

Chapter 15: Statistics

NCERT Exemplar · Class 11 · Maths

Short Answer Questions

Question 1

Find the mean deviation about the mean of the distribution:

Size (x)	20	21	22	23	24
Frequency (f)	6	4	5	1	4

Solution

Ans: Given: A discrete frequency distribution

For a discrete frequency distribution, mean deviation about mean is given by

$$M. D. (\bar{x}) = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{\sum_{i=1}^n f_i}$$

where

$$\bar{x}$$

is the mean of distribution.

Size (x)	Frequency (f)	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
20	6	120	1.65	9.9
21	4	84	0.65	2.6
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.4
	$\sum_{i=1}^n f_i = 20$	$\sum_{i=1}^n f_i x_i = 433$		$\sum_{i=1}^n f_i x_i - \bar{x} = 25$

$$\Rightarrow \bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i} = \frac{433}{20} = 21.65$$

$$\Rightarrow M.D.(\bar{x}) = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{\sum_{i=1}^n f_i} = \frac{25}{20} = 1.25$$

Question 2

Find the mean deviation about the median of the following distribution:

Size (x)	10	11	12	13	14
Frequency (f)	2	3	8	3	4

Solution

Ans: Given: A discrete frequency distribution

For a discrete frequency distribution, mean deviation about mean is given by

$$M.D.(M) = \frac{\sum_{i=1}^n f_i |x_i - M|}{\sum_{i=1}^n f_i}$$

,where

$$M$$

is the mean of distribution

Size (x)	Frequency (f)	Cumulative frequency	$ x_i - M $	$f_i x_i - M $
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
	$\sum_{i=1}^n f_i = 20$			$\sum_{i=1}^n f_i x_i - M = 25$

So,

$$N = 20$$

(even)

Median =

$$\begin{aligned} & \frac{1}{2} \times \left(\left(\frac{n}{2} \right)^{th} \text{ observation} + \left(\frac{n}{2} + 1 \right)^{th} \text{ observation} \right) \\ \Rightarrow M &= \frac{1}{2} \times \left((10)^{th} \text{ observation} + (11)^{th} \text{ observation} \right) \\ \Rightarrow M &= \frac{1}{2} \times (12 + 12) = 12 \\ \Rightarrow M.D.(M) &= \frac{\sum_{i=1}^n f_i |x_i - M|}{\sum_{i=1}^n f_i} = \frac{25}{20} = 1.25 \end{aligned}$$

Question 3

Calculate the mean deviation about the mean of the set of first

n

natural numbers when

n

is an odd number.

Solution

Ans: Mean deviation about mean is given by

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$$

where

$\bar{x}$

is the mean of distribution.

$$\begin{aligned} \bar{x} &= \frac{1 + 2 + 3 + \dots + n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2} \\ M.D.(\bar{x}) &= \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n} \end{aligned}$$

$$M.D.(\bar{x}) = \frac{1}{n} \left(\left(1 - \frac{n+1}{2} \right) + \left(2 - \frac{n+1}{2} \right) + \left(3 - \frac{n+1}{2} \right) + \dots + \left(n - \frac{n+1}{2} \right) \right)$$

$$= \frac{1}{n} \left(\left(\frac{1-n}{2} \right) + \left(\frac{3-n}{2} \right) + \left(\frac{5-n}{2} \right) + \dots + \left(\frac{1-n}{2} \right) \right)$$

$$\begin{aligned} \Rightarrow M.D.(\bar{x}) &= \frac{2}{n} \left(1 + 2 + 3 + \dots + \left(\frac{1-n}{2} \right) + \left(\frac{3-n}{2} \right) + \left(\frac{n-1}{2} \right) \right) \\ \Rightarrow M.D.(\bar{x}) &= \frac{2}{n} \times \frac{\left(\frac{n-1}{2} \right) \left(\frac{n-1}{2} + 1 \right)}{2} \end{aligned}$$

$$\begin{aligned}\Rightarrow M.D.(\bar{x}) &= \frac{1}{n} \times \left(\frac{n-1}{2} \right) \times \left(\frac{n+1}{2} \right) \\ \Rightarrow M.D.(\bar{x}) &= \frac{1}{n} \times \left(\frac{n^2-1}{4} \right) \\ \Rightarrow M.D.(\bar{x}) &= \frac{n^2-1}{4n}\end{aligned}$$

Question 4

Calculate the mean deviation about the mean of the set of first

n

natural numbers when

n

is an even number.

Solution

Ans: Given:

n

is an even number.

First we will calculate the mean of the

n

natural numbers. Then, we shall subtract the mean value from every number and evaluate the mean value of their absolute values. Finally, we shall divide it by the sum of

n

natural numbers.

