

Chapter 8: Exponents and Powers

NCERT Exemplar · Class 8 · Maths

Exercise

Question 1

In 2^n , n is known as

- (a) Base
- (b) Constant
- (c) Exponent
- (d) Variable

Solution

Ans: Option (c) is correct.

2 is a rational number as base in expression, and n is the power of that base which is called an exponent or power or index.

Question 2

For a fixed base, if the exponent decreases by 1, the number becomes

- (a) One-tenth of the previous number.
- (b) Ten times of the previous number.
- (c) Hundredth of the previous number.
- (d) Hundred times of the previous number.

Solution

Ans: Option (a) is correct.

If the exponent is decreased by 1, then for the

fixed base, the number becomes one-tenth of the previous number.

e.g.- For 10^4 , exponent decreases by 1

$$10^{4-1} = 10^3$$

$$\Rightarrow \frac{10^3}{10^4} = \frac{1}{10}$$

Question 3

3^{-2} can be written as

(a) 32

(b) $\frac{1}{3^2}$

(c) $\frac{1}{3^{-2}}$

(d) $-\frac{2}{3}$

Solution

Ans: Option (b) is correct.

Use, law of exponent $a^{-m} = \frac{1}{a^m}$ where a is a non zero integer.

$$\text{Thus, } 3^{-2} = \frac{1}{3^2}$$

Question 4

The value of $\frac{1}{4^{-2}}$ is

(a) 16

(b) 8

(c) $\frac{1}{16}$

(d) $\frac{1}{8}$

Solution

Ans: Option (a) is correct.

Use, law of exponent $a^{-m} = \frac{1}{a^m}$ where a is a non zero integer.

$$\text{Thus, } \frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 4^2 = 16$$

Question 5

The value of $35 \div 3-6$ is

(a) 35

(b) 3-6

(c) 311

(d) 3-11

Solution

Ans: Option (c) is correct.

Use, law of exponent $a^m \div a^n = a^{m-n}$ where a is a non zero integer.

$$\text{Thus, } 3^5 \div 3^{-6} = 3^{5-(-6)} = 3^{5+6} = 3^{11}.$$

Question 6

The value of $\left(\frac{2}{5}\right)^{-2}$ is

(a) $\frac{4}{5}$

(b) $\frac{4}{25}$

(c) $\frac{25}{4}$

(d) $\frac{5}{2}$

Solution

Ans: Option (c) is correct.

Use, law of exponent $a^{-m} = \frac{1}{a^m}$ where a is a non zero integer.

$$\text{Thus, } \left(\frac{2}{5}\right)^{-2} = \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{1}{\frac{4}{25}} = \frac{25}{4}$$

Question 7

The value of $\left(\frac{2}{5}\right)^{-1}$ is

(a) $\frac{2}{5}$

(b) $\frac{5}{2}$

(c) $-\frac{5}{2}$

(d) $-\frac{2}{5}$

Solution

Ans: Option (b) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

$$\text{Thus, } \left(\frac{2}{5}\right)^{-1} = \frac{1}{\left(\frac{2}{5}\right)^1} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$$

Question 8

The multiplicative inverse of 10^{-100} is

- (a) 10
- (b) 100
- (c) 10^{100}
- (d) 10^{-100}

Solution

Ans: Option (c) is correct.

For multiplicative inverse, let a be the multiplicative of 10^{-100}

So,

$$a \times 10^{-100} = 1$$

$$\Rightarrow a = \frac{1}{10^{-100}} = 10^{100}$$

Question 9

The value of $(-2)^{2 \times 3 - 1}$ is

- (a) 32
- (b) 64
- (c) -32
- (d) -64

Solution

Ans: Option (c) is correct.

For $(-a)^m$, if m is odd, then $(-a)^m$ is negative

$$(-2)^{2 \times 3 - 1} = (-2)^{6-1} = (-2)^5$$

$$= (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$$

Question 10

The value of $\left(\frac{-2}{3}\right)^4$ is equal to

(a) $\left(\frac{16}{81}\right)$

(b) $\left(\frac{81}{16}\right)$

(c) $\left(\frac{-16}{81}\right)$

(d) $\left(\frac{81}{-16}\right)$

Solution

Ans: Option (a) is correct.

For $(-a)^m$, if m is even, then $(-a)^m$ is positive.

$$\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) = \frac{16}{81}$$

Question 11

The multiplicative inverse of $\left(-\frac{5}{9}\right)^{-99}$ is

(a) $\left(-\frac{5}{9}\right)^{99}$

(b) $\left(\frac{5}{9}\right)^{99}$

(c) $\left(\frac{9}{-5}\right)^{99}$

(d) $\left(\frac{9}{5}\right)^{99}$

Solution

Ans: Option (a) is correct.

For multiplicative inverse, a is called multiplicative inverse of b, if $a \times b = 1$.

Thus,

$$\Rightarrow a \times \left(\frac{-5}{9}\right)^{-99} = 1$$

$$\Rightarrow a = \frac{1}{\left(\frac{-5}{9}\right)^{-99}} = \left(-\frac{5}{9}\right)^{99}$$

Question 12

If x be any non-zero integer and m, n be negative integers, then $x^m \times x^n$ is equal to

(a) x^m

(b) x^{m+n}

(c) x^n

(d) x^{m-n}

Solution

Ans: Option (b) is correct.

Using law of exponents, $a^m \times a^n = (a)^{m+n}$ (Where, a is a non-zero integer)

Similarly, $x^m \times x^n = (x)^{m+n}$

Question 13

If y be any non-zero integer, then y^0 is equal to

- (a) 1
- (b) 0
- (c) -1
- (d) Not defined

Solution

Ans: Option (a) is correct.

Using law of exponents, $a^0 = 1$

Similarly, $y^0 = 1$.

Question 14

If x be any non-zero integer, then x^{-1} is equal to

- (a) x
- (b) $\frac{1}{x}$
- (c) $-x$
- (d) $\frac{-1}{x}$

Solution

Ans: Option (b) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

$$\text{Thus, } x^{-1} = \frac{1}{x^1} = \frac{1}{x}$$

Question 15

If x be any integer different from zero and m be any positive integer, then x^{-m} is equal to

(a) xm

(b) $-xm$

(c) $\frac{1}{x^m}$

(d) $\frac{-1}{x^m}$

Solution

Ans: Option (c) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

Thus, $x^{-m} = \frac{1}{x^m}$

Question 16

If x be any integer different from zero and m, n be any integers, then

$(x^m)^n$ is equal to

(a) x^{m+n}

(b) x^{mn}

(c) $x^{\frac{m}{n}}$

(d) x^{m-n}

Solution

Ans: Option (b) is correct.

Use the law of exponent $(a^m)^n = (a)^{m \times n}$, where a is a non zero integer.

Thus, $(x^m)^n = (x)^{m \times n} = x^{mn}$.

Question 17

Which of the following is equal to $\left(-\frac{3}{4}\right)^{-3}$?

(a) $\left(\frac{3}{4}\right)^{-3}$

(b) $-\left(\frac{3}{4}\right)^{-3}$

(c) $\left(\frac{4}{3}\right)^3$

(d) $\left(-\frac{4}{3}\right)^3$

Solution

Ans: Option (d) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

$$\text{Thus, } \left(\frac{-3}{4}\right)^{-3} = \frac{1}{\left(\frac{-3}{4}\right)^3} = \left(-\frac{4}{3}\right)^3$$

Question 18

$\left(-\frac{5}{7}\right)^{-5}$ is equal to

(a) $\left(\frac{5}{7}\right)^{-5}$

(b) $\left(\frac{7}{5}\right)^5$

(c) $\left(\frac{7}{5}\right)^5$

(d) $\left(-\frac{7}{5}\right)^5$

Solution

Ans: Option (d) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

$$\text{Thus, } \left(\frac{-5}{7}\right)^{-5} = \frac{1}{\left(\frac{-5}{7}\right)^5} = \left(-\frac{7}{5}\right)^5$$

Question 19

$\left(\frac{-7}{5}\right)^{-1}$ is equal to

(a) $\frac{5}{7}$

(b) $-\frac{5}{7}$

(c) $\frac{7}{5}$

(d) $\frac{-7}{5}$

Solution

Ans: Option (b) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

$$\text{Thus, } \left(\frac{-7}{5}\right)^{-1} = \frac{1}{\left(\frac{-7}{5}\right)^1} = \left(-\frac{5}{7}\right)$$

Question 20

$(-9)^3 \div (-9)^8$ is equal to

- (a) $(9)^5$
- (b) $(9)^{-5}$
- (c) $(-9)^5$
- (d) $(-9)^{-5}$

Solution

Ans: Option (d) is correct.

Use law of exponent $\frac{a^m}{a^n} = (a)^{m-n}$, where a is a non zero integer.

$$\text{Thus, } \frac{(-9)^3}{(-9)^8} = (-9)^{3-8} = (-9)^{-5}$$

Question 21

For a non-zero integer x, $x^7 \div x^{12}$ is equal to

- (a) x^5
- (b) x^{19}
- (c) x^{-5}
- (d) x^{-19}

Solution

Ans: Option (c) is correct.

Use law of exponent $\frac{a^m}{a^n} = (a)^{m-n}$, where a is a non zero integer.

$$\text{Thus, } \frac{x^7}{x^{12}} = x^{7-12} = x^{-5}.$$

Question 22

For a non-zero integer x , $(x^4)^{-3}$ is equal to

- (a) x^{12}
- (b) x^{-12}
- (c) x^{64}
- (d) x^{-64}

Solution

Ans: Option (b) is correct.

Use law of exponent $(x^m)^n = (x)^{m \times n} = (x)^{mn}$, where a is a non zero integer.

$$\text{Thus, } (x^4)^{-3} = x^{4 \times (-3)} = x^{-12}$$

Question 23

The value of $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$ is

- (a) 44
- (b) 56
- (c) 68
- (d) 12

Solution

Ans: Option (a) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

Thus,

$$\begin{aligned} \left(\frac{1}{7} - \frac{1}{8}\right)^{-1} - \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} &= \left(\frac{8-7}{56}\right)^{-1} - \left(\frac{4-3}{12}\right)^{-1} \\ &= \left(\frac{1}{56}\right)^{-1} - \left(\frac{1}{12}\right)^{-1} = 56 - 12 = 44 \end{aligned}$$

Question 24

The standard form for 0.000064 is

- (a) 64×10^4
- (b) 64×10^{-4}
- (c) 6.4×10^5
- (d) 6.4×10^{-5}

Solution

Ans: Option (d) is correct.

Given, $0.000064 = 0.64 \times 10^{-4} = 6.4 \times 10^{-5}$

Question 25

The standard form for 234000000 is

- (a) 2.34×10^8
- (b) 0.234×10^9
- (c) 2.34×10^{-8}
- (d) 0.234×10^{-9}

Solution

Ans: Option (a) is correct.

$234000000 = 234 \times 10^6 = 2.34 \times 10^{2+6}$

$= 2.34 \times 10^8$

Question 26

The usual form for 2.03×10^{-5}

- (a) 0.203
- (b) 0.00203
- (c) 203000
- (d) 0.0000203

Solution

Ans: Option (d) is correct.

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

Thus,

$$2.03 \times 10^{-5} = \frac{2.03}{10^5} = 0.0000203$$

Question 27

$\left(\frac{1}{10}\right)^0$ is equal to

- (a) 0
- (b) $\frac{1}{10}$
- (c) 1
- (d) 10

Solution

Ans: Option (c) is correct.

Use the law of exponent $a^0 = 1$, where a is a non zero integer.

Thus,

$$\left(\frac{1}{10}\right)^0 = 1$$

Question 28

$\left(\frac{3}{4}\right)^5 \div \left(\frac{5}{3}\right)^5$ is equal to

(a) $\left(\frac{3}{4} \div \frac{5}{3}\right)^5$

(b) $\left(\frac{3}{4} \div \frac{5}{3}\right)^1$

(c) $\left(\frac{3}{4} \div \frac{5}{3}\right)^0$

(d) $\left(\frac{3}{4} \div \frac{5}{3}\right)^{10}$

Solution

Ans: Option (a) is correct.

Use the law of exponent $a^m \div b^m = (a \div b)^m$, where a is a non zero integer.

Thus,

$$\left(\frac{3}{4}\right)^5 \div \left(\frac{5}{3}\right)^5 = \left(\frac{3}{4} \div \frac{5}{3}\right)^5$$

Question 29

For any two non-zero rational numbers x and y, $x^4 \div y^4$ is equal to

(a) $(x \div y)^0$

(b) $(x \div y)^1$

(c) $(x \div y)^4$

(d) $(x \div y)^8$

Solution

Ans: Option (c) is correct.

Use the law of exponent $a^m \div b^m = (a \div b)^m$, where a is a non zero integer.

Thus,

$$x^4 \div y^4 = \left(\frac{x}{y}\right)^4 = (x \div y)^4$$

Question 30

For a non-zero rational number p, $p^{13} \div p^8$ is equal to

- (a) p^5
- (b) p^{21}
- (c) p^{-5}
- (d) p^{-19}

Solution

Ans: Option (a) is correct.

Use the law of exponent $a^m \div a^n = (a)^{m-n}$, where a is a non zero integer.

Thus,

$$p^{13} \div p^8 = (p)^{13-8} = p^5$$

Question 31

For a non-zero rational number z, $(z-2)^3$ is equal to

- (a) z^6
- (b) $z-6$
- (c) z^1

(d) z^4

Solution

Ans: Option (b) is correct.

Use the law of exponent $(a^m)^n = (a)^{mn}$, where a is a non zero integer.

Thus,

$$(z^{-2})^3 = z^{-2 \times 3} = z^{-6}$$

Question 32

Cube of $-\frac{1}{2}$ is

(a) $\frac{1}{8}$

(b) $\frac{1}{16}$

(c) $-\frac{1}{8}$

(d) $-\frac{1}{16}$

Solution

Ans: Option (c) is correct.

For $(-a)^m$, if m is odd, then $(-a)^m$ is negative.

$$\left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \left(-\frac{1}{8}\right)$$

Question 33

Which of the following is not the reciprocal of $\left(\frac{2}{3}\right)^4$?

(a) $\left(\frac{3}{2}\right)^4$

(b) $\left(\frac{3}{2}\right)^{-4}$

(c) $\left(\frac{2}{3}\right)^{-4}$

(d) $\frac{3^4}{2^4}$

Solution

Ans: Option (b) is correct.

Reciprocal of a is $\frac{1}{a}$.

$$\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$$

$$\Rightarrow \frac{3^4}{2^4} \neq \left(\frac{3}{2}\right)^{-4}$$

In questions 34 to 65, fill in the blanks to make the statements true.

Question 34

The multiplicative inverse of 1010 is _____.

Solution

Ans: 10-10

Multiplicative inverse is the reciprocal of a number. Thus,

$$10^{10} = \frac{1}{10^{10}} = 10^{-10}$$

Question 35

$$a^3 \times a^{-10} = \underline{\hspace{2cm}}.$$

Solution

$$\text{Ans: } a^3 \times a^{-10} = a^{-7}$$

$$\Rightarrow a^m \times a^n = a^{m+n}$$

$$\Rightarrow a^3 \times a^{-10} = a^{3-10} = a^{-7}$$

Question 36

$$5^0 = \underline{\hspace{2cm}}.$$

Solution

$$\text{Ans: } 1$$

$$\text{As, } a^0 = 1, \text{ so } 5^0 = 1.$$

Question 37

$$5^5 \times 5^{-5} = \underline{\hspace{2cm}}.$$

Solution

$$\text{Ans: } -1$$

$$\text{As, } x^m \times x^n = (x)^{m+n}$$

$$5^5 \times 5^{-5} = (5)^{5-5} = (5)^0 = 1$$

Question 38

$$\text{The value of } \left(\frac{1}{2^3}\right)^2 \text{ is equal to } \underline{\hspace{2cm}}.$$

Solution

$$\text{Ans: } \left(\frac{1}{2^6}\right)$$

As, $(xm)^n = (x)^n m^n$

$$\left(\frac{1}{2^3}\right)^2 = \left(\frac{1^3}{2^3}\right)^2 = \left(\frac{1}{2}\right)^{3 \times 2} = \left(\frac{1}{2}\right)^6 = \frac{1}{2^6}$$

Question 39

The expression for 8^{-2} as a power with the base 2 is _____.

Solution

Ans: $(2)^{-6}$

In 8^{-2} , 8 is the base and -2 is the power.

$$8^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

$$\Rightarrow \frac{1}{64} = \frac{1}{2^6} = 2^{-6}$$

Question 40

Very small numbers can be expressed in standard form by using _____ exponents.

Solution

Ans: negative.

Question 41

Very large numbers can be expressed in standard form by using _____ exponents.

Solution

Ans: positive.

Question 42

By multiplying $(10)^5$ by $(10)^{-10}$ we get _____.

Solution

Ans: $(10)^{-5}$

As, $x^m \times x^n = (x)^{m+n}$

$$10^5 \times 10^{-10} = (10)^{5-10} = (10)^{-5}$$

Question 43

$$\left[\left(\frac{2}{13} \right)^{-6} \div \left(\frac{2}{13} \right)^3 \right]^3 \times \left(\frac{2}{13} \right)^{-9} = \text{---}$$

Solution

Ans: $\left(\frac{2}{13} \right)^{-36}$

Use the law of exponent $a^m \div a^n = (a)^{m-n}$, where a is a non zero integer.

Thus,

$$\begin{aligned} & \left(\left(\frac{2}{13} \right)^{-6} \div \left(\frac{2}{13} \right)^3 \right)^3 \times \left(\frac{2}{13} \right)^{-9} = \left(\left(\frac{2}{13} \right)^{-6-3} \right)^3 \times \left(\frac{2}{13} \right)^{-9} \\ & = \left(\left(\frac{2}{13} \right)^{-9} \right)^3 \times \left(\frac{2}{13} \right)^{-9} \\ & = \left(\frac{2}{13} \right)^{-27} \times \left(\frac{2}{13} \right)^{-9} = \left(\frac{2}{13} \right)^{-27-9} = \left(\frac{2}{13} \right)^{-36} \end{aligned}$$

Question 44

Find the value $[4^{-1} + 3^{-1} + 6^{-2}]^{-1}$.

Solution

Ans: $\frac{18}{11}$

Use the law of exponent $a^{-m} = \frac{1}{a^m}$, where a is a non zero integer.

Thus,

$$\begin{aligned}(4^{-1} + 3^{-1} + 6^{-2})^{-1} &= \left(\frac{1}{4} + \frac{1}{3} + \frac{1}{6^2}\right)^{-1} \\&= \left(\frac{1}{4} + \frac{1}{3} + \frac{1}{36}\right)^{-1} = \left(\frac{9 + 12 + 1}{36}\right)^{-1} \\&= \left(\frac{22}{36}\right)^{-1} = \frac{36}{22} = \frac{18}{11}\end{aligned}$$

Question 45

$[2^{-1} + 3^{-1} + 4^{-1}]^0 = \underline{\hspace{2cm}}$

Solution

Ans: 1

Use the law of exponent $a^0 = 1$, where a is a non zero integer.

Thus,

$$(2^{-1} + 3^{-1} + 4^{-1})^0 = 1$$

Question 46

The standard form of $\left(\frac{1}{100000000}\right)$ is _____.

Solution

Ans: 1×10^{-8}

$$\text{Standard form, } \frac{1}{100000000} = \frac{1}{1 \times 10^8} = 10^{-8}$$

Question 47

The standard form of 12340000 is _____.

Solution

Ans: 1.234×10^7

For standard form of 12340000 = 1234×10^4

$$= 1.234 \times 10^4 \times 10^3 = 1.234 \times 10^7$$

Question 48

The usual form of 3.41×10^6 is _____.

Solution

Ans: 3410000

$$\text{So, } 3.41 \times 10^6 = 3.41 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

$$= 3.41 \times 10^6 = 3410000$$

Question 49

The usual form of 2.39461×10^6 is _____.

Solution

Ans: 2394610

So, $2.39461 \times 10^6 = 2.39461 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$$= 2.39461 \times 10^6 = 2394610$$

Question 50

If $36 = 6 \times 6 = 6^2$, then $\frac{1}{36}$ expressed as a power with the base 6 is _____.

