-3)^3 - 3(-3)^2 - 17(-3) + 30$

$$= 2(-27) - 3(9) - 17(-3) + 30$$

$$= -54 - 27 + 51 + 30 = 0$$

Hence, $(x - 2)(x + 3)$, will be the factors of $f(x)$.

$$\Rightarrow x^2 + x - 6 \text{ is a factor of}$$

$$f(x).$$

Let's divide $f(x) = 2x^3 - 3x^2 - 17x + 30$ by $x^2 + x - 6$ to get the other factors of $f(x)$.

Factors of $f(x)$.

By long division, we have

$$\begin{array}{r}
 2x - 5 \\
 x^2 + x - 6 \overline{) 2x^3 - 3x^2 - 17x + 30} \\
 \underline{2x^3 + 2x^2 - 12x} \\
 -5x^2 - 5x + 30 \\
 \underline{-5x^2 - 5x + 30} \\
 0
 \end{array}$$

$$2x^3 - 3x^2 - 17x + 30 = (x^2 + x - 6)(2x - 5)$$

$$\Rightarrow 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

$$\text{Hence, } 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

Ans: Let $f(x) = x^3 - 6x^2 + 11x - 6$ be the given Polynomial. The factors of the constant term -6 are $\pm 1, \pm 2, \pm 3$ and ± 6 .

$$\text{We } f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 0$$

$$\text{And } f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$$

Hence, $(x - 1)(x - 2)$, will be factors $f(x)$.

$$\Rightarrow (x - 1)(x - 2) = x^2 - 3x + 2 \text{ will be a factor of } f(x).$$

Let us divide $f(x) = x^3 - 6x^2 + 11x - 6$ by $x^2 - 3x + 2$ to get the other factors of $f(x)$.

Factors of $f(x)$.

By long division, we have

$$\begin{array}{r}
 \overline{x^2 - 3x + 2} x^3 - 6x^2 + 11x - 6 \\
 \underline{x^3 - 3x^2 + 2x} \\
 -3x^2 + 9x - 6 \\
 \underline{-3x^2 + 9x - 6} \\
 0
 \end{array}$$

$$x^3 - 6x^2 + 11x - 6 = (x^2 - 3x + 2)(x - 3)$$

$$\Rightarrow x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

$$\text{Hence, } x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

Ans: Let $f(x) = x^3 + x^2 - 4x - 4$ be the given Polynomial. The factors of the constant term -4 are $\pm 1, \pm 2, \pm 4$.

$$\text{We } f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = 0$$

$$\text{And } f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$$

Hence, $(x + 1)$, $(x - 2)$ are factors of $f(x)$.

$$\Rightarrow (x + 1)(x - 2) = x^2 - x - 2 \text{ will be a factor of } f(x).$$

Let's now divide $f(x) = x^3 + x^2 - 4x - 4$ by $x^2 - x - 2$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r}
 \overline{x^2 - x - 2} x^3 + x^2 - 4x - 4 \\
 \underline{x^3 - x^2 - 2x} \\
 2x^2 - 2x - 4 \\
 \underline{2x^2 - 2x - 4} \\
 0
 \end{array}$$

$$x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x + 2)$$

$$\Rightarrow x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$$

$$\text{Hence, } x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$$

Ans: Let $f(x) = 3x^3 - x^2 - 3x + 1$ be the given Polynomial. The factors of a constant term $+1$ are ± 1 .

The factor of coefficient of x^3 is 3 . Hence, possible rational roots of $f(x)$ are $\pm \frac{1}{3}$.

