

Chapter 16: Probability

NCERT Exemplar · Class 11 · Maths

Textbook exercises

Question 1

If the letters of the word ALGORITHM are arranged in a row, what is the probability the letters GOR must remain together.

Solution

Ans: Given: The letters of the word ALGORITHM are arranged.

Find the total number of sample spaces by finding the number of words that can be formed with the given letters of the word.

Now, find the favourable number of arrangements by considering the letters GOR always together.

There are 9 different letters in the word ALGORITHM, so the total number of words that can be formed is the number of sample spaces $n(S) = 9!$.

Now, the letters GOR always remain together. These can be considered as a single unit, so the remaining 6 letters and this single unit makes a total of 7 things which can be arranged in $7!$ ways.

So the number of favourable arrangements is $n(E) = 7!$.

Hence, the required probability

$$\begin{aligned} &= \frac{n(S)}{n(E)} \\ &= \frac{7!}{9!} \\ &= \frac{1}{72} \end{aligned}$$

Question 2

Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

Solution

Ans: Given: Total number of new employees = 6

Total number of desks in the assigned row = 6

A married couple is present among the 6 employees.

Find the total number of sample space by arranging all the six employees without any condition.

Now, find the number of arrangements in which the couple always gets the adjacent seat. Subtract it from the total number of sample spaces to get the favourable sample spaces.

The 6 new employees can be arranged at 6 desks (without any condition) in $6!$ ways, so the number of sample space is $n(S) = 6!$.

Let us consider that the couple gets the adjacent seat, so the two people in this couple can be arranged among themselves in $2!$ ways. This couple can be considered as a single unit, so the remaining four employees and this single unit can be considered as 5 things which can be arranged in $5!$ ways.

The total number of arrangements where the couple is always together = $2! \times 5!$.

Therefore, the number of arrangements where the couple is never together $n(E) = 6! - (2! \times 5!)$.

Hence, the required probability

$$\begin{aligned} &= \frac{n(E)}{n(S)} \\ &= \frac{6! - (2! \times 5!)}{6!} \\ &= \frac{2}{3} \end{aligned}$$

Question 3

Suppose an integer from 1 to 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

Solution

Ans: Given: An integer from 1 to 1000 is chosen at random.

Find the total number of sample space by selecting any one integer out of a total of 1000 integers given.

Find the numbers that are divisible by 2 or 9 and count the number of integers which are divisible by both to get the favourable sample spaces.

The total number of integers is from 1 to 1000 is thousand, so the number of ways in which one can be selected is the total number of sample space $n(S) = {}^{1000}C_1$.

Now, all the even integers are divisible by 2, so there will be 500 even numbers from 1 to 1000 and any one of them can be selected in ${}^{500}C_1$ ways. The integers that are divisible by 9 are 9, 18, 27, 36, ..., 999. There are 111 such integers divisible by 9.

Among these 111 integers there will be integers like 18, 36, 54, ... etc. which are also divisible by 2 and they are already considered earlier. There are 55 such integers. So the number of integers that is to be considered is $(111 - 55) = 56$ and any one of them can be chosen in ${}^{56}C_1$ ways.

Therefore, the number of favourable sample spaces $n(E) = {}^{500}C_1 + {}^{56}C_1$.

Hence, the required probability is

$$\begin{aligned} &= \frac{n(E)}{n(S)} \\ &= \frac{{}^{500}C_1 + {}^{56}C_1}{{}^{1000}C_1} \\ &= 0.556 \end{aligned}$$

Question 4

An experiment consists of rolling a die until a 2 appears.

(i) How many elements of the sample space correspond to the event that 2 appears on the k^{th} roll of the die?

(ii) How many elements of the sample space correspond to the event that 2 appears not later than the k^{th} roll of the die?

Solution

Ans: Given: A die is rolled until a 2 appears.

consider that the first $(k - 1)$ rolls of the die results in any of the five outcomes except 2.

There are 6 outcomes when a die is rolled.

In this case 2 appears on the k^{th} roll, so in the first $(k - 1)$ roll there are 5 possible outcomes other than 2 on each roll.

Therefore, the number of elements $= 5 \times 5 \times 5 \times 5 \times \dots (k - 1) \text{ times} = 5^{k-1}$.

Ans: Given: A die is rolled until a 2 appears.

consider different situations where 2 may appear in the first or second or third time etc. Add the number of elements of different situations.

In this case 2 appears before the k^{th} roll, so it may appear on the first roll or second roll or third roll or ... $(k - 1)^{th}$ roll.

If it appears on the first roll then the number of elements = 1.

If it appears on the second roll that means in the first roll there are 5 outcomes and in the second roll 1, so the number of elements = 5×1 .

If it appears on the third roll that means in the first two rolls there are 5 outcomes each and in the third roll 1, so the number of elements = $5 \times 5 \times 1$.

Observing the pattern, the number of elements in $(k - 1)$ rolls = $5 \times 5 \times 5 \times 5 \times \dots (k - 1) \text{ times} \times 1$.

Hence, the total number of elements of all the different situations = $1 + 5 + 5^2 + 5^3 + \dots 5^{(k-1)}$.

This is a geometric progression whose sum of the terms = $\left(\frac{5^k - 1}{4} \right)$.

Question 5

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution

Ans: Given: Number of times the die is rolled = 1

Probability of occurrence of an odd number is twice that of an even number.

G is the event of occurrence of a number greater than 3 on a single roll.

Find the total number of sample spaces when a die is rolled. Consider the odd numbers likely to occur twice.

Find the favourable sample spaces by considering the numbers 4, 5 or 6 to occur.

When a die is rolled the number of outcomes is 6 which are $\{1, 2, 3, 4, 5, 6\}$. It is said that each odd number is likely to occur twice of the occurrence of each even number.

So the sample spaces can be $\{1, 1, 2, 3, 3, 4, 5, 5, 6\}$. Here $n(S) = 9$.

The numbers greater than 3 are 4, 5 and 6, so the favourable sample spaces to occur are $\{4, 5, 5, 6\}$. Here $n(E) = 4$.

Hence, the required probability $P(G) = \frac{n(S)}{n(E)} = \frac{4}{9}$.

Question 6

In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?

Solution

Ans: Given: Probability that the chosen family owns a colour television set = 0.87

Probability that the chosen family owns a black and white television set = 0.36

Probability that the chosen family owns both the sets = 0.30

Assume $P(C)$ and $P(B)$ as the probabilities that the chosen family owns a colour television set and a black and white television set respectively.

Use the formula $P(B \cup C) = P(B) + P(C) - P(B \cap C)$.

Probability that the chosen family owns a colour television set = $P(C) = 0.87$.

Probability that the chosen family owns a black and white television set = $P(B) = 0.36$.

Probability that the chosen family owns both the sets = $P(B \cap C) = 0.30$.