Solution

Ans: 6^{-2}

Since $36 = 6 \times 6 = 6^2$

$$\text{Then } \frac{1}{36} = \frac{1}{6 \times 6} = \frac{1}{6^2} = 6^{-2}$$

Question 51

By multiplying $\left(\frac{5}{3}\right)^4$ by _____ we get 54.

Solution

Ans: 3^4

Let a to be multiplied.

$$\left(\frac{5}{3}\right)^4 \times a = 5^4$$

$$\Rightarrow \frac{5^4}{3^4} \times a = 5^4$$

$$\Rightarrow a = 3^4$$

Question 52

$3^5 \div 3^{-6}$ can be simplified as _____.

Solution

Ans: 3^{11}

Use the law of exponent $a^m \div a^n = (a)^{m-n}$, where a is a non zero integer.

Thus,

$$3^5 \div 3^{-6} = 3^{5-(-6)} = 3^{5+6} = 3^{11}$$

Question 53

The value of 3×10^{-7} is equal to _____.

Solution

Ans: 0.0000003

Given, $3 \times 10^{-7} = 3.0 \times 10^{-7}$

Now, placing the decimal seven places towards the left of the original position, we get 0.0000003.

Hence, the value of 3×10^{-7} is equal to 0.0000003.

Question 54

To add the numbers given in standard form, we first convert them into numbers with ___ exponents.

Solution

Ans: equal

To add the numbers given in standard form, we first convert them into numbers with equal exponents.

e.g. $2.46 \times 10^6 + 24.6 \times 10^5 = 2.46 \times 10^5 + 2.46 \times 10^6 = 4.92 \times 10^6$

Question 55

The standard form for 32,50,00,00,000 is _____.

Solution

Ans: For standard form, $32500000000 = 3250 \times 10^2 \times 10^2 \times 10^3$

$= 3250 \times 10^7 = 3.250 \times 10^{10}$ or 3.25×10^{10}

Hence, the standard form for 32500000000 is 3.25×10^{10} .

Question 56

The standard form for 0.000000008 is _____.

Solution

Ans: 8.0×10^{-9}

For standard form, $0.000000008 = 0.8 \times 10^{-8} = 8 \times 10^{-9} = 8.0 \times 10^{-9}$

Hence, the standard form for 0.000000008 is 8.0×10^{-9} .

Question 57

The usual form for 2.3×10^{-10} is _____.

Solution

Ans: 0.00000000023

For usual form, $2.3 \times 10^{-10} = 0.23 \times 10^{-11}$

$= 0.00000000023$

Hence, the usual form for 2.3×10^{-10} is 0.00000000023.

Question 58

On dividing 85 by _____ we get 8.

Solution

Ans: 84

Since $\frac{8^5}{8^4} = 8$.

Question 59

On multiplying _____ by 2^{-5} we get 25.

Solution

Ans: 210

As $2^{10} \times 2^{-5} = 2^{10-5} = 2^5$

Question 60

The value of $[3^{-1} \times 4^{-1}]^2$ is _____.

Solution

Ans: $\frac{1}{3^2 \times 4^2}$

As, $(3^{-1} \times 4^{-1})^2 = \left(\frac{1}{3} \times \frac{1}{4}\right)^2 = \frac{1}{3^2 \times 4^2}$

Question 61

The value of $[2-1 \times 3-1]-1$ is _____.

Solution

Ans: $\frac{1}{6}$

$$\text{As, } (2^{-1} \times 3^{-1})^{-1} = \left(\frac{1}{2} \times \frac{1}{3}\right)^{-1} = \left(\frac{1}{6}\right)^{-1} = 6$$

Question 62

By solving $(60 - 70) \times (60 + 70)$ we get _____.

Solution

Ans: 0

$$(6^0 - 7^0) \times (6^0 + 7^0) = (1 - 1) \times (1 + 1) = 0 \times 2 = 0$$

Question 63

The expression for 35 with a negative exponent is _____.

Solution

Ans: $\frac{1}{3^{-5}}$

Expression can be written as $3^5 = \frac{1}{3^{-5}}$

Question 64

The value for $(-7)6 \div 76$ is _____.

Solution

Ans: 1

$$\frac{(-7)^6}{(7)^6} = \frac{(-1 \times 7)^6}{(7)^6} = \frac{(-1)^6 \times (7)^6}{(7)^6} = (-1)^6 = 1$$

Question 65The value of $[1^{-2} + 2^{-2} + 3^{-2}] \times 62$ is _____.**Solution**

$$\text{Ans: } \frac{36}{1} + \frac{36}{4} + \frac{36}{9}$$

$$(1^{-2} + 2^{-2} + 3^{-2}) \times 6^2 = \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \right) \times 36$$

$$= \left(\frac{1}{1} + \frac{1}{4} + \frac{1}{9} \right) \times 36$$

$$= \frac{36}{1} + \frac{36}{4} + \frac{36}{9}$$

$$= 36 + 9 + 4 = 49$$

In questions 66 to 90, state whether the given statements are true (T) or false (F).

Question 66The multiplicative inverse of $(-4)^{-2}$ is $(4)^{-2}$.**Solution**

Ans: False.

$$\text{LHS} = (-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$$

$$\text{RHS} = (4)^{-2} = \frac{1}{(4)^2} = \frac{1}{16}$$

LHS = RHS. Hence, these numbers are the same and not multiplicative.

Question 67

The multiplicative inverse of $\left(\frac{3}{2}\right)^2$ is not equal to $\left(\frac{2}{3}\right)^{-2}$.

Solution

Ans: True.

$$\left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^2$$

Question 68

$$10^{-2} = \frac{1}{100}$$

Solution

Ans: True.

$$\text{Use } x^{-m} = \frac{1}{x^m}.$$

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

Question 69

$$24.58 = 2 \times 10 + 4 \times 1 + 5 \times 10 + 8 \times 100$$

Solution

Ans: False

$$RHS = 2 \times 10 + 4 \times 1 + 5 \times 10 + 8 \times 100 = 20 + 4 + 50 + 800 = 874$$

So, L H S $\neq$ R H S

Question 70

$$329.25 = 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$$

Solution

Ans: True

$$\text{RHS} = 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$$

$$= 3 \times 10 + 2 \times 10 + 9 \times 10 + \frac{2}{10} + \frac{5}{10 \times 10}$$

Therefore, LHS = RHS.

Question 71

$$(-5)^{-2} \times (-5)^{-3} = (-5)^{-6}$$

Solution

Ans: False

$$\text{LHS} = (-5)^{-2} \times (-5)^{-3}$$

Using law of exponents, $a^m \times a^n = a^{m+n}$

$$\text{Therefore, } (-5)^{-2} \times (-5)^{-3} = (-5)^{-5}$$

Question 72

$$(-4)^{-4} \times (4)^{-1} = (4)^5$$

Solution

Ans: False

$$\text{LHS} = (-4)^{-4} \times (4)^{-1}$$

Using law of exponents, $a^m \times a^n = a^{m+n}$

$$(-4)^{-4} \times (4)^{-1} = (4)^{-5}$$

Question 73

$$\left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{10}$$

Solution

Ans: False

$$\text{LHS} = \left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{-2-5} = \left(\frac{2}{3}\right)^{-7}$$

So, L H S $\neq$ R H S

Question 74

$$5^0 = 5$$

Solution

Ans: False.

Using law of exponent $a^0 = 1$

$$5^0 = 1.$$

Question 75

$$(-2)^0 = 2$$

Solution

Ans: False.

Using law of exponent = $a^0 = 1$

$$(-2)^0 = 1.$$

Question 76

$$\left(-\frac{8}{2}\right)^0 = 0$$

Solution

Ans: False.

Using law of exponent = $a^0 = 1$

$$\left(-\frac{8}{2}\right)^0 = 1$$

Question 77

$$(-6)^0 = -1$$

Solution

Ans: False.

Using law of exponent = $a^0 = 1$

$$(-6)^0 = 1.$$

Question 78

$$(-7)^{-4} \times (-7)^2 = (-7)^{-2}$$

Solution

Ans: True.

$$\text{LHS} = (-7)^4 \times (-7)^2$$

Using law of exponents, $a^m \times a^n = a^{m+n}$

$$\text{Therefore, } (-7)^4 \times (-7)^2 = (-7)^{4+2} = (-7)^6$$

$$\text{LHS} = \text{RHS}$$

Question 79

The value of $\frac{1}{4^{-2}}$ is equal to 16.

Solution

Ans: True

Using law of exponents, $x^{-m} = \frac{1}{x^m}$

$$\text{Thus, } \frac{1}{4^{-2}} = 4^2 = 4 \times 4 = 16$$

Question 80

The expression for 4^{-3} as a power with the base 2 is 2^6 .

Solution

Ans: False

Using law of exponents, $x^{-m} = \frac{1}{x^m}$

$$\text{Thus, } 4^{-3} = \frac{1}{4^3} = \frac{1}{(2^2)^3} = \frac{1}{2^6}$$

Question 81

$$a^p \times b^q = (ab)^{pq}$$

Solution

Ans: False.

$$\text{RHS} = (ab)^{pq}$$

Using law of exponents, $a^m \times a^n = a^{m+n}$

$$\text{Therefore, } (ab)^{pq} = (a)^{pq} \times (b)^{pq}$$

$$\text{LHS} \neq \text{RHS}$$

Question 82

$$\frac{x^m}{y^m} = \left(\frac{y}{x}\right)^{-m}$$

Solution

Ans: True

$$\text{RHS} = \left(\frac{y}{x}\right)^{-m} = \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m} = \text{LHS}$$

Question 83

$$a^m = \frac{1}{a^{-m}}$$

Solution

Ans: True

$$\text{Using law of exponents, } x^{-m} = \frac{1}{x^m}$$

$$a^m = \frac{1}{a^{-m}}$$

Question 84

The exponential form for $(-2)^4 \times \left(\frac{5}{2}\right)^4$ is 54.

Solution

Ans: True

$$\begin{aligned} (-2)^4 \times \left(\frac{5}{2}\right)^4 &= (2)^4 \times \left(\frac{5}{2}\right)^4 \\ &= 2^4 \times \frac{5^4}{2^4} = 5^4 \end{aligned}$$

Question 85

The standard form for 0.000037 is 3.7×10^{-5} .

Solution

Ans: True

For standard form, $0.000037 = 0.37 \times 10^{-4} = 3.7 \times 10^{-5}$

Question 86

The standard form for 203000 is 2.03×10^5

Solution

Ans: True

For standard form, $203000 = 203 \times 10 \times 10 \times 10 = 203 \times 10^3$
 $= 2.03 \times 10^2 \times 10^3 = 2.03 \times 10^5$

Question 87

The usual form for 2×10^{-2} is not equal to 0.02.

Solution

Ans: False

$$\text{As, } 2 \times 10^{-2} = 2 \times \frac{1}{10^2} = 2 \times \frac{1}{100} = 0.02$$

Question 88

The value of 5^{-2} is equal to 25.

Solution

Ans: False

$$\text{As, } 5^{-2} = \frac{1}{5^2} = \frac{1}{25} \neq 25$$

Question 89

Large numbers can be expressed in the standard form by using positive exponents.

Solution

Ans: True

$$\begin{aligned} \text{e.g. } 2350000 &= 235 \times 10 \times 10 \times 10 \times 10 = 235 \times 10^4 \\ &= 2.35 \times 10^4 \times 10^2 = 2.35 \times 10^6 \end{aligned}$$

Question 90

$$am \times bm = (ab)m$$

Solution

Ans: True

$$\text{LHS} = am \times bm = (a \times b)m = (ab)m$$

Question 91

Solve the following:

(i) 100^{-10}

(ii) $2^{-2} \times 2^{-3}$

(iii) $\left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3}$

Solution

Ans: Use $x^{-m} = \frac{1}{x^m}$

So, $100^{-10} = \frac{1}{100^{10}}$

Ans: Using law of exponents, $a^m \times a^n = a^{m+n}$

$$2^{-2} \times 2^{-3} = 2^{-2-3} = 2^{-5}$$

Ans: Use $a^m \div a^n = a^{m-n}$

$$\left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3} = \left(\frac{1}{2}\right)^{-2-(-3)} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

Question 92

Express $3^{-5} \times 3^{-4}$ as a power of 3 with positive exponent.

Solution

Ans: Using law of exponents, $a^m \times a^n = a^{m+n}$

$$3^{-5} \times 3^{-4} = 3^{-5-4} = 3^{-9} = \frac{1}{3^9}$$

Question 93

Express 16^{-2} as a power with the base 2.

Solution

Ans: Since $2 \times 2 \times 2 \times 2 = 16 = 2^4$,

$$16^{-2} = (2^4)^{-2} = (2)^{4 \times -2} = 2^{-8}$$

Question 94

Express $\frac{27}{64}$ and $\frac{-27}{64}$ as powers of a rational number.

Solution

Ans: $27 = 3 \times 3 \times 3 = 3^3$

$-27 = -3 \times -3 \times -3 = (-3)^3$

$64 = 4 \times 4 \times 4 = 4^3$

Now use $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$,

$$\frac{27}{64} = \frac{3^3}{4^3} = \left(\frac{3}{4}\right)^3$$

$$\Rightarrow \frac{-27}{64} = \frac{(-3)^3}{4^3} = \left(\frac{-3}{4}\right)^3$$

Question 95

Express $\frac{16}{81}$ and $\frac{-16}{81}$ as powers of a rational number.

Solution

$$\text{Ans: } 16 = 4 \times 4 = (4)^2$$

$$81 = 9 \times 9 = 9^2$$

$$\text{Now use } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},$$

$$\frac{16}{81} = \frac{4^2}{9^2} = \left(\frac{4}{9}\right)^2$$

$$\Rightarrow \frac{-16}{81} = \frac{-(4)^2}{9^2} = -\left(\frac{4}{9}\right)^2$$

Question 96

Express as a power of a rational number with a negative exponent.

$$\text{(a) } \left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3}$$

$$\text{(b) } (25 \div 28) \times 2^{-7}$$

Solution

$$\text{Ans: Use } (a^m)^n = a^{mn} \text{ and } a^{-m} = \frac{1}{a^m}$$

$$\left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3} = \left(\frac{-3}{2}\right)^{-2 \times (-3)} = \left(\frac{-3}{2}\right)^6 = \left(\frac{2}{3}\right)^{-6}$$

$$\text{Ans: Use } a^m \div a^n = a^{m-n} \text{ and } a^m \times a^n = a^{m+n}$$

$$(2^5 \div 2^8) \times 2^{-7} = (2^{5-8}) \times 2^{-7}$$

$$= 2^{-3} \times 2^{-7} = 2^{-3-7} = 2^{-10}$$

Question 97

Find the product of the cube of (-2) and the square of $(+4)$.

Solution

Ans: Since, cube of $(-2) = (-2)^3$

And square of $(+4) = (+4)^2$

Therefore, the product $= (-2)^3 \times (4)^2 = (-8) \times 16 = -128$

Question 98

Simplify:

$$(i) \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}$$

$$(ii) \left(\left(\frac{-2}{3}\right)^{-2}\right)^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$$

$$(iii) \frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} \quad (z \neq 0)$$

$$(iv) (25 \div 28) \times 2^{-7}$$

Solution

Ans: Use $a^{-m} = \frac{1}{a^m}$,

$$\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} = 4^2 + 2^2 + 3^2 = 16 + 4 + 9 = 29$$

Ans: Use $a^m \div a^n = a^{m-n}$

$$\left(\left(\frac{-2}{3}\right)^{-2}\right)^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6} = \left(\frac{-2}{3}\right)^{-2 \times 3} \times 3^4 \times \frac{1}{3} \times \frac{1}{6}$$

$$\begin{aligned}
 &= \left(\frac{-2}{3}\right)^{-6} \times 3^4 \times \frac{1}{3} \times \frac{1}{2 \times 3} \\
 &= \frac{3^6}{2^6} \times 3^4 \times \frac{1}{3} \times \frac{1}{2 \times 3} \\
 &= \frac{3^{10-2}}{2^{6+1}} = \frac{3^8}{2^7}
 \end{aligned}$$

Ans: Use $a^m \div a^n = a^{m-n}$

$$\begin{aligned}
 \frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} &= \frac{7^2 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} = \frac{7^{2+3} \times z^{-3+5}}{10} \\
 &= \frac{7^5 \times z^2}{10} = \frac{7^5}{10} z^2
 \end{aligned}$$

Ans: Use $a^m \div a^n = a^{m-n}$

$$\begin{aligned}
 (2^5 \div 2^8) \times (2)^{-7} &= (2)^{5-8} (2)^{-7} \\
 &= 2^{-3} \times 2^{-7} = 2^{-3-7} = 2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}
 \end{aligned}$$

Question 99

Find the value of x so that

- (i) $\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$
- (ii) $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$
- (iii) $(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

Solution

Ans: Use $a^m \times a^n = a^{m+n}$,

$$\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$$

$$\Rightarrow \left(\frac{5}{3}\right)^{-2-14} = \left(\frac{5}{3}\right)^{8x}$$

$\Rightarrow$ Compare both sides,

$$\Rightarrow -16 = 8x$$

$$\Rightarrow x = -2$$

Ans: Use, $a^m \times a^n = a^{m+n}$

Then, $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

$$(-2)^{3-6} = (-2)^{2x-1}$$

$$(-2)^{-3} = (-2)^{2x-1}$$

Compare both the sides and the result is=

$$-3 = 2x - 1$$

$$\Rightarrow 2x = -3 + 1$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -1$$

Ans: Use $a^{-m} = \frac{1}{a^m}$,

$$(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$$

$$\Rightarrow \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)^x = 1$$

$$\Rightarrow \left(\frac{12 + 6 + 4 + 3}{24}\right)^x = 1$$

$$\Rightarrow \left(\frac{25}{24}\right)^x = 1$$

This can be possible only if $x = 0$ because $a^0 = 1$.

Question 100

Divide 293 by 10,00,000 and express the result in standard form.

Solution

Ans: Use $a^{-m} = \frac{1}{a^m}$,

$$1000000 = 10^6$$

$$\Rightarrow \frac{293}{10^6} = 293 \times 10^{-6}$$

$$= 2.93 \times 10^{-6} \times 10^2 = 2.93 \times 10^{-4}$$

Question 101

Find the value of x^{-3} if $x = (100)^{1-4} \div (100)^0$.

Solution

Ans: Use $a^m \div a^n = a^{m-n}$ and $a^m \times a^n = a^{m+n}$

$$x = (100)^{-3} \div (100)^0$$

$$\Rightarrow x = (100)^{-3-0}$$

$$\Rightarrow x = (100)^{-3}$$

Thus,

$$\Rightarrow x^{-3} = \left((100)^{-3} \right)^{-3} = (100)^9$$

Question 102

By what number should we multiply $(-29)^0$ so that the product becomes $(+29)^0$.

Solution

Ans: Let x be multiplied with $(-29)^0$ to get $(+29)^0$

So,

$$x \times (-29)^0 = (29)^0$$

$$\Rightarrow x \times 1 = 1$$

$$\Rightarrow x = 1$$

Question 103

By what number should $(-15)^{-1}$ be divided so that quotient may be equal to $(-15)^{-1}$?

Solution

Ans: Let $(-15)^{-1}$ be divided by x to get quotient $(-15)^{-1}$.

So,

$$\frac{(-15)^{-1}}{x} = (-15)^{-1}$$

$$\Rightarrow x = \frac{(-15)^{-1}}{(-15)^{-1}}$$

$$\Rightarrow x = 1$$

Question 104

Find the multiplicative inverse of $(-7)^{-2} \div (90)^{-1}$.

Solution

Ans: a is called multiplicative inverse of b , if $a \times b = 1$.

$$(-7)^{-2} \div (90)^{-1} = \frac{1}{(7)^2} \div \frac{1}{(90)^1} = \frac{1}{49} \times 90 = \frac{90}{49}$$

$$\text{Put } b = \frac{90}{49}$$

$$\Rightarrow a \times \frac{90}{49} = 1$$

$$\Rightarrow a = \frac{49}{90}$$

Question 105

If $53x-1 \div 25 = 125$, find the value of x.

