

Chapter 2: Relations and Functions

NCERT Exemplar · Class 11 · Maths

Textbook exercises

Question 1

Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$. Determine

(i). $A \times B$

(ii). $B \times A$

(iii). Is $A \times B = B \times A$?

(iv). Is $n(A \times B) = n(B \times A)$?

Solution

Ans: Given: $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$

Use the Cartesian product.

$$A \times B = \{(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)\}.$$

Ans: Given: $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$

Use the Cartesian product.

$$B \times A = \{(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2), (7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)\}.$$

Ans: Given: $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$

Use the Cartesian product.

$$A \times B \neq B \times A \text{ as the ordered pairs are different.}$$

Ans: Given: $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$

Use the Cartesian product.

Here,

$$n(A \times B) = n(B \times A)$$

$$\Rightarrow n(A) \cdot n(B) = 4 \times 3$$

$$\Rightarrow n(A) \cdot n(B) = 12.$$

Question 2

Find x and y , if

$$(i)(4x + 3, y) = (3x + 5, -2)$$

Solution

Ans: Given: Ordered pairs.

Elements of equal ordered pairs are equal.

Here,

$$(4x + 3, y) = (3x + 5, -2)$$

$$\Rightarrow 4x + 3 = 3x + 5 \text{ and } y = -2$$

$$\Rightarrow x = 2 \text{ and } y = -2$$

$$(ii)(x - y, x + y) = (6, 10)$$

Ans: Given: Ordered pairs.

Elements of equal ordered pairs are equal.

Here,

$$(x - y, x + y) = (6, 10)$$

$$\Rightarrow x - y = 6 \dots\dots\dots (i)$$

$$\Rightarrow x + y = 10 \dots\dots\dots (ii)$$

From (i) and (ii), we get

$$x = 8 \text{ and } y = 2$$

Question 3

If

$$A\{2, 4, 6, 9\} \text{ and } B\{4, 6, 18, 27, 54\}, a \in A, b \in B,$$

find the set of ordered pairs such that 'a' is factor of 'b' and $a < b$.

Solution

Ans: Given:

$$A = \{2, 4, 6, 9\}, a \in A,$$

$$B = \{4, 6, 18, 27, 54\}, b \in B$$

Form ordered pairs such that b' is divisible by 'a'

Required set of ordered pair will be

$$\{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)\}.$$

Question 4

Find the domain and range of the relation

$$R$$

$$\text{given by } R = \{(x, y) : y = x + \frac{6}{x}; \text{ where } x, y \in N \text{ and } x < 6\}.$$

Solution

Ans: Given: A relation

$$R$$

.

Domain and range are values of x and y for which relation is defined.

$$R \text{ is defined only for } x = \{1, 2, 3\}, y \in N$$

$$\therefore \text{Domain of } R = \{1, 2, 3\}$$

$$\text{for, } x = 1, y = 7,$$

$$x = 2, y = 5,$$

$$x = 3, y = 5.$$

$$\therefore \text{Range of } R = \{7, 5\}.$$

Question 5

Is the following relation a function? Justify your answer.

$$(i). R_1 = \left\{ (2, 3), \left(\frac{1}{2}, 0\right), (2, 7), (-4, 6) \right\}$$

Solution

Ans: Given: Relation

$$R_1 \text{ and } R_2.$$

A relation is function only when there is only one image in Range.

Here,

$$\text{Since, } (2, 3) \text{ and } (2, 7) \in R_1$$

$$\Rightarrow R_1 \text{ is not a function}$$

$$(ii). R_2 = \{(x, |x|) \mid x \text{ is a real number}\}$$

Ans: Given: Relation

$$R_1 \text{ and } R_2.$$

A relation is function only when there is only one image in Range.

Here,

for all

$$x \in R,$$

There is a unique image.

Therefore,

$$R_2$$

is a function..

Question 6

Find the domain for which the functions

$$f(x) = 2x^2 - 1 \text{ and } g(x) = 1 - 3x$$

are equal.

Solution

Ans: Given:

$$f(x) = 2x^2 - 1 \text{ and } g(x) = 1 - 3x.$$

Find

$$x, \text{ for } f(x) = g(x).$$

$$\text{for } f(x) = g(x)$$

$$\Rightarrow 2x^2 - 1 = 1 - 3x$$

$$\Rightarrow 2x^2 + 3x - 2 = 0$$

$$\Rightarrow 2x^2 + 4x - x - 2 = 0$$

$$\Rightarrow 2x(x + 2) - 1(x + 2) = 0$$

$$\Rightarrow (2x - 1)(x + 2) = 0$$

$$\Rightarrow x = \frac{1}{2}, -2.$$

$$\therefore \text{Domain for } f(x) = g(x) \text{ is } \left\{ \frac{1}{2}, -2 \right\}.$$

Question 7

Find the domain of each of the following functions:

$$(i). f(x) = \frac{x}{x^2 + 3x + 2}$$

Solution

Ans: Domain is the value of

$$x$$

for which

$$f(x)$$

is defined.

Here,

$f(x)$ is defined, if

$$x^2 + 3x + 2 \neq 0$$

$$\Rightarrow (x + 1)(x + 2) \neq 0$$

$$\Rightarrow x \neq -1, -2$$

$\therefore$ Domain of $f(x) = R - \{-1, -2\}$

$$(ii). f(x) = [x] + x$$

Ans: Domain is the value of

$$x$$

for which

$$f(x)$$

is defined.

Here,

$f(x)$ is defined for all $x \in R$

$\therefore$ Domain of $f(x) = R$

Question 8

Find the range of the following function:

$$(i). f(x) = \frac{|x - 4|}{x - 4}$$

Solution

Ans: Write

x in form of y .

Then, find the range.

Here,

$$f(x) = \frac{|x - 4|}{x - 4}$$

$$\Rightarrow f(x) =$$

$$\frac{x - 4}{x - 4} = 1, x > 4$$

$$\frac{-(x - 4)}{x - 4} = -1, x < 4$$

$\therefore$ Range of $f(x) = \{-1, 1\}$.

$$(ii). f(x) = \sqrt{16 - x^2}$$

Ans: Write

x in form of y .

Then, find the range.

Here,

$$\text{Let } y = f(x),$$

$$\Rightarrow y = \sqrt{16 - x^2}$$

$$\Rightarrow x^2 = 16 - y^2$$

$$\text{Here, } x \in [-4, 4]$$

$\therefore$ Range of $f(x) = [0, 4]$

Question 9

Redefine the function which is given by $f(x) = |x - 1| + |1 + x|, -2 \leq x \leq 2$

Solution

Ans: Given: function

$$f(x) = |x - 1| + |1 + x|, -2 \leq x \leq 2.$$

Split the function over the given intervals.

$$f(x) =$$

$$-x + 1 - 1 - x, -2 \leq x \leq 2$$

$$-x + 1 + x + 1, -1 \leq x < 1$$

$$x - 1 + 1 + x, 1 \leq x \leq 2$$

$$f(x) =$$

$$-2x, -2 \leq x < -1$$

$$2, -1 \leq x < 1$$

$$2x, 1 \leq x \leq 2$$

Question 10

Find the domain of the function

$$f$$

given by

$$f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}.$$

Solution

Ans: Given: Function

$$f(x).$$

find the value of

$$x$$

for which

$$f(x)$$

is defined.

$$f(x) \text{ is defined, if } [x]^2 - [x] - 6 > 0.$$

$$\Rightarrow ([x] - 3)([x] + 2) > 0,$$

$$\Rightarrow [x] < -2 \text{ or } [x] > 3$$

$$\Rightarrow x < -2 \text{ or } x \geq 4$$

$$\therefore \text{Domain} = (-\infty, -2) \cup [4, \infty).$$

Objective Type Question

Question 11

The domain of the function

$$f$$

given by

$$f(x) = \frac{1}{\sqrt{x - |x|}}.$$

(A).

$$R$$

(B).

$$R^+$$

(C).

$$R^-$$

(D).None of these.