Using the formula $P(B \cup C) = P(B) + P(C) - P(B \cap C)$, the probability that the chosen family owns either anyone or both kinds of sets

$$= P(B \cup C)$$

$$= P(B) + P(C) - P(B \cap C)$$

$$= 0.87 + 0.36 - 0.30$$

$$= 0.93$$

Question 7

If A and B are mutually exclusive events, $P(A) = 0.35$, $P(B) = 0.45$, find

(a) $P(A')$

Solution

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

Here $P(A')$ means the probability that the event A does not occur.

$$\Rightarrow P(A') = 1 - P(A)$$

$$\Rightarrow P(A') = 1 - 0.35$$

$$\Rightarrow P(A') = 0.65$$

$$(b) P(B')$$

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

Here $P(B')$ means the probability that the event B does not occur.

$$\Rightarrow P(B') = 1 - P(B)$$

$$\Rightarrow P(B') = 1 - 0.45$$

$$\Rightarrow P(B') = 0.55$$

$$(c) P(A \cup B)$$

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

Here $P(A \cup B)$ is the probability of occurrence of either A or B . Using the formula

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $P(A \cap B)$ is the probability of occurrence of A and B simultaneously.

$$\Rightarrow P(A \cup B) = 0.45 + 0.35 - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.80 - P(A \cap B)$$

Since the events A and B are mutually exclusive that means they cannot occur simultaneously. So $P(A \cap B) = 0$.

$$\Rightarrow P(A \cup B) = 0.80$$

$$(d) P(A \cap B)$$

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

Since the events A and B are mutually exclusive that means they cannot occur simultaneously. So $P(A \cap B) = 0$.

$$(e) P(A \cap B')$$

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

$$\text{Using the formula } P(A \cap B') = P(A) - P(A \cap B),$$

$$\Rightarrow P(A \cap B') = 0.35 - 0$$

$$\Rightarrow P(A \cap B') = 0.35$$

$$(f) P(A' \cap B')$$

Ans: Given: Two mutually exclusive events A and B .

$$P(A) = 0.35$$

$$P(B) = 0.45$$

Use the formulas: -

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = 0 \text{ (for mutually exclusive events)}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A' \cap B') = P(A \cup B)'$$

$$\text{Using the formula } P(A' \cap B') = P(A \cup B)',$$

$$\Rightarrow P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow P(A' \cap B') = 1 - 0.80$$

$$\Rightarrow P(A' \cap B') = 0.20$$

Question 8

A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple, very simple are respectively, 0.15, 0.20, 0.31, 0.26, 0.08. Find the probabilities that a particular surgery will be rated

(a) Complex or very complex.

(b) Neither very complex nor very simple.

(c) Routine or complex

(d) Routine or simple

Solution

Ans: Given: Probabilities that the surgery is very complex = 0.15

Probabilities that the surgery is complex = 0.20

Probabilities that the surgery is routine = 0.31

Probabilities that the surgery is simple = 0.26

Probabilities that the surgery is very simple = 0.08

Assume $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of different surgeries. Use the formulas of set theory to find the probabilities of different combinations.

Assuming $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of surgeries rated very complex, complex, routine, simple and very simple respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

Probability that a particular surgery is rated complex or very complex is $P(A \cup B)$.

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.15 + 0.20 - 0$$

$$\Rightarrow P(A \cup B) = 0.35$$

Ans: Given: Probabilities that the surgery is very complex = 0.15

Probabilities that the surgery is complex = 0.20

Probabilities that the surgery is routine = 0.31

Probabilities that the surgery is simple = 0.26

Probabilities that the surgery is very simple = 0.08

Assume $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of different surgeries. Use the formulas of set theory to find the probabilities of different combinations.

Assuming $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of surgeries rated very complex, complex, routine, simple and very simple respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

Probability that a particular surgery is rated neither very complex nor very simple is $P(A' \cap E')$.

$$\Rightarrow P(A' \cap E') = 1 - P(A \cup E)$$

$$\Rightarrow P(A' \cap E') = 1 - (P(A) + P(E) - P(A \cap E))$$

$$\Rightarrow P(A' \cap E') = 1 - (0.15 + 0.08 - 0)$$

$$\Rightarrow P(A' \cap E') = 0.77$$

Ans: Given: Probabilities that the surgery is very complex = 0.15

Probabilities that the surgery is complex = 0.20

Probabilities that the surgery is routine = 0.31

Probabilities that the surgery is simple = 0.26

Probabilities that the surgery is very simple = 0.08

Assume $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of different surgeries. Use the formulas of set theory to find the probabilities of different combinations.

Assuming $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of surgeries rated very complex, complex, routine, simple and very simple respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

Probability that a particular surgery is rated routine or complex is $P(B \cup C)$.

$$\Rightarrow P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$\Rightarrow P(B \cup C) = 0.20 + 0.31 - 0$$

$$\Rightarrow P(B \cup C) = 0.51$$

Ans: Given: Probabilities that the surgery is very complex = 0.15

Probabilities that the surgery is complex = 0.20

Probabilities that the surgery is routine = 0.31

Probabilities that the surgery is simple = 0.26

Probabilities that the surgery is very simple = 0.08

Assume $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of different surgeries. Use the formulas of set theory to find the probabilities of different combinations.

Assuming $P(A)$, $P(B)$, $P(C)$, $P(D)$ and $P(E)$ as the probabilities of surgeries rated very complex, complex, routine, simple and very simple respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

Probability that a particular surgery is rated routine or simple is $P(C \cup D)$.

$$\Rightarrow P(C \cup D) = P(C) + P(D) - P(C \cap D)$$

$$\Rightarrow P(C \cup D) = 0.31 + 0.26 - 0$$

$$\Rightarrow P(C \cup D) = 0.57$$

Question 9

Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B , B and C are given about the same chance of being selected, while C is twice as likely to be selected as D , what are the probabilities that

(a) C will be selected?

(b) A will not be selected?

Solution

Ans: Given: Total number of candidates = 4 $\{A, B, C, D\}$

Probability of selection of $A = 2 \times$ Probability of selection of B

Probability of selection of $B =$ Probability of selection of C

Probability of selection of $C = 2 \times$ Probability of selection of D

Assume $P(A)$, $P(B)$, $P(C)$ and $P(D)$ as the probabilities of selection of the given candidates. Form relations between them and use the formula $P(A) + P(B) + P(C) + P(D) = 1$ to find their values.

Assuming $P(A)$, $P(B)$, $P(C)$ and $P(D)$ as the probabilities of selection of the candidates A, B, C and D respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

According to the information given,

$$P(A) = 2P(B) \dots (1)$$

$$P(B) = P(C) \dots (2)$$

$$P(C) = 2P(D) \dots (3)$$

Using the formula $P(A) + P(B) + P(C) + P(D) = 1$ and solving the three equations,

$$P(A) = \frac{4}{9}, P(B) = \frac{2}{9}, P(C) = \frac{2}{9} \text{ and } P(D) = \frac{1}{9}$$

Probability that C will be selected is $P(C) = \frac{2}{9}$.

Ans: Given: Total number of candidates = 4 {A, B, C, D}

Probability of selection of A = 2 × Probability of selection of B

Probability of selection of B = Probability of selection of C

Probability of selection of C = 2 × Probability of selection of D

Assume $P(A)$, $P(B)$, $P(C)$ and $P(D)$ as the probabilities of selection of the given candidates. Form relations between them and use the formula $P(A) + P(B) + P(C) + P(D) = 1$ to find their values.

Assuming $P(A)$, $P(B)$, $P(C)$ and $P(D)$ as the probabilities of selection of the candidates A, B, C and D respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

According to the information given,

$$P(A) = 2P(B) \dots (1)$$

$$P(B) = P(C) \dots (2)$$

$$P(C) = 2P(D) \dots (3)$$

Using the formula $P(A) + P(B) + P(C) + P(D) = 1$ and solving the three equations,

$$P(A) = \frac{4}{9}, P(B) = \frac{2}{9}, P(C) = \frac{2}{9} \text{ and } P(D) = \frac{1}{9}$$

Probability that A will not be selected is $P(A') = 1 - P(A)$.

$$\Rightarrow P(A') = 1 - \frac{4}{9}$$

$$\Rightarrow P(A') = \frac{5}{9}$$

Question 10

One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes $S = \{\text{John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}\}$.

