

Chapter 3: Matrices

NCERT Exemplar · Class 12 · Maths

Questions

Question 1

If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Solution

For a given matrix of order $m \times n$, it has mn elements, where m and n are natural numbers.

Here we have, $m \times n = 28$

$$(m, n) = \{(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)\}$$

So, the possible orders are 1×28 , 2×14 , 4×7 , 7×4 , 14×2 , 28×1 .

Also, if it has 13 elements, then $m \times n = 13$

$$(m, n) = \{(1, 13), (13, 1)\}$$

Thus, the possible orders are 1×13 , 13×1 .

$$\begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$$

Question 2

In the matrix $A =$, write :

(i) The order of the matrix A

(ii) The number of elements

(iii) Write elements a_{23} , a_{31} , a_{12}

Solution

For the given matrix,

(i) The order of the matrix A is 3×3 .

(ii) The number of elements of the matrix $= 3 \times 3 = 9$

(iii) Elements: $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$

Question 3

Construct $a_{2 \times 2}$ matrix where

(i) $a_{ij} = (i - 2j)^2 / 2$

(ii) $a_{ij} = |-2i + 3j|$

Solution

We have,

$$A = [a_{ij}]_{2 \times 2}$$

(i) Such that, $a_{ij} = (i - 2j)^2 / 2$; where $1 \leq i \leq 2$; $1 \leq j \leq 2$

So, the terms of the matrix are

$$a_{11} = \frac{(1 - 2)^2}{2} = \frac{1}{2} \qquad a_{12} = \frac{(1 - 2 \times 2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2 - 2 \times 1)^2}{2} = 0 \qquad a_{22} = \frac{(2 - 2 \times 2)^2}{2} = 2$$

$$\text{Therefore, } A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(ii) Here, $a_{ij} = |-2i + 3j|$

So, the terms of the matrix are

$$\begin{aligned} a_{11} &= |-2 \times 1 + 3 \times 1| = 1 & a_{12} &= |-2 \times 1 + 3 \times 2| = 4 \\ a_{21} &= |-2 \times 2 + 3 \times 1| = 1 & a_{22} &= |-2 \times 2 + 3 \times 2| = 2 \end{aligned}$$

Therefore, $A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}$

Question 4

Construct a 3×2 matrix whose elements are given by $a_{ij} = e^{i \cdot x} \sin jx$

Solution

Let A be a 3×2 matrix

Such that, $a_{ij} = e^{i \cdot x} \sin jx$; where $1 \leq i \leq 3$; $1 \leq j \leq 2$

So, the terms are given as

$$\begin{aligned} a_{11} &= e^x \sin x & a_{12} &= e^x \sin 2x \\ a_{21} &= e^{2x} \sin x & a_{22} &= e^{2x} \sin 2x \\ a_{31} &= e^{3x} \sin x & a_{32} &= e^{3x} \sin 2x \end{aligned}$$

Therefore, $A = \begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}$

Question 5

Find values of a and b if $A = B$, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Solution

Given, matrix A = matrix B

Then their corresponding elements are equal.

So, we have

$$a_{11} = b_{11}; a + 4 = 2a + 2 \Rightarrow a = 2$$

$$a_{12} = b_{12}; 3b = b^2 + 2 \Rightarrow b^2 - 3b + 2 = 0 \Rightarrow b = 1, 2$$

$$a_{22} = b_{22}; -6 = b^2 - 5b \Rightarrow b^2 - 5b + 6 = 0 \Rightarrow b = 2, 3$$

Hence, $a = 2$ and $b = 2$ (common value)

Question 6

If possible, find the sum of the matrices A and B, where A =

$$\begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix} \text{ and B =}$$

$$\begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}$$

Solution

The given two matrices, A and B, are of different orders. Two matrices can be added only if the order of both matrices is the same. Thus, the sum of matrices A and B is not possible.

$$X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}, \text{ find}$$

Question 7

If

(i) $X + Y$

(ii) $2X - 3Y$

(iii) A matrix Z such that $X + Y + Z$ is a zero matrix.

Solution

Given, $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}_{2 \times 3}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}_{2 \times 3}$

(i) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

(ii) $2X - 3Y = 2 \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} - 3 \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$
$$= \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $X + Y = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

Also, $X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So, Z is the additive inverse of $(X + Y)$ or negative of $(X + Y)$.