Solution

Ans: Given: Function

$$f(x).$$

Find the value of

$$x$$

for which the function

$$f(x)$$

is defined.

$$x - |x| =$$

$$x - x = 0, \text{ if } x \geq 0$$

$$2x, \text{ if } x < 0$$

Here, $f(x)$ is not defined for any $x \in R$.

$\therefore$ Domain is none

Correct Answer: D

Question 12

If

$$f(x) = x^3 - \frac{1}{x^3}, \text{ then } f(x) + f\left(\frac{1}{x}\right)$$

equals to:

$$(A). 2x^3$$

$$(B). 2\frac{1}{x^3}$$

$$(C). 0$$

$$(D). 1$$

Solution

Ans: Given: Function

$$f(x).$$

Replace the value of

$$x \text{ in } f(x).$$

Then, find the sum.

$$\text{Here, } f(x) = x^3 - \frac{1}{x^3}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$\therefore f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = 0.$$

Correct Answer: C

Question 13

Let A and B be any two sets such that

$$n(B) = p, n(A) = q$$

then the total number of functions is equal to _____

Solution

Ans: Given: A and B be two sets and,

$$n(B) = p, n(A) = q.$$

Total number of function from

$$A \rightarrow B \text{ is } [n(A)]^{n(B)}.$$

Every Element of the set

$$A$$

is related to elements of the set

$$B$$

in

$$p$$

ways. Therefore, the total number of functions

$$f : A \rightarrow B$$

is equal to

$$p^q.$$

Question 14

Let f and g be two functions given by

$$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$$

$$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}$$

Then domain of

$$f + g$$

is _____

(i).

$$A \times B$$

(ii).

$$B \times A$$

(iii).

$$B \times B$$

(iv).

$$A \times A$$

$$(v). \frac{f(t) - f(5)}{t - 5}, \text{ if } t \neq 5$$

(i). For what real numbers

$$x, f(x) = g(x)?$$

(ii). For what real numbers

$$x, f(x) < g(x)?$$

Solution

Ans: Given:

$$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\},$$

$$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}.$$

First, find the domain of

$$f \text{ and } g.$$

Then find the domain of

$$f + g$$

.

$$\text{Domain of } f = \{2, 5, 8, 10\},$$

$$\text{Domain of } g = \{2, 7, 8, 10, 11\}$$

$$\text{Domain of } f + g = (\text{domain of } f \cap \text{domain of } g)$$

$$\therefore \text{Domain of } f + g = \{2, 8, 10\}.$$

EXERCISE QUESTION:

1. If

$$A = \{-1, 2, 3\} \text{ and } B = \{1, 3\}$$

, then determine,

Ans: Given: sets

$$A = \{-1, 2, 3\} \text{ and } B = \{1, 3\}.$$

The terms

$$A \times B$$

represents the Cartesian product and represents the relation of elements in A with B

$$A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$$

Ans: Given: sets

$$A = \{-1, 2, 3\} \text{ and } B = \{1, 3\}.$$

The terms

$$A \times B$$

represents the Cartesian product and represents the relation of elements in A with B

$$B \times A = \{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$$

Ans: Given: sets

$$A = \{-1, 2, 3\} \text{ and } B = \{1, 3\}.$$

The terms

$$A \times B$$

represents the Cartesian product and represents the relation of elements in A with B

$$B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$$

Ans: Given: sets

$$A = \{-1, 2, 3\} \text{ and } B = \{1, 3\}.$$

The terms

$$A \times B$$

represents the Cartesian product and represents the relation of elements in A with B

$$A \times A = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}.$$

2.If

$$P = \{x : x < 3, x \in N\}, Q = \{x : x \leq 2, x \in W\},$$

then find $(P \cup Q) \times (P \cap Q)$ where

$$W$$

is the set of whole numbers.

Ans: Given:

$$P = \{x : x < 3, x \in N\}$$

$$Q = \{x : x \leq 2, x \in W\}$$

Use basic operations of sets like union and intersection of sets and then apply cartesian products. From the sets, we get

$$\Rightarrow P = \{1, 2\}$$

$$\Rightarrow Q = \{0, 1, 2\}$$

$$\Rightarrow P \cup Q = \{0, 1, 2\} \text{ and } P \cap Q = \{1, 2\}$$

$$\Rightarrow (P \cup Q) \times (P \cap Q) = \{0, 1, 2\} \times \{1, 2\}$$

$$\Rightarrow (P \cup Q) \times (P \cap Q) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$$

3.If

$$A = \{x : x \in W, x < 2\}, B = \{x : x \in N, 1 < x < 5\} \text{ and } C = \{3, 5\}$$

then find

$$(i). A \times (B \cup C)$$

Ans: Given: The sets

$$A = \{x : x \in W, x < 2\}, B = \{x : x \in N, 1 < x < 5\} \text{ and } C = \{3, 5\}$$

.

First convert the sets in roster form then use basic set operations and Cartesian products.

The sets in roster form will be,

$$A = \{0, 1\}, B = \{2, 3, 4\} \text{ and } C = \{3, 5\}.$$

$$A \times (B \cap C)$$

$$\Rightarrow A \times (B \cap C) = \{0, 1\} \times \{3\}$$

$$\Rightarrow A \times (B \cap C) = \{(0, 3), (1, 3)\}$$

$$(ii). A \times (B \cup C)$$

Ans: Given: The sets

$$A = \{x : x \in W, x < 2\}, B = \{x : x \in N, 1 < x < 5\} \text{ and } C = \{3, 5\}$$

.

First convert the sets in roster form then use basic set operations and Cartesian products.

The sets in roster form will be,

$$A = \{0, 1\}, B = \{2, 3, 4\} \text{ and } C = \{3, 5\}.$$

$$A \times (B \cup C)$$

$$\Rightarrow A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$$

$$\Rightarrow A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}.$$

4. In each of the following cases, find a and b.

$$(i). (2a + b, a - b) = (8, 3)$$

Ans: Since, two ordered pairs are equal, if their corresponding first and second elements are equal.

$$(2a + b, a - b) = (8, 3)$$

$$\Rightarrow 2a + b = 8 \text{ and } a - b = 3$$

On substituting

$$b = a - 3 \text{ in } 2a + b = 8, \text{ we get}$$

$$\Rightarrow 2a + a - 3 = 8$$

$$\Rightarrow 3a - 3 = 8$$

$$\Rightarrow a = \frac{11}{3}$$

Substituting

$$a = \frac{11}{3} \text{ in } b = a - 3, \text{ we get}$$

$$\Rightarrow b = \frac{11}{3} - 3$$

$$\Rightarrow b = \frac{2}{3}$$

$$\therefore a = \frac{11}{3} \text{ and } b = \frac{2}{3}$$

$$(ii). \left(\frac{a}{4}, a - 2b \right) = (0, 6 + b)$$

Ans: Since, two ordered pairs are equal, if their corresponding first and second elements are equal.

$$\left(\frac{a}{4}, a - 2b \right) = (0, 6 + b)$$

$$\Rightarrow \frac{a}{4} = 0 \text{ and } a - 2b = 6 + b$$

$$\Rightarrow a = 0 \text{ and } 0 - 2b = 6 + b$$

$$\Rightarrow a = 0 \text{ and } b = -2$$

5.

$$A = \{1, 2, 3, 4, 5\}, S = \{(x, y) : x \in A, y \in A\},$$

then find the orders which satisfy the conditions given below.

$$(i). x + y = 5$$

Ans: Given: The sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } S = \{(x, y) : x \in A, y \in A\}.$$

Use the condition for all the elements in sets till the condition is satisfied.