You are told that the chances of John's promotion is the same as that of Gurpreet, Rita's chances of promotion are twice as likely as John's, Aslam's chances are four times that of John.

(a) Determine $P(\text{John promoted})$, $P(\text{Rita promoted})$, $P(\text{Aslam promoted})$, $P(\text{Gurpreet promoted})$?

(b) If $A = (\text{John promoted or Gurpreet promoted})$, find $P(A)$?

Solution

Ans: Given: Total number of persons = 4

Probability of John's promotion = Probability of Gurpreet's promotion

Probability of Rita's promotion $= 2 \times$ Probability of John's promotion

Probability of Aslam's promotion $= 4 \times$ Probability of John's promotion

Assume $P(W)$, $P(X)$, $P(Y)$ and $P(Z)$ as the probabilities of promotion of the four persons. Form relations between them and use the formula $P(W) + P(X) + P(Y) + P(Z) = 1$ to find their values.

Assuming $P(W)$, $P(X)$, $P(Y)$ and $P(Z)$ as the probabilities of promotion of John, Rita, Aslam and Gurpreet respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

According to the information given,

$$P(W) = P(Z) \dots (1)$$

$$P(X) = 2P(W) \dots (2)$$

$$P(Y) = 4P(W) \dots (3)$$

Using the formula $P(W) + P(X) + P(Y) + P(Z) = 1$ and solving the three equations,

$$P(W) = \frac{1}{8}, P(X) = \frac{1}{4}, P(Y) = \frac{1}{2} \text{ and } P(Z) = \frac{1}{8}$$

Probability of promotion of John is $P(W) = \frac{1}{8}$.

Probability of promotion of Rita is $P(X) = \frac{1}{4}$.

Probability of promotion of Aslam is $P(Y) = \frac{1}{2}$.

Probability of promotion of Gurpreet is $P(Z) = \frac{1}{8}$.

Ans: Given: Total number of persons = 4

Probability of John's promotion = Probability of Gurpreet's promotion

Probability of Rita's promotion $= 2 \times$ Probability of John's promotion

Probability of Aslam's promotion $= 4 \times$ Probability of John's promotion

Assume $P(W)$, $P(X)$, $P(Y)$ and $P(Z)$ as the probabilities of promotion of the four persons. Form relations between them and use the formula $P(W) + P(X) + P(Y) + P(Z) = 1$ to find their values.

Assuming $P(W)$, $P(X)$, $P(Y)$ and $P(Z)$ as the probabilities of promotion of John, Rita, Aslam and Gurpreet respectively. All the events are mutually exclusive that means no two of them can occur simultaneously.

According to the information given,

$$P(W) = P(Z) \dots (1)$$

$$P(X) = 2P(W) \dots (2)$$

$$P(Y) = 4P(W) \dots (3)$$

Using the formula $P(W) + P(X) + P(Y) + P(Z) = 1$ and solving the three equations,

$$P(W) = \frac{1}{8}, P(X) = \frac{1}{4}, P(Y) = \frac{1}{2} \text{ and } P(Z) = \frac{1}{8}$$

Probability of promotion of John or Gurpreet $P(A) = P(W \cup Z)$.

$$\Rightarrow P(A) = P(W) + P(Z) - P(W \cap Z)$$

$$\Rightarrow P(A) = \frac{1}{8} + \frac{1}{8} - 0$$

$$\Rightarrow P(A) = \frac{1}{4}$$

Question 11

The accompanying Venn diagram shows three events A , B and C , and also the probabilities of the various intersections (for instance $P(A \cap B) = 0.07$). Determine

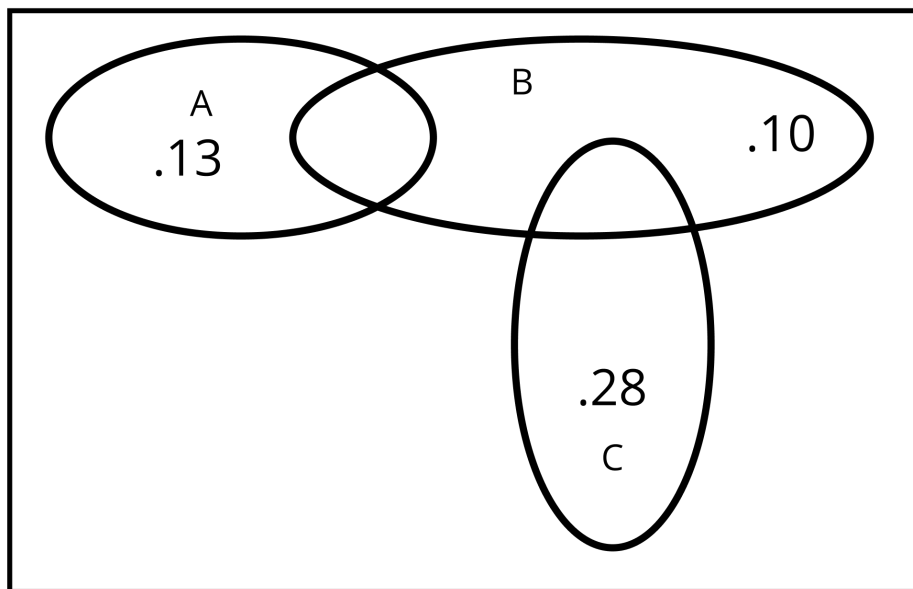


Fig. 16.2

(a) $P(A)$

Solution

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \overline{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

The probability of occurrence of the event A is

$$P(A) = P(\text{only } A) + P(A \cap B)$$

.

$$\Rightarrow P(A) = 0.13 + 0.07$$

$$\Rightarrow P(A) = 0.20$$

$$(b) P(B \cap \overline{C})$$

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \bar{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

Here $(B \cap \bar{C})$ states the event of occurrence of B and not C .

$$\Rightarrow P(B \cap \bar{C}) = P(B) - P(B \cap C)$$

$$\Rightarrow P(B \cap \bar{C}) = (0.10 + 0.07 + 0.15) - 0.15$$

$$\Rightarrow P(B \cap \bar{C}) = 0.17$$

$$(c) P(A \cup B)$$

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \bar{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

Here $P(A \cup B)$ is the probability of occurrence of either A or B .

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = (0.13 + 0.07) + (0.10 + 0.07 + 0.15) - 0.07$$

$$\Rightarrow P(A \cup B) = 0.45$$

$$(d) P(A \cap \bar{B})$$

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \overline{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Here $(A \cap \overline{B})$ states the event of occurrence of A and not B .

$$\Rightarrow P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

$$\Rightarrow P(A \cap \overline{B}) = (0.13 + 0.07) - 0.07$$

$$\Rightarrow P(A \cap \overline{B}) = 0.13$$

$$(e) P(B \cap C)$$

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \overline{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Here $(B \cap C)$ states the event of occurrence of both A and B .

$$\Rightarrow P(B \cap C) = 0.15$$

(f) Probability of exactly one of the three occurs.

Ans: Given:

In the Venn diagram: -

$$P(\text{only } A) = 0.13$$

$$P(\text{only } B) = 0.10$$

$$P(\text{only } C) = 0.28$$

$$P(A \cap B) = 0.07$$

$$P(B \cap C) = 0.15$$

Using the Venn and the formulas of theory solve for each subpart of the question.