Solution

Ans: Given, $53x-1 \div 25 = 125$

Since, $25 = 5 \times 5 = 5^2$

And $125 = 5 \times 5 \times 5 = 5^3$

Therefore, $53x-1 \div (5)^2 = (5)^3$

So, $(5)^{3x-1-2} = (5)^3$ [As $a^m \div a^n = (a)^{m-n}$]

$$(5)^{3x-3} = (5)^3$$

On comparing both sides, we get

$$\Rightarrow 3x - 3 = 3$$

$$\Rightarrow 3x = 6$$

$$\Rightarrow x = 2$$

Question 106

Write 39,00,00,000 in the standard form.

Solution

Ans: For standard form,

$$39,00,00,000 = 39 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

$$= 39 \times 10^7 = 3.9 \times 10^1 \times 10^7$$

Use the law of exponent $a^m \times a^n = a^{m+n}$

$$\text{Thus, } 39,00,00,000 = 3.9 \times 10^8$$

Therefore, the standard form of $39,00,00,000 = 3.9 \times 10^8$

Question 107

Write 0.000005678 in the standard form.

Solution

Ans: For standard form,

$$0.000005678 = 0.5678 \times 10^{-5} = 5.678 \times 10^{-5} \times 10^{-1}$$

$$= 5.678 \times 10^{-6}$$

Hence, 5.678×10^{-6} is the standard form of 0.000005678.

Question 108

Express the product of 3.2×10^6 and 4.1×10^{-1} in the standard form.

Solution

$$\text{Ans: Product of } 3.2 \times 10^6 \text{ and } 4.1 \times 10^{-1} = (3.2 \times 10^6) (4.1 \times 10^{-1})$$

$$= (3.2 \times 4.1) \times 10^6 \times 10^{-1}$$

$$= 13.12 \times 10^5 = 1.312 \times 10^5 \times 10^1 \text{ [As } a^m \times a^n = a^{m+n}]$$

$$= 1.312 \times 10^6$$

Question 109

Express $\frac{1.5 \times 10^6}{2.5 \times 10^{-4}}$ in the standard form.

Solution

Ans: Use $a^m \div a^n = (a)^{m-n}$,

$$\begin{aligned}\frac{1.5 \times 10^6}{2.5 \times 10^{-4}} &= \frac{15}{25} \times 10^{6+4} = \frac{3}{5} \times 10^{10} \\ &= 0.6 \times 10^{10} \\ &= 6 \times 10^{-1} \times 10^{10} = 6 \times 10^9\end{aligned}$$

Question 110

Some migratory birds travel as much as 15,000 km to escape the extreme climatic conditions at home. Write the distance in metres using scientific notation.

Solution

Ans: Total distance traveled by migratory bird = 15000 km.

$$= 15000 \times 1000 \text{ m [As } 1 \text{ km} = 1000 \text{ m]}$$

$$= 15000000 \text{ m}$$

$$= 15 \times 10^6 \text{ m}$$

Scientific notation of $15 \times 10^6 = 1.5 \times 10^7 \text{ m}$.

Question 111

Pluto is 59,1,30,00, 000 m from the sun. Express this in the standard form.

Solution

Ans: Distance between Pluto and Sun = 59,1,30,00, 000

Standard form of 59,1,30,00, 000 = 5913×10^6

= $5.913 \times 10^6 \times 10^3$

Use, $a^m \times a^n = a^{m+n}$

Therefore, $5.913 \times 10^6 \times 10^3 = 5.913 \times 10^9$

Question 112

Special balances can weigh something as 0.00000001 gram. Express this number in the standard form.

Solution

Ans: Weight = 0.00000001 g

Standard form= 0.00000001 = 0.1×10^{-7} g

= $1 \times 10^{-7} \times 10^{-1}$ g [Use, $a^m \times a^n = a^{m+n}$]

= 1.0×10^{-8} g

Question 113

A sugar factory has annual sales of 3 billion 720 million kilograms of sugar. Express this number in the standard form.

Solution

Ans: Annual sales of a sugar factory = 3 billion 720

million kilograms = 3720000 kg

Standard form of-

$3720000 = 372 \times 10 \times 10 \times 10 \times 10 = 372 \times 10^4$ kg

Use $a^m \times a^n = a^{m+n}$

$$= 372 \times 104 \times 102 = 3.72 \times 10^6 \text{ kg.}$$

Question 114

The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red cells in the body? Write the standard form. (1 litre = 1,00,000 mm³)

Solution

Ans: The average body contains 5 L of blood.

Also, the number of red blood cells per cubic millimetre of blood is approximately 5.5 million.

Blood contained by body = 5 L = $5 \times 100000 \text{ mm}^3$

Red blood cells = $5 \times 100000 \text{ mm}^3$

$$\text{Blood} = 5.5 \times 1000000 \times 5 \times 100000 = 55 \times 5 \times 10^5 + 5$$

$$= 275 \times 10^{10} = 2.75 \times 10^{10} \times 10^2 = 2.75 \times 10^{12}$$

Question 115

Express each of the following in standard form:

(a) The mass of a proton in gram is $\frac{1673}{10000000000000000000000000000}$

(b) A Helium atom has a diameter of 0.00000022 cm.

(c) Mass of a molecule of hydrogen gas is about 0.000000000000000000334 tons.

(d) Human body has 1 trillion of cells which vary in shape and size.

Solution

Ans: Standard form = $\frac{1673}{10^{27}} = 1673 \times 10^{-27}g$

$$= 1.673 \times 10^{-27} \times 10^3 g$$

$$= 1.673 \times 10^{-27+3} q$$

$$= 1.673 \times 10^{-24} g$$

Ans: Standard form = $0.22 \times 10^{-7} \text{ cm} = 2.2 \times 10^{-1} \times 10^{-7}$

Using the law of exponent $a^m \times a^n = a^{m+n}$,

$$= 2.2 \times 10^{-1-7} = 2.2 \times 10^{-8} \text{ cm}$$

Therefore, the standard form is $2.2 \times 10^{-8} \text{ cm}$.

Ans: Standard form = $0.334 \times 10^{-20} = 3.34 \times 10^{-20} \times 10^{-1}$

[Use $a^m \times a^n = a^{m+n}$]

$$3.34 \times 10^{-20} \times 10^{-1} = 3.34 \times 10^{-21}$$

Ans: Cells in human body = 1 trillion

As, 1 trillion = 1000000000000

Standard form of 1000000000000 = $10 \times 10 \times 10 \times 10 \times 10 \times$

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^{12}$$

(e) Express 56 km in m.

Ans: Given, 56 km = $56 \times 1000 \text{ m} = 56000 \text{ m}$

Standard form of 56000 m = $56 \times 10^3 = 5.6 \times 10^3 \times 10^1 = 5.6 \times 10^4 \text{ m}$

(f) Express 5 tons in g.

Ans: Given, 5 tons = $5 \times 100 \text{ kg} = 5 \times 100 \times 1000 \text{ g} = 500000 \text{ g}$

Standard form of 500000

$$= 5 \times 10 \times 10 \times 10 \times 10 \times 10 = 5 \times 10^5$$

(g) Express 2 years in seconds.

$$\text{Ans: } 2 \text{ yr} = 2 \times 365 \text{ days}$$

$$= 2 \times 365 \times 24 \text{ h}$$

$$= 2 \times 365 \times 24 \times 60 \text{ min}$$

$$= 2 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$= 63072000 \text{ s}$$

Standard form of

$$63072000 = 63072 \times 10 \times 10 \times 10 = 63072 \times 10^3$$

$$63072 \times 10^3 = 6.3072 \times 10^4 \times 10^3$$

Since, $a^m \times a^n = a^{m+n}$

$$6.3072 \times 10^4 \times 10^3 = 6.3072 \times 10^7 \text{ s}$$

(h) Express 5 hectares in cm^2 (1 hectare = 10000 m^2)

$$\text{Ans: } 5 \text{ hec} = 5 \times 10000 \text{ m}^2$$

$$= 5 \times 10000 \times 100 \times 100 \text{ cm}^2$$

$$= 5 \times 10^{4+2+2} \text{ [Use } a^m \times a^n = a^{m+n}]$$

$$= 5 \times 10^8 \text{ cm}^2$$

Question 116

$$\text{Find } x \text{ so that } \left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$$

Solution

Ans: Use $a^m \times a^n = a^{m+n}$

$$\Rightarrow \left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{2}{9}\right)^{3-6} = \left(\frac{2}{9}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{2}{9}\right)^{-3} = \left(\frac{2}{9}\right)^{2x-1}$$

$$\Rightarrow -3 = 2x - 1$$

$$\Rightarrow 2x = -3 + 1$$

$$\Rightarrow x = \frac{-2}{2} = -1$$

Question 117

By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be

$$\left(\frac{4}{27}\right)^{-2} ?$$

Solution

Ans: Let $\left(\frac{-3}{2}\right)^{-3}$ is divided by x to get quotient $\left(\frac{4}{27}\right)^{-2}$.

Then,

$$\Rightarrow \left(\frac{-3}{2}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$$

$$\Rightarrow \left(\frac{2}{-3}\right)^3 \times \frac{1}{x} = \left(\frac{27}{4}\right)^2$$

$$\Rightarrow x = \frac{2^3 \times 4^2}{(-3)^3 \times (27)^2} = \frac{2^3 \times (2^2)^2}{-(3)^3 \times (3^3)^2} = \frac{2^{3+4}}{-3^{3+6}} = \frac{2^7}{-3^9}$$

In questions 118 and 119, find the value of n .

Question 118

$$\frac{6^n}{6^{-2}} = 6^3$$

Solution

Ans: Use $a^m \div a^n = (a)^{m-n}$

$$\frac{6^n}{6^{-2}} = 6^3$$

$$\Rightarrow 6^{n-(-2)} = 6^3$$

$$\Rightarrow 6^{n+2} = 6^3$$

On comparing both sides

$$n + 2 = 3$$

$$n = 1$$

Therefore, the value of n is 1.

Question 119

$$\frac{2^n \times 2^6}{2^{-3}} = 2^{18}$$

Solution

Ans: Use $a^m \times a^n = (a)^{m+n}$, $a^m \div a^n = (a)^{m-n}$ and $a^{-m} = \frac{1}{a^m}$.

$$\Rightarrow \frac{2^n \times 2^6}{2^{-3}} = 2^{18}$$

$$\Rightarrow 2^{n+6-(-3)} = 2^{18}$$

$$\Rightarrow 2^{n+9} = 2^{18}$$

On comparing both sides,

$$\Rightarrow n + 9 = 18$$

$$\Rightarrow n = 9$$

Question 120

$$\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}}$$

Solution

Ans: Use $a^m \times a^n = (a)^{m+n}$, $a^m \div a^n = (a)^{m-n}$ and $a^{-m} = \frac{1}{a^m}$.

$$\begin{aligned}\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}} &= \frac{5^3 \times x^{-3}}{5^{-3} \times 5^2 \times x^{-6}} \\ &= 5^{3+3-2} \times x^{-3+6} \\ &= 5^4 \times x^3 = 625x^3\end{aligned}$$

Question 121

$$\frac{16 \times 10^2 \times 64}{2^4 \times 4^2}$$

Solution

Ans: Use $a^m \times a^n = (a)^{m+n}$, $a^m \div a^n = (a)^{m-n}$ and $a^{-m} = \frac{1}{a^m}$.

$$\begin{aligned}\frac{16 \times 10^2 \times 64}{2^4 \times 4^2} &= (4)^2 \times 10^2 \times 2^{-4} \times 4^3 \times 4^{-2} \\ &= (2^2)^2 \times 10^2 \times (2^2)^{-2} \times 4^3 \times 4^{-2} \\ &= 4^2 \times 10^2 \times 4^{-2} \times 4^3 \times 4^{-2} \\ &= 4^{2-2+3-2} \times 10^2 \\ &= 4^1 \times 10^2 = 400\end{aligned}$$

Question 122

If $\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$, find m.

Solution

Ans: Use $a^m \times a^n = (a)^{m+n}$, $a^m \div a^n = (a)^{m-n}$ and $a^{-m} = \frac{1}{a^m}$.

$$\Rightarrow 5^{m+3+(-2)-(-5)} = 5^{12}$$

$$\Rightarrow 5^{m+3-2+5} = 5^{12}$$

$$\Rightarrow 5^{m+6} = 5^{12}$$

On comparing both sides,

$$\Rightarrow m + 6 = 12$$

$$\Rightarrow n = 6$$

Question 123

A new born bear weighs 4 kg. How many kilograms might a five-year-old bear weigh if its weight increases by the power of 2 in 5 years?

Solution

Ans: Weight of new born bear = 4 kg

Weight increases by the power of 2 in 5 yr.

Weight of bear in 5 yr = $(4)^2 = 16$ kg

Question 124

The cells of a bacteria double in every 30 minutes. A scientist begins with a single cell. How many cells will be there after-

(a) 12 hours

(b) 24 hours

Solution

Ans: The bacteria cell in every 30 min = 2

So, cell of a bacterium in 1h = 2^2 [As $(2+2)$ in 1h]

cell of a bacterium in 12h = $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2$

$2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = 2^{24}$

Ans: Similarly, bacteria in 24 h = $2^{24} \times 2^{24} = 2^{24+24} = 2^{48}$

Question 125

Planet A is at a distance of 9.35×10^6 km from Earth and planet B is 6.27×10^7 km from Earth. Which planet is nearer to Earth?

Solution

Ans: Distance between planet A and Earth = 9.35×10^6 km Distance between planet B and Earth = 6.27×10^7 km

For finding difference between above two distances, we have to change both in same exponent of 10, i.e. $9.35 \times 10^6 = 0.935 \times 10^7$, clearly 6.27×10^7 is greater.

So, planet A is nearer to Earth.

Question 126

The cells of a bacteria double itself every hour. How many cells will there be after 8 hours, if initially we start with 1 cell. Express the answer in powers.

Solution

Ans: The cell of a bacteria double itself every hour = $1 + 1$

$= 2 = 2^1$

Since, the process started with 1 cell.

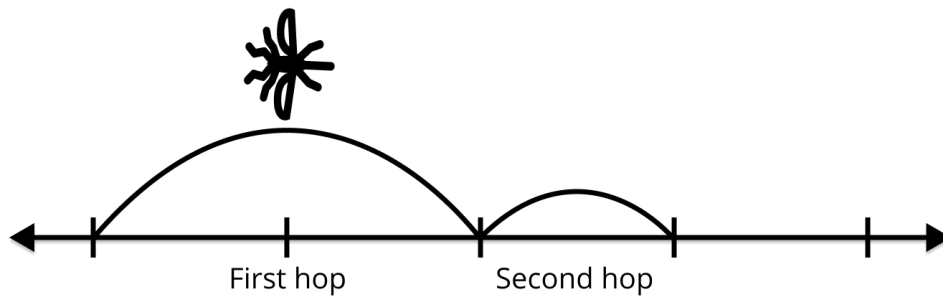
As, $a^m \times a^n = a^{m+n}$

Therefore, total no. of cell in 8h = $21 \times 21 \times 21 \times 21 \times 21 \times 21 \times$

$21 \times 21 = 28$

Question 127

An insect is on the 0 point of a number line, hopping towards 1. She covers half the distance from her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.



- (a) Make a table showing the insect's location for the first 10 hops.
- (b) Where will the insect be after n hops?
- (c) Will the insect ever get to 1? Explain.

Solution

Ans: On the basis of given information in the question, we can arrange the following table which shows the insect's location for the first 10 hops.

No. of Hops	Distance Covered	Distance Left	Distance Covered
1	$\frac{1}{2}$	$\frac{1}{2}$	$1 - \frac{1}{2}$
2	$\frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2}$	$\frac{1}{4}$	$1 - \frac{1}{4}$
3	$\frac{1}{2} \left(\frac{1}{4} \right) + \frac{3}{4}$	$\frac{1}{8}$	$1 - \frac{1}{8}$
4	$\frac{1}{2} \left(\frac{1}{8} \right) + \frac{7}{8}$	$\frac{1}{16}$	$1 - \frac{1}{16}$
5	$\frac{1}{2} \left(\frac{1}{16} \right) + \frac{15}{16}$	$\frac{1}{32}$	$1 - \frac{1}{32}$
6	$\frac{1}{2} \left(\frac{1}{32} \right) + \frac{31}{32}$	$\frac{1}{64}$	$1 - \frac{1}{64}$
7	$\frac{1}{2} \left(\frac{1}{64} \right) + \frac{63}{64}$	$\frac{1}{128}$	$1 - \frac{1}{128}$
8	$\frac{1}{2} \left(\frac{1}{128} \right) + \frac{127}{128}$	$\frac{1}{256}$	$1 - \frac{1}{256}$
9	$\frac{1}{2} \left(\frac{1}{256} \right) + \frac{255}{256}$	$\frac{1}{512}$	$1 - \frac{1}{512}$
10	$\frac{1}{2} \left(\frac{1}{512} \right) + \frac{511}{512}$	$\frac{1}{1024}$	$1 - \frac{1}{1024}$

Ans: If we see the distance covered in each hops

$$\text{Distance covered in 1st hops} = 1 - \frac{1}{2}$$

$$\text{Distance covered in 2nd hops} = 1 - \frac{1}{4}$$

$$\text{Distance covered in 3rd hops} = 1 - \frac{1}{8}$$

$$\text{Distance covered in } n \text{ hops} = 1 - \left(\frac{1}{2}\right)^n$$

Ans: No, because for reaching 1, $\left(\frac{1}{2}\right)^n$ has to be zero for finite n

which is not possible.

Question 128

Predicting the ones digit, copy and complete this table and answer the questions that follow.

x	1x	2x	3x	4x	5x	6x	7x	8x	9x	10x
1	1	2								
2	1	4								
3	1	8								
4	1	16								
5	1	32								
6	1	64								
7	1	128								

8	1	256								
Ones Digits of the Powers	1	2,4,8,6								

(a) Describe patterns you see in the ones digits of the powers.

Solution

Ans: On the basis of given pattern in $1x$ and $2x$, we can make more patterns for $3x$, $4x$, $5x$, $6x$, $7x$, $8x$, $9x$, $10x$.

Thus, we have the following table which shows all details about the patterns.

x	1 x	2x	3x	4x	5x	6x	7x	8x	9x	10x
1	1	2	3	4	5	6	7	8	9	10
2	1	4	9	16	25	36	49	64	81	100
3	1	8	27	64	125	216	343	512	729	1000
4	1	16	81	256	625	1296	2401	4096	6561	10000
5	1	32	243	1024	3125	7776	16807	32768	59049	100000
6	1	64	729	4096	1565	46656	117649	262144	531441	1000000
7	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000
8	1	256	6591	65536	390625	1679616	5764801	16777216	43046721	100000000

Ones Digits of the Powers	1	2,4,8, 6	3, 9, 7, 1	4,6	5	6	7, 9, 3, 1	8, 4, 2, 6	-9.1	0
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(b) Predict the ones digit in the following:

1. 412

Ans: Ones digit in 412 is 6.

2. 920

Ans: Ones digit in 920 is 1.

3. 317

Ans: Ones digit in 317 is 3.

4. 5100

Ans: Ones digit in 5100 is 5.

4. 10500

Ans: Ones digit in 10500 is 0.

(c) Predict the ones digit in the following:

1. 3110

Ans: Ones digit in 3110 is 1.

2. 1210

Ans: Ones digit in 1210 is 4.

3. 1721

Ans: Ones digit in 1721 is 7.

4. 2910

Ans: One digit in 2910 is 1.

Question 129

Astronomy The table shows the mass of the planets, the sun and the moon in our solar system.

Celestial Body	Mass (kg)	Mass (kg) Standard Notation
Sun	1,990,000,000,000,000,000,000,000,000	1.99×10^{30}
Mercury	330,000,000,000,000,000,000,000	
Venus	4,870,000,000,000,000,000,000,000	
Earth	5,970,000,000,000,000,000,000,000	
Mars	642,000,000,000,000,000,000,000,000	
Jupiter	1,900,000,000,000,000,000,000,000,000	
Saturn	568,000,000,000,000,000,000,000,000	
Uranus	86,800,000,000,000,000,000,000,000	
Neptune	102,000,000,000,000,000,000,000,000	

Pluto	12,700,000,000,000,000,000,000	
Moon	73,500,000,000,000,000,000,000	

(a) Write the mass of each planet and the Moon in scientific notation.

(b) Order the planets and the moon by mass, from least to greatest.

(c) Which planet has about the same mass as earth?