Therefore, $Z = -(X + Y) = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}$

Question 8

Find non-zero values of x satisfying the matrix equation:

$$x \begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2 \begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2 \begin{bmatrix} (x^2 + 8) & 24 \\ (10) & 6x \end{bmatrix}.$$

Solution

Given,

$$\begin{aligned} x \begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2 \begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} &= 2 \begin{bmatrix} x^2 + 8 & 24 \\ 10 & 6x \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} &= \begin{bmatrix} 2x^2 + 16 & 48 \\ 20 & 12x \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x^2 + 16 & 2x + 10x \\ 3x + 8 & x^2 + 8x \end{bmatrix} &= \begin{bmatrix} 2x^2 + 16 & 48 \\ 20 & 12x \end{bmatrix} \end{aligned}$$

On comparing the corresponding elements, we get

$$2x + 10x = 48$$

$$12x = 48$$

$$\text{Thus, } x = 4$$

It's also seen that this value of x also satisfies the equation $3x + 8 = 20$ and $x^2 + 8x = 12x$.

Therefore, $x = 4$ (common) is the solution of the given matrix equation.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

Question 9**Show that**

$$(A + B)(A - B) \neq A^2 - B^2$$

Solution

$$\text{Given, } A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\text{So, } (A + B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\text{And, } (A - B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$$

$$(A + B) \cdot (A - B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad \dots (i)$$

$$\text{Also, } A^2 = A \cdot A$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\text{And, } B^2 = B \cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{Therefore, } A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \dots (ii)$$

Given, $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So, $(A + B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$

And, $(A - B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$

$$(A + B) \cdot (A - B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad \dots (i)$$

Also, $A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

And, $B^2 = B \cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Therefore, $A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \dots(ii)$

Hence, from (i) and (ii),

$$(A + B) (A - B) \neq A^2 - B^2$$

Question 10

Find the value of x if

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O.$$

Solution

$$\text{Given, } [1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [1 + 2x + 15 \quad 3 + 5x + 3 \quad 2 + x + 2] \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [16 + 2x \ 5x + 6 \ x + 4] \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

22

$$x^2 + 16x + 28 = 0$$

$$(x + 2)(x + 14) = 0$$

Therefore, $x = -2, -14$

$$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

Question 11Show that satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1} .**Solution**

Given,

$$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\text{So, } A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3 \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

$$\text{And, } 7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\begin{aligned} \text{Hence, } A^2 - 3A - 7I &= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \end{aligned}$$

Now,

$$A^2 - 3A - 7I = 0$$

Multiplying both sides with A^{-1} , we get

$$A^{-1} [A^2 - 3A - 7I] = A^{-1} 0$$

$$A^{-1} \cdot A \cdot A - 3A^{-1} \cdot A - 7A^{-1} \cdot I = 0$$

$$I \cdot A - 3I - 7A^{-1} = 0 \text{ [As } A^{-1} \cdot A = I]$$

$$A - 3I - 7A^{-1} = 0$$

$$7A^{-1} = A - 3I$$

$$7A^{-1} = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$\text{Therefore, } A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

Question 12

Find the matrix A satisfying the matrix equation:

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solution

$$\text{Given, } \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Let } P = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \quad Q = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \quad \text{and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now,

$$P^{-1} PAQ = P^{-1} I$$

$$\text{So, } IAQ = P^{-1}$$

$$AQ = P^{-1}$$

$$AQQ^{-1} = P^{-1} Q^{-1}$$

$$AI = P^{-1} Q^{-1}$$

$$A = P^{-1} Q^{-1}$$

Now adj. $P = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ and $|P| = 1$

Hence, $P^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Also adj. $Q = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$ and $|Q| = -1$

Hence, $Q^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

Thus, $A = P^{-1}Q^{-1}$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 6-5 & 4-3 \\ -9+10 & -6+6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Question 13

Find A, if

Solution

$$\text{Given, } \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Now, let $A = [x \ y \ z]$

$$\text{So, } \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} [xyz] = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

On comparing elements of both sides, we have

$$4x = -4 \Rightarrow x = -1$$

$$4y = 8 \Rightarrow y = 2$$

$$\text{And, } 4z = 4 \Rightarrow z = 1$$

Therefore, $A = [-1 \ 2 \ 1]$

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix},$$

Question 14

If then verify

$$(BA)^2 \neq B^2 A^2$$

Solution

The given matrices A has order 3×2 and B has order 2×3 .

