

Chapter 9: Differential Equations

NCERT Exemplar · Class 12 · Maths

Questions

Question 1

Find the solution of

Solution

Given differential equation,

$$\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x}$$

On separating the variables, we have

$$\frac{dy}{2^y} = \frac{dx}{2^x} \Rightarrow 2^{-y} dy = 2^{-x} dx$$

Now, integrating both the sides, we get

$$\int 2^{-y} dy = \int 2^{-x} dx$$

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + c \Rightarrow -2^{-y} = -2^{-x} + c \log 2$$

$$-2^{-y} + 2^{-x} = c \log 2$$

$$2^{-x} - 2^{-y} = k \quad [\text{where } c \log 2 = k]$$

Thus, the solution of the differential equation is $2^{-x} - 2^{-y} = k$

Question 2

Find the differential equation of all non-vertical lines in a plane.

Solution

We know that, the equation of all non-vertical lines are $y = mx + c$

On differentiating w.r.t. x , we get

$$dy/dx = m$$

Again, on differentiating w.r.t. x , we have

$$d^2y/dx^2 = 0$$

Thus, the required equation is $d^2y/dx^2 = 0$.

$$\frac{dy}{dx} = e^{-2y}$$

Question 3

Given that $y = 0$ when $x = 5$.

Find the value of x when $y = 3$.

Solution

Given equation,

$$\frac{dy}{dx} = e^{-2y}$$

$$\frac{dy}{e^{-2y}} = dx \Rightarrow e^{2y} \cdot dy = dx$$

Integrating both sides, we get

$$\int e^{2y} dy = \int dx \Rightarrow \frac{1}{2} e^{2y} = x + c$$

Put $y = 0$ and $x = 5$

$$\Rightarrow \frac{1}{2} e^0 = 5 + c \Rightarrow c = \frac{1}{2} - 5 = -\frac{9}{2}$$

So, The equation becomes $\frac{1}{2} e^{2y} = x - \frac{9}{2}$

Now putting $y = 3$, we get

$$\frac{1}{2} e^6 = x - \frac{9}{2} \Rightarrow x = \frac{1}{2} e^6 + \frac{9}{2}$$

Thus, the required value of $x = \frac{1}{2} (e^6 + 9)$.

Given equation,

$$\frac{dy}{dx} = e^{-2y}$$

$$\frac{dy}{e^{-2y}} = dx \Rightarrow e^{2y} \cdot dy = dx$$

Integrating both sides, we get

$$\int e^{2y} dy = \int dx \Rightarrow \frac{1}{2} e^{2y} = x + c$$

Put $y = 0$ and $x = 5$

$$\Rightarrow \frac{1}{2} e^0 = 5 + c \Rightarrow c = \frac{1}{2} - 5 = -\frac{9}{2}$$

So, The equation becomes $\frac{1}{2} e^{2y} = x - \frac{9}{2}$

Now putting $y = 3$, we get

$$\frac{1}{2} e^6 = x - \frac{9}{2} \Rightarrow x = \frac{1}{2} e^6 + \frac{9}{2}$$

Thus, the required value of $x = \frac{1}{2} (e^6 + 9)$.

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

Question 4

Solve the differential equation

Solution

Given different equation,

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

On dividing by $(x^2 - 1)$, we have

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{1}{(x^2 - 1)^2}$$

Clearly, it is a linear differential equation of first order and first degree.

$$\text{Now, } P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

$$\text{Integrating factor I.E} = e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = (x^2 - 1).$$

Hence, the solution of the equation is

$$y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x - 1}{x + 1} \right| + C.$$

$$\frac{dy}{dx} + 2xy = y$$

Question 5

Solve the differential equation

Solution

Given differential equation,

$$dy/dx + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy \Rightarrow \frac{dy}{dx} = y(1 - 2x) \Rightarrow \frac{dy}{y} = (1 - 2x) dx$$

Integrating both sides, we have

$$\int \frac{dy}{y} = \int (1 - 2x) dx \Rightarrow \log y = x - 2 \cdot \frac{x^2}{2} + \log c$$

$$\log y = x - x^2 + \log c \Rightarrow \log y - \log c = x - x^2$$

$$\log \frac{y}{c} = x - x^2 \Rightarrow \frac{y}{c} = e^{x-x^2}$$

$$\therefore y = y = c \cdot e^{x-x^2}$$

Thus, the required solution is $y = c \cdot e^{x-x^2}$.

$$\frac{dy}{dx} = y - 2xy \Rightarrow \frac{dy}{dx} = y(1 - 2x) \Rightarrow \frac{dy}{y} = (1 - 2x) dx$$

Integrating both sides, we have

$$\int \frac{dy}{y} = \int (1 - 2x) dx \Rightarrow \log y = x - 2 \cdot \frac{x^2}{2} + \log c$$

$$\log y = x - x^2 + \log c \Rightarrow \log y - \log c = x - x^2$$

$$\log \frac{y}{c} = x - x^2 \Rightarrow \frac{y}{c} = e^{x-x^2}$$

$$\therefore y = y = c \cdot e^{x-x^2}$$

Thus, the required solution is $y = c \cdot e^{x-x^2}$.

$$\frac{dy}{dx} + ay = e^{mx}$$

Question 6

Find the general solution of

Solution

Given equation, $dy/dx + ay = e^{mx}$

Solving for linear differential equation of first order, we have

$P = a$ and $Q = e^{mx}$

So, I.F = $e^{\int P dx} = e^{\int a \cdot dx} = e^{ax}$

Solution of equation is $y \times \text{I.F} = \int Q \text{I.F} dx + c$

$$y \cdot e^{ax} = \int e^{mx} \cdot e^{ax} dx + c \Rightarrow y \cdot e^{ax} = \int e^{(m+a)x} dx + c$$

$$y \cdot e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + c \Rightarrow y = \frac{e^{(m+a)x}}{(m+a)} \cdot e^{-ax} + c \cdot e^{-ax}$$

$$\therefore y = \frac{e^{mx}}{(m+a)} + c \cdot e^{-ax}$$

Thus, the required solution is $y = \frac{e^{mx}}{(m+a)} + c \cdot e^{-ax}$.

$$\frac{dy}{dx} + 1 = e^{x+y}$$

Question 7

Solve the differential equation

Solution

Given differential equation, $dy/dx + 1 = e^{x+y}$

Substituting $x + y = t$ and differentiating w.r.t. x , we have

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} = e^t \Rightarrow \frac{dt}{e^t} = dx \Rightarrow e^{-t} dt = dx$$

Integrating both sides, we have

$$\int e^{-t} dt = \int dx \Rightarrow -e^{-t} = x + c$$

$$-e^{-(x+y)} = x + c \Rightarrow \frac{-1}{e^{x+y}} = x + c \Rightarrow (x + c) e^{x+y} = -1$$

Thus, the required solution is $(x + c) \cdot e^{x+y} + 1 = 0$.

Question 8

Solve: $ydx - xdy = x^2ydx$.

Solution

Given equation, $ydx - xdy = x^2ydx$

$$y dx - x^2y dx = xdy$$

$$y (1 - x^2) dx = xdy$$

$$\left(\frac{1-x^2}{x}\right)dx = \frac{dy}{y} \Rightarrow \left(\frac{1}{x} - x\right)dx = \frac{dy}{y}$$

Integrating both sides we get

$$\int \left(\frac{1}{x} - x\right)dx = \int \frac{dy}{y}$$

$$\log x - \frac{x^2}{2} = \log y + \log c$$

$$\log x - \frac{x^2}{2} = \log yc \Rightarrow \log x - \log c = \frac{x^2}{2} \Rightarrow \log \frac{x}{yc} = \frac{x^2}{2}$$

$$\Rightarrow \frac{x}{yc} = e^{x^2/2} \Rightarrow \frac{yc}{x} = e^{-x^2/2} \Rightarrow yc = xe^{-x^2/2}$$

$$\therefore y = \frac{1}{c} \cdot xe^{-x^2/2} \Rightarrow y = kxe^{-x^2/2} \quad \left[\because k = \frac{1}{c} \right]$$

Thus, the required solution is $y = kxe^{-x^2/2}$

Question 9

Solve the differential equation $dy/dx = 1 + x + y^2 + xy^2$, when $y = 0$, $x = 0$.