The set of ordered pairs satisfying

$$x + y = 5 \text{ is } , \{(1, 4), (2, 3), (3, 2), (4, 1)\}.$$

$$(ii). x + y < 5$$

Ans: Given: The sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } S = \{(x, y) : x \in A, y \in A\}.$$

Use the condition for all the elements in sets till the condition is satisfied.

The set of ordered pairs satisfying

$$x + y < 5 \text{ is , } \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}.$$

$$(iii). x + y > 8$$

Ans: Given: The sets

$$A = \{1, 2, 3, 4, 5\} \text{ and } S = \{(x, y) : x \in A, y \in A\}.$$

Use the condition for all the elements in sets till the condition is satisfied.

The set of ordered pairs satisfying

$$x + y > 8 \text{ is , } \{(4, 5), (5, 4), (5, 5)\}.$$

6.If

$$R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\},$$

then find the domain and range of

$$R.$$

Ans: Given: Relation

$$R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}.$$

First, write the relation in roster form, then find the domain and range of

$$R.$$

$$R = \{(0, 5), (3, 4), (4, 3), (5, 0)\}.$$

Domain of

$$R$$

is set of first element of ordered pairs in

$$R = \{0, 3, 4, 5\}.$$

Range of

$$R$$

is set of second element of ordered pairs in

$$R = \{5, 4, 3, 0\}.$$

7.If

$$R_1 = \{(x, y) | y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}$$

is a relation. Then find the domain and range of

$$R_1.$$

Ans: Given: A relation

$$R_1 = \{(x, y) | y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}.$$

The value of

$$x$$

represents the domain and the values of

$$y \text{ for all } x.$$

Domain of

$$R_1 = \{-5 \leq x \leq 5, x \in R\},$$

$$\Rightarrow R_1 = [-5, 5]$$

.

Range of

$$R_1,$$

$$\text{when } x = -5, \text{ then } y = 2(-5) + 7 = -3$$

$$\text{when } x = 5, \text{ then } y = 2(5) + 7 = 17$$

Range of

$$R_1,$$

$$\Rightarrow R_1 = \{-3 \leq y \leq 17, y \in R\}$$

$$\Rightarrow R_1 = [-3, 17]$$

8.If

$$R_2 = \{(x, y) | x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$$

is a relation, then find the value of

$$R_2.$$

Ans: Given: A relation

$$R_2 = \{(x, y) | x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$$

.

Use the given condition in a relation and then write the set in roster form.

Since,

$$64$$

is the sum of square of

$$0 \text{ and } \pm 8.$$

$$\Rightarrow \text{when } x = 0, \text{ then } y^2 = 64,$$

$$\Rightarrow y = \pm 8$$

$$\Rightarrow x = 8, \text{ then } y^2 = 64 - (8)^2 = 0$$

$$\Rightarrow x = -8, \text{ then } y^2 = 64 - (-8)^2 = 0$$

$$\therefore R_2 = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$$

9.If

$$R_3 = \{(x, |x|) | x\}$$

is a real number is a relation, then find domain and range of

$$R_3.$$

Ans: Given: A relation

$$R_3 = \{(x, |x|) | x\}$$

is a real number.

The value of

$$x$$

represents the domain and the values of

$$y \text{ for all } x.$$

Domain of

$$R_3 = \text{real number}.$$

Since, the image of any real number under

$$R_3$$

is a positive real number or zero.

Range of

$$R_3 = R^+ \cup \{0\} \text{ or } (0, \infty).$$

10. Is the given relation a function? Give reason for your answer.

$$(i). h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$$

Ans: Given: Relations are given in set-builder and roster form.

A function is said to be a relation, only and only if every element in the domain has a unique image.

$$h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$$

Since,

$$3$$

has two images

$$9 \text{ and } 11.$$

So, it is not a function.

$$(ii). f = \{(x, x) | x\}$$

is a real number

Ans: Given: Relations are given in set-builder and roster form.

A function is said to be a relation, only and only if every element in the domain has a unique image.

$$f = \{(x, x) | x\}$$

is a real number

We observe that every element in the domain has a unique image. So, it is a function.

$$(iii). g = \left\{ \left(x, \frac{1}{x} \right) | x \right\}$$

is a positive integer

Ans: Given: Relations are given in set-builder and roster form.

A function is said to be a relation, only and only if every element in the domain has a unique image.

$$g = \left\{ \left(x, \frac{1}{x} \right) | x \text{ is a positive integer} \right\}$$

For every

$$x$$

, it is a positive integer and

$$\frac{1}{x}$$

is unique and distinct. Therefore, every element in the domain has a unique image. So, it is a function.

$$(iv). s = \{(x, x^2) | x\}$$

is a positive integer

Ans: Given: Relations are given in set-builder and roster form.

A function is said to be a relation, only and only if every element in the domain has a unique image.

$$s = \{(x, x^2) | x\}$$

is a positive integer

Since, the square of any positive integer is unique. So, every element in the domain has a unique image. Hence, it is a function.

$$(v).t = \{(x, 3)|x\}$$

is a real number

Ans: Given: Relations are given in set-builder and roster form.

A function is said to be a relation, only and only if every element in the domain has a unique image.

$$t = \{(x, 3)|x\}$$

is a real number

Since, every element in the domain has the image

$$3$$

. So, it is a constant function..

11.If f and g are real functions defined by

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Then find each of the following.

$$(i). f(3) + g(-5)$$

Ans: Given:

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Substitute the value of

$$x$$

in the functions

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

We have,

$$f(3) = (3)^2 + 7 = 16 \text{ and } g(-5) = 3(-5) + 5 = -10$$

$$\Rightarrow f(3) + g(-5) = 16 - 10$$

$$\therefore f(3) + g(-5) = 6$$

$$(ii). f\left(\frac{1}{2}\right) \times g(14)$$

Ans: Given:

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Substitute the value of

$$x$$

in the functions

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

We have,

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 7 = \frac{29}{4}$$

$$g(14) = 3(14) + 5 = 47$$

$$\therefore f\left(\frac{1}{2}\right) \times g(14) = \frac{29}{4} \times 47 = \frac{1363}{4}$$

$$(iii). f(-2) + g(-1)$$

Ans: Given:

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Substitute the value of

$$x$$

in the functions

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

We have,

$$f(-2) = (-2)^2 + 7 = 11,$$

$$g(-1) = 3(-1) + 5 = 2$$

$$\therefore f(-2) + g(-1) = 11 + 2 = 13$$

$$(iv). f(t) - f(-2)$$

Ans: Given:

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Substitute the value of

$$x$$

in the functions

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

We have,

$$f(t) = t^2 + 7,$$

$$f(-2) = (-2)^2 + 7 = 11$$

$$\therefore f(t) - f(-2) = t^2 - 4$$

Ans: Given:

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

Substitute the value of

$$x$$

in the functions

$$f(x) = x^2 + 7 \text{ and } g(x) = 3x + 5.$$

We have,

$$f(t) = t^2 + 7$$

$$f(5) = (5)^2 + 7 = 32$$

$$\therefore \frac{f(t) - f(5)}{t - 5}, \text{ if } t \neq 5$$

$$= \frac{t^2 + 7 - 32}{t - 5}$$

$$= \frac{t^2 - 25}{t - 5}$$

$$\begin{aligned} &= \frac{(t-5)(t+5)}{(t-5)} \\ &= t+5 \end{aligned}$$

12. Let

f and g

be real functions defined by

$$f(x) = 2x + 1 \text{ and } g(x) = 4x - 7.$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 4x - 7.$$

Substitute the real functions according to given condition and obtain the real value of

$$x.$$

We have,

$$\Rightarrow f(x) = g(x)$$

$$\Rightarrow 2x + 1 = 4x - 7$$

$$\Rightarrow x = 4$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 4x - 7.$$

Substitute the real functions according to given condition and obtain the real value of

$$x.$$

We have,

$$\Rightarrow f(x) < g(x)$$

$$\Rightarrow 2x + 1 < 4x - 7$$

$$\Rightarrow -2x + 1 < -7$$

$$\Rightarrow -2x < -8$$

$$\Rightarrow \frac{-2x}{-2} > \frac{-8}{-2}$$

$$\Rightarrow x > 4$$

13.If

f and g

are two real valued functions defined as

$$f(x) = 2x + 1 \text{ and } g(x) = 2x^2 + 1,$$

then find

$$(i). f + g$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 2x^2 + 1.$$

Directly perform the operation according to questions like addition, subtraction, multiplication and division.

We have,

$$\Rightarrow (f + g)(x) = f(x) + g(x)$$

$$\Rightarrow (f + g)(x) = 2x + 1 + x^2 + 1$$

$$\Rightarrow (f + g)(x) = x^2 + 2x + 2$$

$$(ii). f - g$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 2x^2 + 1.$$

Directly perform the operation according to questions like addition, subtraction, multiplication and division.

We have,

$$\Rightarrow (f - g)(x) = f(x) - g(x)$$

$$\Rightarrow (f - g)(x) = (2x + 1) - (x^2 + 1)$$

$$\Rightarrow (f - g)(x) = x(2 - x)$$

$$(iii). fg$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 2x^2 + 1.$$

Directly perform the operation according to questions like addition, subtraction, multiplication and division.

We have,

$$\Rightarrow (fg)(x) = f(x) \cdot g(x)$$

$$\Rightarrow (fg)(x) = (2x + 1)(x^2 + 1)$$

$$\Rightarrow (fg)(x) = 2x^3 + x^2 + 2x + 1$$

$$(iv). \frac{f}{g}$$

Ans: Given:

$$f(x) = 2x + 1 \text{ and } g(x) = 2x^2 + 1.$$

Directly perform the operation according to questions like addition, subtraction, multiplication and division.

We have,

$$\Rightarrow \left(\frac{f}{g} \right) (x) = \frac{f(x)}{g(x)}$$

$$\Rightarrow \left(\frac{f}{g} \right) (x) = \frac{2x + 1}{x^2 + 1}$$

14. Express the following functions as a set of ordered pairs and determine their range.

Ans: Given: A function

$$f : x \rightarrow R, f(x) = x^3 + 1, \text{ where } x = \{-1, 0, 3, 9, 7\}$$

.

Substitute the value of

$$x$$

in the function to obtain values of

y

as range.

We have,

$$f : X \rightarrow R, f(x) = x^3 + 1, \text{ where } x = \{-1, 0, 3, 9, 7\}$$

$$x = -1, \text{ then } f(-1) = (-1)^3 + 1$$

$$\Rightarrow f(-1) = 0$$

$$x = 0, \text{ then } f(0) = (0)^3 + 1$$

$$\Rightarrow f(0) = 1$$

$$\Rightarrow f(3) = 28$$

$$x = 9, \text{ then } f(9) = (9)^3 + 1$$

$$\Rightarrow f(9) = 730$$

$$x = 3, \text{ then } f(3) = (3)^3 + 1$$

$$x = 7, \text{ then } f(7) = (7)^3 + 1$$

$$\Rightarrow f(7) = 344$$

$$\Rightarrow f = \{(-1, 0), (0, 1), (3, 28), (9, 730), (7, 344)\}$$

$$\therefore \text{Range of } f = \{0, 1, 28, 730, 344\}$$

Question 15

Find the value of

 x

for which the functions

$$f(x) = 3x^2 - 1 \text{ and } g(x) = 3 + x$$

are equal.

Solution

Ans: Given:

$$f(x) = g(x)$$

Use the given condition, substitute the functions equal and then find

$$x$$

$$\Rightarrow f(x) = g(x)$$

$$\Rightarrow 3x^2 - 1 = 3 + x$$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow x(3x - 4) + 1(3x - 4)$$

$$\Rightarrow (3x - 4)(x + 1)$$

$$\Rightarrow x = -1, \frac{4}{3}$$

Long Answer Questions

Question 16

Is

$$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$$

a function, justify. If this is described by the relation, $g(x) = \alpha x + \beta$ then what values of should be assigned to α and β

Solution

Ans: Given: A relation

$$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$$

$$\text{and } g(x) = \alpha x + \beta$$

First, find the two equation by substitutions of different values of

$$x \text{ and } g(x).$$

We have,

$$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$$

Since, every element has a unique image under

$$g$$

. So,

$$g$$

is a function.

Now,

$$g(x) = \alpha x + \beta$$

$$x = 1, \text{ then } g(1) = \alpha(1) + \beta$$

$$\Rightarrow 1 = \alpha + \beta \quad (i)$$

$$x = 2, \text{ then } g(2) = \alpha(2) + \beta$$

$$\Rightarrow 3 = 2\alpha + \beta \quad (ii)$$

From (i) and (ii), we get

$$\alpha = 2, \beta = -1$$

Question 17

Find the domain of each of the following functions given by

$$(i). f(x) = \frac{1}{\sqrt{1 - \cos x}}$$

Solution

Ans: The value of

$$x$$

in the function represents the domain of functions.

We have,

$$f(x) = \frac{1}{\sqrt{1-\cos x}}$$

$$\Rightarrow -1 \leq \cos x \leq 1$$

$$\Rightarrow -1 \leq -\cos x \leq 1$$

$$\Rightarrow 0 \leq 1 - \cos x \leq 2$$

$f(x)$ is defined, if $1 - \cos x \neq 0$

$$\Rightarrow \cos x \neq 1$$

$$\Rightarrow \cos x \neq 2n\pi$$

Domain of $f = R - 2n\pi : n \in Z$

$$(ii). f(x) = \frac{1}{\sqrt{x+|x|}}$$

Ans: The value of

$$x$$

in the function represents the domain of functions.