Use the formulas: -

$$P(B \cap \overline{C}) = P(B) - P(B \cap C)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Probability of exactly one of the three occurs means

$$P(\text{only } A) + P(\text{only } B) + P(\text{only } C)$$

.

$$\Rightarrow \text{Required probability} = 0.13 + 0.10 + 0.28$$

$$\Rightarrow \text{Required probability} = 0.51$$

Question 12

One urn contains two black balls (labelled $B1$ and $B2$) and one white ball. A second urn contains one black ball and two white balls (labelled $W1$ and $W2$). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) Write the sample space showing all possible outcomes.

(c) What is the probability that two balls of opposite colour are chosen?

Solution

Ans: Given: Two black balls (labelled $B1$ and $B2$) and one white ball in the first urn.

Two white balls (labelled $W1$ and $W2$) and one black ball in the second urn.

One of the two urns is chosen then a ball is chosen and then a second ball is chosen without replacement.

Find all the possible combinations that can be made in selecting the two balls one after the other and not at once.

Check the favourable number of sample spaces for the subparts.

Let us represent the single white ball in the first urn as W and the single black ball in the second urn as B . The possible combinations of selecting the two balls one after the other from the urns is the total number of sample spaces given as,

$$n(S) = \{B_1B_2, B_2B_1, B_1W, WB_1, B_2W, WB_2, W_1W_2, W_2W_1, BW_1, W_1B, BW_2, W_2B\} = 12$$

(b) What is the probability that two black balls are chosen?

Ans: Given: Two black balls (labelled B_1 and B_2) and one white ball in the first urn.

Two white balls (labelled W_1 and W_2) and one black ball in the second urn.

One of the two urns is chosen then a ball is chosen and then a second ball is chosen without replacement.

Find all the possible combinations that can be made in selecting the two balls one after the other and not at once.

Check the favourable number of sample spaces for the subparts.

The combinations where two black balls are chosen is $n(X) = \{B_1B_2, B_2B_1\} = 2$.

Therefore, the required probability is $\frac{n(X)}{n(S)} = \frac{2}{12} = \frac{1}{6}$.

Ans: Given: Two black balls (labelled B_1 and B_2) and one white ball in the first urn.

Two white balls (labelled W_1 and W_2) and one black ball in the second urn.

One of the two urns is chosen then a ball is chosen and then a second ball is chosen without replacement.

Find all the possible combinations that can be made in selecting the two balls one after the other and not at once.

Check the favourable number of sample spaces for the subparts.

The combinations where two balls of opposite colours are chosen is

$$n(Y) = \{B_1W, WB_1, B_2W, WB_2, BW_1, W_1B, BW_2, W_2B\} = 8.$$

Therefore, the required probability is $\frac{n(Y)}{n(S)} = \frac{8}{12} = \frac{2}{3}$.

Question 13

A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:

(a) All the three balls are white.

(c) One ball is red and two balls are white.

Solution

Ans: Given: Total number of red balls = 8

Total number of white balls = 5

Three balls are drawn at random.

Use the formula ${}^{13}C_3$ to find the total number of sample spaces.

Select the balls using the formula of combination for each subpart.

Since there are a total of 13 balls and any 3 balls can be selected in ${}^{13}C_3$ ways, so the total number of sample space is $n(S) = {}^{13}C_3$.

To select all the three white balls they must be selected from the given 5 white balls, so the number of favourable sample spaces is $n(A) = {}^5C_3$.

Therefore, the required probability is $= \frac{n(A)}{n(S)} = \frac{{}^5C_3}{{}^{13}C_3} = \frac{5}{143}$.

(b) All the three balls are red.

Ans: Given: Total number of red balls = 8

Total number of white balls = 5

Three balls are drawn at random.

Use the formula ${}^{13}C_3$ to find the total number of sample spaces.

Select the balls using the formula of combination for each subpart.

Since there are a total of 13 balls and any 3 balls can be selected in ${}^{13}C_3$ ways, so the total number of sample space is $n(S) = {}^{13}C_3$.

To select all the three red balls they must be selected from the given 8 red balls, so the number of favourable sample spaces is $n(B) = {}^8C_3$.

Therefore, the required probability is $= \frac{n(B)}{n(S)} = \frac{{}^8C_3}{{}^{13}C_3} = \frac{28}{143}$.

Ans: Given: Total number of red balls = 8

Total number of white balls = 5

Three balls are drawn at random.

Use the formula ${}^{13}C_3$ to find the total number of sample spaces.

Select the balls using the formula of combination for each subpart.

Since there are a total of 13 balls and any 3 balls can be selected in ${}^{13}C_3$ ways, so the total number of sample space is $n(S) = {}^{13}C_3$.

To select one red ball the number of ways is

$${}^8C_1$$

and to select two white balls the number of ways is

$${}^5C_2$$

. So the number of favourable sample spaces is $n(C) = {}^8C_1 \times {}^5C_2$.

Therefore, the required probability is $= \frac{n(C)}{n(S)} = \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{40}{143}$.

Question 14

If the letters of the word ASSASSINATION are arranged at random. Find the probability that

- (a) Four S's come consecutively in the word
- (b) Two I's and two N's come together
- (c) All A's are not coming together
- (d) No two A's are coming together.

Solution

Ans: Given: The letters of the word Assassination are arranged.

Find the total number of sample spaces by arranging all the thirteen letters of the word and dividing it by the product of the factorial of the number of times each letter is repeating.

Arrange the letters as per the condition given in the subparts to find the favourable sample spaces.

There are 13 letters in the word ASSASSINATION in which A is repeated 3 times, S is repeated 4 times, I is repeated 2 times and N is repeated 2 times. So the number of different words that can be formed is the total number of sample

$$\text{spaces } n(S) = \frac{13!}{4! \times 3! \times 2! \times 2!}.$$

All the four S's come together so they can be assumed as a single unit. Since, they are identical alphabets, so there is no meaning of arranging them among themselves. The remaining 9 alphabets and this single unit can be considered

as 10 objects and they can be arranged in $n(A) = \frac{10!}{3! \times 2! \times 2!}$ ways.

Therefore, the required probability is $\frac{n(A)}{n(S)} = \frac{2}{143}$.

Ans: Given: The letters of the word ASSASSINATION are arranged.

Find the total number of sample spaces by arranging all the thirteen letters of the word and dividing it by the product of the factorial of the number of times each letter is repeating.

Arrange the letters as per the condition given in the subparts to find the favourable sample spaces.

There are 13 letters in the word ASSASSINATION in which A is repeated 3 times, S is repeated 4 times, I is repeated 2 times and N is repeated 2 times. So the number of different words that can be formed is the total number of sample

$$\text{spaces } n(S) = \frac{13!}{4! \times 3! \times 2! \times 2!}.$$

Here two I's and two N's come together (IINN). These four letters can be arranged in $\frac{4!}{2! \times 2!}$ ways among themselves. The remaining 9 alphabets and this single unit can be considered as 10 objects and they can be arranged in $\frac{10!}{4! \times 3!}$ ways. So the number of favorable arrangements is $n(B) = \frac{4!}{2! \times 2!} \times \frac{10!}{4! \times 3!}$.

Therefore, the required probability is $\frac{n(B)}{n(S)} = \frac{2}{143}$.

Ans: Given: The letters of the word ASSASSINATION are arranged.

Find the total number of sample spaces by arranging all the thirteen letters of the word and dividing it by the product of the factorial of the number of times each letter is repeating.

Arrange the letters as per the condition given in the subparts to find the favourable sample spaces.