Solution

Given equation, $dy/dx = 1 + x + y^2 + xy^2$

$$\frac{dy}{dx} = 1(1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2) \Rightarrow \frac{dy}{1+y^2} = (1+x) dx$$

On integrating both side, we get

$$\int \frac{dy}{1+y^2} = \int (1+x) dx \Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$$

Put $x=0$ and $y=0$, we get $\tan^{-1}(0) = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan \left(x + \frac{x^2}{2} \right)$$

Thus, the required solution is $y = \tan \left(x + \frac{x^2}{2} \right)$.

Question 10

Find the general solution of $(x + 2y^3) dy/dx = y$.

Solution

Given equation, $(x + 2y^3) \frac{dy}{dx} = y$

$$\frac{dy}{dx} = \frac{y}{x + 2y^3} \Rightarrow \frac{dx}{dy} = \frac{x + 2y^3}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + \frac{2y^3}{y} \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

Here $P = -\frac{1}{y}$ and $Q = 2y^2$.

So,

$$\text{Integrating factor I.F.} = e^{\int P dy} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = e^{\log \frac{1}{y}} = \frac{1}{y}$$

So the solution of the equation is

$$x \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dy + c$$

$$x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy + c$$

$$\frac{x}{y} = 2 \int y dy + c \Rightarrow \frac{x}{y} = 2 \cdot \frac{y^2}{2} + c \Rightarrow \frac{x}{y} = y^2 + c$$

$$x = y^3 + cy = y(y^2 + c)$$

Thus, the required solution is $x = y(y^2 + c)$

$$\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$$

Question 11

If $y(x)$ is a solution of and $y(0) = 1$, then find the value of $y(\pi/2)$.

Solution

Given equation,

$$\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$$

$$\left(\frac{2 + \sin x}{\cos x} \right) \frac{dy}{dx} = -(1 + y) \Rightarrow \frac{dy}{(1 + y)} = - \left(\frac{\cos x}{2 + \sin x} \right) dx$$

Integrating both sides, we get

$$\int \frac{dy}{1 + y} = - \int \frac{\cos x}{2 + \sin x} dx$$

$$\log |1 + y| = -\log |2 + \sin x| + \log c$$

$$\log |1 + y| + \log |2 + \sin x| = \log c$$

$$\log (1 + y) (2 + \sin x) = \log c \Rightarrow (1 + y) (2 + \sin x) = c$$

Put $x = 0$ and $y = 1$, we get

$$(1 + 1) (2 + \sin 0) = c \Rightarrow 4 = c$$

So, equation is $(1 + y) (2 + \sin x) = 4$

Now put $x = \frac{\pi}{2}$

$$(1 + y) \left(2 + \sin \frac{\pi}{2} \right) = 4$$

$$(1 + y) (2 + 1) = 4 \Rightarrow 1 + y = \frac{4}{3} \Rightarrow y = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\text{So, } y \left(\frac{\pi}{2} \right) = \frac{1}{3}$$

Thus, the required solution is $y \left(\frac{\pi}{2} \right) = \frac{1}{3}$

Given equation,

$$\left(\frac{2 + \sin x}{1 + y} \right) \frac{dy}{dx} = -\cos x$$

$$\left(\frac{2 + \sin x}{\cos x} \right) \frac{dy}{dx} = -(1 + y) \Rightarrow \frac{dy}{(1 + y)} = - \left(\frac{\cos x}{2 + \sin x} \right) dx$$

Integrating both sides, we get

$$\int \frac{dy}{1 + y} = - \int \frac{\cos x}{2 + \sin x} dx$$

$$\log |1 + y| = -\log |2 + \sin x| + \log c$$

$$\log |1 + y| + \log |2 + \sin x| = \log c$$

$$\log (1 + y) (2 + \sin x) = \log c \Rightarrow (1 + y) (2 + \sin x) = c$$

Put $x = 0$ and $y = 1$, we get

$$(1 + 1) (2 + \sin 0) = c \Rightarrow 4 = c$$

So, equation is $(1 + y) (2 + \sin x) = 4$

Now put $x = \frac{\pi}{2}$

$$(1 + y) \left(2 + \sin \frac{\pi}{2} \right) = 4$$

$$(1 + y) (2 + 1) = 4 \Rightarrow 1 + y = \frac{4}{3} \Rightarrow y = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\text{So, } y\left(\frac{\pi}{2}\right) = \frac{1}{3}$$

Thus, the required solution is $y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

$$(1 + t) \frac{dy}{dt} - ty = 1$$

Question 12

If $y(t)$ is a solution of and $y(0) = -1$, then show that $y(1) = -1/2$.

Solution

Given equation,

$$(1+t) \frac{dy}{dt} - ty = 1 \Rightarrow \frac{dy}{dt} - \left(\frac{t}{1+t} \right) y = \frac{1}{1+t}$$

Here, $P = \frac{-t}{1+t}$ and $Q = \frac{1}{1+t}$

Now,

$$\begin{aligned} \text{Integrating factor I.F.} &= e^{\int P dt} = e^{\int \frac{-t}{1+t} dt} = e^{-\int \frac{1+t-1}{1+t} dt} \\ &= e^{-\int \left(1 - \frac{1}{1+t} \right) dt} = e^{-[t - \log(1+t)]} \\ &= e^{-t + \log(1+t)} = e^{-t} \cdot e^{\log(1+t)} \end{aligned}$$

$$\therefore \text{I.F.} = e^{-t} \cdot (1+t)$$

The required solution of the given different equation is,

$$y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dt + c$$

$$y \cdot e^{-t} (1+t) = \int \frac{1}{(1+t)} \cdot e^{-t} \cdot (1+t) dt + c$$

$$y \cdot e^{-t} (1+t) = \int e^{-t} dt + c$$

$$\Rightarrow y \cdot e^{-t} (1+t) = -e^{-t} + c$$

Putting $t = 0$ and $y = -1$

$$[\because y(0) = -1]$$

$$-1 \cdot e^0 \cdot 1 = -e^0 + c$$

$$-1 = -1 + c \Rightarrow c = 0$$

So the equation becomes

$$ye^{-t} (1+t) = -e^{-t}$$

Now put $t = 1$

$$y \cdot e^{-1} (1+1) = -e^{-1}$$

$$2y = -1 \Rightarrow y = -\frac{1}{2}$$

Thus, $y(1) = -\frac{1}{2}$ is verified.

Question 13

Form the differential equation having $y = (\sin^{-1}x)^2 + A \cos^{-1}x + B$, where A and B are arbitrary constants, as its general solution.

Solution

Given equation is $y = (\sin^{-1}x)^2 + A \cos^{-1}x + B$

$$\frac{dy}{dx} = 2 \sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} + A \cdot \left(\frac{-1}{\sqrt{1-x^2}} \right)$$

On multiplying both sides by $\sqrt{1-x^2}$, we get

$$\sqrt{1-x^2} \frac{dy}{dx} = 2 \sin^{-1}x - A$$

Again differentiating w.r.t x, we get

$$\begin{aligned} \sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1 \times (-2x)}{2\sqrt{1-x^2}} &= \frac{2}{\sqrt{1-x^2}} \\ \Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \frac{dy}{dx} &= \frac{2}{\sqrt{1-x^2}} \end{aligned}$$

On multiplying both sides by $\sqrt{1-x^2}$, we get

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$$

Thus, the above is the required differential equation.

