

Chapter 12: Heron's Formula

NCERT Exemplar · Class 9 · Maths

Exercise 12.1 · Textbook page 113

Question 1

An isosceles right triangle has area 8 cm^2 . The length of its hypotenuse is

(A) $\sqrt{32} \text{ cm}$

(B) $\sqrt{16} \text{ cm}$

(C) $\sqrt{48} \text{ cm}$

(D) $\sqrt{24} \text{ cm}$

Solution

(A) $\sqrt{32} \text{ cm}$

Explanation:

Let height of triangle = h

As the triangle is isosceles,

Let base = height = h

According to the question,

Area of triangle = 8 cm^2

$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Height} = 8$

$\Rightarrow \frac{1}{2} \times h \times h = 8$

$\Rightarrow h^2 = 16$

$\Rightarrow h = 4 \text{ cm}$

Base = Height = 4 cm

Since the triangle is right angled,

$$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Height}^2$$

$$\Rightarrow \text{Hypotenuse}^2 = 4^2 + 4^2$$

$$\Rightarrow \text{Hypotenuse}^2 = 32$$

$$\Rightarrow \text{Hypotenuse} = \sqrt{32}$$

Hence, Options A is the correct answer.

Question 2

The perimeter of an equilateral triangle is 60 m. The area is

(A) $10\sqrt{3} \text{ m}^2$

(B) $15\sqrt{3} \text{ m}^2$

(C) $20\sqrt{3} \text{ m}^2$

(D) $100\sqrt{3} \text{ m}^2$

Solution

(D) $100\sqrt{3} \text{ m}^2$

Explanation:

Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4} a^2$

According to the question,

Perimeter of triangle = 60m

$$\Rightarrow a + a + a = 60$$

$$\Rightarrow 3a = 60$$

$$\Rightarrow a = 20\text{m}$$

Area of the triangle = $\frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} (20)^2$$

$$= \frac{\sqrt{3}}{4} (400)$$

$$= 100\sqrt{3}$$

Hence, Options D is the correct answer.

Question 3

The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm^2

(B) 1311 cm^2

(C) 1344 cm^2

(D) 1392 cm^2

Solution

(C) 1344 cm^2

Explanation:

According to the question,

Sides of a triangle,

$$a = 56, b = 60, c = 52$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (56 + 60 + 52)/2$$

$$= 168/2 = 84.$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{84(84-56)(84-60)(84-52)}$$

$$= \sqrt{84 \times 28 \times 24 \times 32}$$

$$= 1344 \text{ cm}^2$$

Hence, Options C is the correct answer.

Question 4

The area of an equilateral triangle with side $2\sqrt{3}$ cm is

(A) 5.196 cm^2

(B) 0.866 cm^2

(C) 3.496 cm^2

(D) 1.732 cm^2

Solution

(A) 5.196 cm^2

Explanation:

Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4} a^2$

According to the question,

$$a = 2\sqrt{3}$$

$$\text{Area of triangle} = \left(\frac{\sqrt{3}}{4}\right) a^2$$

$$= \left(\frac{\sqrt{3}}{4}\right) (2\sqrt{3})^2$$

$$= \left(\frac{\sqrt{3}}{4}\right)(12)$$

$$= 3\sqrt{3}$$

$$= 5.196$$

Hence, Options A is the correct answer.

Exercise 12.2 · Textbook page 115

Question 1

The area of a triangle with base 4 cm and height 6 cm is 24 cm^2 .

Solution

False

Justification:

$$\text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times 4 \times 6$$

$$= 12\text{cm}^2$$

Hence, the statement “the area of a triangle with base 4 cm and height 6 cm is 24 cm^2 ” is False.

Question 2

The area of $\triangle ABC$ is 8 cm^2 in which $AB = AC = 4\text{ cm}$ and $\angle A = 90^\circ$.

Solution

True

Justification:

Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$= \frac{1}{2} \times 4 \times 4$$

$$= 8\text{cm}^2$$

Hence, the statement is “area of $\triangle ABC$ is 8 cm^2 in which $AB = AC = 4\text{ cm}$ and $\angle A = 90^\circ$ ” is True.