We have,

$$f(x) = \frac{1}{\sqrt{x+|x|}}$$

$$\Rightarrow x + |x| = x - x$$

$$\Rightarrow 0, x < 0$$

$$\Rightarrow x + x = 2x, x \geq 0$$

Hence, domain of

$$f = R^+.$$

$$(iii). f(x) = x|x|$$

Ans: The value of

$$x$$

in the function represents the domain of functions.

We have,

$$f(x) = x|x|$$

Clearly, function is defined for all real numbers.

Domain of

$$f = R.$$

$$(iv). f(x) = \frac{x^3 - x + 3}{x^2 - 1}$$

Ans: The value of

$$x$$

in the function represents the domain of functions.

We have,

$$f(x) = \frac{x^3 - x + 3}{x^2 - 1}$$

$f(x)$ is not defined, if $x^2 - 1 = 0$

$$\Rightarrow (x - 1)(x + 1) = 0$$

$$\Rightarrow x = -1, 1$$

Domain of $f = R - \{-1, 1\}$.

$$(v). f(x) = \frac{3x}{28 - x}$$

Ans: The value of

$$x$$

in the function represents the domain of functions.

We have,

$$f(x) = \frac{3x}{28 - x}$$

$f(x)$ is defined, if $28 - x \neq 0$

$$\Rightarrow x \neq 28$$

Domain of $f = R - \{28\}$.

Question 18

Find the range of the following functions is given by

$$(i). f(x) = \frac{3}{2-x^2}$$

Solution

Ans: First, find the value of

$$x$$

in terms of

$$y, \text{ where } y = f(x).$$

Then, find the values of

$$y$$

for which

$$x$$

attain real values.

We have,

$$f(x) = \frac{3}{2-x^2}$$

$$\text{Let } y = f(x)$$

$$\text{then, } y = \frac{3}{2-x^2}$$

$$\Rightarrow x = \sqrt{\frac{2y-3}{y}}$$

$$\Rightarrow 2y - 3 \geq 0 \text{ and } y > 0$$

$$\Rightarrow y \geq \frac{3}{2}$$

$$\text{Range of } f = \left[\frac{3}{2}, \infty \right).$$

$$(ii). f(x) = 1 - |x - 2|$$

Ans: First, find the value of

$$x$$

in terms of

$$y, \text{ where } y = f(x).$$

Then, find the values of

$$y$$

for which

$$x$$

attain real values.

We have,

$$f(x) = 1 - |x - 2|$$

$$\Rightarrow |x - 2| \geq 0$$

$$\Rightarrow -|x - 2| \leq 0$$

$$\Rightarrow 1 - |x - 2| \leq 1$$

$$\Rightarrow f(x) \leq 1$$

$$\text{Range of } f = (-\infty, 1].$$

$$(iii). f(x) = |x - 3|$$

Ans: First, find the value of

$$x$$

in terms of

$$y, \text{ where } y = f(x).$$

Then, find the values of

$$y$$

for which

$$x$$

attain real values.

We have,

$$f(x) = |x - 3|$$

$$\Rightarrow |x - 3| \geq 0$$

$$\Rightarrow f(x) \geq 0$$

$$\text{Range of } f = [0, \infty).$$

$$(iv). f(x) = 1 + 3\cos 2x$$

Ans: First, find the value of

$$x$$

in terms of

$$y, \text{ where } y = f(x).$$

Then, find the values of

$$y$$

for which

$$x$$

attain real values.

We have,

$$f(x) = 1 + 3\cos 2x$$

$$\Rightarrow -1 \leq \cos 2x \leq 1$$

$$\Rightarrow -3 \leq 3\cos 2x \leq 3$$

$$\Rightarrow 1 - 3 \leq 1 + 3\cos 2x \leq 1 + 3$$

$$\Rightarrow -2 \leq f(x) \leq 4$$

Range of $f = [-2, 4]$.

Question 19

Redefine the function,

$$f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3.$$

Solution

Ans: Given: Function,

$$f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3.$$

First, find the interval in which

$$|x - 2| \text{ and } |2 + x|$$

is defined, then find the value of

$$f(x)$$

in that interval.

$$\text{Since, } |x - 2| = -(x - 2), x < 2$$

$$\Rightarrow x - 2, x \geq 2$$

$$\text{and, } |2 + x| = -(2 + x), x < -2$$

$$\Rightarrow (2 + x), x \geq -2$$

$$\therefore f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3$$

$$-(x - 2) - (2 + x), -3 \leq x < -2$$

$$-(x - 2) + 2 + x, -2 \leq x < 2$$

$$x - 2 + 2 + x, 2 \leq x \leq 3$$

$$-2x, -3 \leq x < -2$$

$$4, -2 \leq x < 2$$

$$2x, 2 \leq x \leq 3$$

Question 20

If

$$f(x) = \frac{x-1}{x+1},$$

then show that

$$(i). f\left(\frac{1}{x}\right) = -f(x)$$

Solution

Ans: Given:

$$f(x) = \frac{x-1}{x+1}.$$

Replace the value of

$$x$$

according to functions and then substitute them equal

Taking L.H.S,

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{-(x-1)}{1+x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = -f(x)$$

$$(ii). f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Ans: Given:

$$f(x) = \frac{x-1}{x+1}.$$

Replace the value of

$$x$$

according to functions and then substitute them equal.

Taking L.H.S

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1}$$

$$\Rightarrow f\left(-\frac{1}{x}\right) = \frac{-(x+1)}{x-1}$$

Now,

$$\frac{-1}{f(x)} = \frac{-1}{\frac{x-1}{x+1}}$$

$$\Rightarrow \frac{-1}{f(x)} = \frac{-(x+1)}{x-1}$$

$$\therefore f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Question 21

If

$$f(x) = \sqrt{x} \text{ and } g(x) = x$$

be two functions defined in the domain

$$R^+ \cup \{0\},$$

then find the value of

$$(i). (f+g)(x)$$

Solution

Ans: Given:

$$f(x) = \sqrt{x} \text{ and } g(x) = x$$

Use the two functions according to operations like addition, subtraction, multiplication and division.

Value of

$$(f + g)(x),$$

$$(f + g)(x) = f(x) + g(x)$$

$$\Rightarrow (f + g)(x) = \sqrt{x} + x$$

$$(ii). (f - g)(x)$$

Ans: Given:

$$f(x) = \sqrt{x} \text{ and } g(x) = x$$

Use the two functions according to operations like addition, subtraction, multiplication and division.

Value of

$$(f - g)(x),$$

$$(f - g)(x) = f(x) - g(x)$$

$$\Rightarrow (f - g)(x) = \sqrt{x} - x$$

$$(iii). (fg)(x)$$

Ans: Given:

$$f(x) = \sqrt{x} \text{ and } g(x) = x$$

Use the two functions according to operations like addition, subtraction, multiplication and division.

Value of

$$(fg)(x),$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\Rightarrow (fg)(x) = \sqrt{x} \cdot x$$

$$\Rightarrow (fg)(x) = x^{\frac{3}{2}}$$

$$(iv). \left(\frac{f}{g}\right)(x)$$

Ans: Given:

$$f(x) = \sqrt{x} \text{ and } g(x) = x$$

Use the two functions according to operations like addition, subtraction, multiplication and division.