We have,

$$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1 = 0$$

$$\text{And } f(-1) = 3(-1)^3 - (-1)^2 - 3(-1) + 1 = -3 - 1 + 3 + 1 = 0$$

Hence, $(x - 1)(x + 1)$, will be factors of $f(x)$.

$$\Rightarrow (x - 1)(x + 2) = x^2 - 1 \text{ will be factor of } f(x).$$

Let's now divide $f(x) = 3x^3 - x^2 - 3x + 1$ by $x^2 - 1$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r} \overline{3x-1} \\ x^2-1 \overline{) 3x^3 - x^2 - 3x + 1} \\ \underline{3x^3 +} \\ -x^2 + 1 \\ \underline{-x^2 + 1} \\ + \\ \hline 0 \end{array}$$

$$3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$$

$$\Rightarrow 3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$$

$$\text{Hence, } 3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$$

Question 25

Using suitable identify, evaluate the following:

(i) 103^3

(ii) 101×102

(iii) 999^2

Solution

Ans: $103^3 = (100 + 3)^3$

Now using identify $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$, we have

$$\begin{aligned}(100 + 3)^3 &= (100)^3 + (3)^3 + 3(100)(3)(100 + 3) \\&= 1000000 + 27 + 900(100 + 3) \\&= 1000000 + 27 + 90000 + 2700 \\&= 1092727\end{aligned}$$

Ans: 101×102

$$= (100 + 1)(100 + 2) \text{ (using } (x + a)(x + b) = x^2 + (a + b)x + ab \text{)}$$

We have,

$$\begin{aligned}&= (100)^2 + (1 + 2)100 + (1)(2) \\&= 10000 + (3)100 + 2 = 10000 + 300 + 2 = 10302\end{aligned}$$

Ans: 999^2

Ans: $(1000 - 1)^2$

$$\begin{aligned}&= (1000)^2 - 2(1000) \times 1 + 1^2 \\&= 1000000 - 2000 + 1 = 998001\end{aligned}$$

Question 26

Factorise the following:

(i) $4x^2 + 20x + 25$

(ii) $9y^2 - 66yz + 121z^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Solution

Ans: We have,

$$\begin{aligned} 4x^2 + 20x + 25 &= (2x)^2 + 2(2x)5 + (5)^2 \\ &= (2x + 5)^2 \quad [a^2 + 2ab + b^2 = (a + b)^2] \\ &= (2x + 5)(2x + 5) \end{aligned}$$

Ans: We have,

$$\begin{aligned} 9y^2 - 66yz + 121z^2 &= (3y)^2 - 2(3y)(11z) + (-11z)^2 \\ &= (3y - 11z)^2 \quad [a^2 + b^2 - 2ab = (a - b)^2] \\ &= (3y - 11)(3y - 11) \end{aligned}$$

Ans: $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Now, using identity $a^2 - b^2 = (a + b)(a - b)$

$$\begin{aligned} &= \left[\left(2x + \frac{1}{3}\right) + \left(x - \frac{1}{2}\right)\right] \left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right] \\ &= \left(2x + \frac{1}{3} + x - \frac{1}{2}\right) \left(2x + \frac{1}{3} - x + \frac{1}{2}\right) = \left(3x - \frac{1}{6}\right) \left(x + \frac{5}{6}\right) \end{aligned}$$

Question 27

Factorise the following:

(i) $9x^2 - 12x + 3$

(ii) $9x^2 - 12x + 4$

Solution

Ans: we have,

$$9x^2 - 12x + 3 = 9x^2 - 9x - 3x + 3$$

$$= 9x(x-1) - 3(x-1)$$

$$= (9x-3)(x-1)$$

$$= 3(3x-1)(x-1)$$

Ans: We have,

$$9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$$

$$= (3x-2)^2 \left[a^2 - 2ab + b^2 = (a-b)^2 \right]$$

$$= (3x-2)(3x-2)$$

Question 28

Expand the following:

(i) $(4a - b + 2c)^2$

(ii) $(3a - 5b - c)^2$

(iii) $(-x + 2y - 3z)^2$

Solution

Ans: We have,

$$(4a - b + 2c)^2 = (4a)^2 + (-b)^2 + (2c)^2 + 2(4a)(-b) + 2(-b)(2c) + 2(2c)(4a)$$

$$\text{Using } \left[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2 \right]$$

$$= 16a^2 + b^2 + 4c^2 - 8ab - 4bc + 16ca$$

Ans: We have,

$$(3a - 5b - c)^2 = (3a)^2 + (-5b)^2 + (-c)^2 + 2(3a)(-5b) + 2(-5b)(-c) + 2(-c)(3a)$$

$$\text{Using } \left[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2 \right]$$

$$= 9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ca$$

Ans: We have

$$(-x + 2y - 3z)^2 = (-x)^2 + (2y)^2 + (-3z)^2 + 2(-x)(2y) + 2(2y)(-3z) + 2(-3z)(-x)$$

$$= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz$$

Question 29

Factorise the following:

(i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

(iii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

Solution

Ans: We have,

$$\begin{aligned} &\Rightarrow (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x) \\ &\Rightarrow \{3x + 2y + (-4z)\}^2 \left[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2 \right] \\ &\Rightarrow (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z) \end{aligned}$$

Ans: We have,

$$\begin{aligned} &\Rightarrow (-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) + 2(4y)(2z) + 2(2z)(-5x) \\ &\Rightarrow \{-5x + 4y + 2z\}^2 \left[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2 \right] \end{aligned}$$

Ans: We have,

$$\begin{aligned} &\Rightarrow (4x)^2 + (-2y)^2 + (3z)^2 + 2(4x)(-2y) + 2(-2y)(3z) + 2(3z)(4x) \\ &\Rightarrow \{4x - 2y + 3z\}^2 \left[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2 \right] \\ &\Rightarrow (4x - 2y + 3z)^2 = (4x - 2y + 3z)(4x - 2y + 3z) \end{aligned}$$

Question 30If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.**Solution**

Ans: We know that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

Now, on putting the values, we get

$$\Rightarrow (9)^2 = (a^2 + b^2 + c^2) + 2(26)$$

$$\Rightarrow a^2 + b^2 + c^2 = 81 - 52 = 29$$

Question 31

Expand the following:

(i) $(3a - 2b)^3$

(ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

(iii) $\left(4 - \frac{1}{3x}\right)^3$

Solution

Ans: We have

$$(3a - 2b)^3 = (3a)^3 - (2b)^3 - 3(3a)(2b)(3a - 2b)$$

Now, using identity $\left[(a - b)^3 = a^3 - b^3 - 3ab(a - b)\right]$

$$= 27a^3 - 8b^3 - 54a^2b + 36ab^2$$

Ans: Here, using the identity $\left[(a + b)^3 = a^3 + b^3 + 3ab(a + b)\right]$

$$\left(\frac{1}{x} + \frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3 \times \frac{1}{x} \times \frac{y}{3} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$\Rightarrow \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$\Rightarrow \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

Ans: We have

$$\left(4 - \frac{1}{3x}\right)^3 = (4)^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4 - \frac{1}{3x}\right)$$

Now, using identity $\left[(a - b)^3 = a^3 - b^3 - 3ab(a - b)\right]$

$$\begin{aligned}
 &= 64 - \frac{1}{27x^3} - \frac{4}{x} \left(4 - \frac{1}{3x} \right) \\
 &= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}
 \end{aligned}$$

Question 32

Factorise the following:

(i) $1 - 64a^3 - 12a + 48a^2$

(ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

Solution

Ans: We have,

$$\begin{aligned}
 1 - 64a^3 - 12a + 48a^2 &= (1)^3 - (4a)^3 - 3(1)(4a)(1 - 4a) \\
 &= (1 - 4a)^3 \left[a^3 - b^3 - 3ab(a - b) = (a - b)^3 \right] \\
 &= (1 - 4a)(1 - 4a)(1 - 4a)
 \end{aligned}$$