There are 13 letters in the word ASSASSINATION in which A is repeated 3 times, S is repeated 4 times, I is repeated 2 times and N is repeated 2 times. So the number of different words that can be formed is the total number of sample

$$\text{spaces } n(S) = \frac{13!}{4! \times 3! \times 2! \times 2!}.$$

Let us assume that all the three A's come together so they can be assumed as a single unit. They are identical alphabets. The remaining 10 alphabets and this single unit can be considered as 11 objects and they can be arranged

$$\text{in } n(C) = \frac{11!}{4! \times 2! \times 2!} \text{ ways.}$$

Therefore, the probability that all the A's comes together is $\frac{n(C)}{n(S)} = \frac{1}{26}$.

Hence, the probability that all the A's does not come together is $\left(1 - \frac{n(C)}{n(S)}\right) = \frac{25}{26}$.

Ans: Given: The letters of the word ASSASSINATION are arranged.

Find the total number of sample spaces by arranging all the thirteen letters of the word and dividing it by the product of the factorial of the number of times each letter is repeating.

Arrange the letters as per the condition given in the subparts to find the favourable sample spaces.

There are 13 letters in the word ASSASSINATION in which A is repeated 3 times, S is repeated 4 times, I is repeated 2 times and N is repeated 2 times. So the number of different words that can be formed is the total number of sample

$$\text{spaces } n(S) = \frac{13!}{4! \times 3! \times 2! \times 2!}.$$

Here no two A's come together. Let us arrange the remaining letters first.

_ S _ S _ S _ S _ I _ N _ T _ I _ O _ N _

The three A's can be filled at any 3 of the 11 blank spaces in ${}^{11}C_3$ ways (arrangement not required as they are identical). The remaining 10 letters can be arranged in $\frac{10!}{4! \times 2! \times 2!}$ ways. So the number of favourable

$$\text{arrangements is } n(D) = {}^{11}C_3 \times \frac{10!}{4! \times 2! \times 2!}.$$

$$\text{Therefore, the required probability is } \frac{n(D)}{n(S)} = \frac{15}{26}.$$

Question 15

A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution

Ans: Given: A deck of 52 cards and one card is drawn.

Assume $P(K)$, $P(H)$ and $P(R)$ as the probabilities of drawing a king, a heart and a red card respectively.

Use the formula

$$P(K \cup H \cup R) = P(K) + P(H) + P(R) - P(K \cap H) - P(H \cap R) - P(K \cap R) + P(K \cap H \cap R)$$

to solve for the L, H, S .

Let us assume $P(K)$, $P(H)$ and $P(R)$ as the probabilities of drawing a king, a heart and a red card respectively.

The number of ways to draw a 1 card from 52 cards is $n(S) = {}^{52}C_1$.

There are four kings, thirteen hearts cards and 26 red cards.

Now, there is 1 card which is both king and red, 13 cards which are both red and hearts, 2 cards which are both red and king.

There is only one card which is king, hearts and red.

So the probability of drawing a king, a heart or a red card is $P(K \cup H \cup R)$ which is

$$\begin{aligned}
&= P(K) + P(H) + P(R) - P(K \cap H) - P(H \cap R) - P(K \cap R) + P(K \cap H \cap R) \\
&= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} \\
&= \frac{28}{52} \\
&= \frac{7}{13}
\end{aligned}$$

Question 16

A sample space consists of 9 elementary outcomes $e_1, e_2, \dots, e_9$ whose probabilities are

$$P(e_1) = P(e_2) = 0.08, P(e_3) = P(e_4) = P(e_5) = 0.1$$

$$P(e_6) = P(e_7) = 0.2, P(e_8) = P(e_9) = 0.07$$

Suppose $A = \{e_1, e_2, e_8\}$ and $B = \{e_2, e_5, e_8, e_9\}$.

(a) Calculate $P(A)$, $P(B)$, and $P(A \cap B)$.

(b) Using the additional law of probability, calculate $P(A \cup B)$.

(c) List the composition of the event $A'EB$, and calculate $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

(d) Calculate $P(\overline{B})$ from $P(B)$, also calculate $P(\overline{B})$ directly from the elementary outcomes of $\overline{B}$.

Solution

Ans: Given: The 9 elementary outcomes $e_1, e_2, \dots, e_9$ and their probabilities.

$$A = \{e_1, e_2, e_8\}$$

$$B = \{e_2, e_5, e_8, e_9\}$$

Use the formulas: -

$$P(A) = P(e_1) + P(e_2) + P(e_8) \text{ and } P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\overline{B}) = 1 - P(B)$$

Since $A = \{e_1, e_2, e_8\}$, therefore,

$$\Rightarrow P(A) = P(e_1) + P(e_2) + P(e_8)$$

$$\Rightarrow P(A) = 0.08 + 0.08 + 0.07$$

$$\Rightarrow P(A) = 0.23$$

Since $B = \{e_2, e_5, e_8, e_9\}$, therefore,

$$\Rightarrow P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$\Rightarrow P(B) = 0.08 + 0.1 + 0.07 + 0.07$$

$$\Rightarrow P(B) = 0.32$$

Now, $(A \cap B)$ is the set of common elementary outcomes of A and B . So $(A \cap B) = \{e_5, e_8\}$, therefore,

$$\Rightarrow P(A \cap B) = P(e_5) + P(e_8)$$

$$\Rightarrow P(A \cap B) = 0.1 + 0.07$$

$$\Rightarrow P(A \cap B) = 0.17$$

Ans: Given: The 9 elementary outcomes $e_1, e_2, \dots, e_9$ and their probabilities.

$$A = \{e_1, e_2, e_8\}$$

$$B = \{e_2, e_5, e_8, e_9\}$$

Use the formulas: -

$$P(A) = P(e_1) + P(e_2) + P(e_8) \text{ and } P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\overline{B}) = 1 - P(B)$$

According to the addition law of probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,

$$\Rightarrow P(A \cup B) = 0.23 + 0.25 - 0.17$$

$$\Rightarrow P(A \cup B) = 0.23 + 0.32 - 0.40$$

Ans: Given: The 9 elementary outcomes $e_1, e_2, \dots, e_9$ and their probabilities.

$$A = \{e_1, e_2, e_8\}$$

$$B = \{e_2, e_5, e_8, e_9\}$$

Use the formulas: -

$$P(A) = P(e_1) + P(e_2) + P(e_8) \text{ and } P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\overline{B}) = 1 - P(B)$$

The event $(A \cup B)$ is the set of all the elements present in either A or B . So the composition of $(A \cup B)$ is $\{e_1, e_2, e_5, e_8, e_9\}$, therefore,

$$\Rightarrow P(A \cup B) = P(e_1) + P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$\Rightarrow P(A \cup B) = 0.08 + 0.08 + 0.1 + 0.07 + 0.07$$

$$\Rightarrow P(A \cup B) = 0.40$$

Ans: Given: The 9 elementary outcomes $e_1, e_2, \dots, e_9$ and their probabilities.

$$A = \{e_1, e_2, e_8\}$$

$$B = \{e_2, e_5, e_8, e_9\}$$

Use the formulas: -

$$P(A) = P(e_1) + P(e_2) + P(e_8) \text{ and } P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\overline{B}) = 1 - P(B)$$

Using $P(B)$ the value of $P(\overline{B})$ is given as $1 - P(B)$.

$$\Rightarrow P(\overline{B}) = 1 - P(B)$$

$$\Rightarrow P(\overline{B}) = 1 - 0.32$$

$$\Rightarrow P(\overline{B}) = 0.68$$

Since $B = \{e_2, e_5, e_8, e_9\}$, so $\overline{B} = \{e_1, e_3, e_4, e_6, e_7\}$, therefore,

$$\Rightarrow P(\overline{B}) = P(e_1) + P(e_3) + P(e_4) + P(e_6) + P(e_7)$$

$$\Rightarrow P(\overline{B}) = 0.08 + 0.1 + 0.1 + 0.2 + 0.2$$

$$\Rightarrow P(\overline{B}) = 0.68$$

Question 17

Determine the probability p for each of the following events.