Question 3

The area of the isosceles triangle is $\frac{5}{4}\sqrt{11}\text{cm}^2$, if the perimeter is 11 cm and the base is 5 cm.

Solution

True

Justification:

According to the question,

Perimeter = 11cm

And base, $a = 5$

As the triangle is isosceles, $b = c$

Perimeter = 11

$$\Rightarrow a + b + c = 11$$

$$\Rightarrow 5 + b + b = 11$$

$$\Rightarrow 5 + 2b = 11$$

$$\Rightarrow 2b = 6$$

$$\Rightarrow b = 3$$

So, we have,

$$a = 5, b = 3, c = 3$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (5 + 3 + 3)/2 = 11/2$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{11}{2} \left(\frac{11}{2} - 5 \right) \left(\frac{11}{2} - 3 \right) \left(\frac{11}{2} - 3 \right)}$$

$$= \sqrt{\frac{11}{2} \left(\frac{1}{2} \right) \left(\frac{5}{2} \right) \left(\frac{5}{2} \right)}$$

$$\Rightarrow \text{Area of triangle} = (5\sqrt{11})/4 \text{ cm}^2$$

Hence, the statement “The area of the isosceles triangle is $5/4 \sqrt{11} \text{ cm}^2$, if the perimeter is 11 cm and the base is 5 cm” is True.

Question 4

The area of the equilateral triangle is $20\sqrt{3} \text{ cm}^2$ whose each side is 8 cm.

Solution

False

Justification:

$$\text{Area of an equilateral triangle of side } a = \frac{\sqrt{3}}{4} a^2$$

According to the question,

$$\text{Area of a triangle} = 20\sqrt{3} \text{ cm}^2$$

$$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 20\sqrt{3}$$

$$\Rightarrow a^2 = 20 \times 4$$

$$\Rightarrow a^2 = 80$$

$$\Rightarrow a = 4\sqrt{5} \text{ cm}$$

Hence, the statement “the area of the equilateral triangle is $20\sqrt{3} \text{ cm}^2$ whose each side is 8 cm” is False.

Exercise 12.3 · Textbook page 117

Question 1

Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2 .

Solution

According to the question,

Sides of the triangular field are 50 m, 65 m and 65 m.

Cost of laying grass in a triangular field = Rs 7 per m^2

Let $a = 50$, $b = 65$, $c = 65$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (50 + 65 + 65)/2$$

$$= 180/2$$

$$= 90.$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{90(90-50)(90-65)(90-65)}$$

$$= \sqrt{90 \times 40 \times 25 \times 25}$$

$$= 1500\text{m}^2$$

Cost of laying grass = Area of triangle $\times$ Cost per m^2

$$= 1500 \times 7$$

$$= \text{Rs. } 10500$$

Question 2

The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m^2 a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution

According to the question,

The sides of the triangle are 13 m, 14 m and 15 m

$$\text{Let } a = 13, b = 14, c = 15$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (13 + 14 + 15)/2$$

$$= 42/2$$

$$= 21.$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}$$

$$= 84\text{m}^2$$

$$\text{Cost of advertisements for a year} = \text{Area of triangle} \times \text{Cost per m}^2$$

$$= 84 \times 2000$$

$$= \text{Rs. } 168000$$

Since the board is rented for only 6 months:

$$\text{Cost of advertisements for 6 months} = (6/12) \times 168000$$

$$= \text{Rs. } 84000$$

Question 3

From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Solution

According to the question,

The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm.

We know that,

Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4} a^2$

We divide the triangle into three triangles,

Area of triangle = $(\frac{1}{2} \times a \times 14) + (\frac{1}{2} \times a \times 10) + (\frac{1}{2} \times a \times 6)$

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times (14 + 10 + 6)$$

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times 30$$

$$a = 60/\sqrt{3}$$

$$= 20\sqrt{3}$$

Area of the triangle = $\frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} (20\sqrt{3})^2$$

$$= 300\sqrt{3} \text{ cm}^2$$

Question 4

The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Solution

According to the question,

Perimeter of the isosceles triangle = 32 cm

It is also given that,

Ratio of equal side to base = 3 : 2

Let the equal side = $3x$

So, base = $2x$

Perimeter of the triangle = 32

$$\Rightarrow 3x + 3x + 2x = 32$$

$$\Rightarrow 8x = 32$$

$$\Rightarrow x = 4.$$

Equal side = $3x = 3 \times 4 = 12$

Base = $2x = 2 \times 4 = 8$

The sides of the triangle = 12cm, 12cm and 8cm.