Ans: We have,

$$\begin{aligned}
 8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125} &= (2p)^3 + 3 \times (2p)^2 \times \frac{1}{5} + 3 \times (2p) \times \left(\frac{1}{5} \right)^2 + \left(\frac{1}{5} \right)^3 \\
 &= (2p)^3 + \left(\frac{1}{5} \right)^3 + 3 \times (2p) \times \frac{1}{5} \left[2p + \frac{1}{5} \right] \\
 &= \left(2p + \frac{1}{5} \right)^3 = \left(2p + \frac{1}{5} \right) \left(2p + \frac{1}{5} \right) \left(2p + \frac{1}{5} \right)
 \end{aligned}$$

Question 33

Find the following products:

(i) $\left(\frac{x}{2} + 2y \right) \left(\frac{x^2}{4} - xy + 4y^2 \right)$

(ii) $(x^2 - 1)(x^4 + x^2 + 1)$

Solution

Ans: We have,

$$\begin{aligned} \left(\frac{x}{2} + 2y\right) \left(\frac{x^2}{4} - xy + 4y^2\right) &= \left(\frac{x}{y} + 2y\right) \left(\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right) \\ &= \left(\frac{x}{2}\right)^3 + (2y)^3 [(a+b)(a^2 - ab + b^2) = a^3 + b^3] \\ &= \frac{x^3}{8} + 8y^3 \end{aligned}$$

Ans: We have,

$$\begin{aligned} ((x^2)^2 - 1)(x^4 + x^2 + 1) &= (x^2 - 1)((x^2)^2 + (x^2)(1) + (1)^2) \\ &= (x^2)^3 - (1)^3 [(a-b)(a^2 + ab + b^2) = a^3 - b^3] \\ &= x^6 - 1 \end{aligned}$$

Question 34

Factorise:

(i) $1 + 64x^3$

(ii) $a^3 - 2\sqrt{2}b^3$

Solution

Ans: We have,

$$\begin{aligned} 1 + 64x^3 &= (1)^3 + (4x)^3 \\ &= (1 + 4x) \left((1)^2 - (1)(4x) + (4x)^2 \right) \\ &= (1 + 4x) (1 - 4x + 16x^2) \end{aligned}$$

Ans: We have,

$$\begin{aligned} a^3 - 2\sqrt{2}b^3 &= (a)^3 - (\sqrt{2}b)^3 \\ &= (a - \sqrt{2}b) \left((a)^2 + (a)(\sqrt{2}b) + (\sqrt{2}b)^2 \right) \end{aligned}$$

$$= (a - \sqrt{2}b) (a^2 + \sqrt{2}ab + 2b^2)$$

Question 35

Find the following product:

$$(2x - y + 3z) (4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$$

Solution

Ans: We have,

$$\begin{aligned} & (2x - y + 3z) (4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz) \\ &= \{2x + (-y) + 3z\} \left\{ (2x)^2 + (-y)^2 + (3z)^2 - 2x(-y) - (-y)(3z) - (-y)(3z) - (3z)(2x) \right\} \\ &= (2x)^3 + (-y)^3 + (3z)^3 - 3(2x)(-y)(3z) \end{aligned}$$

$$\text{using identity } [(a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

$$= 8x^3 - y^3 + 27z^2 + 18xyz$$

Question 36

Factorise:

$$(i) a^3 - 8b^3 - 64c^3 - 24abc$$

$$(ii) 2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$$

Solution

Ans: We have,

$$\begin{aligned} & a^3 - 8b^3 - 64c^3 - 24abc \\ &= \left\{ (a)^3 + (-2b)^3 + (-4c)^3 - 3(a)(-2b)(-4c) \right\} \\ &= \{a + (-2b) + (-4c)\} \left\{ a^2 + (-2b)^2 + (-4c)^2 - a(-2b) - (-2b)(-4c) - (-4c)a \right\} \end{aligned}$$

$$\text{Using identity } [(a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