Question 14

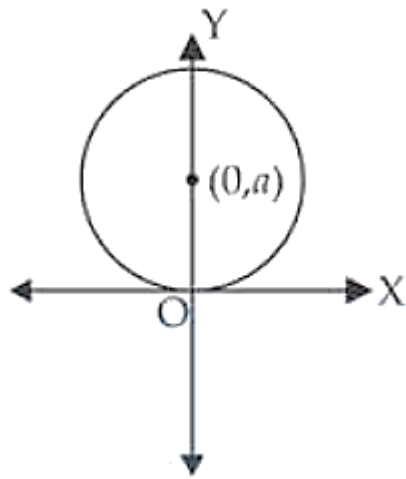
Form the differential equation of all circles which pass through origin and whose centres lie on y-axis.

Solution

The equation of circles which pass through the origin and whose centre lies on the y-axis is given by,

$$(x-0)^2 + (y-a)^2 = a^2, \text{ where } (0, a) \text{ is the centre}$$

$$x^2 + y^2 + a^2 - 2ay = a^2$$



$$x^2 + y^2 - 2ay = 0 \dots\dots\dots (i)$$

Differentiating both sides w.r.t. x, we get

$$2x + 2y \cdot \frac{dy}{dx} - 2a \cdot \frac{dy}{dx} = 0$$

$$x + y \frac{dy}{dx} - a \cdot \frac{dy}{dx} = 0 \Rightarrow x + (y - a) \cdot \frac{dy}{dx} = 0$$

$$y - a = \frac{x}{\frac{dy}{dx}}$$

$$\therefore a = y + \frac{-x}{\frac{dy}{dx}}$$

Putting the value of a in eq. (i), we get

$$x^2 + y^2 - 2 \left(y + \frac{x}{\frac{dy}{dx}} \right) y = 0$$

$$\Rightarrow x^2 + y^2 - 2y^2 - \frac{2xy}{\frac{dy}{dx}} = 0 \Rightarrow x^2 - y^2 = \frac{2xy}{\frac{dy}{dx}}$$

$$\therefore (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Thus, the required differential equation is

$$(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

$$2x + 2y \cdot \frac{dy}{dx} - 2a \cdot \frac{dy}{dx} = 0$$

$$x + y \frac{dy}{dx} - a \cdot \frac{dy}{dx} = 0 \Rightarrow x + (y - a) \cdot \frac{dy}{dx} = 0$$

$$y - a = \frac{x}{\frac{dy}{dx}}$$

$$\therefore a = y + \frac{-x}{\frac{dy}{dx}}$$

Putting the value of a in eq. (i), we get

$$x^2 + y^2 - 2 \left(y + \frac{x}{\frac{dy}{dx}} \right) y = 0$$

$$\Rightarrow x^2 + y^2 - 2y^2 - \frac{2xy}{\frac{dy}{dx}} = 0 \Rightarrow x^2 - y^2 = \frac{2xy}{\frac{dy}{dx}}$$

$$\therefore (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Thus, the required differential equation is

$$(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Question 15

Find the equation of a curve passing through origin and satisfying the differential equation

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$$

Solution

Given equation,

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

Here, $P = \frac{2x}{1+x^2}$ and $Q = \frac{4x^2}{1+x^2}$

Integrating factor I.F. = $e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$

So, the solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + c$$

$$y(1+x^2) = \int \frac{4x^2}{1+x^2} \times (1+x^2) dx + c$$

$$y(1+x^2) = \int 4x^2 dx + c$$

$$\Rightarrow y(1+x^2) = \frac{4}{3} x^3 + c \quad \dots(i)$$

Since the curve is passing through origin *i.e.*, (0, 0)

Putting $y = 0$ and $x = 0$ in eq. (i)

$$0(1+0) = \frac{4}{3} (0)^3 + c \Rightarrow c = 0$$

Now,

$$\text{Equation is } y(1+x^2) = \frac{4}{3} x^3 \Rightarrow y = \frac{4x^3}{3(1+x^2)}$$

Thus, the required solution is $y = \frac{4x^3}{3(1+x^2)}$

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2.$$

Question 16

Solve:

Solution

Given equation,

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Now, taking $y = vx$ [As it is a homogeneous differential equation]

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

So,

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + vx^2 + v^2 x^2}{x^2}$$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2(1 + v + v^2)}{x^2}$$

$$v + x \cdot \frac{dv}{dx} = 1 + v + v^2 \Rightarrow x \cdot \frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow x \cdot \frac{dv}{dx} = 1 + v^2 \Rightarrow \frac{dv}{1 + v^2} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{1 + v^2} = \int \frac{dx}{x}$$

$$\Rightarrow \tan^{-1}v = \log x + c \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + c$$

Thus, the required solution is $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$.

$$(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0.$$

Question 17

Find the general solution of the differential equation

Solution

Given differential equation,

$$(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$$

$$(x - e^{\tan^{-1} y}) \frac{dy}{dx} = -(1 + y^2) \Rightarrow \frac{dy}{dx} = \frac{-(1 + y^2)}{x - e^{\tan^{-1} y}}$$

$$\frac{dx}{dy} = \frac{x - e^{\tan^{-1} y}}{-(1 + y^2)} \Rightarrow \frac{dx}{dy} = -\frac{x}{(1 + y^2)} + \frac{e^{\tan^{-1} y}}{1 + y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1 + y^2)} = \frac{e^{\tan^{-1} y}}{1 + y^2} \quad \text{[First order linear differential equation in } y\text{]}$$

$$\text{Here, } P = \frac{1}{1 + y^2} \text{ and } Q = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

So,

$$\text{Integrating factor I.F.} = e^{\int P dy} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1} y}$$

And, the solution is

$$x \cdot \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dy + c$$

$$\Rightarrow x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2} \cdot e^{\tan^{-1} y} dy + c$$

Put $e^{\tan^{-1} y} = t$, we have

$$e^{\tan^{-1} y} \cdot \frac{1}{1 + y^2} dy = dt$$

$$x \cdot e^{\tan^{-1} y} = \int t \cdot dt + c$$

$$x \cdot e^{\tan^{-1} y} = \frac{1}{2} t^2 + c$$

$$x \cdot e^{\tan^{-1} y} = \frac{1}{2} (e^{\tan^{-1} y})^2 + c \Rightarrow x = \frac{1}{2} (e^{\tan^{-1} y}) + \frac{c}{e^{\tan^{-1} y}}$$

$$2x = e^{\tan^{-1} y} + \frac{2c}{e^{\tan^{-1} y}}$$

$$\Rightarrow 2x \cdot e^{\tan^{-1} y} = (e^{\tan^{-1} y})^2 + 2c$$

$$(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$$

$$(x - e^{\tan^{-1} y}) \frac{dy}{dx} = -(1 + y^2) \Rightarrow \frac{dy}{dx} = \frac{-(1 + y^2)}{x - e^{\tan^{-1} y}}$$

$$\frac{dx}{dy} = \frac{x - e^{\tan^{-1} y}}{-(1 + y^2)} \Rightarrow \frac{dx}{dy} = -\frac{x}{(1 + y^2)} + \frac{e^{\tan^{-1} y}}{1 + y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1 + y^2)} = \frac{e^{\tan^{-1} y}}{1 + y^2} \quad \text{[First order linear differential equation in } y\text{]}$$

$$\text{Here, } P = \frac{1}{1 + y^2} \text{ and } Q = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

So,

$$\text{Integrating factor I.F.} = e^{\int P dy} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1} y}$$

And, the solution is

$$x \cdot \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dy + c$$

$$\Rightarrow x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2} \cdot e^{\tan^{-1} y} dy + c$$

Put $e^{\tan^{-1} y} = t$, we have

$$e^{\tan^{-1} y} \cdot \frac{1}{1 + y^2} dy = dt$$

$$x \cdot e^{\tan^{-1} y} = \int t \cdot dt + c$$

$$x \cdot e^{\tan^{-1} y} = \frac{1}{2} t^2 + c$$

$$x \cdot e^{\tan^{-1} y} = \frac{1}{2} (e^{\tan^{-1} y})^2 + c \Rightarrow x = \frac{1}{2} (e^{\tan^{-1} y}) + \frac{c}{e^{\tan^{-1} y}}$$

$$2x = e^{\tan^{-1} y} + \frac{2c}{e^{\tan^{-1} y}}$$

$$\Rightarrow 2x \cdot e^{\tan^{-1} y} = (e^{\tan^{-1} y})^2 + 2c$$

Thus, the above is the required solution of the given differential equation.