(a) An odd number appears in a single toss of a fair die.

(b) At least one head appears in two tosses of a fair coin.

(c) A king, 9 of hearts, or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.

(d) The sum of 6 appears in a single toss of a pair of fair dice.

Solution

Ans: write all the sample space and find the favourable events among them.

When the fair die is tossed one time then the sample spaces are $n(S) = \{1, 2, 3, 4, 5, 6\} = 6$. There are three odd numbers, so the favourable events are $n(A) = \{1, 3, 5\} = 3$.

Therefore, the probability p is $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

Ans: write all the sample space and find the favourable events among them.

When two fair coins are tossed the sample spaces are $n(S) = \{HH, HT, TH, TT\} = 4$.

There are three outcomes where at least one head appears, so $n(B) = \{HH, HT, TH\} = 3$.

Therefore, the probability p is $\frac{n(B)}{n(S)} = \frac{3}{4}$.

Ans: consider that there are four kings, one 9 of hearts and one 3 of spades in a pack.

In a deck of 52 cards the number of ways to draw a card is the total number of sample spaces given as $n(S) = {}^{52}C_1 = 52$.

There are four kings (one in each suit), one 9 of hearts and one 3 of spades in a pack. So the number of favourable outcomes is $n(C) = (4 + 1 + 1) = 6$.

Therefore, the probability p is $\frac{n(C)}{n(S)} = \frac{6}{52} = \frac{3}{26}$.

Ans: write all the sample space and find the favourable events among them.

When a pair of dice is tossed one time then there are $6^2 = 36$ combinations of outcomes possible. So the number of sample spaces is $n(S) = 36$.

The sum of 6 appears in the cases (1, 5), (5, 1), (2, 4), (4, 2) and (3, 3), so the number of favourable outcomes is $n(D) = 5$.

Therefore, the probability p is $\frac{n(D)}{n(S)} = \frac{5}{36}$.

Objective Type Questions:

Question 18

In a non-leap year, the probability of having

53

Tuesday or

53

Wednesday is

(A)

$\frac{1}{7}$

(B)

$\frac{2}{7}$

(C)

$\frac{3}{7}$

(D) None of these.

Solution

Ans: Given: Non-leap year, There are

365

days that have

52

weeks and

1

days.

$$P(A) = \frac{\{\text{Tuesday}\}}{\{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}}$$

$$P(B) = \frac{\{\text{Wednesday}\}}{\{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}}$$

If this day is a Tuesday or Wednesday then, the year will have

53

Tuesday

Or

$$53$$

Wednesday.

$$\therefore$$

Required Probability

$$= \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$$

Correct Answer: B

Question 19

Three numbers are chosen from

$$1$$

to

$$20$$

. Find the probability that they are not consecutive

(A)

$$\frac{186}{190}$$

(B)

$$\frac{187}{190}$$

(C)

$$\frac{188}{190}$$

(D)

$$\frac{18}{{}^{20}C_3}$$

Solution

Ans: Given: Three numbers are chosen from

$$1$$

to

$$20$$

The probability that the numbers are not a consecutive

$$= 1 -$$

probability of the numbers being consecutive.

Since the set of consecutive numbers are from 1–20

$P(1, 2, 3), (2, 3, 4), (3, 4, 5) \dots \dots (18, 19, 20)$ i.e., 18.

$$P(\text{consecutive numbers}) = \frac{18}{{}^{20}C_3}$$

$$= \frac{18}{\frac{20 \times 19 \times 18}{3!}} = \frac{3}{19}$$

$$P(\text{three number are not consecutive}) = 1 - \frac{3}{190} = \frac{187}{190}$$

Correct Answer: B

Question 20

While shuffling a pack of

52

Playing cards and

2

got accidentally dropped. Find the probability that the missing cards to be of different colours is

(A)

$$\frac{29}{52}$$

(B)

$$\frac{1}{2}$$

(C)

$$\frac{26}{51}$$

(D)

$$\frac{27}{51}$$

Solution

Ans: Given: While shuffling a pack,

52

Playing cards and

2

got accidentally dropped.

$$P(\text{both cards of opposite colour}) = \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2}$$

On the back of

52

cards,

26

red colour and

26

black colour.

$$P(\text{both cards of opposite colour}) = \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26 \times 26}{\frac{52 \times 51}{2}} = \frac{26}{51}$$

Correct answer: C

Question 21

Seven persons are to be seated in a row. The probability that two particular persons sit next to each other

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{2}{7}$

(D) $\frac{1}{2}$

Solution

Ans: Given: Seven persons are to be seated in a row

With the help of the basic formulas,

$$n! = n \times (n - 1)!$$

$$\text{Required probability} = \frac{\text{Number of arrangement}}{\text{Total number of arrangement of 7 persons}}$$

If two particular persons sit next to each other, then consider two people a group.

Now we have to arrange

$$6$$

persons.

Number of arrangements

$$= 2! \times 6!$$

Total number of arrangement of

$$7$$

persons

$$= 7!$$

Required probability

$$\begin{aligned} &= \frac{2! \times 6!}{7!} \\ &= \frac{2}{7} \end{aligned}$$

Correct Answer: C

Question 22

Without repetition of the numbers, four-digit numbers are formed with the numbers

$$0, 2, 3, 5.$$

the probability of such a number divisible by

$$5$$

is

(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $\frac{1}{30}$

(D) $\frac{5}{9}$

Solution

Ans: Given: four-digit numbers are formed with the numbers

$$0, 2, 3, 5.$$

With the help of the formulas,

$$n! = n \times (n - 1)!$$

Required probability

$$= \frac{\text{total number of numbers divisible by 5}}{\text{total number of numbers formed from 0, 2, 3, 5}}$$

Number divisible by 5 has unit place digit $\rightarrow$ '0' or '5'

If a unit place is '0', the first three places can be filled in $3!$ ways.

If a unit place is '5', then the first place can be filled in two ways.

The second and third places can be filled in $2!$ ways.

So number ending with the digit '5' $= 2 \times 2! = 4$

Total numbers formed from 0, 2, 3, 5 are divisible by 5 $= 3! + 4 = 10 = n(E)$

$$\text{Required probability} = \frac{10}{18} = \frac{5}{9}$$

Correct Answer: D

Question 23

If A and B is mutually exclusive events, then

(A)

$$P(A) \leq P(\overline{B})$$

(B)

$$P(A) \geq P(\overline{B})$$

(C)

$$P(A) < P(B)$$

(D) None of these

Solution

Ans: For mutually exclusive events,

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A) + P(B) \leq 1$$

$$P(A) \leq 1 - P(B)$$

$$P(A) \leq P(\overline{B})$$

Correct Answer: A

Question 24

If

$$P(A \cup B) = P(A \cap B)$$

. For any two events A and B, then

$$(A) P(A) = P(B)$$

$$(B) P(A) > P(B)$$

$$(C) P(A) < P(B)$$

(D) None of these.