So, BA is defined and will have order 3×3 .

But, A^2 and B^2 are not defined as the orders don't satisfy the multiplication condition.

Hence, $(BA)^2 \neq B^2 A^2$

Question 15

If possible, find BA and AB where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

Solution

Given, $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2}$

So, AB and BA both are defined

Now, $AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$

And, $BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$
 $= \begin{bmatrix} 8+1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$

Question 16

Show by an example that for $A \neq 0$, $B \neq 0$, $AB = 0$.

Solution

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \neq O$ and $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \neq O$

So, the product $AB = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

– Hence Proved

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}.$$

Question 17

Given Is $(AB)' = B' A'$?

Solution

$$\text{Given, } A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2 \times 3} \text{ and } B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3 \times 2}$$

So, their product is

$$AB = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

$$\text{And, } (AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \dots(i)$$

$$\text{Also, } B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2 \times 3} \text{ and } A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3 \times 2}$$

$$\begin{aligned} \text{Therefore, } B'A' &= \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we have

$$(AB)' = B' A'$$

Question 18

Solve for x and y:

$$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O$$

Solution

$$\text{Given, } x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O$$

$$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3y \\ 5y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = O \quad \text{(Multiplying the variables with the matrices)}$$

$$\text{So, } \begin{bmatrix} 2x + 3y - 8 \\ x + 5y - 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{(Addition of matrices)}$$

Now, we have

$$2x + 3y - 8 = 0 \dots (1) \text{ and}$$

$$x + 5y - 11 = 0 \dots (2)$$

On solving the equations (1) and (2), we get

$$x = 1 \text{ and } y = 2$$

Question 19

If X and Y are 2 x 2 matrices, then solve the following matrix equations for X and Y

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}.$$

Solution

Given, $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$... (i)

and $3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$... (ii)

On subtractiong equations (i) and (ii), we get

$$(3X + 2Y) - (2X + 3Y) = \begin{bmatrix} -2 - 2 & 2 - 3 \\ 1 - 4 & -5 - 0 \end{bmatrix}$$

Thus, $X - Y = \begin{bmatrix} -4 & -1 \\ -3 & -5 \end{bmatrix}$... (iii)

On adding equations (i) and (ii), we get

$$5X + 5Y = \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix}$$
$$X + Y = \frac{1}{5} \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \dots \text{(iv)}$$

On adding equations (iii) and (iv), we get

$$(X - Y) + (X + Y) = \begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

$$2X = \begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

Thus, $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$

From equation (iv), we get

$$\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix} + Y = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Thus, $Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

Question 20

If $A = [3 \ 5]$, $B = [7 \ 3]$, then find a non-zero matrix C such that $AB = AC$.

Solution

Given, $A = [3 \ 5]_{1 \times 2}$ and $B = [7 \ 3]_{1 \times 2}$

For $AC = BC$

We have order of $C = 2 \times n$

For $n = 1$, let $C = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore AC = [3 \ 5] \begin{bmatrix} x \\ y \end{bmatrix} = [3x + 5y]$$

$$\text{And } BC = [7 \ 3] \begin{bmatrix} x \\ y \end{bmatrix} = [7x + 3y]$$

For $AC = BC$,

$$[3x + 5y] = [7x + 3y]$$

$$3x + 5y = 7x + 3y$$

$$4x = 2y$$

$$x = \frac{1}{2}y$$

$$y = 2x$$

Hence, $C = \begin{bmatrix} x \\ 2x \end{bmatrix}$

It's seen that on taking C of order 2×1 , 2×2 , 2×3 , ..., we get

$$C = \begin{bmatrix} x \\ 2x \end{bmatrix}, \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix}, \begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix} \dots$$

In general,

$$C = \begin{bmatrix} k \\ 2k \end{bmatrix}, \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix} \text{ etc ...}$$

Question 21

Give an example of matrices A , B and C such that $AB = AC$, where A is non-zero

matrix, but $B \neq C$.

Solution

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ [$\because B \neq C$]

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \dots(i)$$

and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \dots(ii)$

From (i) and (ii),

Hence, $AB = AC$ but $B \neq C$.

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix},$$

Question 22

If

(i) $(AB)C = A(BC)$ (ii) $A(B + C) = AB + AC$.