Question 18

Find the general solution of $y^2 dx + (x^2 - xy + y^2) dy = 0$.

Solution

Given equation, $y^2 dx + (x^2 - xy + y^2) dy = 0$

$$y^2 dx = -(x^2 - xy + y^2) dy$$

$$\Rightarrow \frac{dx}{dy} = -\frac{x^2 - xy + y^2}{y^2}$$

Since it is a homogeneous differential equation

Putting, $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$

So, $v + y \cdot \frac{dv}{dy} = -\left(\frac{v^2 y^2 - vy^2 + y^2}{y^2}\right)$

$$v + y \cdot \frac{dv}{dy} = -\frac{y^2(v^2 - v + 1)}{y^2}$$

$$v + y \cdot \frac{dv}{dy} = (-v^2 + v - 1) \Rightarrow y \cdot \frac{dv}{dy} = -v^2 + v - 1 - v$$

$$y \cdot \frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{(v^2 + 1)} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\int \frac{dv}{(v^2 + 1)} = -\int \frac{dy}{y} \Rightarrow \tan^{-1} v = -\log y + c$$

$$\Rightarrow \tan^{-1} \left(\frac{x}{y} \right) + \log y = c$$

Thus, the required solution is $\tan^{-1} \left(\frac{x}{y} \right) + \log y = c$.

$$y^2 dx = -(x^2 - xy + y^2) dy$$

$$\Rightarrow \frac{dx}{dy} = -\frac{x^2 - xy + y^2}{y^2}$$

Since it is a homogeneous differential equation

Putting, $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$

So, $v + y \cdot \frac{dv}{dy} = -\left(\frac{v^2 y^2 - vy^2 + y^2}{y^2}\right)$

$$v + y \cdot \frac{dv}{dy} = -\frac{y^2(v^2 - v + 1)}{y^2}$$

$$v + y \cdot \frac{dv}{dy} = (-v^2 + v - 1) \Rightarrow y \cdot \frac{dv}{dy} = -v^2 + v - 1 - v$$

$$y \cdot \frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{(v^2 + 1)} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\int \frac{dv}{(v^2 + 1)} = -\int \frac{dy}{y} \Rightarrow \tan^{-1} v = -\log y + c$$

$$\Rightarrow \tan^{-1} \left(\frac{x}{y} \right) + \log y = c$$

Thus, the required solution is $\tan^{-1} \left(\frac{x}{y} \right) + \log y = c$.

Question 19

Solve : $(x + y)(dx - dy) = dx + dy$. [Hint: Substitute $x + y = z$ after separating dx and dy]

Solution

Given differential equation, $(x + y)(dx - dy) = dx + dy$

$$(x + y) dx - (x - y) dy = dx + dy$$

$$-(x+y) dy - dy = dx - (x+y) dx$$

$$-(x+y+1) dy = -(x+y-1) dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Putting $x+y=z$, we have

$$\text{So, } 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\text{Now, } \frac{dz}{dx} - 1 = \frac{z-1}{z+1}$$

$$\frac{dz}{dx} = \frac{z-1}{z+1} + 1 \Rightarrow \frac{dz}{dx} = \frac{z-1+z+1}{z+1}$$

$$\frac{dz}{dx} = \frac{2z}{z+1} \Rightarrow \frac{z+1}{z} dz = 2 \cdot dx$$

Integrating both sides, we get

$$\int \frac{z+1}{z} dz = 2 \int dx$$

$$\int \left(1 + \frac{1}{z}\right) dz = 2 \int dx$$

$$z + \log |z| = 2x + \log |c|$$

$$x+y + \log |x+y| = 2x + \log |c|$$

$$y + \log |x+y| = x + \log |c|$$

$$\log |x+y| = x - y + \log |c|$$

$$\log |x+y| - \log |c| = (x-y)$$

$$\Rightarrow \log \left| \frac{x+y}{c} \right| = (x-y) \Rightarrow \frac{x+y}{c} = e^{x-y}$$

$$\therefore x+y = c \cdot e^{x-y}$$

Thus, the required solution is $x+y = c \cdot e^{x-y}$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Putting $x+y=z$, we have

$$\text{So, } 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\text{Now, } \frac{dz}{dx} - 1 = \frac{z-1}{z+1}$$

$$\frac{dz}{dx} = \frac{z-1}{z+1} + 1 \Rightarrow \frac{dz}{dx} = \frac{z-1+z+1}{z+1}$$

$$\frac{dz}{dx} = \frac{2z}{z+1} \Rightarrow \frac{z+1}{z} dz = 2 \cdot dx$$

Integrating both sides, we get

$$\int \frac{z+1}{z} dz = 2 \int dx$$

$$\int \left(1 + \frac{1}{z}\right) dz = 2 \int dx$$

$$z + \log |z| = 2x + \log |c|$$

$$x+y + \log |x+y| = 2x + \log |c|$$

$$y + \log |x+y| = x + \log |c|$$

$$\log |x+y| = x - y + \log |c|$$

$$\log |x+y| - \log |c| = (x-y)$$

$$\Rightarrow \log \left| \frac{x+y}{c} \right| = (x-y) \Rightarrow \frac{x+y}{c} = e^{x-y}$$

$$\therefore x+y = c \cdot e^{x-y}$$

Thus, the required solution is $x+y = c \cdot e^{x-y}$

Question 20

Solve : $2(y+3) - xy \, dy/dx = 0$, given that $y(1) = -2$.

Solution

Given differential equation, $2(y + 3) - xy \, dy/dx = 0$

$$2(y+3) - xy \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{dy}{dx} = 2y + 6$$

$$\left(\frac{y}{2y+6}\right) dy = \frac{dx}{x} \Rightarrow \frac{1}{2} \left(\frac{y}{y+3}\right) dy = \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{1}{2} \int \frac{y}{y+3} \cdot dy = \int \frac{dx}{x} \Rightarrow \frac{1}{2} \int \frac{y+3-3}{y+3} dy = \int \frac{dx}{x}$$

$$\frac{1}{2} \int \left(1 - \frac{3}{y+3}\right) dy = \int \frac{dx}{x}$$

$$\frac{1}{2} \int 1 \cdot dy - \frac{3}{2} \int \frac{1}{y+3} dy = \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} y - \frac{3}{2} \log |y+3| = \log x + c$$

Putting $x = 1$ and $y = -2$, we have

$$\Rightarrow \frac{1}{2}(-2) - \frac{3}{2} \log |-2+3| = \log(1) + c$$

$$-1 - \frac{3}{2} \log(1) = \log(1) + c$$

$$-1 - 0 = 0 + c$$

$$[\because \log(1) = 0]$$

$$\therefore c = -1$$

Now, the equation is

$$\frac{1}{2} y - \frac{3}{2} \log |y+3| = \log x - 1$$

$$y - 3 \log |y+3| = 2 \log x - 2$$

$$y - \log |(y+3)^3| = \log x^2 - 2$$

$$\log |(y+3)^3| + \log x^2 = y + 2$$

$$\log |x^2 (y+3)^3| = y + 2 \Rightarrow x^2 (y+3)^3 = e^{y+2}$$

Thus, the required solution is $x^2 (y+3)^3 = e^{y+2}$.