Solution

Ans: Using law of exponents $a^m \times a^n = a^{m+n}$

$$\text{Sun} = 199 \times 10^{28} = 1.99 \times 10^{28} \times 10^2 = 1.99 \times 10^{30}$$

$$\text{Mercury} = 33 \times 10^{22} = 3.3 \times 10^{22} \times 10 = 3.3 \times 10^{23}$$

$$\text{Venus} = 487 \times 10^{22} = 4.87 \times 10^{22} \times 10^2 = 4.87 \times 10^{24}$$

$$\text{Earth} = 597 \times 10^{22} = 5.97 \times 10^{22} \times 10^2 = 5.97 \times 10^{24}$$

$$\text{Mars} = 642 \times 10^{27} = 6.42 \times 10^{27} \times 10^2 = 6.42 \times 10^{29}$$

$$\text{Jupiter} = 19 \times 10^{26} = 1.9 \times 10^{26} \times 10 = 1.9 \times 10^{27}$$

$$\text{Saturn} = 568 \times 10^{24} = 5.68 \times 10^{24} \times 10^2 = 5.68 \times 10^{26}$$

$$\text{Uranus} = 868 \times 10^{23} = 8.68 \times 10^{23} \times 10^2 = 8.68 \times 10^{25}$$

$$\text{Neptune} = 102 \times 10^{24} = 1.02 \times 10^{24} \times 10^2 = 1.02 \times 10^{26}$$

$$\text{Pluto} = 127 \times 10^{24} = 1.27 \times 10^{20} \times 10^2 = 1.27 \times 10^{22}$$

$$\text{Moon} = 795 \times 10^{20} = 7.95 \times 10^{20} \times 10^2 = 7.95 \times 10^{22}$$

Ans: Order of mass of all planets and Moon from least to greatest.

Pluto < Moon < Mercury < Venus < Earth < Uranus < Neptune < Saturn < Jupiter < Mars

Ans: Venus has the mass similar to Earth.

Question 130

Investigating the Solar System The table shows the average distance from each planet in our solar system to the sun.

Planet	Distance from Sun (km)	Distance from Sun (km) Standard Notation
Earth	149,600,000	1.496×10^8
Jupiter	778,300,000	
Mars	227,900,000	
Mercury	57,900,000	
Neptune	4,497,000,000	
Pluto	5,900,000,000	
Saturn	1,427,000,000	
Uranus	2,870,000,000	
Venus	108,200,000	

(a) Complete the table by expressing the distance from each planet to the Sun in scientific notation.

(b) Order the planets from closest to the sun to farthest from the sun.

Solution

Ans: Scientific notation of distance from Sun to

Earth= 149600000 = $1496 \times 10^5 = 1.496 \times 10^8$

$$\text{Jupiter} = 778300000 = 7783 \times 10^5 = 7.783 \times 10^8$$

$$\text{Mars} = 227900000 = 2279 \times 10^5 = 2.279 \times 10^8$$

$$\text{Mercury} = 57900000 = 579 \times 10^5 = 5.97 \times 10^7$$

$$\text{Neptune} = 4497000000 = 4497 \times 10^5 = 4.497 \times 10^8$$

$$\text{Pluto} = 5900000000 = 59 \times 10^8 = 5.9 \times 10^9$$

$$\text{Saturn} = 1427000000 = 1427 \times 10^6 = 1.427 \times 10^9$$

$$\text{Venus} = 108200000 = 1082 \times 10^5 = 1.082 \times 10^8$$

Ans: Order of planet from closest to the Sun to farthest from the sun is given by

Mercury < Venus < Earth < Mars < Jupiter < Saturn < Uranus
< Neptune < Pluto.

Question 131

This table shows the mass of one atom for five chemical elements. Use it to answer the question given.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}
Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

(a) Which is the heaviest element?

(b) Which element is lighter, Silver or Titanium?

(c) List all five elements in order from lightest to heaviest.

Solution

Ans: The heaviest element is Lead = 34.4×10^{-26}

Ans: Silver = 17.9×10^{-26} and Titanium = 7.95×10^{-26} , therefore,

Titanium is lighter.

Ans: Arrangement of elements from lightest to heaviest-

Hydrogen < Lithium < Titanium < Silver < Lead

Question 132

The planet Uranus is approximately 2,896,819,200,000 metres away from the Sun. What is this distance in standard form?

Solution

Ans: Distance between the planet Uranus and the Sun is 2896819200000 m.

Standard form of 2896819200000 = $28968192 \times 10 \times 10 \times 10 \times 10 \times 10$

= $28968192 \times 10^5 = 2.8968192 \times 10^{12}$ m

Question 133

An inch is approximately equal to 0.02543 metres. Write this distance in standard form.

Solution

Ans: Standard form of 0.02543 m = 0.2543×10^{-1} m = 2.543×10^{-2} m Hence, 'standard form of 0.02543 is 2.543×10^{-2} m.

Question 134

The volume of the Earth is approximately 7.67×10^{-7} times the volume of the Sun. Express this figure in usual form.

Solution

Ans: Given, volume of the Earth = 7.67×10^{-7} times the volume of the Sun.

Usual form of $7.67 \times 10^{-7} = 0.000000767$

We have placed decimal 7 places towards the left of the original position.

Question 135

An electron's mass is approximately $9.1093826 \times 10^{-31}$ kilograms. What is this mass in grams?

Solution

Ans: Mass of electron = $9.1093826 \times 10^{-31}$

= $9.1093826 \times 10^{-31} \times 1000$ g

(Use, $a^m \times a^n = a^{m+n}$)

= $9.1093826 \times 10^{-31} \times 10^3 = 9.1093826 \times 10^{-28}$ g.

Question 136

At the end of the 20th century, the world population was approximately 6.1×10^9 people. Express this population in usual form. How would you say this number in words?

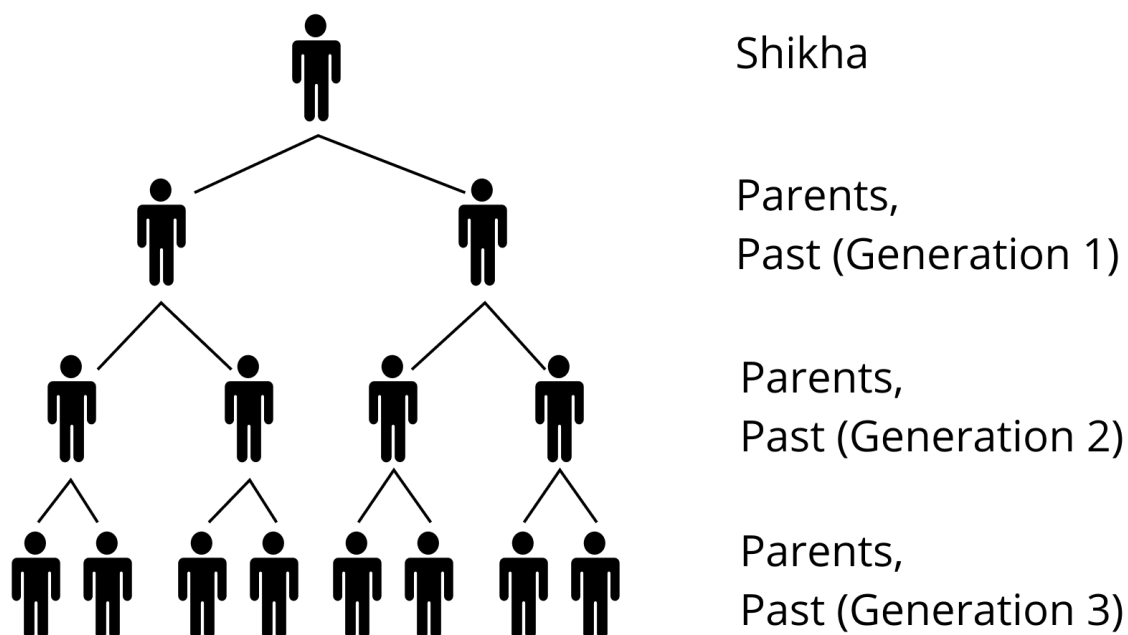
Solution

Ans: Given, at the end of the 20th century, the world population was 6.1×10^9 (approx). People population in usual form = $6.1 \times 10^9 = 6100000000$

Hence, the population in its usual form was six thousand one hundred million.

Question 137

While studying her family's history, Shikha discovers records of ancestors 12 generations back. She wonders how many ancestors she has had in the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



(a) Make a table and a graph showing the number of ancestors in each of the 12 generations.

(b) Write an equation for the number of ancestors in a given generation n .

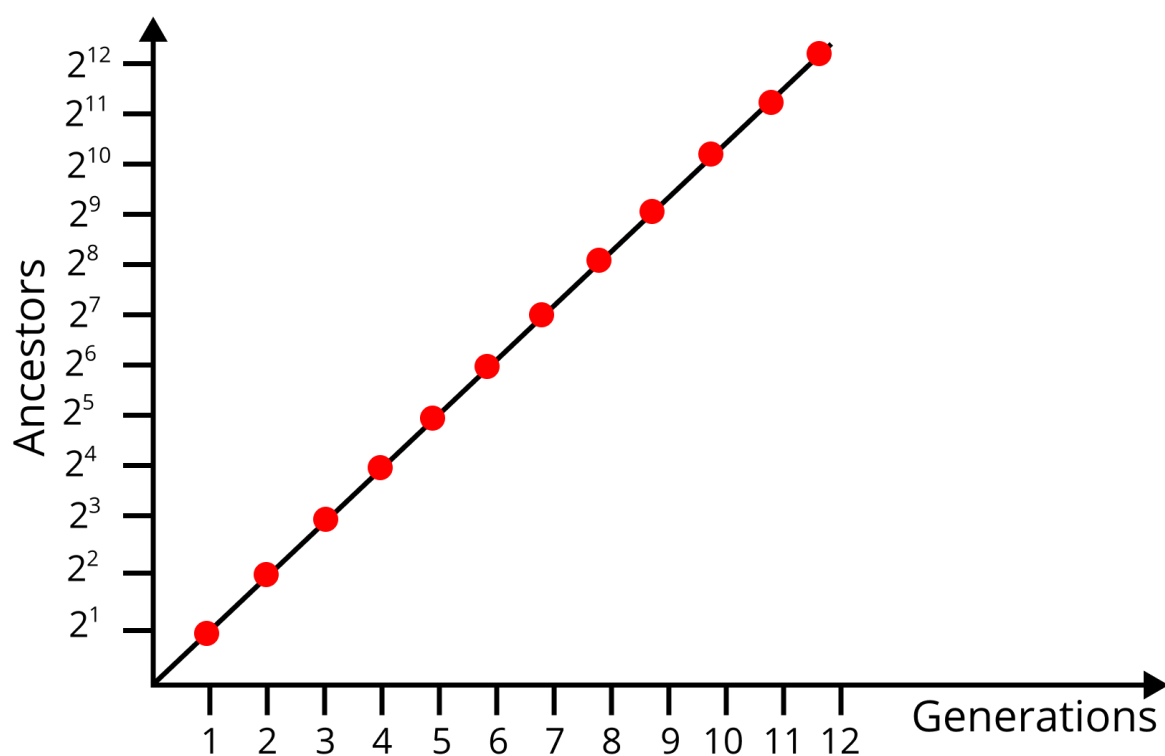
Solution

Ans: On the basis of the given diagram, we can make a table that shows the number of ancestors in each of the 12 generations.

Generation	Ancestors
1st	2

2nd	22
3rd	23
:	:
4th	24

Hence, we can also make a graph that shows the relation between generation and ancestor.



Ans: On the basis of the generation-ancestor graph, the numbers of ancestors in n generations will be 2^n .

Question 138

About 230 billion litres of water flows through a river each day. How many litres of water flows through that river in a week? How many litres of water flows through the river in an year? Write your answer in standard notation.

Solution

Ans: Water flows through a river in each day = 230000000000 = 230 billion

Water flows through the river in a week = 7×230000000000

= 1610000000000 = 1610 billion = 1.61×10^{12} L

Water flows through the river in a year = 230000000000×365

= 83950000000000 = 8.395×10^{13} L.

Question 139

A half-life is the amount of time that it takes for a radioactive substance to decay to one half of its original quantity.

Suppose radioactive decay causes 300 grams of a substance to decrease to 300×2^{-3} grams after 3 half-lives. Evaluate 300×2^{-3} to determine how many grams of the substance are left.

Explain why the expression 300×2^{-n} can be used to find the amount of the substance that remains after n half-lives.

Solution

Ans: Since, 300 g of a substance decreases to 300×2^{-3} g

after 3 half-lives.

So, we have to evaluate $300 \times 2^{-3} = \frac{300}{8} = 37.5$ g

Question 140

Consider a quantity of a radioactive substance. The fraction of this quantity that remains after t half-lives can be found by using the expression 2^{-t} .

(a) What fraction of substance remains after 7 half-lives?

(b) After how many half-lives will the fraction be $\frac{1}{243}$ of the original?

Solution

Ans: Since, 3-t expression is used for finding the fraction of the quality that remains after t half-lives.

Hence, the fraction of substance remains after 7 half-lives will be equal to 3-7 i.e., $\frac{1}{3^7}$.

Ans: Given, t half-lives = 3-t

$$\text{So, } \frac{1}{3^5} = \frac{1}{3^t}$$

And t = 5.

On comparing both sides, we get t = 5 half-lives.

Question 141

One Fermi is equal to 10⁻¹⁵ metre. The radius of a proton is 1.3 Fermis. Write the radius of a proton in metres in standard form.

Solution

Ans: The radius of a proton is 1.3 fermi.

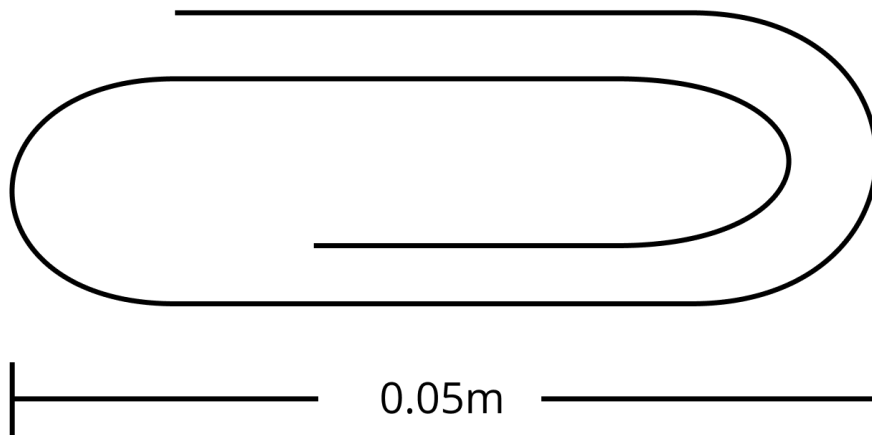
One fermi is equal to 10⁻¹⁵ m.

So, the radius of the proton is 1.3 x 10⁻¹⁵ m.

Hence, standard form of radius of the proton is 1.3 x 10⁻¹⁵ m.

Question 142

The paper clip below has the indicated length. What is the length in standard form?

**Solution**

Ans: Length of the paper clip = 0.05 m

In standard form, $0.05 \text{ m} = 0.5 \times 10^{-1} = 5.0 \times 10^{-2} \text{ m}$

Hence, the length of the paper clip in standard form is $5.0 \times 10^{-2} \text{ m}$

Question 143

Use the properties of exponents to verify that each statement is true.

(a) $\frac{1}{4}(2^n) = 2^{n-2}$

(b) $4^{n-1} = \frac{1}{4}(4)^n$

(c) $25(5n-2) = 5n$

Solution

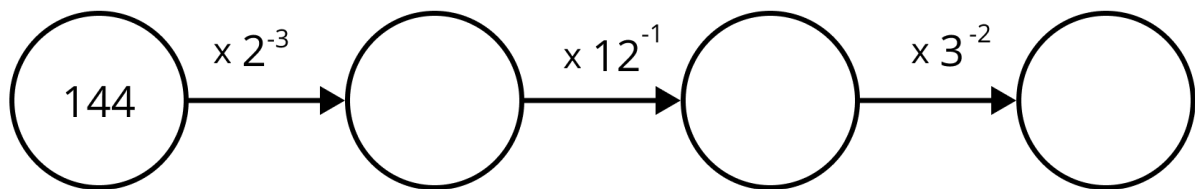
Ans: LHS = $\frac{1}{4}(2^n) = \frac{1}{2^2} \times 2^n = 2^{n-2} = \text{RHS}$

$$\text{Ans: RHS} = \frac{1}{4}(4)^n = \frac{1}{4} \times 4^n = 4^{n-1} = \text{LHS}$$

$$\text{Ans: LHS} = 25(5^{n-2}) = 5^2 5^{n-2} = 5^{2+n-2} = 5^n = \text{RHS}$$

Question 144

Fill in the blanks

**Solution**

$$\text{Ans: } 144 \times 2^{-3} = 144 \times \frac{1}{8} = 18$$

$$18 \times 12^{-1} = 18 \times \frac{1}{12} = \frac{3}{2}$$

$$\frac{3}{2} \times 3^{-2} = \frac{3}{2} \times \frac{1}{3^2} = \frac{1}{2 \times 3} = \frac{1}{6}$$

Question 145

There are 864,00 seconds in a day. How many days long is a second? Express your answer in scientific notation.

Solution

Ans: Total seconds in a day = 86400

So, a second is long as $\frac{1}{86400} = 0.000011574$

Scientific notation of 0.000011574 = 1.1574×10^{-5} days

Question 146

The given table shows the crop production of a State in the year 2008 and 2009. Observe the table given below and answer the given questions.

Crop	2008 Harvest (Hectare)	Increase/Decrease (Hectare) in 2009
Bajra	1.4×10^3	– 100
Jowar	1.7×10^6	– 440,000
Rice	3.7×10^3	– 100
Wheat	5.1×10^5	+ 190,000

(a) For which crop(s) did the production decrease?

(b) Write the production of all the crops in 2009 in their standard form.

(c) Assuming the same decrease in rice production each year as in 2009, how many acres will be harvested in 2015? Write in standard form.

Solution

Ans: From the given table we can figure out that bajra, jowar and rice crop's production decreased.

Ans: The production of all crops in 2009

$$\text{Bajra} = 1.4 \times 10^3 - 0.1 \times 10^3 = 1.3 \times 10^3$$

$$\text{Jowar} = 1.7 \times 10^6 - 44 \times 10^4$$

$$= 1.7 \times 10^6 - 0.44 \times 10^6 = 1.26 \times 10^6$$

$$\text{Rice} = 3.7 \times 10^3 - 0.1 \times 10^3 = 3.6 \times 10^3$$

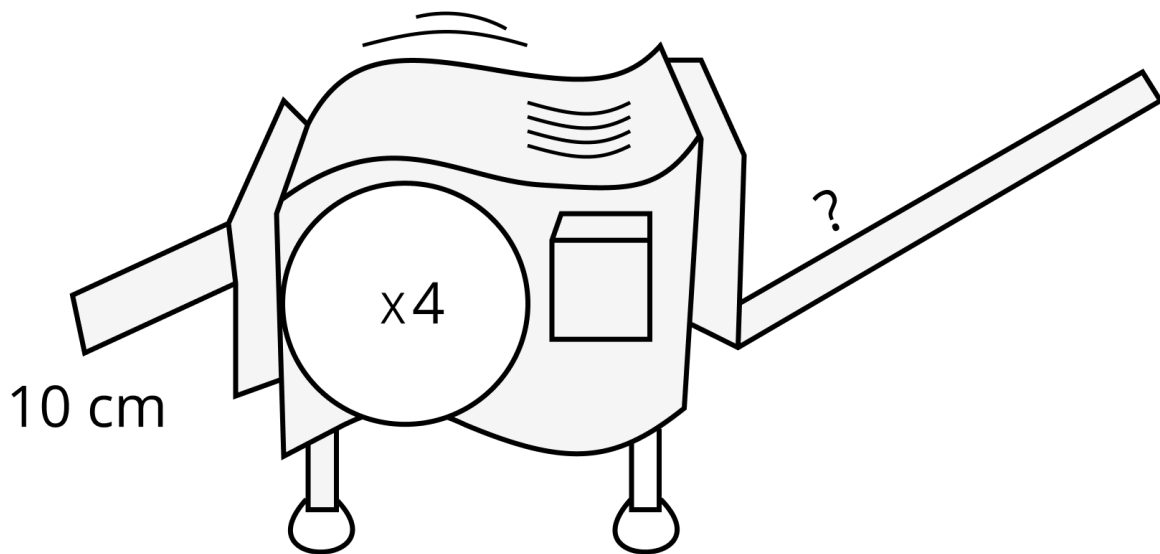
$$\text{Wheat} = 5.1 \times 10^5 + 19 \times 10^4$$

$$= 5.1 \times 10^5 + 1.9 \times 10^5 = 7 \times 10^5$$

Ans: Incomplete information.