Value of

$$\left(\frac{f}{g}\right)(x),$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\Rightarrow \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x}$$

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$$

Question 22

Find the domain and range of the function

$$f(x) = \frac{1}{\sqrt{x-5}}.$$

Solution

Ans: Given:

$$f(x) = \frac{1}{\sqrt{x-5}}.$$

The value of

$$x$$

represents the domain of function and the value of

$$y \text{ at } x.$$

$f(x)$ is defined, if $x - 5 > 0$

Domain of $f = (5, \infty)$.

Let, $f(x) = y$

$$\Rightarrow y = \frac{1}{\sqrt{x-5}}$$

$$\Rightarrow x = \frac{1}{y^2} + 5$$

Range of $f = \mathbb{R}^+$.

Question 23

If

$$f(x) = y = \frac{ax - b}{cx - a},$$

then prove that

$$f(y) = x.$$

Solution

Ans: Given:

$$f(x) = y = \frac{ax - b}{cx - a}.$$

Use the given condition in question and substitute the value of

$$y$$

in the function.

We have,

$$f(x) = y = \frac{ax - b}{cx - a}$$

$$f(y) = \frac{ay - b}{cy - a}$$

$$\Rightarrow f(y) = \frac{a \left(\frac{ax - b}{cx - a} \right) - b}{c \left(\frac{ax - b}{cx - a} \right) - a}$$

$$\Rightarrow f(y) = \frac{a^2x - ab - bcx + ab}{acx - bc - acx + a^2}$$

$$\Rightarrow f(y) = \frac{a^2x - bcx}{a^2 - bc}$$

$$\Rightarrow f(y) = \frac{x(a^2 - bc)}{(a^2 - bc)}$$

$$\Rightarrow f(y) = x.$$

Hence proved.

Objective Type Questions:

Question 24

Let

$$n(A) = m \text{ and } n(B) = n.$$

Then, total number of non-empty relations that can be defined from

A to B

is

(A).

$$m^n$$

(B).

$$n^m - 1$$

(C).

$$mn - 1$$

(D).

$$2^{mn} - 1$$

Solution

Ans: Given:

$$n(A) = m \text{ and } n(B) = n.$$

First, find the number of elements in

$$A \times B$$

and then find the number of relation by using

$$2^{n(A \times B)} - 1.$$

We have,

$$n(A) = m \text{ and } n(B) = n$$

$$n(A \times B) = n(A) \cdot n(B)$$

$$n(A \times B) = mn$$

Total number of relations from

$$A \text{ to } B = 2^{mn} - 1.$$

Correct Answer: D

Question 25

If

$$[x]^2 - 5[x] + 6 = 0, \text{ where } [.]$$

represents the greatest integer function, then

$$(A) x \in [3, 4]$$

$$(B) x \in (2, 3]$$

$$(C) x \in [2, 3]$$

$$(D) x \in [2, 4)$$

Solution

Ans: Given:

$$[x]^2 - 5[x] + 6 = 0, \text{ where } [.]$$

represents Greatest integer function.

If

$$a \text{ and } b$$

are two successive positive integer and

$$[x] = a, b, \text{ then } x \in [a, b].$$

We have,

$$[x]^2 - 5[x] + 6 = 0$$

$$\Rightarrow [x]^2 - 3[x] + 2[x] + 6 = 0$$

$$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$$

$$\Rightarrow ([x] - 3)([x] - 2) = 0$$

$$\Rightarrow [x] = 2, 3$$

$$\therefore x \in [2, 3]$$

Correct Answer: C

Question 26

Range of

$$f(x) = \frac{1}{1 - 2\cos x}$$

is

$$(A) \left[\frac{1}{3}, 1 \right]$$

$$(B) \left[-1, \frac{1}{3} \right]$$

$$(C)(-\infty, -1] \cup \frac{1}{3}, \infty)$$

$$(D) \left[-\frac{1}{3}, 1\right]$$

Solution

Ans: Given:

$$f(x) = \frac{1}{1-2\cos x}$$

Range of function represents the value the function attains. Start with the range of functions you know.

Range of cosine function is,

$$\Rightarrow -1 \leq -\cos x \leq 1$$

$$\Rightarrow -2 \leq -2\cos x \leq 2$$

$$\Rightarrow 1 - 2 \leq 1 - 2\cos x \leq 1 + 2$$

$$\Rightarrow -1 \leq 1 - 2\cos x \leq 3$$

$$\Rightarrow -1 \leq \frac{1}{1-\cos 2x} \leq \frac{1}{3}$$

$$\Rightarrow -1 \leq f(x) \leq \frac{1}{3}$$

$$\text{Range of } f = \left[-1, \frac{1}{3}\right].$$

Correct Answer: B

Question 27

Let

$$f(x) = \sqrt{1+x^2},$$

then

$$(A) f(xy) = f(x) \cdot f(y)$$

$$(B) f(xy) \geq f(x) \cdot f(y)$$

$$(C) f(xy) \leq f(x) \cdot f(y)$$

(D) None of these

Solution

Ans: Given: Function

$$f(x) = \sqrt{1+x^2}.$$

Replace the variables of function as

$$x \rightarrow xy$$

and then compare.

We have,

$$f(x) = \sqrt{1+x^2}$$

$$f(xy) = \sqrt{1+x^2y^2}$$

$$f(x) \cdot f(y) = \sqrt{1+x^2} \cdot \sqrt{1+y^2}$$

$$f(x) \cdot f(y) = \sqrt{(1+x^2)(1+y^2)}$$

$$f(x) \cdot f(y) = \sqrt{1+x^2+y^2+x^2y^2}$$

$$\sqrt{1+x^2y^2} \leq \sqrt{1+x^2} \cdot \sqrt{1+y^2}$$

$$\Rightarrow f(xy) \leq f(x) \cdot f(y).$$

Correct Answer: C

Question 28

Domain of

$$\sqrt{a^2 - x^2} (a > 0) \text{ is}$$

$$(A) (-a, a)$$

$$(B) [-a, a]$$

$$(C) [0, a]$$

$$(D) (-a, 0]$$

Solution

Ans: Given:

$$f(x) = \sqrt{a^2 - x^2}$$

Domain of the function is the value of

$$x$$

for which the function is defined.

$$\text{Let } f(x) = \sqrt{a^2 - x^2}$$

$$f(x) \text{ is defined, if } a^2 - x^2 \geq 0$$

$$\Rightarrow x^2 - a^2 \leq 0$$

$$\Rightarrow (x - a)(x + a) \leq 0$$

$$\Rightarrow -a \leq x \leq a$$

$$\therefore \text{Domain of } f = [-a, a].$$

Correct Answer: B

Question 29

If

$$f(x) = ax + b,$$

where **a** and **b** are integers,

$$f(-1) = -5 \text{ and } f(3) = 3$$

, then **a** and **b** are equal to

$$(A) a = -3, b = -1$$

$$(B) a = 2, b = -3$$

$$(C) a = 0, b = 2$$

$$(D) a = 2, b = 3$$

Solution

Ans: Given:

$$f(x) = ax + b$$

$$f(-1) = -5$$

$$f(3) = 3$$

First, substitute the values in functions. Solve equations for

a and b .

$$\text{for, } f(x) = ax + b$$

$$\Rightarrow f(-1) = a(-1) + b$$

$$\Rightarrow -5 = -a + b \quad (i)$$

and ,

$$f(3) = a(3) + b$$

$$\Rightarrow 3 = 3a + b \quad (ii)$$

from (i) and (ii), we get

$$a = 2 \text{ and } b = -3$$

Correct answer: B

Question 30

The domain of the function

f

defined by

$$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

is equal to

$$(A) (-\infty, -1) \cup (1, 4]$$

$$(B)(-\infty, -1] \cup (1, 4]$$

$$(C)(-\infty, 1) \cup [1, 4]$$

$$(D)(-\infty, 1) \cup [1, 4)$$

Solution

Ans: Given: Function

$$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

Domain of the function is the value of

$$x$$

for which the function is defined.