Question 21

Solve the differential equation $dy = \cos x (2 - y \operatorname{cosec} x) dx$ given that $y = 2$ when $x = \pi/2$.

Solution

Given differential equation, $dy = \cos x (2 - y \operatorname{cosec} x) dx$

$$\frac{dy}{dx} = \cos x (2 - y \operatorname{cosec} x) \Rightarrow \frac{dy}{dx} = 2 \cos x - y \cos x \cdot \operatorname{cosec} x$$

$$\frac{dy}{dx} = 2 \cos x - y \cot x \Rightarrow \frac{dy}{dx} + y \cot x = 2 \cos x$$

Here, $P = \cot x$ and $Q = 2 \cos x$.

So,

Integrating factor I.F. = $e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

And,

Required solution is $y \times \text{I.F.} = \int Q \times \text{I.F.} dx + c$

$$y \cdot \sin x = \int 2 \cos x \cdot \sin x dx + c$$

$$y \cdot \sin x = \int \sin 2x dx + c \Rightarrow y \cdot \sin x = -\frac{1}{2} \cos 2x + c$$

Put $x = \frac{\pi}{2}$ and $y = 2$, we get

$$2 \sin \frac{\pi}{2} = -\frac{1}{2} \cos \pi + c$$

$$2(1) = -\frac{1}{2}(-1) + c \Rightarrow 2 = \frac{1}{2} + c \Rightarrow c = 2 - \frac{1}{2} = \frac{3}{2}$$

Thus, the equation is $y \sin x = -\frac{1}{2} \cos 2x + \frac{3}{2}$.

Question 22

Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

Solution

Given, $Ax^2 + By^2 = 1$

Differentiating w.r.t. x , we get

$$2A \cdot x + 2By \frac{dy}{dx} = 0$$

$$Ax + By \cdot \frac{dy}{dx} = 0 \Rightarrow By \cdot \frac{dy}{dx} = -Ax$$

$$\Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

Differentiating both sides again w.r.t. x , we have

$$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \left(\frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2} \right) = 0$$

$$\frac{yx^2}{x} \cdot \frac{d^2y}{dx^2} + x \cdot \left(\frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{d^2y}{dx^2} + x \cdot \left(\frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0 \Rightarrow xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$

Thus, the required equation is

$$xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$

$$2A \cdot x + 2By \frac{dy}{dx} = 0$$

$$Ax + By \cdot \frac{dy}{dx} = 0 \Rightarrow By \cdot \frac{dy}{dx} = -Ax$$

$$\Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

Differentiating both sides again w.r.t. x , we have

$$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \left(\frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2} \right) = 0$$

$$\frac{yx^2}{x} \cdot \frac{d^2y}{dx^2} + x \cdot \left(\frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{d^2y}{dx^2} + x \cdot \left(\frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0 \Rightarrow xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$

Thus, the required equation is

$$xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$

Question 23

Solve the differential equation $(1 + y^2) \tan^{-1}x \, dx + 2y(1 + x^2) \, dy = 0$.

Solution

Given differential equation, $(1 + y^2) \tan^{-1}x \, dx + 2y(1 + x^2) \, dy = 0$

$$2y(1 + x^2) \, dy = -(1 + y^2) \tan^{-1}x \, dx$$

$$2y(1+x^2)dy = -(1+y^2) \cdot \tan^{-1}x \cdot dx$$

$$\frac{2y}{1+y^2}dy = -\frac{\tan^{-1}x}{1+x^2} \cdot dx$$

Integrating both sides, we get

$$\int \frac{2y}{1+y^2} dy = -\int \frac{\tan^{-1}x}{1+x^2} \cdot dx$$

$$\log|1+y^2| = -\frac{1}{2}(\tan^{-1}x)^2 + c$$

$$\Rightarrow \frac{1}{2}(\tan^{-1}x)^2 + \log|1+y^2| = c$$

Thus, the above equation is the required solution.

Question 24

Find the differential equation of system of concentric circles with centre (1, 2).

Solution

The family of concentric circles with centre (1, 2) and radius 'r' is given by

$$(x-1)^2 + (y-2)^2 = r^2$$

Differentiating both sides w.r.t, x we get

$$2(x-1) + 2(y-2)\frac{dy}{dx} = 0 \Rightarrow (x-1) + (y-2)\frac{dy}{dx} = 0$$

Thus, the above is the required equation.

Long Answer (L.A.)

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

Question 25**Solve :****Solution**

Given differential equation,

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$y + x \cdot \frac{dy}{dx} + y = x(\sin x + \log x)$$

$$x \frac{dy}{dx} = x(\sin x + \log x) - 2y$$

$$\Rightarrow \frac{dy}{dx} = (\sin x + \log x) - \frac{2y}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x}y = (\sin x + \log x)$$

$$\text{Here, } P = \frac{2}{x} \text{ and } Q = (\sin x + \log x)$$

$$\text{Integrating factor I.F.} = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$$

Now, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \, dx + c$$

$$\Rightarrow y \cdot x^2 = \int (\sin x + \log x) x^2 dx + c \quad \dots(1)$$

$$\begin{aligned} \text{Let } I &= \int (\sin x + \log x) x^2 dx \\ &= \int \underset{\text{II}}{x^2} \underset{\text{I}}{\sin x} dx + \int \underset{\text{II}}{x^2} \underset{\text{I}}{\log x} dx \\ &= \left[x^2 \cdot \int \sin x dx - \int \left(D(x^2) \cdot \int \sin x dx \right) dx \right] + \\ &\quad \left[\log x \cdot \int x^2 dx - \int \left(D(\log x) \cdot \int x^2 dx \right) dx \right] \\ &= \left[x^2(-\cos x) - 2 \int -x \cos x dx \right] + \left[\log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \right] \\ &= \left[-x^2 \cos x + 2 \left(x \sin x - \int 1 \cdot \sin x dx \right) \right] + \left[\frac{x^3}{3} \log x - \frac{1}{3} \int x^2 dx \right] \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 \end{aligned}$$

Now from eq (1) we get,

$$y \cdot x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 + c$$

$$\therefore y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x} + \frac{x \log x}{3} - \frac{1}{9} x + c \cdot x^{-2}$$

Thus, the required solution is

$$y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9}x + c \cdot x^{-2}$$

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$y + x \cdot \frac{dy}{dx} + y = x(\sin x + \log x)$$

$$x \frac{dy}{dx} = x(\sin x + \log x) - 2y$$

$$\Rightarrow \frac{dy}{dx} = (\sin x + \log x) - \frac{2y}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x}y = (\sin x + \log x)$$

$$\text{Here, } P = \frac{2}{x} \text{ and } Q = (\sin x + \log x)$$

$$\text{Integrating factor I.F.} = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$$

Now, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \, dx + c$$

$$\Rightarrow y \cdot x^2 = \int (\sin x + \log x) x^2 dx + c \quad \dots(1)$$

$$\begin{aligned} \text{Let } I &= \int (\sin x + \log x) x^2 dx \\ &= \int \underset{\text{II}}{x^2} \underset{\text{I}}{\sin x} dx + \int \underset{\text{II}}{x^2} \underset{\text{I}}{\log x} dx \\ &= \left[x^2 \cdot \int \sin x dx - \int \left(D(x^2) \cdot \int \sin x dx \right) dx \right] + \\ &\quad \left[\log x \cdot \int x^2 dx - \int \left(D(\log x) \cdot \int x^2 dx \right) dx \right] \\ &= \left[x^2(-\cos x) - 2 \int -x \cos x dx \right] + \left[\log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \right] \\ &= \left[-x^2 \cos x + 2 \left(x \sin x - \int 1 \cdot \sin x dx \right) \right] + \left[\frac{x^3}{3} \log x - \frac{1}{3} \int x^2 dx \right] \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 \end{aligned}$$

Now from eq (1) we get,

$$y \cdot x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 + c$$

$$\therefore y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x} + \frac{x \log x}{3} - \frac{1}{9} x + c \cdot x^{-2}$$

Thus, the required solution is

$$y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9}x + c \cdot x^{-2}$$

Question 26

Find the general solution of $(1 + \tan y)(dx - dy) + 2xdy = 0$.