Solution

Given, $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

$$(i) \quad AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

$$\text{and } (AB)C = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 8+5 & 0 \\ -1+10 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix} \quad \dots(i)$$

$$\text{Again, } (BC) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2-3 & 0 \\ 3+4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$$

$$\text{And } A(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} -1+14 & 0 \\ 2+7 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get

Hence, $(AB)C = A(BC)$

$$(ii) \quad B + C = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$$

Now,

$$\begin{aligned} A \cdot (B + C) &= \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix} \\ &= \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad \dots(iii) \end{aligned}$$

$$AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

$$\text{and, } AC = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1-2 & 0 \\ -2-1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$$

$$\text{Thus, } AB + AC = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad \dots(iv)$$

Hence from equation (iii) and (iv), we have

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$\text{If } P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \text{ and } Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}, \text{ prove that } PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

23.

Given, $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

It's seen that both P and Q are diagonal matrices.

We know that, for diagonal matrices elements of product matrix are obtained by multiplying elements of matrices in the principal diagonal.

Hence,

$$\begin{aligned} PQ &= \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \\ &= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix} = QP \end{aligned}$$

Therefore, $PQ = QP$

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Question 23

If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, prove that $PQ = QP$.

Solution

Solution coming soon.

Question 24

If: = A, find A.

Solution

$$\text{Given, } [2 \ 1 \ 3]_{1 \times 3} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}_{3 \times 1} = A$$

$$\text{So, } [2 \ 1 \ 3]_{1 \times 3} \begin{bmatrix} -1+0+1 \\ -1+0+0 \\ 0+0-1 \end{bmatrix}_{3 \times 1} = A$$

$$[2 \ 1 \ 3]_{1 \times 3} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}_{3 \times 1} = A$$

$$[0 - 1 - 3] = A$$

$$\text{Thus, } A = [-4]$$

$$A = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix},$$

Question 25

If verify that

$$A(B + C) = (AB + AC).$$

Solution

Given, $A = [2 \ 1], B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

Now,

$$\begin{aligned} A(B + C) &= [2 \ 1] \begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix} \\ &= [2 \ 1] \begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix} \\ &= [8 + 9 \ 10 + 7 \ 10 + 8] \\ &= [17 \ 17 \ 18] \end{aligned} \quad \dots(i)$$

And,

$$AB = [2 \ 1] \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix} = [10 + 8 \ 6 + 7 \ 8 + 6] = [18 \ 13 \ 14]$$

And,

$$AC = [2 \ 1] \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = [-2 + 1 \ 4 + 0 \ 2 + 2] = [-1 \ 4 \ 4]$$

$$\begin{aligned} \text{So, } AB + AC &= [18 \ 13 \ 14] + [-1 \ 4 \ 4] \\ &= [17 \ 17 \ 18] \end{aligned} \quad \dots(ii)$$

From equations (i) and (ii),

$$A(B + C) = (AB + AC)$$

Hence, verified

Given, $A = [2 \ 1], B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

Now,

$$\begin{aligned} A(B + C) &= [2 \ 1] \begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix} \\ &= [2 \ 1] \begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix} \\ &= [8+9 \ 10+7 \ 10+8] \\ &= [17 \ 17 \ 18] \end{aligned} \quad \dots(i)$$

And,

$$AB = [2 \ 1] \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix} = [10+8 \ 6+7 \ 8+6] = [18 \ 13 \ 14]$$

And,

$$AC = [2 \ 1] \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = [-2+1 \ 4+0 \ 2+2] = [-1 \ 4 \ 4]$$

$$\begin{aligned} \text{So, } AB + AC &= [18 \ 13 \ 14] + [-1 \ 4 \ 4] \\ &= [17 \ 17 \ 18] \end{aligned} \quad \dots(ii)$$

From equations (i) and (ii),

$$A(B + C) = (AB + AC)$$

Hence, verified

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix},$$

Question 26

If then verify that $A^2 + A = A(A + I)$, where I is 3×3 unit matrix.