Question 147

Stretching Machine Suppose you have a stretching machine which could stretch almost anything. For example, if you put a 5 metre stick into a ($\times 4$) stretching machine (as shown below), you get a 20 metre stick. Now if you put a 10 cm carrot into a ($\times 4$) machine, how long will it be when it comes out?



Solution

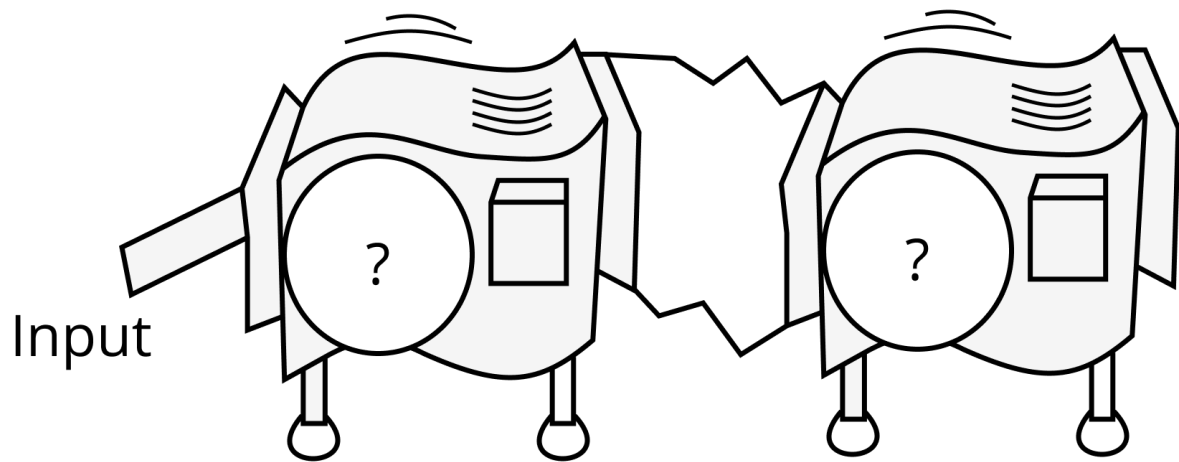
Ans: According to the question, if we put a 5 m stick into a (x 4) stretching machine, then the machine produces 20 m sticks.

Similarly, if we put 10 cm carrots into a (x 4) stretching machine, then the machine produces $10 \times 4 = 40$ cm sticks.

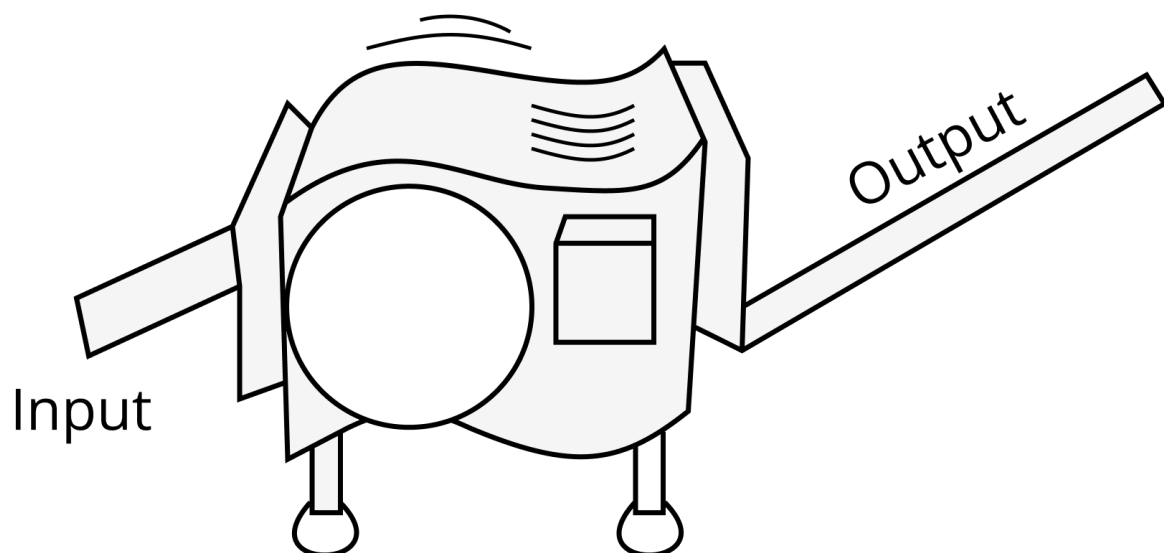
Question 148

Two machines can be hooked together. When something is sent through this hook up, the output from the first machine becomes the input for the second.

(a) Which two machines hooked together do the same work a ($\times 102$) machine does? Is there more than one arrangement of two machines that will work?



(b) Which stretching machine does the same work as two ($\times 2$) machines hooked together?



Solution

Ans:

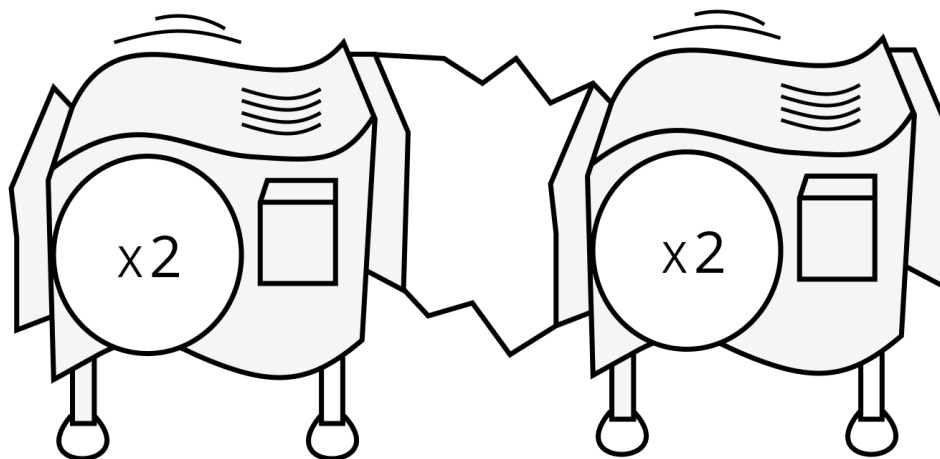
To get the same work in (x102) machine, hook (x 22) and (x 52) machine.

So, $x 102 = x 100$

Now, we get,

$$22 \times 52 = 4 \times 25 = 100$$

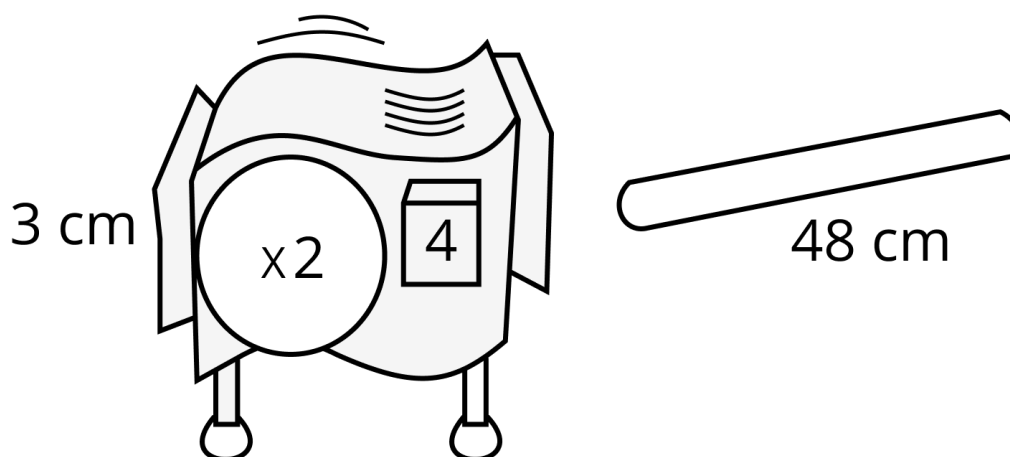
Ans:



The same work would be produced by a (x 4) single machine when two machines (x 2) and (x 2) are hooked together.

Question 149

Repeater Machine Similarly, a repeater machine is a hypothetical machine which automatically enlarges items several times. For example, sending a piece of wire through a ($\times 24$) machine is the same as putting it through a ($\times 2$) machine four times. So, if you send a 3 cm piece of wire through a ($\times 24$) machine, its length becomes $3 \times 2 \times 2 \times 2 \times 2 = 48$ cm. It can also be written that a base (2) machine is being applied 4 times.

**Solution**

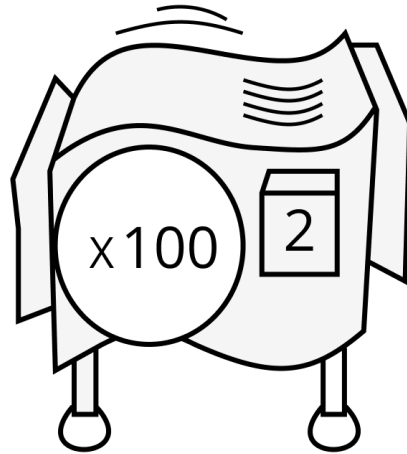
Ans: According to the question, if we put a 3 cm piece of wire through a ($\times 24$) machine, its length becomes $3 \times 2 \times 2 \times 2 \times 2 = 48$ cm.

Similarly, a 4 cm long strip becomes $4 \times 2 \times 2 \times 2 \times 2 = 64$ cm.

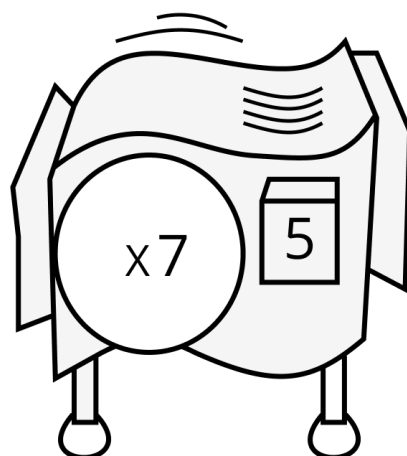
Question 150

For the following repeater machines, how many times the base machine is applied and how much the total stretch is?

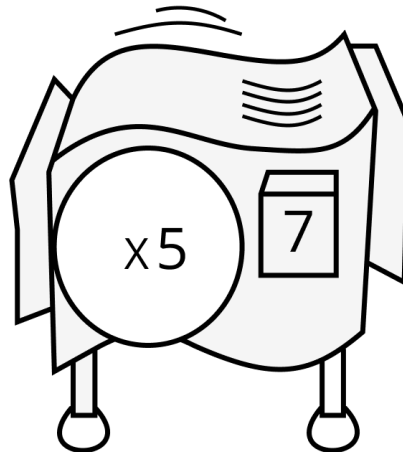
(a)



(b)



(c)



Solution

Ans: In the first machine that is (a), $(\times 1002) = 10000$ stretch. Because it is twice the base machine.

Ans: In machine (b), $(\times 75) = 16807$ stretch.

Since, it is fair times the base machine.

Ans: In machine (c), $(\times 57) = 78125$ stretch.

Since, it is 7 times the base machine.

Question 151

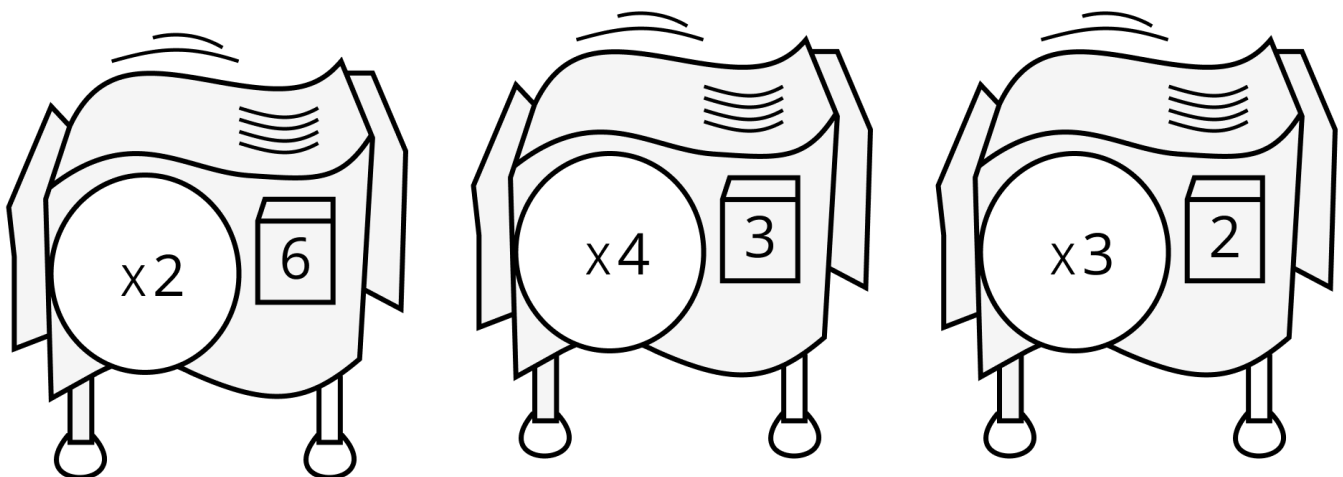
Find three repeater machines that will do the same work as a ($\times 64$) machine. Draw them, or describe them using exponents.

Solution

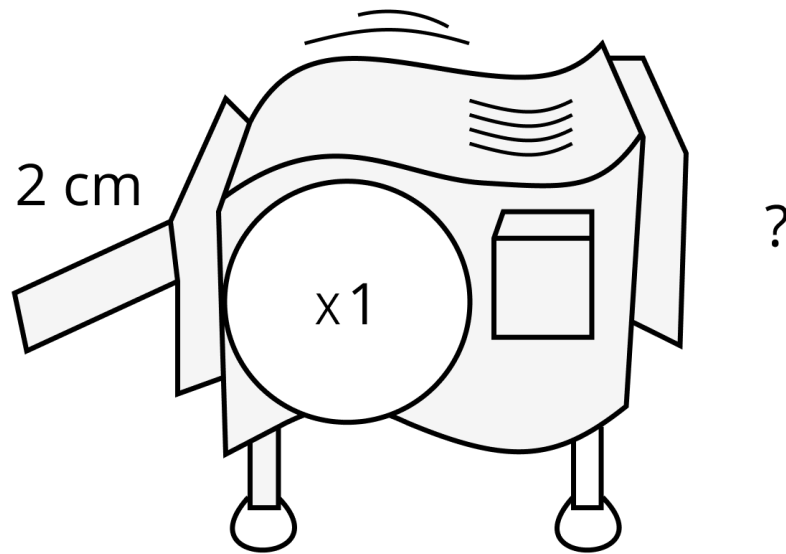
Ans: We know that, the possible factors of 64 are 2, 4, 8. :

If $2^6 = 64$, $4^3 = 64$ and $8^2 = 64$

Hence, three repeater machines that would work as a ($\times 64$) will be ($\times 2$), ($\times 4$) and ($\times 8$). The diagram of ($\times 2$), ($\times 4$) and ($\times 8$) is given below.

**Question 152**

What will the following machine do to a 2 cm long piece of chalk?

**Solution**

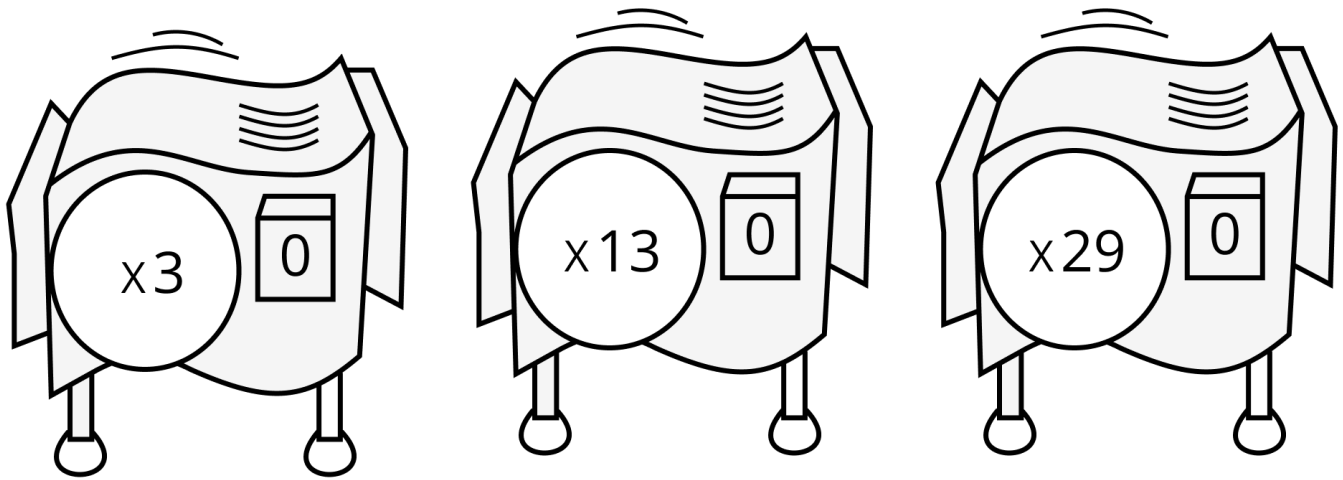
Ans: The machine produce $x 1100 = 1$

So, if we insert a 2 cm long piece of chalk in that machine, the piece of chalk remains the same.

Question 153

In a repeater machine with 0 as an exponent, the base machine is applied 0 times.

(a) What do these machines do to a piece of chalk?



(b) What do you think the value of 60 is?

Solution

Ans: Use the law of exponent $a^0 = 1$.

Since, $3^0 = 1$, $13^0 = 1$, $29^0 = 1$

So, machine (x 30), (x 130) and (x 290) produce nothing on not changing the piece 7 chalk.

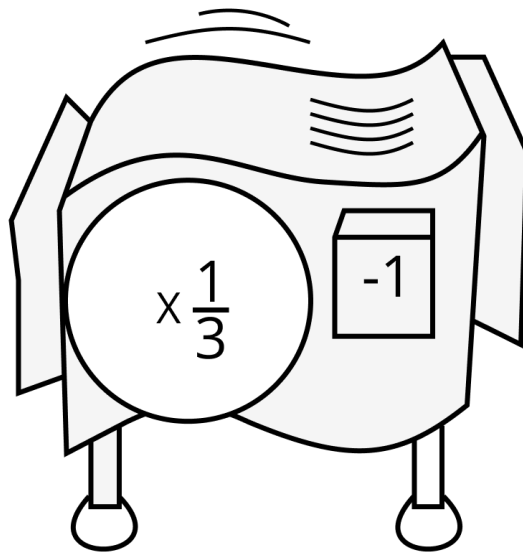
Ans: Use law of exponent $a^0 = 1$,

Similarly, (x 60) machine does not change the piece.

Question 154

Shrinking Machine

In a shrinking machine, a piece of stick is compressed to reduce its length. If 9 cm long sandwich is put into the shrinking machine below, how many cm long will it be when it emerges?