Solution

Given, $(1 + \tan y)(dx - dy) + 2xdy = 0$

$$(1 + \tan y) dx - (1 + \tan y) dy + 2x dy = 0$$

$$(1 + \tan y) dx - (1 + \tan y - 2x) dy = 0$$

$$(1 + \tan y) \frac{dx}{dy} = (1 + \tan y - 2x) \Rightarrow \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$$

$$\frac{dx}{dy} = 1 - \frac{2x}{1 + \tan y} \Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Here, $P = \frac{2}{1 + \tan y}$ and $Q = 1$

Integrating factor I.F.

$$= e^{\int \frac{2}{1 + \tan y} dy} = e^{\int \frac{2 \cos y}{\sin y + \cos y} dy}$$

$$= e^{\int \frac{\sin y + \cos y - \sin y + \cos y}{(\sin y + \cos y)} dy} = e^{\int \left(1 + \frac{\cos y - \sin y}{\sin y + \cos y}\right) dy}$$

$$= e^{\int 1 dy} \cdot e^{\int \frac{\cos y - \sin y}{\sin y + \cos y} dy}$$

$$= e^y \cdot e^{\log(\sin y + \cos y)} = e^y \cdot (\sin y + \cos y)$$

Now, the solution is $x \times \text{I.F.} = \int Q \times \text{I.F.} dy + c$

$$\Rightarrow x \cdot e^y (\sin y + \cos y) = \int 1 \cdot e^y (\sin y + \cos y) dy + c$$

$$x \cdot e^y (\sin y + \cos y) = e^y \cdot \sin y + c$$

$$\left[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c \right]$$

$$x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$$

Thus, the required solution is $x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$.

$$(1 + \tan y) dx - (1 + \tan y) dy + 2x dy = 0$$

$$(1 + \tan y) dx - (1 + \tan y - 2x) dy = 0$$

$$(1 + \tan y) \frac{dx}{dy} = (1 + \tan y - 2x) \Rightarrow \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$$

$$\frac{dx}{dy} = 1 - \frac{2x}{1 + \tan y} \Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Here, $P = \frac{2}{1 + \tan y}$ and $Q = 1$

Integrating factor I.F.

$$= e^{\int \frac{2}{1 + \tan y} dy} = e^{\int \frac{2 \cos y}{\sin y + \cos y} dy}$$

$$= e^{\int \frac{\sin y + \cos y - \sin y + \cos y}{(\sin y + \cos y)} dy} = e^{\int \left(1 + \frac{\cos y - \sin y}{\sin y + \cos y}\right) dy}$$

$$= e^{\int 1 dy} \cdot e^{\int \frac{\cos y - \sin y}{\sin y + \cos y} dy}$$

$$= e^y \cdot e^{\log(\sin y + \cos y)} = e^y \cdot (\sin y + \cos y)$$

Now, the solution is $x \times \text{I.F.} = \int Q \times \text{I.F.} dy + c$

$$\Rightarrow x \cdot e^y (\sin y + \cos y) = \int 1 \cdot e^y (\sin y + \cos y) dy + c$$

$$x \cdot e^y (\sin y + \cos y) = e^y \cdot \sin y + c$$

$$\left[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c \right]$$

$$x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$$

Thus, the required solution is $x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$.

Question 27

Solve: $dy/dx = \cos(x + y) + \sin(x + y)$. [Hint: Substitute $x + y = z$]

Solution

Putting $x + y = v$ and on differentiating w.r.t. x , we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\frac{dv}{dx} = \cos v + \sin v + 1$$

So,

$$\frac{dv}{\cos v + \sin v + 1} = dx$$

Integrating both sides, we have

$$\int \frac{dv}{\cos v + \sin v + 1} = \int 1 \cdot dx$$

$$\int \frac{dv}{\left(\frac{1 - \tan^2 \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + \frac{2 \tan \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + 1 \right)} = \int 1 \cdot dx$$

$$\int \frac{\left(1 + \tan^2 \frac{v}{2} \right)}{1 - \tan^2 \frac{v}{2} + 2 \tan \frac{v}{2} + 1 + \tan^2 \frac{v}{2}} dv = \int 1 \cdot dx$$

$$\int \frac{\sec^2 \frac{v}{2}}{2 + 2 \tan \frac{v}{2}} dv = \int 1 \cdot dx$$

Now, putting $2 + 2 \tan \frac{v}{2} = t$

Differentiating, $2 \cdot \frac{1}{2} \sec^2 \frac{v}{2} dv = dt \Rightarrow \sec^2 \frac{v}{2} dv = dt$

$$\int \frac{dt}{t} = \int 1 \cdot dx$$

$$\log |t| = x + c$$

$$\log \left| 2 + 2 \tan \frac{v}{2} \right| = x + c$$

$$\log \left| 2 + 2 \tan \left(\frac{x+y}{2} \right) \right| = x + c \Rightarrow \log 2 \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\log 2 + \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\Rightarrow \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c - \log 2$$

Thus, the required solution is

$$\log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + K \quad [c - \log 2 = K]$$

Putting $x + y = v$ and on differentiating w.r.t. x , we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\frac{dv}{dx} = \cos v + \sin v + 1$$

So,

$$\frac{dv}{\cos v + \sin v + 1} = dx$$

Integrating both sides, we have

$$\int \frac{dv}{\cos v + \sin v + 1} = \int 1 \cdot dx$$

$$\int \frac{dv}{\left(\frac{1 - \tan^2 \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + \frac{2 \tan \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + 1 \right)} = \int 1 \cdot dx$$

$$\int \frac{\left(1 + \tan^2 \frac{v}{2} \right)}{1 - \tan^2 \frac{v}{2} + 2 \tan \frac{v}{2} + 1 + \tan^2 \frac{v}{2}} dv = \int 1 \cdot dx$$

$$\int \frac{\sec^2 \frac{v}{2}}{2 + 2 \tan \frac{v}{2}} dv = \int 1 \cdot dx$$

Now, putting $2 + 2 \tan \frac{v}{2} = t$

Differentiating, $2 \cdot \frac{1}{2} \sec^2 \frac{v}{2} dv = dt \Rightarrow \sec^2 \frac{v}{2} dv = dt$

$$\int \frac{dt}{t} = \int 1 \cdot dx$$

$$\log |t| = x + c$$

$$\log \left| 2 + 2 \tan \frac{v}{2} \right| = x + c$$

$$\log \left| 2 + 2 \tan \left(\frac{x+y}{2} \right) \right| = x + c \Rightarrow \log 2 \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\log 2 + \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c$$

$$\Rightarrow \log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + c - \log 2$$

Thus, the required solution is

$$\log \left[1 + \tan \left(\frac{x+y}{2} \right) \right] = x + K \quad [c - \log 2 = K]$$

Question 28

Find the general solution of $dy/dx - 3y = \sin 2x$.