Let $a = 12$, $b = 12$, $c = 8$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (12 + 12 + 8)/2$$

$$= 32/2$$

$$= 16.$$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-12)(16-12)(16-8)}$$

$$= \sqrt{16 \times 4 \times 4 \times 8}$$

$$= 32\sqrt{2} \text{ cm}^2$$

Question 5

Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.

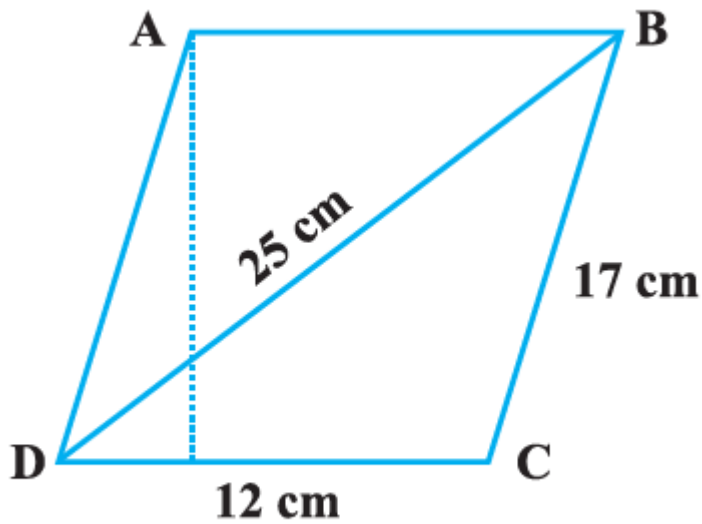


Fig. 12.2

Solution

We know that,

$$\text{Area of parallelogram(ABCD)} = \text{Area}(\triangle BCD) + \text{Area}(\triangle ABD)$$

For Area ($\triangle BCD$),

We have,

$$a = 12, b = 17, c = 25$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (12 + 17 + 25)/2 = 54/2 = 27.$$

$$\text{Area of } (\triangle BCD) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27(27-12)(27-17)(27-25)}$$

$$= \sqrt{27 \times 15 \times 10 \times 2}$$

$$= 90 \text{ cm}^2$$

Since, ABCD is a parallelogram,

$$\text{Area}(\triangle BCD) = \text{Area}(\triangle ABD)$$

$$\text{Area of parallelogram(ABCD)} = \text{Area}(\triangle BCD) + \text{Area}(\triangle ABD)$$

$$= 90 + 90$$

$$= 180 \text{ cm}^2$$

Let altitude from A be = x

Also, Area of parallelogram(ABCD) = $CD \times (\text{Altitude from A})$

$$\Rightarrow 180 = 12 \times x$$

$$\Rightarrow x = 15 \text{ cm}$$

Exercise 12.4 · Textbook page 118

Question 1

How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.