Solution

Given, $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

So, $A^2 = A \cdot A$

Thus,
$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0-1 & -1+0-1 \\ 2+2+0 & 0+1+3 & -2+3+3 \\ 0+2+0 & 0+1+1 & 0+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \dots(i)$$

Now,
$$A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

So,
$$A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & 0+0-1 & -1+0-2 \\ 4+2+0 & 0+2+3 & -2+3+6 \\ 0+2+0 & 0+2+1 & 0+3+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get

$$A^2 + A = A(A + I)$$

Given, $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

So, $A^2 = A \cdot A$

Thus,
$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0-1 & -1+0-1 \\ 2+2+0 & 0+1+3 & -2+3+3 \\ 0+2+0 & 0+1+1 & 0+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \dots(i)$$

Now,
$$A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

So,
$$A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & 0+0-1 & -1+0-2 \\ 4+2+0 & 0+2+3 & -2+3+6 \\ 0+2+0 & 0+2+1 & 0+3+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get

$$A^2 + A = A(A + I)$$

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix},$$

Question 27

If then verify that:

(i) $(A\phi)\phi = A$

(ii) $(AB)\phi = B\phi A\phi$

(iii) $(kA)\phi = (kA\phi).$

Solution

Given, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(i) We have to verify that, $(A')' = A$

So,

$$A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

And, $(A')' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$

(ii) We have to verify that, $(AB)' = B'A'$

So,

$$AB = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

and, $B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix} = (AB)'$

Hence proved.

(iii) We have to verify that, $(kA)' = (kA)'$

$$\text{Now, } (kA) = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$$

$$\text{And, } (kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$$

$$\text{Also, } kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix} = (kA)'$$

Hence proved.

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix},$$

Question 28

If then verify that:

(i) $(2A + B)\phi = 2A\phi + B\phi$

(ii) $(A - B)\phi = A\phi - B\phi.$

Solution

$$\text{Given, } A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$$

$$(i) \quad (2A + B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$$

$$\text{And, } (2A + B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$$

$$\text{Also, } 2A' + B'$$

$$= \begin{bmatrix} 2 & 8 & 10 \\ 4 & 2 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix} = (2A + B)'$$

$$\text{Hence, } 2A' + B' = (2A + B)'$$

$$(ii) \quad A - B = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$$

$$\text{And, } (A - B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$$

$$\text{Also, } A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix} = (A - B)'$$

$$\text{Thus, } A' - B' = (A - B)'$$

- Hence proved

Question 29

Show that $A^t A$ and AA^t are both symmetric matrices for any matrix A .

Solution

Let $P = A^t A$

So, $P^t = (A^t A)^t$

$$= A^t (A^t)^t [As (AB)^t = B^t A^t]$$

Hence, $A^t A$ is the symmetric matrix for any matrix A .

Now, let $Q = AA^t$

$$\text{So, } Q^t = (AA^t)^t = (A^t)^t A = AA^t = Q$$

Hence, AA^t is the symmetric matrix for any matrix A .

Question 30

Let A and B be square matrices of the order 3×3 . Is $(AB)^2 = A^2 B^2$? Give reasons.

Solution

As, A and B be square matrices of order 3×3 .

$$\text{We have, } (AB)^2 = AB \cdot AB$$

$$= A(BA)B$$

$$= A(AB)B \text{ [If } AB = BA]$$

$$= AAB B$$

$$= A^2 B^2$$

Thus, $(AB)^2 = A^2 B^2$ is true only if $AB = BA$.

Question 31

Show that if A and B are square matrices such that $AB = BA$, then

$$(A + B)^2 = A^2 + 2AB + B^2.$$

Solution

Given, A and B are square matrices such that $AB = BA$.

$$\text{So, } (A + B)^2 = (A + B) \cdot (A + B)$$

$$= A^2 + AB + BA + B^2$$

$$= A^2 + AB + AB + B^2 \text{ [Since, } AB = BA]$$

$$= A^2 + 2AB + B^2$$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

Question 32

Let $a = 4$, $b = -2$.

Show that:

(a) $A + (B + C) = (A + B) + C$

(b) $A(BC) = (AB)C$

(c) $(a + b)B = aB + bB$

(d) $a(C - A) = aC - aA$

(e) $(A^T)^T = A$

(f) $(bA)^T = bA^T$

(g) $(AB)^T = B^T A^T$

(h) $(A - B)C = AC - BC$

(i) $(A - B)^T = A^T - B^T$

Solution

$$(a) \quad A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$$

And,

$$(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$$

$$(b) \quad (BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$\text{And, } A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix} = \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$$

$$\text{Thus, } (AB)C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix} = A(BC)$$

Hence proved.

$$(c) \quad (a + b)B = (4 - 2) \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \quad [\because \text{given } a = 4, b = -2]$$

$$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

Also,

$$aB + bB = 4B - 2B = \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix} = (a + b)B$$

Hence proved.