Mean of first n natural numbers, where n is even:

$$x = \frac{1+2+3+\dots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

Hence, mean deviation about the mean will be:

$$\begin{aligned}M.D. &= \frac{\left| 1 - \frac{n+1}{2} \right| + \left| 2 - \frac{n+1}{2} \right| + \left| 3 - \frac{n+1}{2} \right| + \dots + \left| \frac{n-2}{2} - \frac{n+1}{2} \right| + \left| \frac{n}{2} - \frac{n+1}{2} \right| + \left| \frac{n+2}{2} - \frac{n+1}{2} \right| + \dots + \left| n - \frac{n+1}{2} \right|}{n} \\ &= \frac{\left| \frac{1-n}{2} \right| + \left| \frac{3-n}{2} \right| + \left| \frac{5-n}{2} \right| + \dots + \left| \frac{-3}{2} \right| + \left| \frac{-1}{2} \right| + \left| \frac{1}{2} \right| + \dots + \left| \frac{n-1}{2} \right|}{n} \\ &= \frac{\frac{n-1}{2} + \frac{n-3}{2} + \frac{n-5}{2} + \dots + \frac{3}{2} + \frac{1}{2} + \frac{1}{2} + \dots + \frac{n-1}{2}}{n} \\ &= \frac{2 \times \frac{n}{2} \left(\frac{n-1}{2} + \frac{1}{2} \right)}{n} = \frac{n}{4}\end{aligned}$$

Question 5

Find the standard deviation of first

n

natural numbers.

Solution

Ans: Sum of first n natural numbers:

$$\sum x_i = 1 + 2 + \dots + n = \frac{n}{2}(n+1)$$

Sum of the squares of natural numbers:

$$\sum x_i^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Standard deviation:

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \\ &= \sqrt{\frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{\frac{n(n+1)}{2}}{n}\right)^2} = \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}} \\ &= \sqrt{\frac{2n^2 + 3n + 1}{6} - \frac{n^2 + 2n + 1}{4}} = \sqrt{\frac{4n^2 + 6n + 2 - 3n^2 - 6n - 3}{12}} = \sqrt{\frac{n^2 - 1}{12}}\end{aligned}$$

Question 6

The mean and standard deviation of same data for the time taken to complete a test are calculated with following results:

Number of observations =

25

, mean =

18.2seconds

, standard deviation =

3.25seconds

.

Further another set of

15

observations

$x_1, x_2, \dots, x_{15}$

, also in seconds, is now available and we have

$$\sum_{i=1}^{15} x_i = 279$$

and

$$\sum_{i=1}^{15} x_i^2 = 5524$$

. Calculate the standard deviation based on all 40 observations.

Solution

Ans: Given: Number of observations =

25

, mean =

18.2 seconds

, standard deviation =

3.25 seconds

.

$$\sum_{i=1}^{15} x_i = 279$$

$$\sum_{i=1}^{15} x_i^2 = 5524$$

In the first set,

$$\text{Mean} = \bar{x}_i = \frac{\sum x_i}{25} = 18.2$$

$$\sum x_i = 25 \times 18.2 = 455$$

$$\sigma_1^2 = \frac{\sum x_i^2}{25} - \left(\frac{\sum x_i}{25} \right)^2 = \frac{\sum x_i^2}{25} - (18.2)^2$$

$$S.D. = \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

For 40 observations,

$$\sum x_i^2 = 8545.06 + 5524 = 14069.06$$

$$\sum x_i = 455 + 279 = 734$$

$$\therefore \sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2} = \sqrt{\frac{14069.06}{40} - \left(\frac{734}{40} \right)^2} = \sqrt{15.004} = 3.87$$

Question 7

The mean and standard deviation of a set of

n_1

observations are

$\bar{x}_1$

and

s_1

, respectively while the mean and standard deviation of another set of

n_2

observation are

$\bar{x}_2$

and

s_2

respectively. Show that the standard deviation of the combined set of

$(n_1 + n_2)$

observations is given by

$$[S.D. = \sqrt{\frac{\{n_1\{s_1\}^2 + n_2\{s_2\}^2\}}{\{n_1 + n_2\}} + \frac{\{n_1 n_2\}(\overline{x_1} - \overline{x_2})^2}{\{n_1 + n_2\}^2}}]$$

Solution

Ans: Given: The mean and standard deviation of a set of

$$n_1$$

observations are

$$\overline{x_1}$$

and

$$s_1$$

, respectively. The mean and standard deviation of another set of

$$n_2$$

observation are

$$\overline{x_2}$$

and

$$s_2$$

respectively.