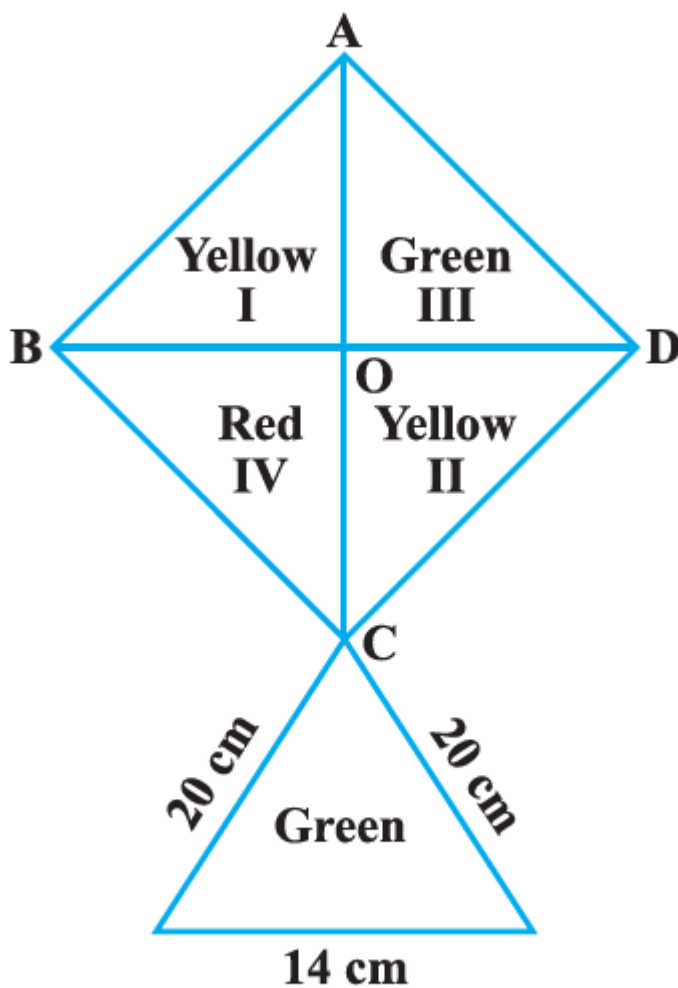


Fig. 12.4

Solution

According to the figure,

$$AC = BD = 44\text{cm}$$

$$AO = 44/2 = 22\text{cm}$$

$$BO = 44/2 = 22\text{cm}$$

From $\triangle AOB$,

$$AB^2 = AO^2 + BO^2$$

$$\Rightarrow AB^2 = 22^2 + 22^2$$

$$\Rightarrow AB^2 = 2 \times 22^2$$

$$\Rightarrow AB = 22\sqrt{2} \text{ cm}$$

$$\text{Area of square} = (\text{Side})^2$$

$$= (22\sqrt{2})^2$$

$$= 968 \text{ cm}^2$$

$$\text{Area of each triangle (I, II, III, IV)} = \text{Area of square} / 4$$

$$= 968 / 4$$

$$= 242 \text{ cm}^2$$

To find area of lower triangle,

$$\text{Let } a = 28, b = 28, c = 14$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (28 + 28 + 14)/2 = 70/2 = 35.$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{35(35-28)(35-28)(35-14)}$$

$$= \sqrt{35 \times 7 \times 7 \times 21}$$

$$= 49\sqrt{15} = 189.77\text{cm}^2$$

Therefore,

We get,

Area of Red = Area of IV

$$= 242 \text{ cm}^2$$

Area of Yellow = Area of I + Area of II

$$= 242 + 242$$

$$= 484 \text{ cm}^2$$

Area of Green = Area of III + Area of lower triangle

$$= 242 + 189.77$$

$$= 431.77 \text{ cm}^2$$

Question 2

The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution

Let the smaller side be = x cm

Then, larger side = $(x + 4)$ cm

And, third side = $(2x - 6)$ cm

Given that,

Perimeter = 50 cm

$$\Rightarrow x + (x + 4) + (2x - 6) = 50$$

$$\Rightarrow 4x - 2 = 50$$

$$\Rightarrow 4x = 52$$

$$\Rightarrow x = 13$$

Therefore, smaller side = 13cm

$$\text{Larger side} = x + 4 = 13 + 4 = 17\text{cm}$$

$$\text{Third side} = 2x - 6 = 2 \times 13 - 6 = 26 - 6 = 20\text{cm}$$

To find area of triangle,

$$\text{Let } a = 13, b = 17, c = 20$$

$$s = (a + b + c)/2$$

$$\Rightarrow s = (13 + 17 + 20)/2 = 50/2 = 25.$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{25(25-13)(25-17)(25-20)}$$