$$= (a - 2b - 4c) (a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$$

Ans: We have,

$$\begin{aligned}
 & 2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc \\
 &= \left\{ \left(\sqrt{2}a\right)^3 + (2b)^3 + (-3c)^3 - 3\left(\sqrt{2}a\right)(2b)(-3c) \right\} \\
 &= \left\{ \sqrt{2}a + 2b + (-3c) \right\} \left\{ \left(\sqrt{2}a\right)^2 + (2b)^2 + (-3c)^2 - \left(\sqrt{2}a\right)(2b) - (2b)(-3c) - (-3c)\left(\sqrt{2}a\right) \right\}
 \end{aligned}$$

Using identity $[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$

$$= (\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ca)$$

Question 37

Without actually calculating the cubes, find the value of:

(i) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

(ii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

Solution

Ans: Let $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = -\frac{5}{6}$

$$\begin{aligned}
 a + b + c &= \frac{1}{2} + \frac{1}{3} - \frac{5}{6} \\
 &= \frac{3 + 2 - 5}{6} = \frac{0}{6} = 0
 \end{aligned}$$

$$[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

Therefore,

$$\begin{aligned}
 &\Rightarrow a^3 + b^3 + c^3 = 3abc \\
 &\Rightarrow 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6} \\
 &\Rightarrow -\frac{5}{12}
 \end{aligned}$$

Ans: We have,

$$(0.2)^3 - (0.3)^3 + (0.1)^3 = (0.2)^3 + (-0.3)^3 + (0.1)^3$$

$a = 0.2$, $b = -0.3$, $c = 0.1$. Then,

$$a + b + c = 0.2 - 0.3 + 0.1 = 0.$$

$$[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

Therefore,

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow 3 \times 0.2 \times (-0.3) \times (0.1)$$

$$\Rightarrow -0.018$$

$$\text{Hence, } (0.2)^3 - (0.3)^3 + (0.1)^3 = -0.018$$

Question 38

Without finding the cubes, factorise

$$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$$

Solution

Ans: We have,

$$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$$

$a = x - 2y$, $b = 2y - 3z$, $c = 3z - x$. Then,

$$a + b + c = x - 2y + 2y - 3z + 3z - x = 0.$$

$$[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$$

Therefore,

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow 3 \times (x - 2y) \times (2y - 3z) \times (3z - x)$$

$$\Rightarrow 3(x - 2y)(2y - 3z)(3z - x)$$

Question 39**Find the value of**

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

(ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

Solution

Ans: $x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3xy(4)$

$$= (x + y + 4)(x^2 + y^2 + 4^2 - xy - 4y - 4x)$$

$$[x + y = -4]$$

$$= (0)(x^2 + y^2 + 4^2 - xy - 4y - 4x) = 0$$

Ans: $x^3 - 8y^3 - 36xy - 216 = x^3 + (-2y)^3 + (-6)^3 - 3x(-2y)(-6)$

$$= (x - 2y - 6)(x^2 + (-2y)^2 + (-6)^2 - x(-2y) - (-2y)(-6) - x(-6))$$

$$= (x - (2y + 6))(x^2 + (-2y)^2 + (-6)^2 - x(-2y) - (-2y)(-6) - x(-6))$$

$$[x = 2y + 6]$$

$$= (0)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) = 0$$

Question 40**Give possible experiments for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$** **Solution**

Ans: Area $4a^2 + 4a - 3$.

Here we will use a method of splitting the middle term.

Here we have to split the middle term as $4a = 6a - 2a$

Now,

$$\Rightarrow 4a^2 + (6a - 2a) - 3$$

$$\Rightarrow 4a^2 + 6a - 2a - 3$$

$$\Rightarrow 2a(2a + 3) - 1(2a + 3)$$

$$\Rightarrow (2a - 1)(2a + 3)$$

We know that, area of rectangle

$$= 4a^2 + 4a - 3$$

Here we also know that, area of a rectangle = $length \times breadth$ and

$$4a^2 + 4a - 3 = (2a - 1)(2a + 3).$$

Therefore, its possible length and breadth = $(2a - 1)$ and $(2a + 3)$ or, we can say that $length = (2a + 3)$ and $breadth = (2a - 1)$.