For the first and second set,

$$\overline{x_1} = \frac{1}{n_1} \sum_{i=1}^n x_i, \overline{x_2} = \frac{1}{n_2} \sum_{j=1}^n y_j$$

Mean of the combines series,

$$\overline{x} = \frac{1}{n_1 + n_2} \left[\sum_{i=1}^n x_i + \sum_{j=1}^n y_j \right] = \frac{n_1 \overline{x_1} + n_2 \overline{x_2}}{n_1 + n_2} \quad \text{--- (1)}$$

$$\sigma_1^2 = \frac{1}{n_1} \sum_{i=1}^n (x_i - \overline{x})^2, \sigma_2^2 = \frac{1}{n_2} \sum_{j=1}^n (y_j - \overline{x})^2$$

For the total set,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 = \frac{1}{n_1 + n_2} \left(\sum_{i=1}^n (x_i - \overline{x})^2 + \sum_{j=1}^n (y_j - \overline{x})^2 \right) \quad \text{--- (2)}$$

$$\sum_{i=1}^n (x_i - \overline{x})^2 = \sum_{i=1}^n (x_i - \overline{x_j})^2 + n_1 (\overline{x_j} - \overline{x})^2 + 2(\overline{x_j} - \overline{x}) \sum_{i=1}^n (x_i - \overline{x_j})^2$$

$$\text{Since, } \sum_{i=1}^n (x_i - \overline{x_j})^2 = 0$$

$$\sum_{i=1}^n (x_i - \overline{x})^2 = n_1 s_1^2 + n_1 (\overline{x_j} - \overline{x})^2 \quad \text{--- (3)}$$

$$\therefore d_1 = \overline{x_1} - \overline{x}$$

From (1),

$$d_1 = \overline{x_1} - \frac{n_1 \overline{x_1} + n_2 \overline{x_2}}{n_1 + n_2} = \frac{n_1 \overline{x_1} + n_2 \overline{x_1} - n_1 \overline{x_1} - n_2 \overline{x_2}}{n_1 + n_2} = \frac{n_2 (\overline{x_1} - \overline{x_2})}{n_1 + n_2}$$

$$\text{Finally, } \sum_{i=1}^n (x_i - \overline{x})^2 = n_1 s_1^2 + \frac{n_1 n_2^2 (\overline{x_1} - \overline{x_2})^2}{(n_1 + n_2)^2}$$

Similarly,

$$\sum_{j=1}^n (y_j - \overline{x})^2 = n_2 s_2^2 + \frac{n_2 n_1^2 (\overline{x_2} - \overline{x_1})^2}{(n_1 + n_2)^2}$$

$$\begin{aligned}
\therefore \sigma^2 &= \sigma_1^2 + \sigma_2^2 = \frac{1}{n_1 + n_2} \left(\sum_{i=1}^n (x_i - \bar{x})^2 + \sum_{j=1}^n (y_j - \bar{x})^2 \right) \\
&= \frac{1}{n_1 + n_2} \left(n_1 s_1^2 + \frac{n_1 n_2^2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2} + n_2 s_2^2 + \frac{n_2 n_1^2 (\bar{x}_2 - \bar{x}_1)^2}{(n_1 + n_2)^2} \right) \\
&= \frac{1}{n_1 + n_2} \left(n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2^2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2} + \frac{n_2 n_1^2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2} \right) \\
&= \frac{1}{n_1 + n_2} \left(n_1 s_1^2 + n_2 s_2^2 + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)} (n_2 + n_1) \right) \\
\text{Hence, } \sigma^2 &= \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}
\end{aligned}$$

Question 8

Two sets of

20

observations have the same standard deviation

5

. The first set has a mean

17

and the second a mean

22

. Determine the standard deviation of the set obtained by combining the given

2

sets.

Solution

Ans: Given:

$$n_1 = n_2 = 20$$

,

$$\bar{x}_1 = 17$$

,

$$\bar{x}_2 = 22$$

,

$$\sigma_1 = \sigma_2 = 5$$

$$\begin{aligned}
\sigma &= \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}} \\
&= \sqrt{\frac{20 \times 25 + 20 \times 25}{40} + \frac{400(-5)^2}{40^2}} = \sqrt{25 + \frac{25}{4}} = \sqrt{31.25} = 5.59
\end{aligned}$$

Question 9

The frequency distribution:

x	A	$2A$	$3A$	$4A$	$5A$	$6A$
f	2	1	1	1	1	1

where A is a positive integer, has a variance of 160. Determine the value of A .**Solution**

Ans: Given: Variance =

$$160$$

$$\text{Variance} = \sigma^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n} \right)^2$$

$$160 = \frac{2A^2 + 1.(4A^2) + 1.(9A^2) + 1.(16A^2) + 1.(25A^2) + 1.(36A^2)}{7} - \left(\frac{2A + 2A + 3A + 4A + 5A + 6A}{7} \right)^2$$

$$\Rightarrow 160 = \frac{92A^2}{7} - \frac{484A^2}{49}$$

On Solving, we get

$$A = 7$$

Question 10

For the frequency distribution:

x	2	3	4	5	6	7
y	4	9	16	14	11	6

Find the standard distribution.