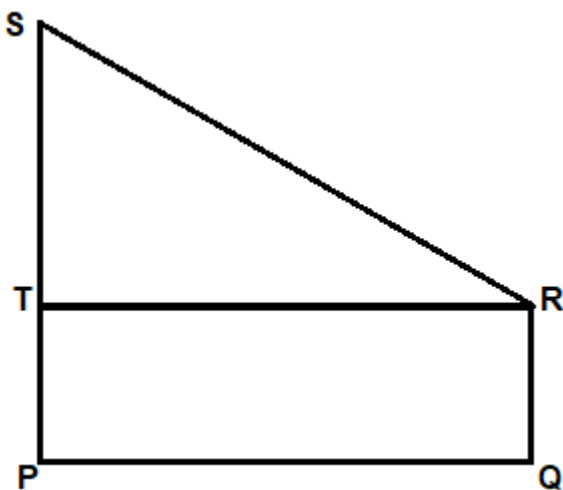
$$= \sqrt{25 \times 12 \times 8 \times 5}$$

$$= 20\sqrt{30} \text{ cm}^2$$

Question 3

The area of a trapezium is 475 cm^2 and the height is 19 cm . Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Solution



Let the given trapezium be PQRS, given in the figure.

According to the question,

$$PQ = 19\text{cm}$$

Let $RQ = x$ cm

Then,

$$PS = (x + 4)\text{cm}$$

Construction:

Draw a perpendicular from R on PS which will also be parallel to PQ.

Now,

We get,

PQRT is a rectangle,

$$\text{Area of rectangle PQRT} = PQ \times QR$$

$$\Rightarrow \text{Area(PQRT)} = 19 \times x = 19x$$

Now,

$$PS = PT + TS$$

$$\text{Since } PT = QR = x \text{ cm}$$

$$(x + 4) = x + TS$$

$$\Rightarrow TS = 4\text{cm}$$

$$\text{Area of triangle RST} = \frac{1}{2} \times RT \times ST$$

$$\text{Since } RT = PQ = 19\text{cm}$$

$$\Rightarrow \text{Area}(\triangle RST) = \frac{1}{2} \times 19 \times 4$$

$$= 38\text{cm}^2$$

$$\text{Area(PQRS)} = \text{Area(PQRT)} + \text{Area}(\triangle RST)$$

$$\Rightarrow 475 = 19x + 38$$

$$\Rightarrow 19x = 475 - 38$$

$$\Rightarrow 19x = 437$$

$$\Rightarrow x = 23 \text{ cm}$$

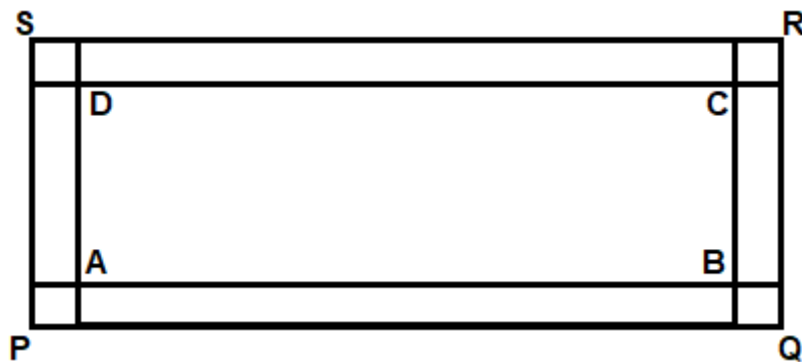
$$(x + 4) = 23 + 4 = 27\text{cm}$$

Therefore, lengths of parallel sides is 23cm and 27cm.

Question 4

A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Solution



Let the given rectangle be rectangle PQRS,

According to the question,

$$PQ = 40\text{m and } QR = 15\text{m}$$

As 3m is left in both front and back,

$$AB = PQ - 3 - 3$$

$$\Rightarrow AB = 40 - 6$$

$$\Rightarrow AB = 34\text{m}$$

Also,

Given that 2m has to be left at both the sides,

$$BC = QR - 2 - 2$$

$$\Rightarrow BC = 15 - 4$$

$$\Rightarrow BC = 11\text{m}$$

Now, Area left for house construction is area of ABCD.

Hence,

$$\text{Area(ABCD)} = AB \times CD$$

$$= 34 \times 11$$

$$= 374 \text{ m}^2$$