$$(d) \quad C - A = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$$

$$\text{And,} \quad a(C - A) = 4(C - A) \\ = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$

$$\text{Also, } aC - aA = 4C - 4A \\ = \begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix} = a(C - A)$$

Hence proved.

$$(e) \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\text{Thus,} \quad (A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A$$

Hence proved.

$$(f) \quad (bA)^T = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T \quad [\because b = -2] \\ = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$$

$$\text{And,} \quad A^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\text{Thus,} \quad bA^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix} = (bA)^T$$

Hence proved.

$$(g) \quad AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$$

$$\text{So,} \quad (AB)^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$$

$$\text{Now, } B^T A^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4+2 & -4+3 \\ 0+10 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix} = (AB)^T$$

Hence proved.

$$(h) \quad (A - B) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}$$

$$\text{Thus, } (A - B)C = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

Now,

$$AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix}$$

And,

$$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

Therefore,

$$AC - BC = \begin{bmatrix} 4-8 & -4-0 \\ 1-7 & -6+10 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix} = (A - B)C$$

Hence proved.

$$(i) \quad (A - B)^T = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$$

$$\text{Now, } A^T - B^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix} = (A - B)^T$$

Hence proved.

$$33. \text{ If } A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \text{ then show that } A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}.$$

Given, $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

Now, $A^2 = A \cdot A$

$$\begin{aligned} &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta - \sin^2 \theta & \cos \theta \sin \theta + \sin \theta \cos \theta \\ -\sin \theta \cos \theta - \cos \theta \sin \theta & -\sin^2 \theta + \cos^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\theta & 2 \sin \theta \cos \theta \\ 2 \sin \theta \cos \theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \end{aligned}$$

Hence proved.

$$A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Question 33

If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

Solution

Solution coming soon.

Question 34

If and $x^2 = -1$, then show that $(A + B)^2 = A^2 + B^2$

Solution

Given, $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$

So, $(A + B) = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$

And, $(A + B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \quad \dots(i)$

Also, $A^2 = A \cdot A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix}$

And, $B^2 = B \cdot B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Thus, $A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \quad \dots(ii)$

Hence, from equations (i) and (ii), we have

$$(A + B)^2 = A^2 + B^2$$

$$\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}.$$

Question 35

Verify that $A^2 = I$ when $A =$

Solution

Given, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

So, $A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 0+4-3 & 0-3+3 & 0+4-4 \\ 0-12+12 & 4+9-12 & -4-12+16 \\ 0-12+12 & 3+9-12 & -3-12+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence proved.

Question 36

Prove by Mathematical Induction that $(A\phi)^n = (A^n)\phi$, where $n \in \mathbb{N}$ for any square matrix A .

Solution

Let $P(n): (A\phi)^n = (A^n)\phi$

So, $P(1): (A\phi) = (A)\phi$

$$A\phi = A\phi$$

Hence, $P(1)$ is true.

Now, let $P(k) = (A\phi)^k = (A^k)\phi$, where $k \in \mathbb{N}$

And,

$$P(k+1): (A\phi)^{k+1} = (A\phi)^k A\phi$$

$$= (A^k)\phi A\phi$$

$$= (AA^k)\phi$$

$$= (A^{k+1}) \quad \text{♣}$$

Hence, $P(1)$ is true and whenever $P(k)$ is true $P(k + 1)$ is true.

Therefore, $P(n)$ is true for all $n \in \mathbb{N}$.

Question 37

Find the inverse, by elementary row operations (if possible), of the following matrices

$$(i) \quad \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} \qquad (ii) \quad \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}.$$

Solution

$$(i) \quad \text{Let } A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A \quad [\because R_2 \rightarrow R_2 + 5R_1]$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A \quad \left[\because R_2 \rightarrow \frac{1}{22} R_2 \right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A \quad [\because R_1 \rightarrow R_1 - 3R_2]$$

$I = BA$, where B is the inverse of A .

Hence,

$$B = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

Now, $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$$\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A \quad [\because R_2 \rightarrow R_2 + 2R_1]$$

As we obtain all the zeroes in a row of the matrix A on the L.H.S., A^{-1} does not exist.

$$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix},$$

Question 38

If then find values of x, y, z and w.