Solution

Given equation, $dy/dx - 3y = \sin 2x$

It's a first order linear differential equation

Here $P = -3$ and $Q = \sin 2x$

So,

$$\text{Integrating factor I.F.} = e^{\int P dx} = e^{\int -3 dx} = e^{-3x}$$

And, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \, dx + c$$

$$\Rightarrow y \cdot e^{-3x} = \int \sin 2x \cdot e^{-3x} \, dx + c$$

Now let, $I = \int \sin 2x \cdot e^{-3x} \, dx$

$$I = \sin 2x \cdot \int e^{-3x} \, dx - \int (D(\sin 2x) \cdot \int e^{-3x} \, dx) \, dx$$

$$I = \sin 2x \cdot \frac{e^{-3x}}{-3} - \int 2 \cos 2x \cdot \frac{e^{-3x}}{-3} \, dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \int \cos 2x \cdot e^{-3x} \, dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[\cos 2x \cdot \int e^{-3x} \, dx - \int [D \cos 2x \cdot \int e^{-3x} \, dx] \, dx \right]$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[\cos 2x \cdot \frac{e^{-3x}}{-3} - 2 \sin 2x \cdot \frac{e^{-3x}}{-3} \right] dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} \cos 2x \cdot e^{-3x} - \frac{4}{9} \int \sin 2x \cdot e^{-3x} \, dx$$

$$= \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x - \frac{4}{9} I$$

$$I + \frac{4}{9} I = \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x$$

$$\frac{13 I}{9} = -\frac{1}{9} [3 e^{-3x} \sin 2x + 2 e^{-3x} \cos 2x]$$

$$1 = -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x]$$

Hence, the equation becomes

$$y \cdot e^{-3x} = -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x] + c$$

$$\therefore y = -\frac{1}{13} [3 \sin 2x + 2 \cos 2x] + c \cdot e^{3x}$$

Thus, the required solution is

$$y = -\left[\frac{3 \sin 2x + 2 \cos 2x}{13}\right] + c \cdot e^{3x}$$

So,

$$\text{Integrating factor I.F.} = e^{\int P dx} = e^{\int -3 dx} = e^{-3x}$$

And, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \, dx + c$$

$$\Rightarrow y \cdot e^{-3x} = \int \sin 2x \cdot e^{-3x} \, dx + c$$

Now let, $I = \int \sin 2x \cdot e^{-3x} \, dx$

$$I = \sin 2x \cdot \int e^{-3x} \, dx - \int (D(\sin 2x) \cdot \int e^{-3x} \, dx) \, dx$$

$$I = \sin 2x \cdot \frac{e^{-3x}}{-3} - \int 2 \cos 2x \cdot \frac{e^{-3x}}{-3} \, dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \int \cos 2x \cdot e^{-3x} \, dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[\cos 2x \cdot \int e^{-3x} \, dx - \int [D \cos 2x \cdot \int e^{-3x} \, dx] \, dx \right]$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[\cos 2x \cdot \frac{e^{-3x}}{-3} - 2 \sin 2x \cdot \frac{e^{-3x}}{-3} \right] dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} \cos 2x \cdot e^{-3x} - \frac{4}{9} \int \sin 2x \cdot e^{-3x} \, dx$$

$$= \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x - \frac{4}{9} I$$

$$I + \frac{4}{9} I = \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x$$

$$\frac{13 I}{9} = -\frac{1}{9} [3 e^{-3x} \sin 2x + 2 e^{-3x} \cos 2x]$$

$$1 = -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x]$$

Hence, the equation becomes

$$y \cdot e^{-3x} = -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x] + c$$

$$\therefore y = -\frac{1}{13} [3 \sin 2x + 2 \cos 2x] + c \cdot e^{3x}$$

Thus, the required solution is

$$y = -\left[\frac{3 \sin 2x + 2 \cos 2x}{13}\right] + c \cdot e^{3x}$$

Question 29

Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $(x^2 + y^2)/2xy$.

Solution

Given that the slope of tangent to a curve at (x, y) is $(x^2 + y^2)/2xy$.

It's a homogeneous differential function

So, putting $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2x \cdot vx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1 + v^2}{2v} - v \Rightarrow x \cdot \frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1 - v^2}{2v} \Rightarrow \frac{2v}{1 - v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v}{1 - v^2} dv = \int \frac{dx}{x} \Rightarrow -\log |1 - v^2| = \log x + \log c$$

$$-\log \left| 1 - \frac{y^2}{x^2} \right| = \log x + \log c \Rightarrow -\log \left| \frac{x^2 - y^2}{x^2} \right| = \log x + \log c$$

$$\log \left| \frac{x^2}{x^2 - y^2} \right| = \log |xc| \Rightarrow \frac{x^2}{x^2 - y^2} = xc$$

As, the curve is passing through the point (2, 1)

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = 2c \Rightarrow \frac{4}{3} = 2c \Rightarrow c = \frac{2}{3}$$

Thus, the required equation is

$$\frac{x^2}{x^2 - y^2} = \frac{2}{3}x \Rightarrow 2(x^2 - y^2) = 3x$$

So, putting $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2x \cdot vx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1 + v^2}{2v} - v \Rightarrow x \cdot \frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1 - v^2}{2v} \Rightarrow \frac{2v}{1 - v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v}{1 - v^2} dv = \int \frac{dx}{x} \Rightarrow -\log |1 - v^2| = \log x + \log c$$

$$-\log \left| 1 - \frac{y^2}{x^2} \right| = \log x + \log c \Rightarrow -\log \left| \frac{x^2 - y^2}{x^2} \right| = \log x + \log c$$

$$\log \left| \frac{x^2}{x^2 - y^2} \right| = \log |xc| \Rightarrow \frac{x^2}{x^2 - y^2} = xc$$

As, the curve is passing through the point (2, 1)

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = 2c \Rightarrow \frac{4}{3} = 2c \Rightarrow c = \frac{2}{3}$$

Thus, the required equation is

$$\frac{x^2}{x^2 - y^2} = \frac{2}{3}x \Rightarrow 2(x^2 - y^2) = 3x$$

Question 30

Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (x, y) is $(y - 1)/(x^2 + x)$

Solution

Given that the slope of the tangent to the curve at (x, y) is

$$\frac{dy}{dx} = \frac{y-1}{x^2+x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

Integrating both sides, we have

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}} \quad [\text{making perfect square}]$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\log |y-1| = \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}} \right| + \log c$$

$$\log |y-1| = \log \left| \frac{x}{x+1} \right| + \log c$$

$$\log |y-1| = \log \left| c \left(\frac{x}{x+1} \right) \right|$$

$$y-1 = \frac{cx}{x+1} \Rightarrow (y-1)(x+1) = cx$$

$$\frac{dy}{dx} = \frac{y-1}{x^2+x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

Integrating both sides, we have

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}} \text{ [making perfect square]}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\log |y-1| = \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}} \right| + \log c$$

$$\log |y-1| = \log \left| \frac{x}{x+1} \right| + \log c$$

$$\log |y-1| = \log \left| c \left(\frac{x}{x+1} \right) \right|$$

$$y-1 = \frac{cx}{x+1} \Rightarrow (y-1)(x+1) = cx$$

As, the line is passing through the point (1, 0), then $(0-1)(1+1) = c(1) \Rightarrow c = 2$

Thus, the required solution is $(y-1)(x+1) = 2x$.

Question 31

Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.

Solution

We know that,

The slope of the tangent of the curve = dy/dx

And the difference between the abscissa and ordinate = $x - y$

So, as given in the question we have

$$dy/dx = (x - y)^2$$

Taking, $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So, the equation becomes

$$1 - \frac{dv}{dx} = v^2 \Rightarrow \frac{dv}{dx} = 1 - v^2 \Rightarrow \frac{dv}{1 - v^2} = dx$$

Integrating both sides, we get

$$\int \frac{dv}{1 - v^2} = \int dx$$

$$\frac{1}{2} \log \left| \frac{1+v}{1-v} \right| = x + c \Rightarrow \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| = x + c \quad \dots(1)$$

As, the curve is passing through (0, 0)

$$\text{then } \frac{1}{2} \log \left| \frac{1+0-0}{1-0+0} \right| = 0 + c \Rightarrow c = 0$$

$\therefore$ On putting $c = 0$ in eq. (1) we get

$$\frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| = x \Rightarrow \log \left| \frac{1+x-y}{1-x+y} \right| = 2x$$

$$\frac{1+x-y}{1-x+y} = e^{2x}$$

$$\Rightarrow (1+x-y) = e^{2x} (1-x+y)$$

Thus, the required equation is $(1+x-y) = e^{2x} (1-x+y)$

Question 32

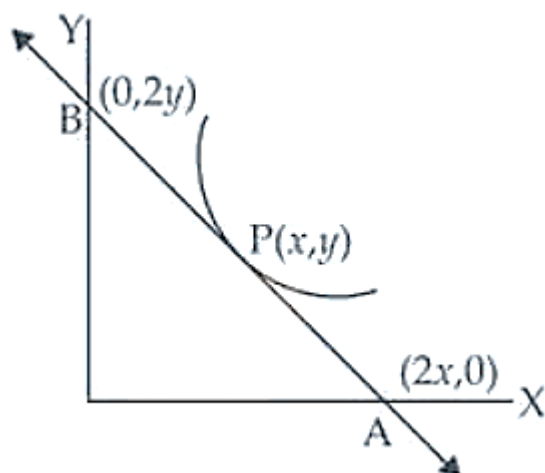
Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P (x, y) on the curve meets the co-ordinate axes at A and B such that P is the mid-point of AB.