Solution

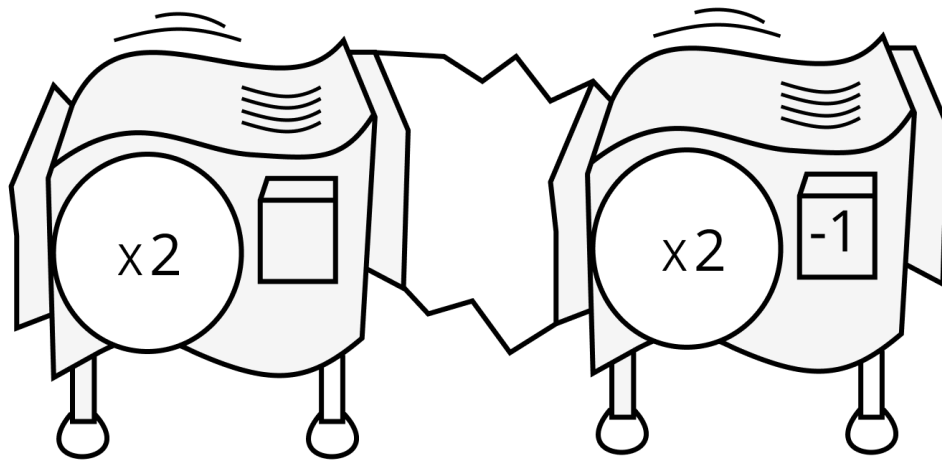
Ans: According to the question, in a shrinking machine, a piece of stick is compressed to reduce its length. If a 9 cm long sandwich is put into the shrinking machine, then the length of the sandwich will be

$$9 \times \frac{1}{3^{-1}} = 9 \times 3 = 27 \text{ cm.}$$

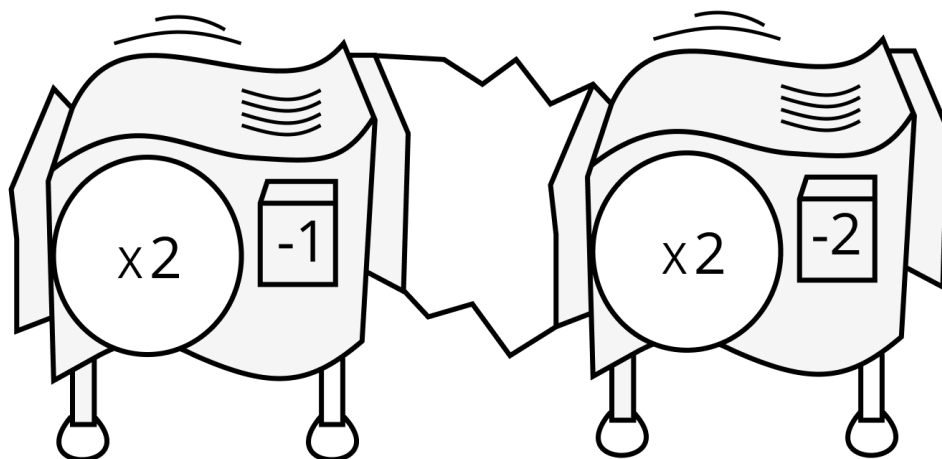
Question 155

What happens when 1 cm worms are sent through these hook-ups?

(i)



(ii)



Solution

Ans: If you put 1 cm worms through (x2) and (x2-1)

machine, the result will be $= 1 \times 2 \times \frac{1}{2} = 1\text{cm}$.

Ans: If we send 1 cm worms through hooked machines (x 2-1) and (x 2)-2, result will be-

$$1 \times \frac{1}{2} \times \frac{1}{2 \times 2} = \frac{1}{2 \times 4} = \frac{1}{8} \text{ cm} = 0.125 \text{ cm}$$

Question 156

Sanchay put a 1cm stick of gum through a $(1 \times 3-2)$ machine. How long was the stick when it came out?

Solution

Ans: Length of the stick of gum that Sanchay put through the $(1 \times 3-2)$ machine = 1 cm

Negation (-) sign in power states that the given machine is a

shrinking machine.

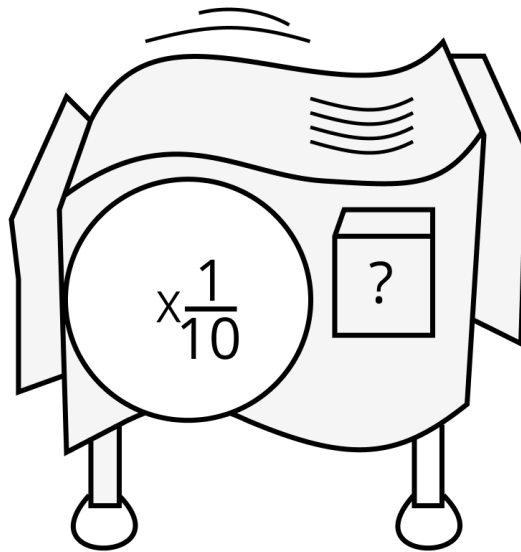
$$\text{Hence } 1 \times \frac{1}{3^2} = 1 \times \frac{1}{9} = \frac{1}{9} \text{ cm}$$

Therefore, the length of the stick when it came out is $\frac{1}{9}$ cm.

Question 157

Ajay had a 1cm piece of gum. He put it through the repeater machine given below and it came out

$\frac{1}{100,000}$ cm long. What is the missing value?



Solution

Ans: The 1 cm piece of gum that Ajay put came out as $\frac{1}{100000}$.

From this we can conclude that the machine is a shrinking machine.

Therefore, the type of shrinking machine is $\left(\times \frac{1}{10}\right)^5$.

Hence, the value missing is 5.

Question 158

Find a single machine that will do the same job as the given hook-up.

(a) a $(\times 23)$ machine followed by $(\times 2-2)$ machine.

(b) a $(\times 24)$ machine followed by $\left(\times \left(\frac{1}{2}\right)^2\right)$ machine.

(c) a ($\times 599$) machine followed by a ($5-100$) machine.

Solution

Ans: If ($\times 23$) machine is followed by a ($\times 2-2$) machine,

$$\text{the result will be- } 2 \times 2 \times 2 \times \frac{1}{2} \times \frac{1}{2} = 2^1$$

Thus, a single machine ($\times 21$) will also work as the given hook-up.

Ans: A ($\times 24$) machine is followed by a $\left[\left(\frac{1}{2}\right)^2\right]$ machine.

$$\text{Therefore, the result will be } 2 \times 2 \times 2 \times 2 \times \frac{1}{2} \times \frac{1}{2} = 4$$

Ans: A ($\times 599$) machine is followed by a ($\times 5-100$) machine.

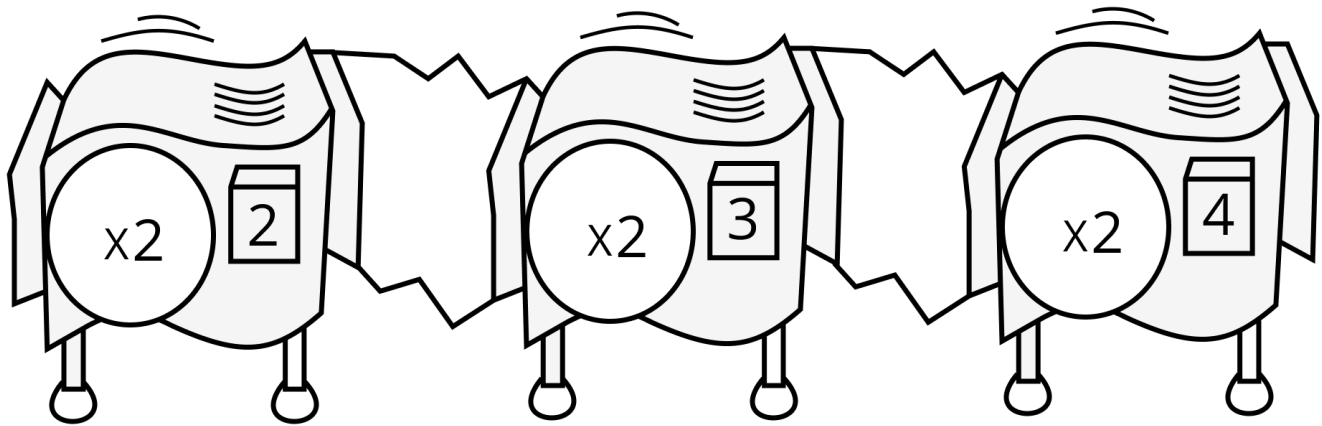
$$\text{Therefore, the result will be } 5^{99} \times \frac{1}{5^{100}} = \frac{1}{5}$$

Thus, a $\left(\times \frac{1}{5}\right)$ machine will work as the given hook-up.

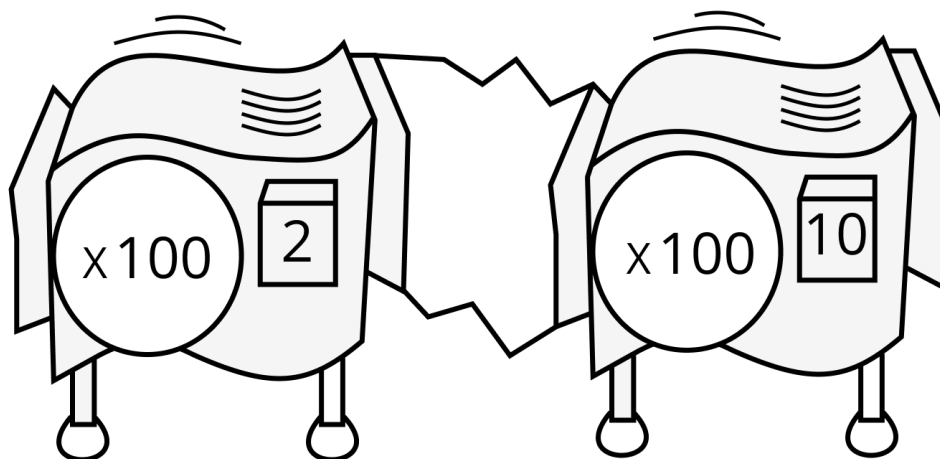
Question 159

Find a single repeater machine that will do the same work as each hook-up.

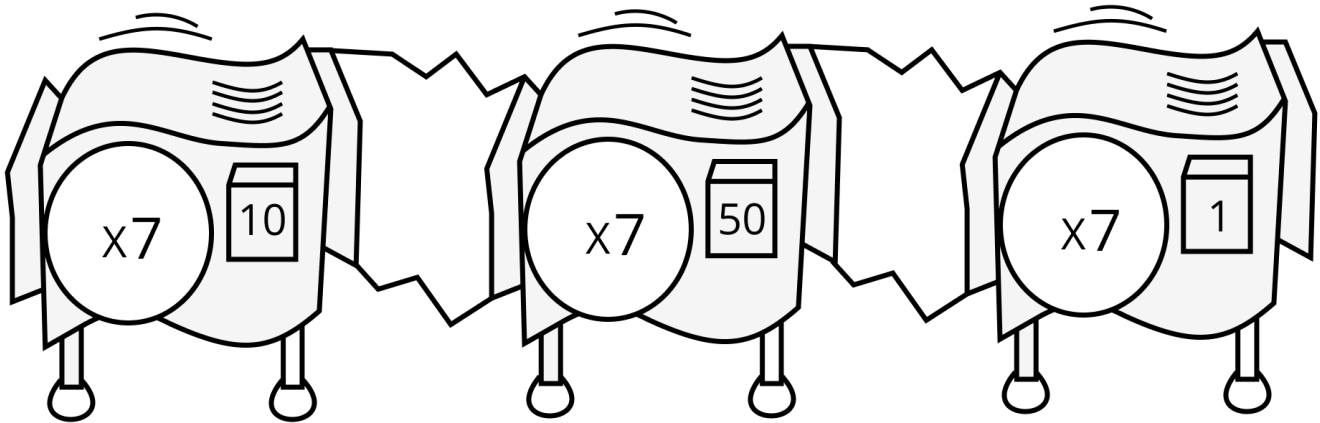
(a)



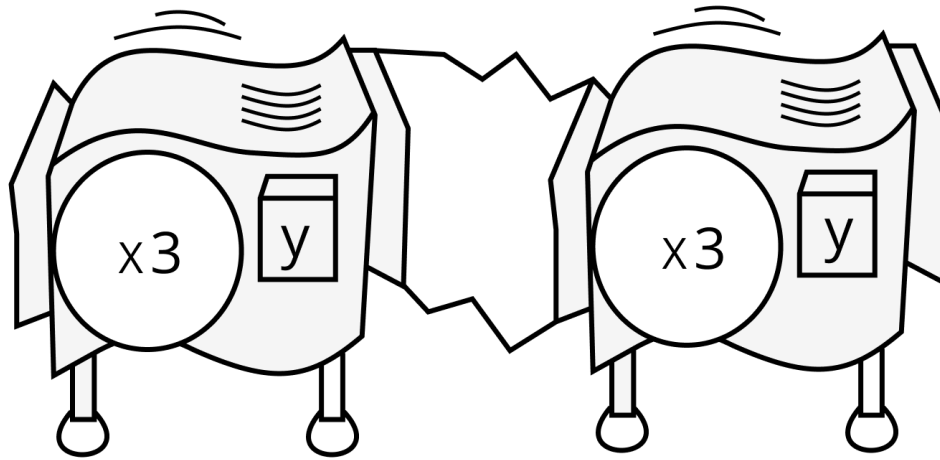
(b)



(c)



(d)



Solution

Ans: The work that can be performed by the repeater machine is

equal to $22 \times 23 \times 24 = 29$.

Hence, $(\times 29)$ single machine can perform the same amount of work.

Ans: The work that can be performed by the repeater machine is equal to 1002×10010 .

Hence, $(\times 10012)$ single machine can perform the same amount of work.

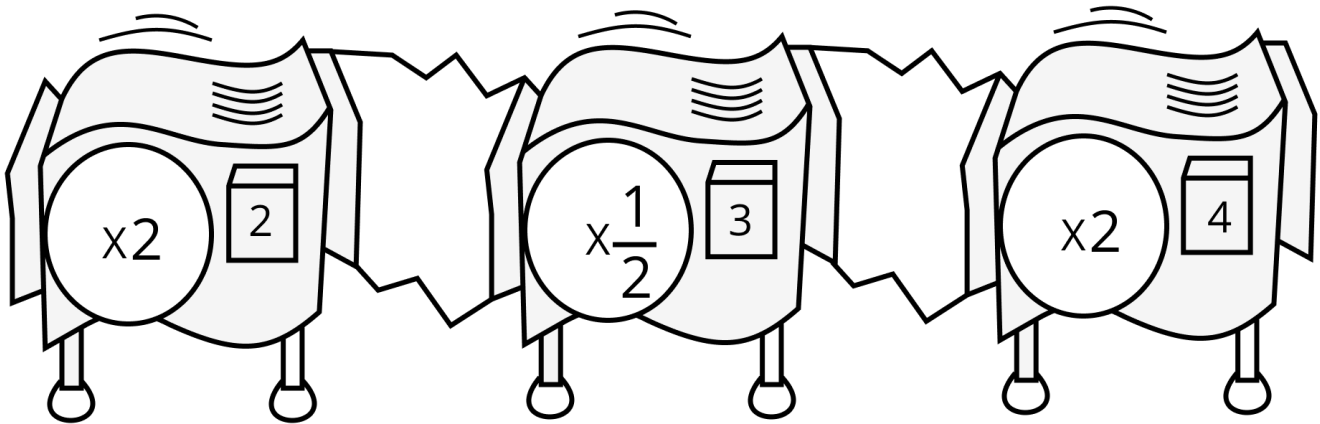
Ans: The work that can be performed by the repeater machine is equal to $710 \times 750 \times 71 = 761$

Hence, $(\times 761)$ single machine can perform the same amount of work.

Ans: The work that can be performed by the repeater machine is equal to $3y \times 3y = 32y$

Hence, $(32y)$ single machine can perform the same amount of work.

(e)

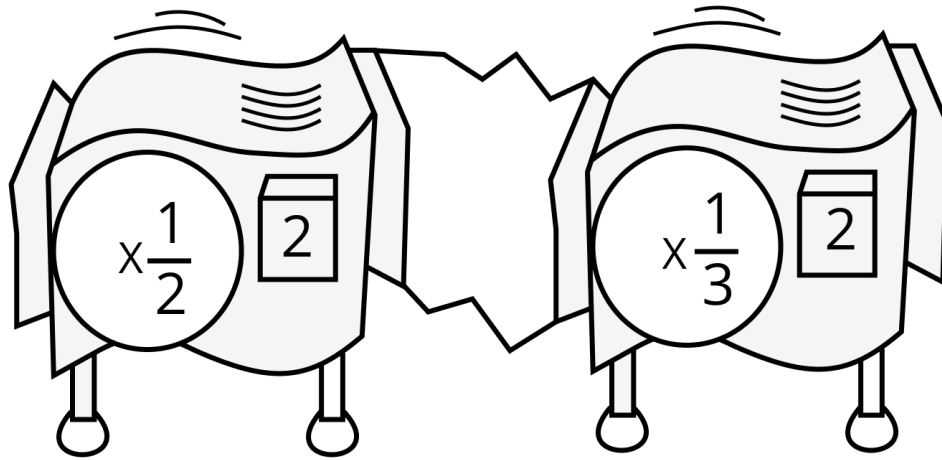


Ans: The work that can be performed by the repeater machine is equal to

$$2^2 \times \left(\frac{1}{2}\right)^3 \times 2^4 = 2^6 \times \frac{1}{2^3} = 2^3$$

Hence, (x23) single machines can perform the same amount of work.

(f)



Ans: The work that can be performed by the repeater machine is equal to

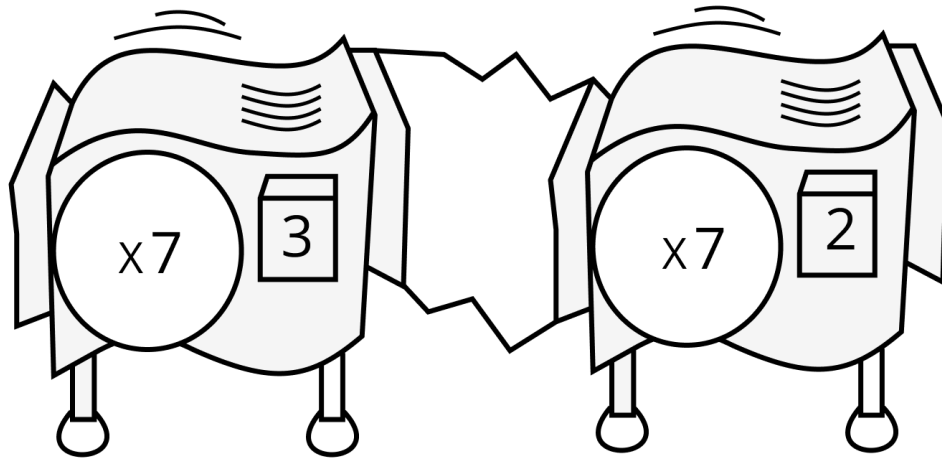
$$\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{3}\right)^2 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{2 \times 3 \times 2 \times 3} = \frac{1}{6^2}$$

Hence, $\left[\left(\times \frac{1}{6}\right)^2\right]$ single machine can perform the same amount of work.

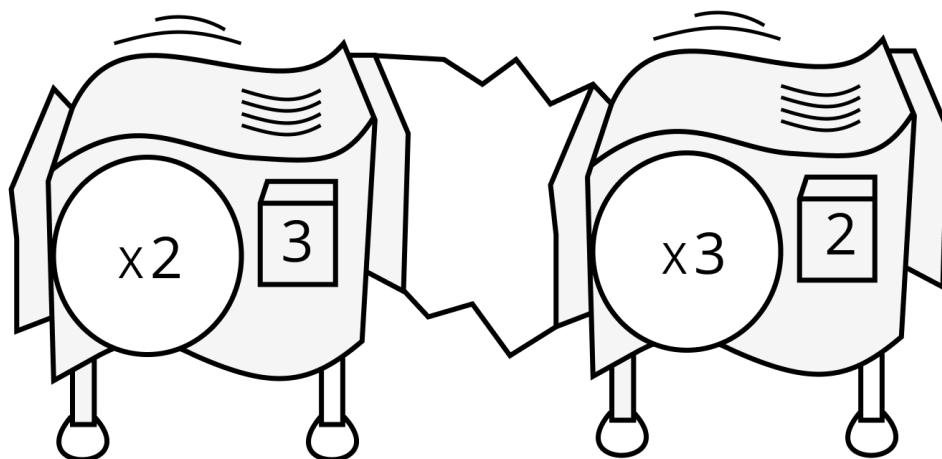
Question 160

For each hook-up, determine whether there is a single repeater machine that will do the same work. If so, describe or draw it.