Exercise 2.4

Question 1

If the Polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided $z - 3$, by Find the value of a.

Solution

Ans: We have,

$$p(z) = az^3 + 4z^2 + 3z - 4$$

$$\text{And } q(z) = z^3 - 4z + a$$

As it is given that both the Polynomials leave the same remainder when divided by $z - 3$.

Therefore,

$$p(3) = q(3)$$

$$\Rightarrow a(3)^3 + 4(3)^2 + 3(3) - 4 = (3)^3 - 4(3) + a$$

$$\Rightarrow 27a + 4 \times 9 + 9 - 4 = 27 - 12 + a$$

$$\Rightarrow 27a + 36 + 5 = 15 + a$$

$$\Rightarrow 27a - a = 15 - 41$$

$$\Rightarrow 26a = -26$$

$$\Rightarrow a = -1$$

Hence, the required value of a is -1 .

Question 2

The Polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ **when divided by** $x + 1$ **leave remainder** 19 . **Also,** **find the remainder when** $p(x)$ **is divided by** $x + 2$.

Solution

Ans: We know that when $p(x)$ is divided by $x + b$, then the remainder $= p(-b)$

Now, $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ is divided by $x + 1$, then the remainder $= p(-1)$.

Also, we know that the remainder is 19 .

Therefore,

$$p(-1) = 19$$

$$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 = 19$$

$$\Rightarrow 1 + 2 + 3 + a + 3a - 7 = 19$$

$$\Rightarrow 4a - 1 = 19$$

$$\Rightarrow 4a = 20$$

$$\Rightarrow a = 5$$

$$\text{Now, } p(x) = x^4 - 2x^3 + 3x^2 - 5x + 3(5) - 7 = x^4 - 2x^3 + 3x^2 - 5x + 8$$

Again, when $p(x)$ is divided by $x + 2$, then

$$\text{Remainder} = p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$$

$$= 16 + 16 + 12 + 10 + 8$$

$$= 62$$

Question 3

If both

$$(x - 2)$$

and $\left(x - \frac{1}{2}\right)$ **are factors of** $px^2 + 5x + r$, **Show that** $p = r$.

Solution

$$\text{Ans: Let } q(x) = px^2 + 5x + r$$

As $(x - 2)$ is a factor of $q(x)$

Then, $q(2) = 0$

$$\Rightarrow p(2)^2 + 5(2) + r = 0$$

$$\Rightarrow 4p + 10 + r = 0$$

$$\Rightarrow -4p - r = 10 \dots \dots \dots (1)$$

Again, $\left(x - \frac{1}{2}\right)$ is a factor of $q(x)$.

Then, $q\left(\frac{1}{2}\right) = 0$

$$\Rightarrow p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$$

$$\Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0$$

$$\Rightarrow p + 10 + 4r = 0$$

Now, using equation 1

$$\Rightarrow p - 4p - r + 4r = 0$$

$$\Rightarrow -3p + 3r = 0$$

$$\Rightarrow p = r$$

Hence, proved.

Question 4

Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

Solution

Ans: We have,

$$x^2 - 3x + 2$$

We can also write it as

$$\Rightarrow x^2 - 3x + 2 = x^2 - 2x - x + 2$$

$$\Rightarrow x(x - 2) - 1(x - 2) = (x - 1)(x - 2)$$

Now, we have

$$p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$$

Put $x = 1$

$$p(1) = 2(1)^4 - 5(1)^3 + 2(1)^2 - (1) + 2$$

$$p(1) = 2 - 5 + 2 + 1 = 0$$

$$p(1) = 0$$

Therefore, $x - 1$ divides $p(x)$.