$f(x)$ is defined, if

$$4 - x \geq 0 \text{ or } x^2 - 1 > 0$$

$$x - 4 \leq 0 \text{ or } (x + 1)(x - 1) > 0$$

$$x \leq 4 \text{ or } x < -1 \text{ and } x > 1$$

$$\therefore \text{Domain of } f = (-\infty, -1) \cup (1, 4].$$

Correct Answer: A

Question 31

The domain and range of the real functions

$$f$$

defined by

$$f(x) = \frac{4-x}{x-4}$$

is given by

$$(A) \text{Domain} = R, \text{Range} = \{-1, 1\}$$

$$(B) \text{Domain} = R - \{1\}, \text{Range} = R$$

$$(C) \text{Domain} = R - \{4\}, \text{Range} = \{-1\}$$

$$(D) \text{Domain} = R - \{-4\}, \text{Range} = \{-1, 1\}$$

Solution

Ans: Given:

$$f(x) = \frac{4-x}{x-4}$$

.

A rational function

$$\frac{f(x)}{g(x)}$$

is defined if

$$g(x) \neq 0.$$

$f(x)$ is defined, if $x - 4 \neq 0$ or $x \neq 4$

$\therefore$ Domain of $f = R - \{4\}$.

$$\text{Let } f(x) = y$$

$$\Rightarrow y = \frac{4-x}{x-4}$$

$$\Rightarrow xy - 4y = 4 - x$$

$$\Rightarrow xy + x = 4 + 4y$$

$$\Rightarrow x(y+1) = 4(1+y)$$

$$\Rightarrow x = \frac{4(1+y)}{y+1}$$

x is defined, if $y + 1 \neq 0, y \neq -1$

$\therefore$ Range of $f = R - \{-1\}$.

Correct Answer: C

Question 32

The domain and range of real function

$$f$$

defined by

$$f(x) = \sqrt{x-1}$$

is given by

$$(A) \text{ Domain} = (1, \infty), \text{ Range} = (0, \infty)$$

$$(B) \text{ Domain} = [1, \infty), \text{ Range} = (0, \infty)$$

$$(C) \text{ Domain} = (1, \infty), \text{ Range} = [0, \infty)$$

$$(D) \text{ Domain} = [1, \infty), \text{ Range} = [0, \infty)$$

Solution

Ans: Given:

$$f(x) = \sqrt{x-1}$$

.

A square root function

$$f(x) = \sqrt{x}$$

is defined by

$$x \geq 0.$$

$$f(x) \text{ is defined, if } x - 1 \geq 0$$

$$\Rightarrow x \geq 1$$

$$\therefore \text{ Domain of } f = [1, \infty).$$

$$\text{Let } f(x) = y$$

$$\Rightarrow y = \sqrt{x-1}$$

$$\Rightarrow y^2 = x - 1$$

$$\Rightarrow x = y^2 + 1$$

$$x \text{ is defined for } y \in \mathbb{R}.$$

$$\text{but, } y \geq 0$$

$$\therefore \text{ Range of } f = [0, \infty).$$

Correct Answer: D

Question 33

The domain of function

$$f$$

given by

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - x - 6}.$$

$$(A) R - \{3, -2\}$$

$$(B) R - \{-3, 2\}$$

$$(C) R - [3, -2]$$

$$(D) R - (3, -2)$$

Solution

Ans: Given:

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - x - 6}.$$

The domain is the value of

$$x$$

for which function is defined.

$$f(x) \text{ is defined, if } x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x - 3) + 2(x - 3) = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3, -2$$

$\therefore$ Domain of $f = R - \{3, -2\}$.

Correct Answer: A

Question 34

The domain and range of the function

f

given by

$$f(x) = 2 - |x - 5|$$

is

(A) Domain = R^+ , Range = $(-\infty, 1]$

(B) Domain = R , Range = $(-\infty, 2]$

(C) Domain = R , Range = $(-\infty, 2)$

(D) Domain = R^+ , Range = $(-\infty, 2]$

Solution

Ans: Given:

$$f(x) = 2 - |x - 5|.$$

The absolute modulus function

$$|x| \geq 0.$$

$f(x)$ is defined for all $x \in R$

$\therefore$ Domain of $f = R$.

$$\text{Now, } |x - 5| \geq 0$$

$$\Rightarrow -|x - 5| \leq 0$$

$$\Rightarrow 2 - |x - 5| \leq 2$$

$$\Rightarrow f(x) \leq 2$$

$\therefore$ Range of $f = (-\infty, 2]$.

Correct Answer: B

Question 35

The domain for which the functions defined by

$$f(x) = 3x^2 - 1 \text{ and } g(x) = 3 + x$$

are equal to

$$(A) \left[-1, \frac{4}{3} \right]$$

$$(B) \left[1, \frac{4}{3} \right]$$

$$(C) \left[-1, -\frac{4}{3} \right]$$

$$(D) \left[-2, -\frac{4}{3} \right]$$

Solution

Ans: Given:

$$f(x) = 3x^2 - 1 \text{ and } g(x) = 3 + x.$$

Substitute the functions equal and then find

$$x.$$

$$\text{For, } f(x) = g(x)$$

$$\Rightarrow 3x^2 - 1 = 3 + x$$

$$\Rightarrow 3x^2 - x - 4 = 0$$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow x(3x - 4) + 1(x + 1) = 0$$

$$\Rightarrow (3x - 4)(x + 1) = 0$$

$$\Rightarrow x = -1, \frac{4}{3}$$

$\therefore$ Domain for $f(x) = g(x)$ is $\left[-1, \frac{4}{3}\right]$.

Correct Answer: A

Fill in the blanks:

Question 36

Let

f and g

be two real functions given by

$$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\} \text{ and } g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\},$$

then the domain of

$f \cdot g$

is given by

Solution

Ans: Given:

$$f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)\},$$

$$g = \{(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)\}.$$

First, find the domain of

f

and domain of

g .

Then, domain of

$$f \cdot g = \text{Domain of } f \cap \text{Domain of } g.$$

$$\text{Domain of } f = \{0, 2, 3, 4, 5\},$$

$$\text{Domain of } g = \{1, 2, 3, 4, 5\}$$

$$\therefore \text{Domain of } (f \cdot g) = \text{Domain of } f \cap \text{Domain of } g$$

$$\Rightarrow \text{Domain of } (f \cdot g) = \{2, 3, 4, 4\}.$$

Question 37

Let

$$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\} \text{ and } g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$$

be two real functions. Then match the following.

(a) $f - g$	(i) $\left\{ \left(2, \frac{4}{5} \right), \left(8, \frac{-1}{4} \right), \left(10, \frac{-3}{13} \right) \right\}$
(b) $f + g$	(ii) $\{(2, 20), (8, -4), (10, -39)\}$
(c) $f \cdot g$	(iii) $\{(2, -1), (8, -5), (10, -16)\}$
(d) $\frac{f}{g}$	(iv) $\{(2, 9), (8, 3), (10, -10)\}$

The domain of

$$f - g, f + g, f \cdot g, \frac{f}{g}$$

is domain of

$$f \cap$$

domain of

$$g.$$

Then, find their images.