Solution

Ans: We have,

$$P(A \cup B) = P(A \cap B)$$

$$P(A) + P(B) - P(A \cap B) = P(A \cap B)$$

$$[P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$$

$$[P(A) - P(A \cap B)] \geq 0$$

$$[P(B) - P(A \cap B)] \geq 0$$

$$[P(A) - P(A \cap B)] = 0$$

Correct answer: A

Question 25

6

boys and

6

girls sit in a row at random. The probability that all the girls sit together is

(A) $\frac{1}{432}$

(B) $\frac{12}{431}$

(C) $\frac{1}{132}$

(D) None of these

Solution

Ans: All girls sit together in a group.

Number of arrangements $6 + 1 = 7$ And girls interchange their seats in

6!

Ways

Probability

$$= \frac{6! \times 7!}{12!} = \frac{1}{132}$$

Correct Answer: C

Question 26

A single letter is selected at random from the word 'PROBABILITY' the probability that it is a vowel is

(A)

$$\frac{1}{3}$$

(B)

$$\frac{4}{11}$$

(C)

$$\frac{2}{11}$$

(D)

$$\frac{3}{11}$$

Solution

Ans: Total number of alphabets in the word = 11

Vowels

$$= 4(\text{O,A,I,I})$$

$$P(\text{letter is vowel}) = \frac{4}{11}$$

Correct Answer: B

Question 27

If the probabilities for A to fail in an examination are

$$0.2$$

and that for B is

$$0.3$$

, then the probability that either A or B fails is

(A)

$$> .5$$

(B)

$$.5$$

(C)

$$\leq .5$$

(D) 0

Solution

Ans:

$$P(A \text{ fail}) = 0.2$$

$$P(B \text{ fail}) = 0.3$$

$$P(\text{Either A Or B fail}) \leq P(\text{A fail}) + P(\text{B fail})$$

$$\leq 0.2 + 0.3$$

$$\leq 0.5$$

Correct Answer: C

Question 28

The probability that at least one of the events A and B occur is

$$0.6.$$

If A and B occur simultaneously with probability

$$0.2$$

, then

$$P(A) + P(B)$$

is

(A)

$$0.4$$

(B)

$$0.8$$

(C)

$$1.2$$

(D)

$$1.6$$

Solution

Ans:

$$P(A \cup B) = 0.6$$

$$P(A \cap B) = 0.2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = P(A) + P(B) - 0.2$$

$$P(A) + P(B) = 0.8$$

$$P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B) = 2 - [P(A) + P(B)]$$

$$= 2 - 0.8 = 1.2$$

Correct Answer: C

Question 29

If M and N are any two events, the probability that at least one of them occurs is

(A)

$$P(M) + P(N) - 2P(M \cap N)$$

(B)

$$P(M) + P(N) - P(M \cap N)$$

(C)

$$P(M) + P(N) + P(M \cap N)$$

(D)

$$P(M) + P(N) + 2P(M \cap N)$$

Solution

Ans: If M and N are any two events.

$$\therefore P(M \cup N) = P(M) + P(N) - P(M \cap N)$$

Correct Answer: B

State Whether the Statements are True or False

Question 30

The probability that a person visiting a zoo will see the giraffe is

$$0.72$$

, the probability that he will see the bears are

$$0.84$$

and the probability that he will see both is

$$0.52.$$

Solution

Ans: Let Giraffe = G

Bear = B

$$P(G) = 0.72$$

$$P(B) = 0.84$$

$$P(G \cup B) = 0.52$$

$$P(G \cup B) = P(G) + P(B) - P(G \cap B)$$

$$P(G \cap B) = 0.72 + 0.84 - 0.52 = 1.04$$

(This is not possible).

Correct Answer: False

Question 31

The probability that a student will pass his examination is

$$0.73,$$

the probability of the student getting a compartment is

$$0.13$$

, and the probability that the student will either pass or get compartment is

$$0.96.$$

Solution

Ans: Given: Student will pass his examination

$$= 0.73$$

Student getting a compartment

$$= 0.13$$

Student will either pass or get compartment

$$0.96.$$

The probability,

$$P(A) = 0.73,$$

$$P(B) = 0.13$$

$$P(A \cup B) = 0.96$$

Find

$$P(A \cap B)$$

By General addition rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute the values to general addition rule,

$$\Rightarrow P(A \cup B) = 0.73 + 0.13 - 0$$

$$\therefore A$$

and

$$B$$

are independent events

$$\Rightarrow P(A \cup B) = 0.86$$

But

$$P(A \cup B) = 0.96$$

Hence, the given statement is false.

Question 32

The probabilities that a typist will make

$$0, 1, 2, 3, 4, 5$$

or more mistakes in typing a report are, respectively

$$, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.$$

Solution

Ans: Given: Probabilities that a typist will make

$$0, 1, 2, 3, 4, 5$$

Mistakes in typing a report are

$$, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.$$

Sum of all probabilities

$$= 1$$

$$\therefore P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 0.12 + 0.25 + 0.36 + 0.14 + 0.08 + 0.11$$

$$= 1.06 > 1$$

Hence, the given statement is False.

Question 33

If

$$A$$

and

$$B$$

are two candidates seeking admission in an engineering College. The probability that

$$A$$

is selected is

$$0.5$$

and the probability that both A and B are selected is at most .

$$3$$

. Is it possible that the probability of

$$B$$

getting selected is

$$0.7?$$

Solution

Ans: Given:

$$P(A) = 0.5$$

and

$$P(A \cap B) \leq 0.3$$

The probability,

$$P(A) = 0.5$$

$$P(A \cap B) \leq 0.3$$

$$P(A) \times P(B) \leq 0.3$$

Substitute,

$$\Rightarrow 0.5 \times P(B) \leq 0.3$$

$$\Rightarrow P(B) \leq \frac{0.3}{0.5}$$

$$\Rightarrow P(B) \leq 0.6$$

Hence the given statement is False.

Question 34

The probability of intersection of two events

 A

and

 B

is always less than or equal to those favourable to the event

 A

.

Solution

Ans: Given: The probability of intersection of two events

 A

and

 B
 A

and

 B

are two events,

Here,

$$A \cap B \subset A$$

$$\Rightarrow P(A \cap B) \leq P(A)$$

This is always true.

Hence, the given statement is true.

Question 35

The probability of an occurrence of event

 A

is

 $.7$

and that of the occurrence of event

$$B$$

is

$$.3$$

and the probability of occurrence of both is

$$.4.$$

Solution

Ans: Given:

$$P(A)$$

$$= .7$$

$$P(B) = .3$$

The probability of an occurrence of both

$$= .4$$

The probability,

$$P(A) = 0.7$$

$$P(B) = 0.3$$

$$\therefore P(A \cap B) = P(A) \times P(B)$$

Substitute the values of

$$P(A)$$

and

$$P(B)$$

,

$$P(A \cap B) = 0.7 \times 0.3$$

$$= 0.21$$

But the given probability is

$$0.4$$

.

Hence, the given statement is False.

Question 36

The sum of probabilities of two students getting distinction in their final examinations is

$$1.2.$$

Solution

Ans: Given: Distinction in their final examinations

$$= 1.2.$$

The probability of each student getting distinction in their final examination is less than or equal to

$$1$$

Since, the two given events are not related to the given sample space.

Therefore,

The sum of the probabilities of two may be

$$1.2$$

.

Hence, the given statement is true.

Question 37

The probability that the home team will win an upcoming football game is

$$0.77$$

, the probability that it will tie the game is

$$0.08$$

, and the probability that it will lose the game is ____.