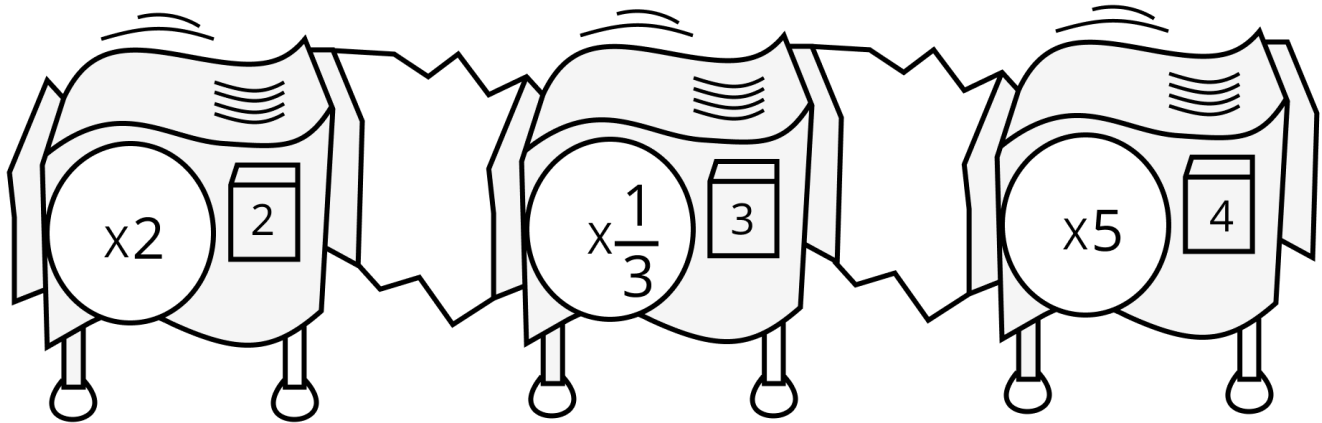
(a)



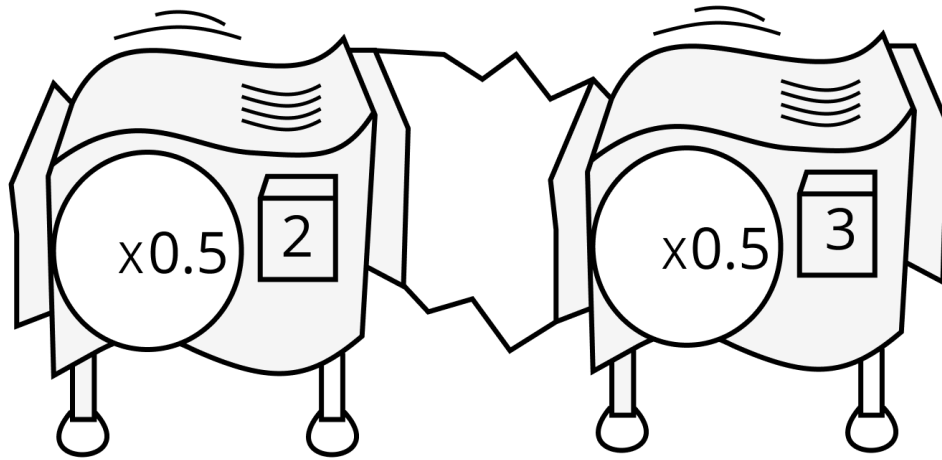
(b)



(c)



(d)

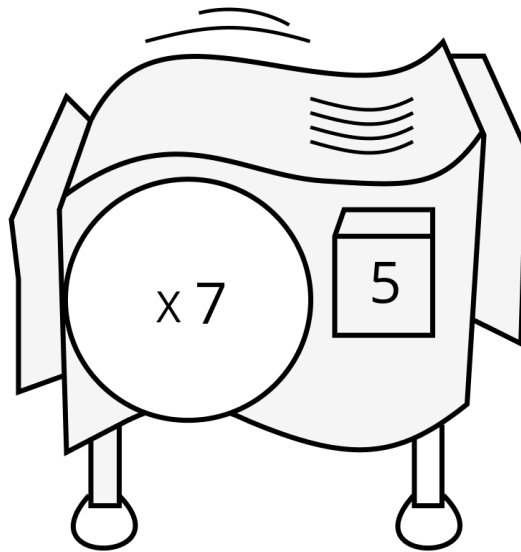


Solution

Ans: The work that can be performed by the hook-up machine = $73 \times 72 = 75$

So, (x 75) single machine can perform the same amount of work.

The diagram of a (x 75) single machine can be shown as below:



Ans: The work that can be performed by the hook-up machine = $23 \times 33 = 8 \times 9 = 72$

Hence, it is not possible that a single machine to perform the same amount of work as $(x 82) = 64$ and $(x 92) = 81$.

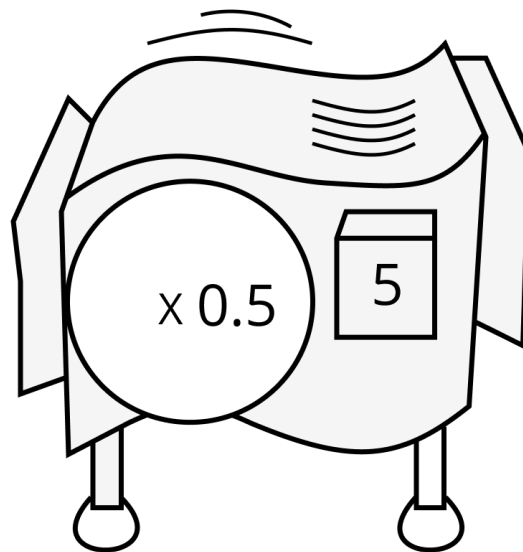
$$\begin{aligned} \text{Ans: The work that can be performed by the hook-up machine} &= 2^2 \times \left(\frac{1}{3}\right)^3 \times 5^4 = 4 \times \frac{1}{27} \times 625 \\ &= \frac{2500}{27} = 92.59 \end{aligned}$$

Hence, it is not possible that a single machine to perform the same amount of work.

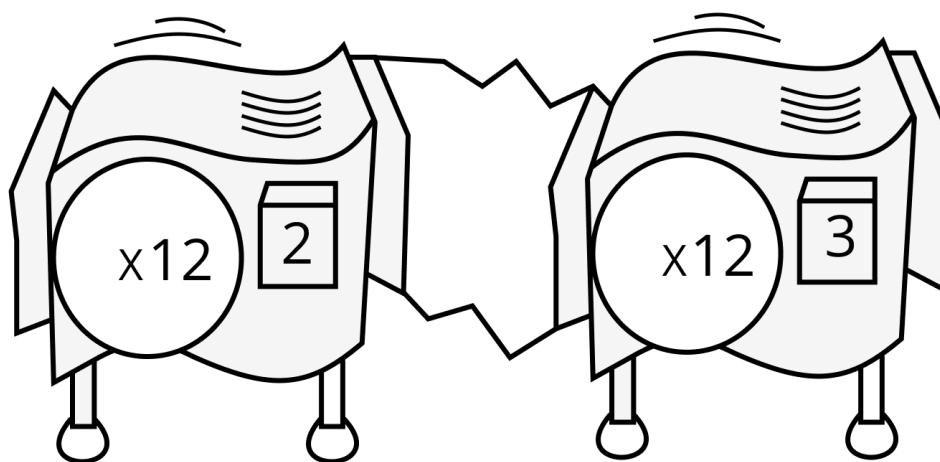
Ans: The work that can be performed by the hook-up machine = $(0.5)^2 \times (0.5)^3 = (0.5)^5$

So, $(x (0.5)^5)$ a single machine can perform the same amount of work.

The diagram of a $(x 0.5)^5$ single machine can be shown as below:



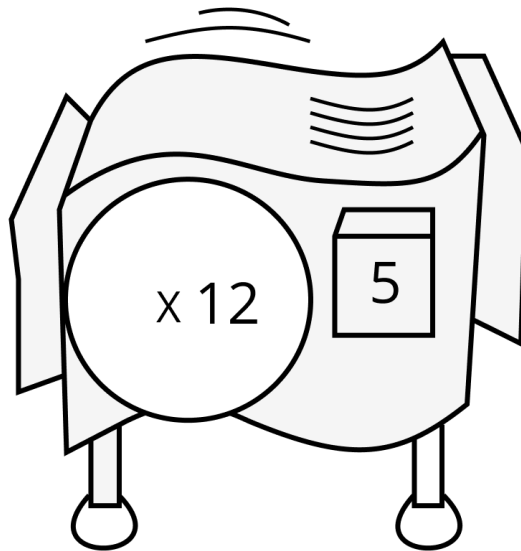
(e)



Ans: The work that can be performed by the hook-up machine = $122 \times 123 = 125$

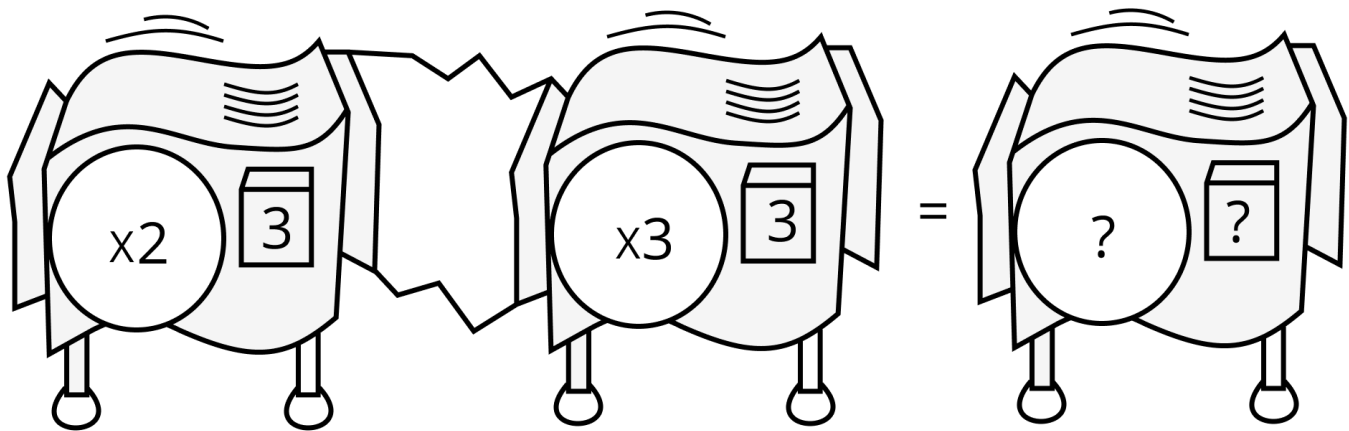
So, $(\times 125)$ single machine can perform the same amount of work.

The diagram of a $(\times 125)$ single machine can be shown as below:



Question 161

Shikha has an order from a golf course designer to put palm trees through a $(\times 23)$ machine and then through a $(\times 33)$ machine. She thinks she can do the job with a single repeater machine. What single repeater machine should she use?

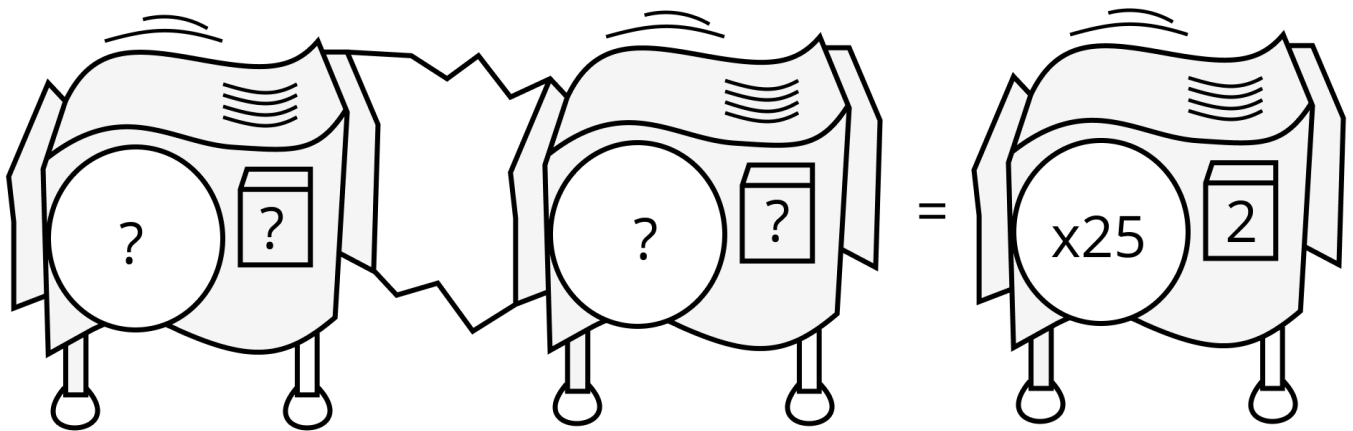


Solution

Ans: The work done by hook-up machine is equal to $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216 = 63$ So, she should use (x 63) single machine for the purpose.

Question 162

Neha needs to stretch some sticks to 252 times their original lengths, but her (x25) machine is broken. Find a hook-up of two repeater machines that will do the same work as a (x252) machine. To get started, think about the hookup you could use to replace the (x25) machine.



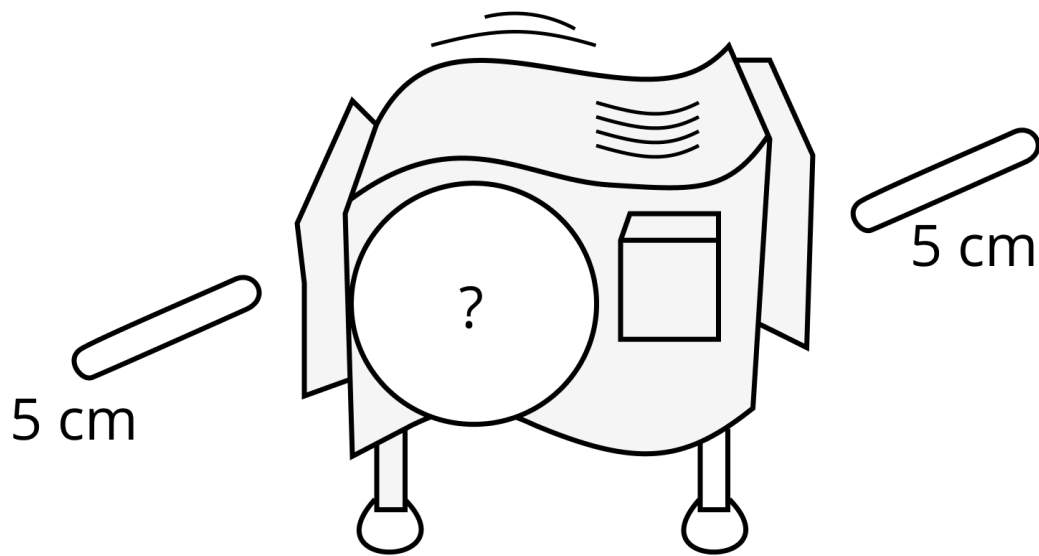
Solution

Ans: Work done by single machine $(x \ 252) = 25 \times 25 = 625$ or $5 \times 5 \times 5 \times 5$ or 52×52 Hence, $(x \ 52)$ and $(x \ 52)$ hook-up machine can replace the $(x \ 25)$ machine.

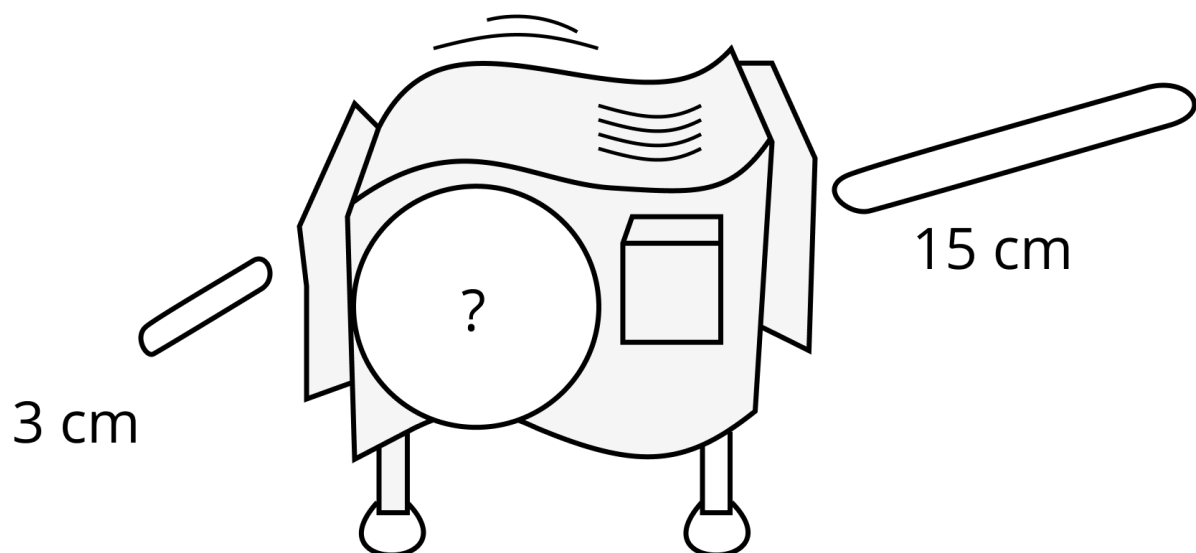
Question 163

Supply the missing information for each diagram.

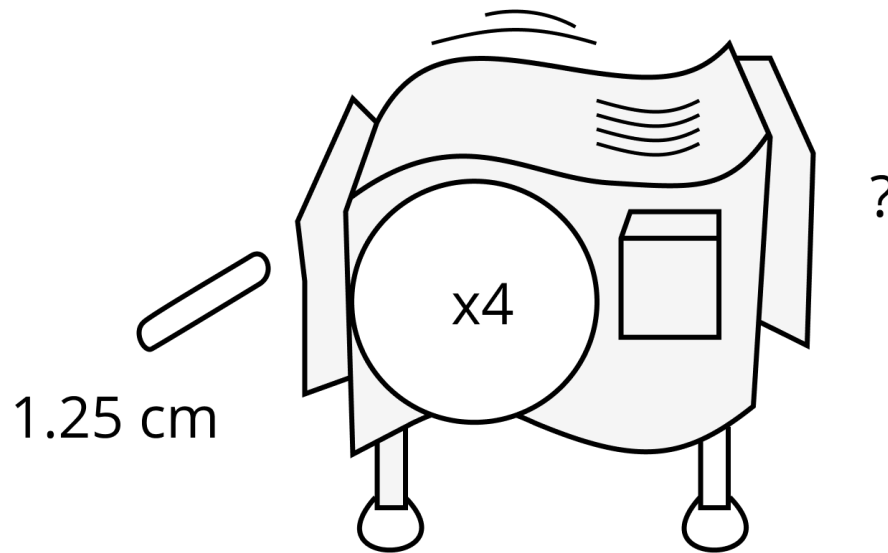
(a)



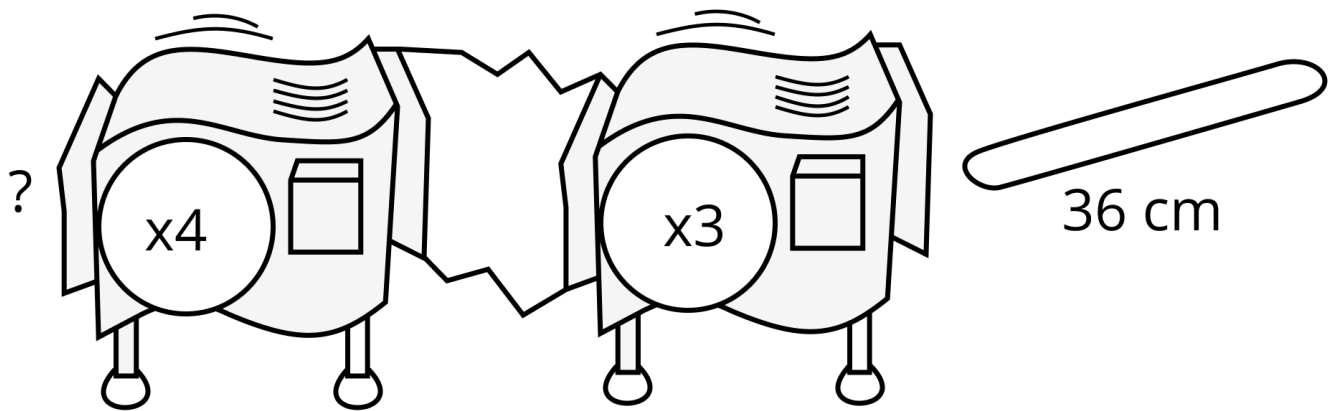
(b)



(c)



(d)



Solution

Ans: When a piece of length 5 cm is inserted in a single machine, it produces a piece having the same length of 5 cm.