Solution

Let's take P(x, y) be any point on the curve and AB be the tangent to the given curve at P.

Also given, P is the mid-point of AB

So, the coordinates of A and B are $(2x, 0)$ and $(0, 2y)$ respectively.



The slope of the tangent AB = $(2y - 0) / (0 - 2x) = -y/x$

$$\text{So, } \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\log y + \log x = \log c \Rightarrow \log yx = \log c$$

$$\therefore yx = c$$

As, the curve passes through $(1, 1)$

$$1 \times 1 = c \quad \therefore c = 1$$

$$\Rightarrow yx = 1$$

Thus, the required equation is $xy = 1$

$$x \frac{dy}{dx} = y (\log y - \log x + 1)$$

Question 33

Solve :

Solution

Given that: $x \frac{dy}{dx} = y (\log y - \log x + 1)$

$$\Rightarrow x \frac{dy}{dx} = y \left[\log \left(\frac{y}{x} \right) + 1 \right] \Rightarrow \frac{dy}{dx} = \frac{y}{x} \left[\log \left(\frac{y}{x} \right) + 1 \right]$$

As its a homogeneous different equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x \cdot \frac{dv}{dx} = \frac{vx}{x} \left[\log \left(\frac{vx}{x} \right) + 1 \right]$$

$$v + x \cdot \frac{dv}{dx} = v [\log v + 1]$$

$$x \cdot \frac{dv}{dx} = v [\log v + 1] - v \Rightarrow x \cdot \frac{dv}{dx} = v [\log v + 1 - 1]$$

$$\Rightarrow x \cdot \frac{dv}{dx} = v \cdot \log v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

Now, put $\log v = t$ on L.H.S.

$$\frac{1}{v} dv = dt$$

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\log |t| = \log |x| + \log c$$

$$\log |\log v| = \log xc \Rightarrow \log v = xc$$

$$\Rightarrow \log \left(\frac{y}{x} \right) = xc$$

Thus, the required solution is $\log \left(\frac{y}{x} \right) = xc$.

Given that: $x \frac{dy}{dx} = y (\log y - \log x + 1)$

$$\Rightarrow x \frac{dy}{dx} = y \left[\log \left(\frac{y}{x} \right) + 1 \right] \Rightarrow \frac{dy}{dx} = \frac{y}{x} \left[\log \left(\frac{y}{x} \right) + 1 \right]$$

As its a homogeneous different equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x \cdot \frac{dv}{dx} = \frac{vx}{x} \left[\log \left(\frac{vx}{x} \right) + 1 \right]$$

$$v + x \cdot \frac{dv}{dx} = v [\log v + 1]$$

$$x \cdot \frac{dv}{dx} = v [\log v + 1] - v \Rightarrow x \cdot \frac{dv}{dx} = v [\log v + 1 - 1]$$

$$\Rightarrow x \cdot \frac{dv}{dx} = v \cdot \log v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

Now, put $\log v = t$ on L.H.S.

$$\frac{1}{v} dv = dt$$

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\log |t| = \log |x| + \log c$$

$$\log |\log v| = \log xc \Rightarrow \log v = xc$$

$$\Rightarrow \log \left(\frac{y}{x} \right) = xc$$

Thus, the required solution is $\log \left(\frac{y}{x} \right) = xc$.

Objective Type

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q)

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right) \text{ is:}$$

Question 34

The degree of the differential equation

(A) 1 (B) 2 (C) 3 (D) not defined

Solution

Correct option is (D) not defined.

Since the value of $\sin(dy/dx)$ on expansion will be in increasing power of dy/dx , the degree of the given differential equation is not defined.

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2} \text{ is}$$

Question 35

The degree of the differential equation

(A) 4 (B) 3/2 (C) not defined (D) 2

Solution

Correct option is (D) 2.

Given differential equation is

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left(\frac{d^2y}{dx^2}\right)$$

Squaring both sides, we have

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Thus, the degree of the given differential equation is 2.

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0,$$

Question 36

The order and degree of the differential equation respectively, are

(A) 2 and not defined (B) 2 and 2 (C) 2 and 3 (D) 3 and 3

Solution

Correct option is (A) 2 and not defined.

Given differential equation is

$$\begin{aligned} \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} &= 0 \\ \Rightarrow \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} &= -x^{\frac{1}{5}} \end{aligned}$$

As the degree of dy/dx is in fraction its undefined and the degree is 2.

Question 37

If $y = e^{-x}$ (A $\cos x$ + B $\sin x$), then y is a solution of

$$(A) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} = 0$$

$$(B) \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

$$(C) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$$

$$(D) \frac{d^2 y}{dx^2} + 2y = 0$$

Solution

Correct option is (C).

Given equation, $y = e^{-x} (A \cos x + B \sin x)$

Differentiating on both the sides, w.r.t. x , we get

$$\frac{dy}{dx} = e^{-x} (-A \sin x + B \cos x) - e^{-x} (A \cos x + B \sin x)$$

$$\frac{dy}{dx} = e^{-x} (-A \sin x + B \cos x) - y$$

Again differentiating w.r.t. x , we get

$$\frac{d^2 y}{dx^2} = e^{-x} (-A \cos x - B \sin x) - e^{-x} (-A \sin x + B \cos x) - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -e^{-x} (A \cos x + B \sin x) - \left[\frac{dy}{dx} + y \right] - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -y - \frac{dy}{dx} - y - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -2 \frac{dy}{dx} - 2y \Rightarrow \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$$

Question 38

The differential equation for $y = A \cos ax + B \sin ax$, where A and B are arbitrary constants is

$$(A) \frac{d^2 y}{dx^2} - \alpha^2 y = 0$$

$$(B) \frac{d^2 y}{dx^2} + \alpha^2 y = 0$$

$$(C) \frac{d^2 y}{dx^2} + \alpha y = 0$$

$$(D) \frac{d^2 y}{dx^2} - \alpha y = 0$$

Solution

Correct option is (B).

Given equation is $y = A \cos \alpha x + B \sin \alpha x$

Differentiating both sides w.r.t. x , we have

$$\begin{aligned}\frac{dy}{dx} &= -A \sin \alpha x \cdot \alpha + B \cos \alpha x \cdot \alpha \\ &= -A \alpha \sin \alpha x + B \alpha \cos \alpha x\end{aligned}$$

Again differentiating w.r.t. x , we get

$$\frac{d^2 y}{dx^2} = -A \alpha^2 \cos \alpha x - B \alpha^2 \sin \alpha x$$

$$\frac{d^2 y}{dx^2} = -\alpha^2 (A \cos \alpha x + B \sin \alpha x)$$

$$\frac{d^2 y}{dx^2} = -\alpha^2 y \Rightarrow \frac{d^2 y}{dx^2} + \alpha^2 y = 0$$