Solution

In the given matrix equation,

Comparing the corresponding elements, we get

$$x + y = 6,$$

$$xy = 8,$$

$$z + 6 = 0 \text{ and}$$

$$w = 4$$

From the first two equations, we have

$$(6 - y) \cdot y = 8$$

$$y^2 - 6y + 8 = 0$$

$$(y - 2)(y - 4) = 0$$

$$y = 2 \text{ or } y = 4$$

Hence, $x = 4$ and $x = 2$

Also, $z + 6 = 0$

$z = -6$ and $w = 4$

Therefore,

$x = 2, y = 4$ or $x = 4, y = 2, z = -6$ and $w = 4$

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix},$$

Question 39

If find a matrix C such that $3A + 5B + 2C$ is a null matrix.

Solution

Let's consider a matrix C , such that

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$3A + 5B + 2C = O$$

$$\begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 48 + 2a & 20 + 2b \\ 56 + 2c & 76 + 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the terms,

$$2a + 48 = 0 \Rightarrow a = -24$$

$$20 + 2b = 0 \Rightarrow b = -10$$

$$56 + 2c = 0 \Rightarrow c = -28$$

And,

$$76 + 2d = 0 \Rightarrow d = -38$$

Therefore, the matrix C is

$$C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Question 40

If , then find $A^2 - 5A - 14I$. Hence, obtain A^3 .

Solution

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$... (i)

Now, $A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$

$$A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So, $A^2 - 5A - 14I = O$

$$A \cdot A^2 - 5A \cdot A = 14A I = O$$

$$A^3 - 5A^2 - 14A = O$$

$$A^3 = 5A^2 + 14A$$

$$= 5 \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14 \begin{bmatrix} 3 & -5 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

Hence, $A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

Question 41

Find the value of a, b, c and d, if

$$3 \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}.$$

Solution

Given,

$$3 \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$
$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ -1+c+d & 2d+3 \end{bmatrix}$$

Now,

$$3a = a + 4 \Rightarrow a = 2$$

$$3b = 6 + a + b$$

$$3b - b = 8$$

$$\Rightarrow b = 4$$

And,

$$3d = 3 + 2d$$

$$\Rightarrow d = 3$$

And,

$$3c = c + d - 1$$

$$2c = 3 - 1 = 2$$

$$\Rightarrow c = 1$$

Hence,

$$a = 2, b = 4, c = 1 \text{ and } d = 3$$

Question 42

Find the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}.$$

Solution

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a & b & c \\ -3a + 4d & -3b + 4e & -3c + 4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Let,

Now, by equality of matrices, we get

$$a = 1, b = -2, c = -5$$

And,

$$2a - d = -1 \Rightarrow d = 2a + 1 = 3;$$

$$2b - e = -8 \Rightarrow e = 2(-2) + 8 = 4$$

$$2c - f = -10 \Rightarrow f = 2c + 10 = 0$$

Thus,

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix},$$

Question 43

If $A =$ find $A^2 + 2A + 7I$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

$$\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix},$$

Question 44

If $A =$ and $A^{-1} = A'$, find the value of a .

Solution**Given,**

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Also,

$$A^{-1} = A'$$

$$AA^{-1} = AA'$$

$$I = AA'$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

By using the equality of matrices, we get

 $\cos^2 \alpha + \sin^2 \alpha = 1$, which is true for all real values of α .

$$\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

Question 45

If the matrix is a skew symmetric matrix, find the values of a, b and c.

Solution

$$\text{Let } A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

As, A is skew - symmetric matrix.

So, we have

$$A' = -A$$

Then,

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & 1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get

$$a = -2, c = -3 \text{ and } b = -b \Rightarrow b = 0$$

Hence,

$$a = -2, b = 0 \text{ and } c = -3$$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix},$$

Question 46

If $P(x) =$ then show that

$$P(x) \cdot P(y) = P(x+y) = P(y) \cdot P(x).$$

Solution

Given,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

So, $P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

Now,

$$\begin{aligned} P(x) \cdot P(y) &= \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cdot \cos y - \sin x \cdot \sin y & \cos x \cdot \sin y + \sin x \cdot \cos y \\ -\sin x \cdot \cos y - \cos x \cdot \sin y & -\sin x \cdot \sin y + \cos x \cdot \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \\ &= P(x+y) \end{aligned} \quad \dots(i)$$

Also,

$$\begin{aligned} P(y) \cdot P(x) &= \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \\ &= \begin{bmatrix} \cos y \cdot \cos x - \sin y \cdot \sin x & \cos y \cdot \sin x + \sin y \cdot \cos x \\ -\sin y \cdot \cos x - \cos y \cdot \sin x & -\sin y \cdot \sin x + \cos y \cdot \cos x \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \end{aligned} \quad \dots(ii)$$