Hence, it is a (x 1) repeated machine.

Therefore, the value of ? = 1.

Ans: When a piece of length 3 cm is inserted in a single machine, it produces a piece having the length 15 cm.

Hence, it is a (x 5) repeated machine.

Therefore, the value of ? = 5.

Ans: When a piece of length 1.25 cm is inserted in a (x 4) repeated machine, it will produce a piece that is $1.25 \times 4 = 10$ cm in length.

Therefore, the value of ? = 10 cm

Ans: When a piece of length x cm is inserted in $(\times 4)$ and $(\times 3)$

hooked machine, it will produce a piece that is 36 cm in

length.

Hence, $x \times 4 \times 3 = 36$

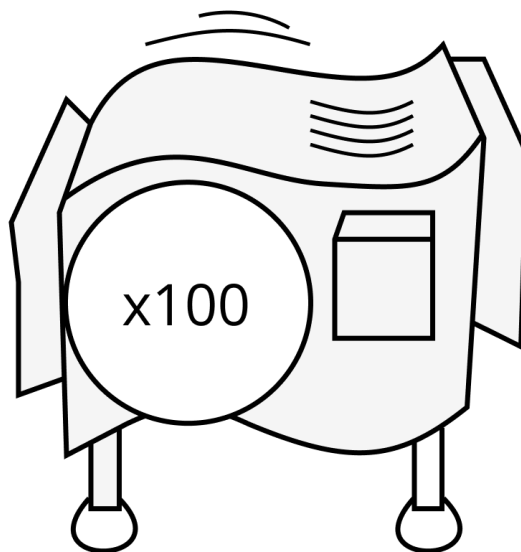
$$\frac{\text{Output}}{\text{Input}} = 12$$

$$\frac{36}{12} = 3 \text{ cm}$$

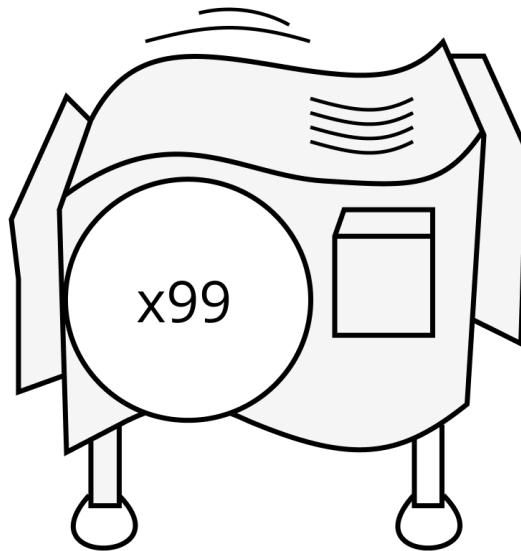
Question 164

If possible, find a hook-up of a prime base number machine that will do the same work as the given stretching machine. Do not use $(\times 1)$ machines.

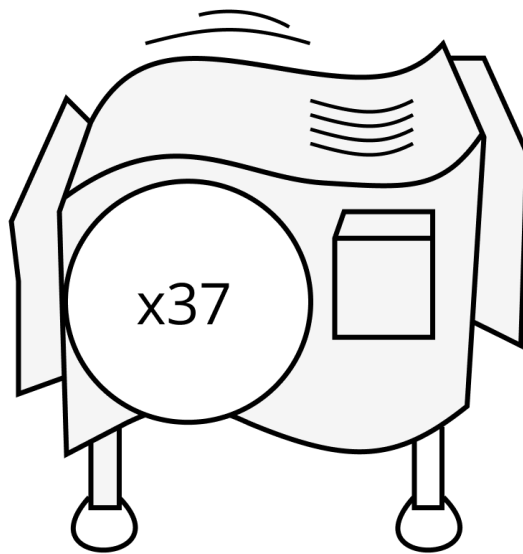
(a)



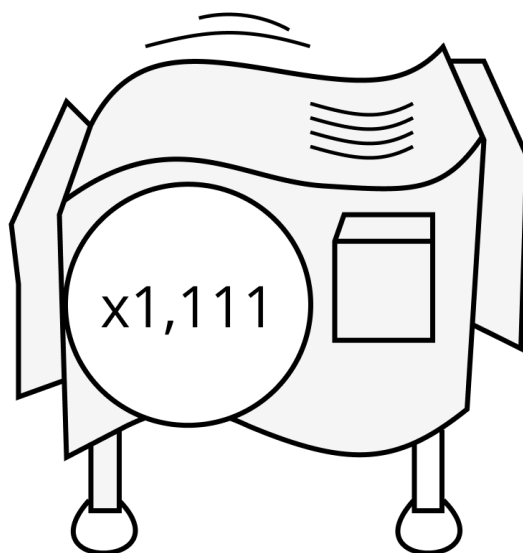
(b)



(c)



(d)



Solution

Ans: Single machine work = 100

Hook-up machine of prime base number that do the same work down by x 100

$$= 22 \times 52$$

$$= 4 \times 25$$

$$= 100$$

Ans: x 99 = 32 x 111 hook-up machine.

Ans: x 37 machine cannot do the same work.

Ans: x 1111 = 101 x 11 hook-up machine.

Question 165

Find two repeater machines that will do the same work as a (x 81) machine.

Solution

Ans: Two repeater machines that do the same work as (x 81) are (x 34) and (x 92).

Since, factor of 81 are 3 and 9.

Question 166

Find a repeater machine that will do the same work as a $\left(\times \frac{1}{8}\right)$ machine.

Solution

Ans: The repeater machine that performs the same

amount of work: $\frac{1}{8}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

Hence, $\frac{1}{8} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3$

Thus, $\left(\times \frac{1}{2^3}\right)$ repeater machine can perform the same work as a $\left(\times \frac{1}{8}\right)$ machine.

Question 167

Find three machines that can be replaced with hook-ups of ($\times 5$) machines.

Solution

Ans: Since, $5^2 = 25$, $5^3 = 125$, $5^4 = 625$

Hence, ($\times 5^2$), ($\times 5^3$) and ($\times 5^4$) machines can replace ($\times 5$) hook-up machines.

Question 168

The left column of the chart lists the lengths of input pieces of ribbon. Stretching machines are listed across the top. The other entries are the outputs for sending the input ribbon from that row through the machine from that column. Copy and complete the chart.

Input Length	Machine			
	$\times 2$			
	1	5		
3				15
	14		7	

Solution

Ans: In the given table, the left column of the chart list is the length of the input piece of ribbon. Thus, the outputs for sending the input ribbon are given in the following table.

Input Length	Machine			
	$\times 2$	$\times 10$	$\times 1$	$\times 5$
5	1	5	0.5	2.5
3	6	30	3	15
7	14	70	7	35

Question 169

The left column of the chart lists the lengths of input chains of gold. Repeater machines are listed across the top. The other entries are the outputs you get when you send the input chain from that row through the repeater machine from that column. Copy and complete the chart.

Input Length	Repeater Machine		
	$\times 23$		
	40		125
2		162	
		81	

Solution

Ans: In the given table, the left column of the chart lists the length of input chains of gold. Thus, the output we get when we send the input chain from the row through the repeater machine are detailed in the following table.

Input Length	Repeater Machine
--------------	------------------

	$\times 23$	$\times 34$	$\times 52$
13.3	40	405	125
2	16	162	50
1	8	81	25

Question 170

Long back in ancient times, a farmer saved the life of a king's daughter. The king decided to reward the farmer with whatever he wished. The farmer, who was a chess champion, made an unusual request:

"I would like you to place 1 rupee on the first square of my chessboard, 2 rupees on the second square, 4 on the third square, 8 on the fourth square, and so on, until you have covered all 64 squares.

Each square should have twice as many rupees as the previous square." The king thought this to be too less and asked the farmer to think of some better reward, but the farmer didn't agree. How much money has the farmer earned?

Hint: The following table may help you. What is the first square on which the king will place at least Rs 10 lakh?

Position of Square on chess board	Amount (in Rs)
1st square	1
2nd square	2
3rd square	4

Solution

Ans: A grid of dimensions 8×8 is provided

We find the sum of each row-

e.g. The sum of the 1st row = $20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 =$

255

The sum of the 2nd row= $28 + 29 + 210 + 211 + 212 + 213 + 214 +$

215

$$= 28 (20 + 21 + 22 + 23 + 24 + 25 + 26 + 27)$$

$$= 28 \times 255 = 256 \times 255 = 65280$$

The sum of the 3rd row= $216 \times 255 = 16711680$

Question 171

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Compare their diameters by division.

Solution

Ans: Given,

Sun's diameter = 1.4×10^9 m

Earth's diameter = 1.2756×10^7 m

For comparing, we need to change the diameters of both sun

and earth in the same powers of 10, i.e. $1.2756 \times 10^7 = 0.012756 \times 10^9$

$$\text{Now, if the diameter of sun is divided by the diameter of the earth} = \frac{1.4 \times 10^9}{0.012756 \times 10^9} = 110$$

Therefore, the sun's diameter is 110 times the earth's diameter.

Question 172

Mass of Mars is 6.42×10^{29} kg and the mass of the Sun is 1.99×10^{30} kg. What is the total mass?

Solution

Ans: Mass of Mars = 6.42×10^{29} kg

Mass of the Sun = 1.99×10^{30} kg

Total mass of Mars and Sun together = $6.42 \times 10^{29} + 1.99 \times 10^{30}$

$$= 6.42 \times 10^{29} + 19.9 \times 10^{29} = 26.32 \times 10^{29} \text{ kg}$$

Question 173

The distance between the Sun and the Earth is 1.496×10^8 km and the distance between the Earth and the Moon is 3.84×10^8 m. During a solar eclipse the Moon comes in between the Earth and the Sun. What is the distance between the Moon and the Sun at that particular time?

Solution

Ans: The distance between the Sun and the Earth is 1.496×10^8 km

$$= 1.496 \times 10^8 \times 10^3 \text{ m} = 1496 \times 10^8 \text{ m}$$

The distance between the Earth and the Moon is 3.84×10^8 m.

The distance between the Moon and the Sun at particular time (solar eclipse) = $(1496 \times 10^8 - 3.84 \times 10^8) \text{ m} = 1492.16 \times 10^8 \text{ m}$

Question 174

A particular star is at a distance of about 8.1×10^{13} km from the Earth. Assuming that light travels at 3×10^8 m per second, find how long does light take from that star to reach the Earth.

Solution

Ans: Total distance between the earth and the star = 8.1×10^{13} km = $8.1 \times 10^{13} \times 10^3$ m

Speed of light = 3×10^8 m per second.

$$\text{Hence, time that light would take to reach the earth} = \frac{8.1 \times 10^{13} \times 10^3}{3 \times 10^8}$$

$$= \frac{8.1 \times 10^{16}}{3 \times 10^8}$$

$$= \frac{8.1}{3} \times 10^8$$
$$= 2.7 \times 10^8 s$$

Question 175

By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Solution

Ans: Let the number be x .

Hence,

$$(-15)^{-1} \div x = (-5)^{-1}$$

$$\Rightarrow \frac{1}{-15} \times \frac{1}{x} = \frac{1}{-5}$$

$$\Rightarrow x = \frac{-15}{-5} = 3$$

Therefore, the required number is 3.

Question 176

By what number should $(-8)^{-3}$ be multiplied so that the product may be equal to $(-6)^{-3}$?

Solution

Ans: Let the number be x .

Hence,

$$x \times (-8)^{-3} = (-6)^{-3}$$

$$\Rightarrow x = \frac{(-6)^{-3}}{(-8)^{-3}} = \frac{(-8)^3}{(-6)^3} = \frac{-8^3}{-6^3}$$

$$= \frac{512}{216} = \frac{64}{27}$$

Therefore, the required number is $\frac{64}{27}$.

Question 177

Find x

$$(1) \left(-\frac{1}{7}\right)^{-5} \div \left(-\frac{1}{7}\right)^{-7} = (-7)^x$$

Solution

Ans: Use law of exponents: $a^m \div a^n = a^{m-n}$,

$$\left(-\frac{1}{7}\right)^{-5} \div \left(-\frac{1}{7}\right)^{-7} = (-7)^x$$

$$\Rightarrow \left(-\frac{1}{7}\right)^{-5-(-7)} = (-7)^x$$

$$\Rightarrow \left(-\frac{1}{7}\right)^{-5+7} = (-7)^x$$

$$\Rightarrow \left(-\frac{1}{7}\right)^2 = (-7)^x$$

$$\Rightarrow (-7)^{-2} = (-7)^x$$

Compare both sides,

$$x = -2$$

$$(2) \left(\frac{2}{5}\right)^{2x+6} \times \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^{x+2}$$

Ans: Use law of exponents: $a^m \times a^n = a^{m+n}$,

$$\left(\frac{2}{5}\right)^{2x+6} \times \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^{x+2}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2x+6+3} = \left(\frac{2}{5}\right)^{x+2}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2x+9} = \left(\frac{2}{5}\right)^{x+2}$$

Compare both sides,

$$2x + 9 = x + 2$$

$$x = 2 - 9 = -7$$

$$(3) 2^x + 2^x + 2^x = 192$$

$$\text{Ans: } 2^x (1 + 1 + 1) = 192$$

$$\Rightarrow 2^x \times 3 = 192$$

$$\Rightarrow 2^x = \frac{192}{3} \Rightarrow 2^x = 64$$

$$\Rightarrow 2^x = 2^6$$

Compare both sides,

$$x = 6$$

$$(4) \left(\frac{-6}{7}\right)^{x-7} = 1$$

Ans: Using law of exponents $x^0 = 1$

$$\text{So, } \left(\frac{-6}{7}\right)^{x-7} = 1 \text{ is possible if } x - 7 = 0$$

$$\text{Thus, } x = 7$$

$$(5) 2^{3x} = 8^{2x+1}$$

$$\text{Ans: } 2^{3x} = 8^{2x+1}$$

$$\Rightarrow 2^{3x} = (2^3)^{2x+1}$$

$$\Rightarrow 2^{3x} = 2^{3(2x+1)}$$

$$\Rightarrow 2^{3x} = 2^{6x+3}$$

Compare both sides,

$$\Rightarrow x = -1$$

$$(6) 5^x + 5^{x-1} = 750$$

$$\text{Ans: } 5^x + 5^{x-1} = 750$$

$$\Rightarrow 5^x + 5^x \cdot 5^{-1} = 750$$

$$\Rightarrow 5^x (1 + 5^{-1}) = 750$$

$$\Rightarrow 5^x \left(1 + \frac{1}{5}\right) = 750$$

$$\Rightarrow 5^x \left(\frac{6}{5}\right) = 750$$

$$\Rightarrow 5^x = \frac{750 \times 5}{6} \Rightarrow 5^x = 125 \times 5$$

$$\Rightarrow 5^x = 5^3 \times 5$$

$$\Rightarrow 5^x = 5^4 \Rightarrow x = 4$$

Question 178

If $a = -1$, $b = 2$, then find the value of the following:

(1) $ab + ba$

Solution

$$\text{Ans: } a^b + b^a = (-1)^2 + (2)^{-1}$$

$$= 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$$

(2) $ab - ba$

$$\begin{aligned}\text{Ans: } a^b - b^a &= (-1)^2 - (2)^{-1} \\ &= 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}\end{aligned}$$

(3) $ab \times b^2$

$$\begin{aligned}\text{Ans: } a^b \times b^a &= (-1)^2 \times (2)^{-1} \\ &= 1 \times \frac{1}{2} = \frac{1}{2}\end{aligned}$$

(4) $ab \div ba$

$$\begin{aligned}\text{Ans: } a^b \div b^a &= (-1)^2 \div (2)^{-1} \\ &= 1 \div \frac{1}{2} = 1 \times 2 = 2\end{aligned}$$

Question 179

Express each of the following in exponential form:

(1) $\frac{-1296}{14641}$

Solution

$$\text{Ans: } \frac{-1296}{14641} = -\frac{(6)^4}{(11)^4} = -\left(\frac{6}{11}\right)^4$$

(2) $\frac{-125}{343}$

$$\text{Ans: } \frac{-125}{343} = -\frac{(5)^3}{(7)^3} = -\left(\frac{5}{7}\right)^3$$

$$(3) \frac{400}{3969}$$

$$\text{Ans: } \frac{400}{3969} = \frac{(20)^2}{(63)^2} = \left(\frac{20}{63}\right)^2$$

$$(4) \frac{-625}{10000}$$

$$\text{Ans: } \frac{-625}{10000} = \frac{-(5)^4}{(10)^4} = -\left(\frac{1}{2}\right)^4$$

Question 180

Simplify:

$$(1) \left(\left(\frac{1}{2} \right)^2 - \left(\frac{1}{4} \right)^3 \right)^{-1} \times 2^{-3}$$

Solution

Ans: This equation can be simplified as:

$$\begin{aligned} \left(\left(\frac{1}{2} \right)^2 - \left(\frac{1}{4} \right)^3 \right)^{-1} \times 2^{-3} &= \left(\frac{1}{4} - \frac{1}{64} \right)^{-1} \times \left(\frac{1}{2} \right)^3 \\ &= \left(\frac{15}{64} \right)^{-1} \times \left(\frac{1}{8} \right) = \frac{64}{15} \times \frac{1}{8} = \frac{8}{15} \end{aligned}$$

$$(2) \left(\left(\frac{4}{3} \right)^{-2} - \left(\frac{3}{4} \right)^2 \right)^{-2}$$

Ans: This equation can be simplified as:

$$\left(\left(\frac{4}{3} \right)^{-2} - \left(\frac{3}{4} \right)^2 \right)^{-2} = \left(\left(\frac{3}{4} \right)^2 - \left(\frac{3}{4} \right)^2 \right)^{-2} = (0)^{-2} = 0$$

$$(3) \left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3$$

Ans: This equation can be simplified as:

$$\begin{aligned} \left(\frac{4}{13}\right)^4 \times \left(\frac{13}{7}\right)^2 \times \left(\frac{7}{4}\right)^3 &= 4^4 \times 13^{-4} \times 13^2 \times 7^{-2} \times 7^3 \times 4^{-3} \\ &= 4^{4-3} \times 13^{2-4} \times 7^{3-2} \\ &= 4^1 \times 13^{-2} \times 7^1 \\ &= \frac{4 \times 7}{13^2} = \frac{28}{169} \end{aligned}$$

$$(4) \left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{-28} \times \left(\frac{1}{5}\right)^{-43}$$

Ans: This equation can be simplified as:

$$\begin{aligned} \left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{28} \times \left(\frac{1}{5}\right)^{-43} \\ &= 5^{-45} \times 5^{60} - 5^{-28} \times 5^{43} \\ &= 5^{-45+60} - 5^{-28+43} = 5^{15} - 5^{15} = 0 \end{aligned}$$

$$(5) \frac{(9)^3 \times 27 \times t^4}{(3)^{-2} \times (3)^4 \times t^2}$$

Ans: This equation can be simplified as:

$$\begin{aligned} \frac{(3^2)^3 \times (3)^3 \times t^4}{(3)^{-2} \times (3)^4 \times t^2} &= 3^6 \times 3^3 \times 3^2 \times 3^{-4} \times t^4 \times t^{-2} \\ &= (3)^{11-4} \times t^{4-2} = 3^7 \times t^2 \end{aligned}$$

$$(6) \frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3}$$

Ans: This equation can be simplified as:

$$\begin{aligned}\frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3} &= \frac{3^{-4} \times 5^{-6} \times t^{-6}}{3^{-10} \times 5^{-6} \times t^{-12}} \\&= 3^{-4+10} \times t^{-6+12} \\&= 3^6 \times t^6\end{aligned}$$

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