Put $x = 2$

$$p(2) = 2(2)^4 - 5(2)^3 + 2(2)^2 - (2) + 2$$

$$p(2) = 32 - 40 + 8 - 2 + 2 = 0$$

$$p(2) = 0$$

Therefore, $x - 2$ divides $p(x)$.

Hence, $x^2 - 3x + 2$ divides $2x^4 - 5x^3 + 2x^2 - x + 2$.

Question 5

Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

Solution

Ans: We have,

$$= (2x - 5y)^3 - (2x + 5y)^3$$

Now, using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \{(2x - 5y) - (2x + 5y)\} \left\{ (2x - 5y)^2 + (2x - 5y)(2x + 5y) + (2x + 5y)^2 \right\}$$

$$= (-10y)(2x^2 + 25y^2)$$

$$= -120x^2y - 250y^3$$

Question 6

Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ **by** $(-z + x - 2y)$.

Solution

Ans: We have,

$$\begin{aligned}
 & (-z + x - 2y)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz) \\
 &= \{x + (-2y) + (-z)\} \left\{ (x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (x)(-z) - (-2y)(-z) \right\} \\
 &= x^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z) \\
 &= x^3 - 8y^3 - z^3 - 6xyz
 \end{aligned}$$

Question 7

If a, b, c are all non-zero and

$$a + b + c = 0$$

, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Solution

Ans: Here, a, b, c are all non-zero and

$$a + b + c = 0$$

.

Therefore,

$$a^3 + b^3 + c^3 = 3abc$$

Now, we will divide the complete equation by abc.

$$\begin{aligned}
 \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} &= \frac{3abc}{abc} \\
 \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} &= 3.
 \end{aligned}$$

Question 8

If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Solution

Ans: We know that,

$$\begin{aligned}a^3 + b^3 + c^3 - 3abc &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\&= (a + b + c)(a^2 + b^2 + c^2 - (ab + bc + ca))\end{aligned}$$

It is given that $a + b + c = 5$ and $ab + bc + ca = 10$.

$$= 5(a^2 + b^2 + c^2 - 10)$$

Now, we have

$$a + b + c = 5$$

Squaring both sides, we get

$$(a + b + c)^2 = (5)^2$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$$

$$a^2 + b^2 + c^2 + 2(10) = 25$$

$$a^2 + b^2 + c^2 = 25 - 20 = 5$$

Now, put the above value in equation $= 5(a^2 + b^2 + c^2 - 10)$

$$= 5(5 - 10)$$

$$= 5 \times -5 = -25$$

Hence, proved.

Question 9

Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

Solution

$$\text{Ans: } (a + b + c)^3 = [a + (b + c)]^3$$

$$= a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3$$

$$= a^3 + 3a^2b + 3a^2c + 3a(b^2 + 2bc + c^2) + (b^3 + 3b^2c + 3bc^2 + c^3)$$

$$= a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3$$

$$= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + 3b^2c + 3bc^2$$

We can also write it as,

$$= a^3 + b^3 + c^3 + 3abc + 3a^2b + 3ac^2 + 3a^2c + 3b^2c + 3b^2a + 3bc^2 + 3abc$$

$$= a^3 + b^3 + c^3 + 3ab(a + c) + 3ac(a + c) + 3b^2(a + c) + 3bc(a + c)$$

$$= a^3 + b^3 + c^3 + (a + c)(3ab + 3ac + 3b^2 + 3bc)$$

$$= a^3 + b^3 + c^3 + (a + c)(3a(b + c) + 3b(b + c))$$

$$= a^3 + b^3 + c^3 + (a + c)(b + c)(3a + 3b)$$

$$= a^3 + b^3 + c^3 + 3(a + c)(b + c)(a + b)$$

Now, on transposing we get

$$(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$$

Hence, proved.