Therefore, from (i) and (ii), we get

$$P(x) \cdot P(y) = P(x+y) = P(y) \cdot P(x)$$

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

So, $P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

Now,

$$\begin{aligned} P(x) \cdot P(y) &= \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cdot \cos y - \sin x \cdot \sin y & \cos x \cdot \sin y + \sin x \cdot \cos y \\ -\sin x \cdot \cos y - \cos x \cdot \sin y & -\sin x \cdot \sin y + \cos x \cdot \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \\ &= P(x+y) \end{aligned} \quad \dots(i)$$

Also,

$$\begin{aligned} P(y) \cdot P(x) &= \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \\ &= \begin{bmatrix} \cos y \cdot \cos x - \sin y \cdot \sin x & \cos y \cdot \sin x + \sin y \cdot \cos x \\ -\sin y \cdot \cos x - \sin x \cdot \cos y & -\sin y \cdot \sin x + \cos y \cdot \cos x \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \end{aligned} \quad \dots(ii)$$

Therefore, from (i) and (ii), we get

$$P(x) \cdot P(y) = P(x+y) = P(y) \cdot P(x)$$

Question 47

If A is square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

Solution

We know that,

$$A \cdot I = I \cdot A$$

So, A and I are commutative.

Thus, we can expand $(I + A)^3$ like real numbers expansion.

$$\text{So, } (I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$$

$$= I + 3IA + 3A^2 + AA^2 \text{ (As } I^n = I, n \in \mathbb{N})$$

$$= I + 3A + 3A + AA$$

$$= I + 3A + 3A + A^2 = I + 3A + 3A + A = I + 7A$$

Question 48

If A, B are square matrices of same order and B is a skew-symmetric matrix, show that $A\phi BA$ is skew symmetric.

Solution

Given, A and B are square matrices such that B is a skew-symmetric matrix

$$\text{So, } B\phi = -B$$

Now, we have to prove that $A\phi BA$ is a skew-symmetric matrix.

$$(A\phi BA)\phi = A\phi B\phi (A\phi)\phi \text{ [Since, } (AB)\phi = B\phi A\phi]$$

$$= A\phi (-B)A$$

$$= -A\phi BA$$

Hence, $A\phi BA$ is a skew-symmetric matrix.

Long Answer (L.A)**Question 49**

If $AB = BA$ for any two square matrices, prove by mathematical induction that $(AB)^n = A^n B^n$.

Solution

Let $P(n) : (AB)^n = A^n B^n$

So, $P(1) : (AB)^1 = A^1 B^1$

$AB = AB$

So, $P(1)$ is true.

Let $P(n)$ is true for some $k \in \mathbb{N}$

Now,

$$\begin{aligned}
 (AB)^{k+1} &= (AB)^k (AB) && \text{(using (i))} \\
 &= A^k B^k (AB) \\
 &= A^k B^{k-1} (BA) B \\
 &= A^k B^{k-1} (AB) B && \text{(as given } AB = BA) \\
 &= A^k B^{k-1} AB^2 \\
 &= A^k B^{k-2} (BA) B^2 \\
 &= A^k B^{k-2} ABB^2 \\
 &= A^k B^{k-2} AB^3 \\
 &\dots\dots \\
 &\dots\dots \\
 &= A^{k+1} B^{k+1}
 \end{aligned}$$

Hence, $P(1)$ is true and whenever $P(k)$ is true $P(k+1)$ is true.

Thus, $P(n)$ is true for all $n \in \mathbb{N}$.

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

Question 50

Find x, y, z if $A =$ satisfies $A\phi = A^{-1}$.

Solution

Matrix A is such that $A\phi = A^{-1}$

$$AA\phi = I$$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 + z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4y^2 + z^2 = 1$$

$$2y^2 - z^2 = 0$$

$$x^2 + y^2 + z^2 = 1$$

$$x^2 - y^2 - z^2 = 0$$

$$y^2 = 1/6, z^2 = 1/3, x^2 = 1/2$$

So, the roots are:

$$x = \pm \frac{1}{\sqrt{2}}$$

$$y = \pm \frac{1}{\sqrt{6}}$$

And,

$$z = \pm \frac{1}{\sqrt{3}}$$