Solution

Ans: Given:

$$f = \{(2, 4), (5, 6), (8, -1), (10, -3)\},$$

$$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, 5)\}$$

First, find the domain of

$$f - g, f + g, f \cdot g, \frac{f}{g}.$$

Then, find their images by using a domain.

Domain of $f - g, f + g, f \cdot g, \frac{f}{g}$ (is domain of $f \cap$ domain of g).

$$\Rightarrow \{2, 5, 8, 10\} \cap \{2, 7, 8, 10, 11\} = \{2, 8, 10\}.$$

$$(a)(f - g)(2) = f(2) - g(2)$$

$$(f - g)(2) = 4 - 5$$

$$\Rightarrow (f - g)(2) = -1$$

$$(f - g)(8) = f(8) - g(8)$$

$$(f - g)(8) = -1 - 4$$

$$\Rightarrow (f - g)(8) = -5$$

$$(f - g)(10) = f(10) - g(10)$$

$$(f - g)(10) = -3 - 13$$

$$\Rightarrow (f - g)(10) = -16$$

$$\therefore f - g = \{(2, -1), (8, -5), (10, -16)\}.$$

$$(b)(f + g)(2) = f(2) + g(2)$$

$$(f + g)(2) = 4 + 5$$

$$\Rightarrow (f + g)(2) = 9$$

$$(f + g)(8) = f(8) + g(8)$$

$$(f + g)(8) = -1 + 4$$

$$\Rightarrow (f + g)(8) = 3$$

$$(f + g)(10) = f(10) + g(10)$$

$$(f + g)(10) = -3 + 13$$

$$\Rightarrow (f + g)(10) = 10$$

$$\therefore f + g = \{(2, 9), (8, 3), (10, 10)\}.$$

$$(c)(f \cdot g)(2) = f(2) \cdot g(2)$$

$$(f \cdot g)(2) = 4 \times 5$$

$$\Rightarrow (f \cdot g)(2) = 20$$

$$(f \cdot g)(8) = f(8) \cdot g(8)$$

$$(f \cdot g)(8) = -1 \times 4$$

$$\Rightarrow (f \cdot g)(8) = -4$$

$$(f \cdot g)(10) = f(10) \cdot g(10)$$

$$(f \cdot g)(10) = -3 \times 13$$

$$\Rightarrow (f \cdot g)(10) = -39$$

$$\therefore f \cdot g = \{(2, 20), (8, -4), (10, -39)\}.$$

$$(d) \left(\frac{f}{g} \right) (2) = \frac{f(2)}{g(2)}$$

$$\Rightarrow \left(\frac{f}{g} \right) (2) = \frac{4}{5}$$

$$\left(\frac{f}{g} \right) (8) = \frac{f(8)}{g(8)}$$

$$\Rightarrow \left(\frac{f}{g} \right) (8) = \frac{-1}{4}$$

$$\left(\frac{f}{g} \right) (10) = \frac{f(10)}{g(10)}$$

$$\Rightarrow \left(\frac{f}{g} \right) (10) = \frac{-3}{13}$$

$$\therefore \frac{f}{g} = \{(2, \frac{4}{5}), (8, \frac{-1}{4}), (10, \frac{-3}{13})\}.$$

Hence, the correct matches are

$$(a) \rightarrow (iii), (b) \rightarrow (iv), (c) \rightarrow (ii), (d) \rightarrow (i).$$

State True or False for the following statements given in Exercises 38 to 42:

Question 38

The ordered pair

$$(5, 2)$$

belongs to the relation

$$R = \{(x, y) : y = x - 5, x, y \in \mathbb{Z}\}.$$

Solution

Ans: Given: Ordered pair

$$(5, 2).$$

Relation

$$R = \{(x, y) : y = x - 5, x, y \in \mathbb{Z}\}.$$

The ordered pair must satisfy the relation.

$$R = \{(x, y) : y = x - 5, x, y \in \mathbb{Z}\}$$

If $x = 5$, then

$$y = 5 - 5 = 0.$$

Hence,

$$(5, 2)$$

does not belong to

$$R.$$

Question 39

If

$$P = \{1, 2\},$$

then

$$P \times P \times P = \{(1, 1, 1), (2, 2, 2), (1, 2, 2), (2, 1, 1)\}.$$

Solution

Ans: Given:

$$P = \{1, 2\}.$$

First, find the total number of elements

$$n(P \times P \times P).$$

Then, compare.

$$P = \{1, 2\} \text{ and } n(P) = 2$$

$$\therefore n(P \times P \times P) = 8$$

But there are 4 elements

Therefore, False

Question 40

If

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\},$$

then

$$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

Solution

Ans: Given:

$$A = \{1, 2, 3\},$$

$$B = \{3, 4\},$$

$$C = \{4, 5, 6\}.$$

First, find

$$A \times B \text{ and } A \times C,$$

then find

$$(A \times B) \cup (A \times C).$$

$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\},$$

$$A \times C = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\},$$

$$(A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}.$$

Therefore, True.

Question 41

If

$$(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$$

are two equal ordered pairs, then

$$x = 4, y = \frac{-14}{3}.$$

Solution

Ans: Given: Ordered Pair

$$(x - 2, y + 5) = \left(-2, \frac{1}{3}\right).$$

Compare the elements of two ordered pairs. Then , find

x and y .

$$(x - 2, y + 5) = \left(-2, \frac{1}{3}\right)$$

$$\Rightarrow x - 2 = -2 \text{ and } y + 5 = \frac{1}{3}$$

$$\Rightarrow x = 0 \text{ and } y = \frac{-14}{3}.$$

Therefore, False.

Question 42

If

$$A \times B = \{(a, x), (a, y), (b, x), (b, y)\},$$

then

$$A = \{a, b\} \text{ and } B = \{x, y\}.$$

Solution

Ans: Given:

$$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}.$$

First element and second element in

$$A \times B$$

forms set

$$A \text{ and } B.$$

$$A =$$

Set of first elements of ordered pair in

$$A \times B = \{a, b\}.$$

$$B =$$

Set of second elements of ordered pair in

$$A \times B = \{x, y\}.$$

Therefore, True.

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Some Solved Examples

1. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$. Determine

(i) $A \times B$

(ii) $B \times A$

(iii) $B \times B$

(iv) $A \times A$

Given that: $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$

(i) $A \times B = \{-1, 2, 3\} \times \{1, 3\} = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

(ii) $B \times A = \{1, 3\} \times \{-1, 2, 3\} = \{(1, -1), (3, -1), (1, 2), (3, 2), (1, 3), (3, 3)\}$

(iii) $B \times B = \{1, 3\} \times \{1, 3\} = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

(iv) $A \times A = \{-1, 2, 3\} \times \{-1, 2, 3\} = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

2. Given $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$, find the domain and range of R .

Given that: $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$

So, the ordered pairs satisfying the given condition $x^2 + y^2 = 25$ are $(0, 5), (3, 4), (5, 0), (4, 3)$

Hence, the domain = $\{0, 3, 4, 5\}$ and the range = $\{0, 3, 4, 5\}$.

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