Solution

Ans: Given:

$$P(A) = 0.77$$

$$P(B) = 0.08$$

The probability that it will tie the game

$$= 0.08$$

,

A

be the event that home team will win the football game

 B

be the event that home team will tie the game and,

 C

be the event that they will lose the game

Then,

$$P(A) = 0.77$$

$$P(B) = 0.08$$

Find the

 P

or

$$P(C)$$

Sum of all Probabilities

$$= 1$$

$$\therefore P(A) + P(B) + P(C) = 1$$

Substitute the values,

$$\Rightarrow 0.77 + 0.08 + P(C) = 1$$

$$\Rightarrow P(C) = 1 - (0.77 + 0.08)$$

$$P(C)$$

$$= 0.15$$

.

The probability that the home team will win an upcoming football game is

$$0.77$$

, the probability that it will tie the game is

$$0.08$$

, and the probability that it will lose the game is

$$0.15.$$

Question 38

If

$$e_1, e_2, e_3, e_4$$

are the four elementary outcomes in a sample space

$$P(e_1) = .1, P(e_2) = .5, P(e_3) = .1,$$

then the probability of

$$e_4$$

is _____.

Solution

Ans: Given: Four elementary outcomes

$$= e_1, e_2, e_3, e_4$$

$$P(e_1) = .1,$$

$$P(e_2) = .5,$$

$$P(e_3) = .1$$

Sum of all probabilities

$$= 1$$

$$\therefore P(e_1) + P(e_2) + P(e_3) + P(e_4) = 1$$

Substitute the values,

$$\Rightarrow 0.1 + 0.5 + 0.1 + P(e_4) = 1$$

$$\Rightarrow P(e_4) = 1 - 0.7$$

$$\Rightarrow P(e_4) = 0.3$$

Therefore ,

If

$$e_1, e_2, e_3, e_4$$

are the four elementary outcomes in a sample space

$$P(e_1) = .1, P(e_2) = .5, P(e_3) = .1,$$

then the probability of

$$e_4$$

is

$$0.3.$$

Question 39

Let

$$S = \{1, 2, 3, 4, 5, 6\}$$

and

$$E = \{1, 3, 5\},$$

then

$$\overline{E}$$

is _____.

Solution

Ans: Given:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 3, 5\},$$

Given that,

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 3, 5\}$$

$$\overline{E} = S - E$$

$$= \{1, 2, 3, 4, 5, 6\}$$

$$- \{1, 3, 5\}$$

$$= \{2, 4, 6\}$$

Hence, the value of the filler is

$$\{2, 4, 6\}.$$

Therefore,

Let

$$S = \{1, 2, 3, 4, 5, 6\}$$

and

$$E = \{1, 3, 5\},$$

then

$$\overline{E}$$

is

$$\{2, 4, 6\}.$$

Question 40

If

$$A$$

and

$$B$$

are two events associated with a random experiment such that

$$P(A) = 0.3, P(B) = 0.2$$

and

$$P(A \cap B) = 0.1,$$

then the value of

$$P(A \cap \overline{B})$$

is_____.

Solution

Ans: Given:

$$P(A) = 0.3,$$

$$P(B) = 0.2$$

$$P(A \cap B) = 0.1,$$

Given that,

$$P(A) = 0.3,$$

$$P(B) = 0.2$$

$$P(A \cap B) = 0.1$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Substitute the values,

$$P(A)$$

and

$$P(B)$$

$$P(A \cap B)$$

$$= 0.3 - 0.1$$

$$= 0.2$$

Hence, the value of the filler is

$$0.2.$$

Therefore the statement is,

If

$$A$$

and

$$B$$

are two events associated with a random experiment such that

$$P(A) = 0.3, P(B) = 0.2$$

and

$$P(A \cap B) = 0.1,$$

then the value of

$$P(A \cap \overline{B})$$

is

$$0.2$$

.

Question 41

The probability of happening of an event

$$A$$

is

$$0.5$$

and that of

$$B$$

is

$$0.3$$

. If

$$A$$

and

$$B$$

are mutually exclusive events, then the probability of neither

$$A$$

nor

$$B$$

is _____.

Solution

Ans: Given: Probability of happening of an event

$$A$$

is

$$0.5$$

and that of

$$B$$

is

$$0.3$$

.

Given that,

$$P(A) = 0.5,$$

$$P(B) = 0.3$$

and

$$P(A \cap B) = 0$$

$$\therefore A$$

and

B

are mutually exclusive events

$$\begin{aligned} P(\overline{A} \cap \overline{B}) &= P(\overline{A \cup B}) \\ &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B)] \end{aligned}$$

Substitute the values,

$$\begin{aligned} P(\overline{A} \cap \overline{B}) \\ &= 1 - (0.5 + 0.3) \end{aligned}$$

$$= 1 - 0.8$$

$$= 0.2$$

Hence, the value of the filler is

$$0.2.$$

The probability of happening of an event

A

is

$$0.5$$

and that of

B

is

$$0.3$$

. If A and B are mutually exclusive events, then the probability of neither A nor B is

$$0.2.$$

Question 42

Match the proposed probability under Column

C_1

with the appropriate written description under column

C_2

.

C_1	C_2
Probability	Written description
(a) 0.95	(i) An incorrect assignment
(b) 0.02	(ii) No chance of happening
(c) -0.3	(iii) As much chance of happening as not
(d) 0.5	(iv) Very likely to happen
(e) 0	(v) Very little chance of happening

Solution

Ans:

$$0.95 =$$

Very likely to happen, so it is close to

$$1.$$

$$0.02 =$$

Very little chance of happening as the probability is very low.

$$-0.3 =$$

an incorrect assignment because probability is never negative.

$$0.5 =$$

as much chance of happening as not because sum of chances of happening and not happening is one.

$$0 =$$

no chance of happening.

Hence,

$$(a) \leftrightarrow (iv),$$

$$(b) \leftrightarrow (v),$$

$$(c) \leftrightarrow (i),$$

$$(d) \leftrightarrow (iii),$$

(e) $\leftrightarrow (ii)$ **Question 43**

Match the following

(a) If E_1 and E_2 are the two mutually exclusive events	(i) $E_1 \cap E_2 = E_1$
(b) If E_1 and E_2 are the mutually exclusive and exhaustive events	(ii) $(E_1 - E_2) \cup (E_1 \cup E_2) = E_1$
(c) If E_1 and E_2 have common outcomes, then	(iii) $E_1 \cap E_2 = \varphi, E_1 \cup E_2 = S$
(d) If E_1 and E_2 are two events such that $E_1 \subset E_2$	(iv) $E_1 \cap E_2 = \varphi$

Solution

Ans: (a) If

$$E_1$$

and

$$E_2$$

are mutually exclusive events, then

$$E_1 \cap E_2 = \phi.$$

(b) If

$$E_1$$

and

$$E_2$$

are mutually exclusive and exhaustive events then

$$E_1 \cap E_2 = \phi$$

and

$$E_1 \cup E_2 = S$$

(c) If

$$E_1$$

and

$$E_2$$

have common outcomes, then

$$(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$$

(d) If

$$E_1$$

and

$$E_2$$

are two events such that

$$E_1 \subset E_2 \Rightarrow E_1 \cap E_2 = E_1$$

Hence,

$$(a) \leftrightarrow (iv), (b) \leftrightarrow (iii), (c) \leftrightarrow (ii), (d) \leftrightarrow (i)$$

The Important Topics For Chapter 16 Probability Class 11 Maths

The important topics are described as the following:

- Random experiment
- Outcome
- Sample Space
- Event
- Types of events
- Impossible and Sure Events
- Simple or Elementary Event
- Compound Event
- Mutually exclusive event
- Exhaustive events
- An axiomatic approach to probability
- Probabilities of equally likely outcomes
- Addition rule of probability
- Addition rule for a mutually exclusive event