Solution

Ans: For standard deviation:

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n} \right)^2}$$

$$= \sqrt{\frac{(4 \times 2^2 + 9 \times 3^2 + 16 \times 4^2 + 14 \times 5^2 + 11 \times 6^2 + 6 \times 7^2)}{4 + 9 + 16 + 14 + 11 + 6} - \left(\frac{8 + 27 + 64 + 70 + 66 + 42}{60} \right)^2}$$

$$= \sqrt{\frac{1393}{60} - \frac{277^2}{3600}} = 1.379 \approx 1.38$$

Question 11

There are 60 students in a class. The following is the frequency distribution of marks obtained by the students in a test:

Marks	0	1	2	3	4	5
Frequency	$x - 2$	x	x^2	$(x + 1)^2$	$2x$	$x + 1$

Where x , is positive integer. Determine the mean and standard deviation of the marks.

Solution

Ans: $\sum f_i = 60$

$$\Rightarrow (x - 2) + x + x^2 + (x + 1)^2 + 2x + x + 1 = 60$$

$$\Rightarrow 2x^2 + 7x - 60 = 0$$

Solving the above equation, we get $x = -\frac{15}{2}, 4$

$\therefore x = 4$.

Let the mean is 3.

Marks	Frequency	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
0	2	$0 - 3 = -3$	-6	18
1	4	$1 - 3 = -2$	-8	16
2	16	$2 - 3 = -1$	-16	16
3	25	$3 - 3 = 0$	0	0
4	8	$4 - 3 = 1$	8	8
5	5	$5 - 3 = 2$	10	20
	N=60		$\sum f_i d_i = -12$	$\sum f_i d_i^2 = 78$

Mean:

$$\bar{x} = A + \left(\frac{\sum f_i d_i}{n} \right) = 3 + \left(\frac{-12}{60} \right) = 3 - 0.2 = 2.8$$

So the mean is 2.8.

Standard deviation:

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n} \right)^2} \\ &= \sqrt{\frac{78}{60} - \left(\frac{-12}{60} \right)^2} = \sqrt{1.3 - \frac{144}{3600}} = \sqrt{1.26} = 1.122 \end{aligned}$$

So the Standard deviation = 1.122

Question 12

The mean life of a sample of

60

bulbs was

650

hours and the standard deviation was

8

hours. A second sample of

80

bulbs has a mean life of

650

hours and a standard deviation of

7

hours. Find the overall standard deviation.

Solution

Ans: Given: Mean life of a sample of

60

bulbs was

650

hours and the standard deviation was

8

hours is 8 hours.

With the help of the below formula,

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

Then,

$$n_1 = 60, \bar{x}_1 = 650, s_1 = 8; n_2 = 80, \bar{x}_2 = 660, s_2 = 7$$

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$\Rightarrow \sigma = \sqrt{\frac{60 \times 8^2 + 80 \times 7^2}{60 + 80} + \frac{60 \times 80 (650 - 660)^2}{(60 + 80)^2}}$$

$$\text{Since, } = \frac{60 \times 64 + 80 \times 49}{140} = 20 \frac{[3(64) + 4(49)]}{140}$$

$$= \frac{[3(64) + 4(49)]}{7} = \frac{192 + 196}{7}$$

$$\sigma = \sqrt{\frac{60 \times 64 + 80 \times 49}{140} + \frac{60 \times 80 (650 - 660)^2}{140^2}}$$

$$\Rightarrow \sigma = \sqrt{\frac{60 \times 64 + 80 \times 49}{140} + \frac{60 \times 80 (650 - 660)^2}{140 \times 140}}$$

$$\frac{60 \times 80 (650 - 660)^2}{140 \times 140} = \frac{60 \times 80 \times 100}{140 \times 140}$$

$$\begin{aligned}
 &= \frac{1200}{49} \\
 \Rightarrow \sigma &= \sqrt{\frac{192 + 196}{7} + \frac{1200}{49}} = \sqrt{\frac{2716 + 1200}{49}} \\
 &= \sqrt{\frac{3916}{49}} = \frac{62.58}{7} = 8.9
 \end{aligned}$$

Question 13

Mean and standard deviation of

100

items are

50

and

4

respectively. Find the sum of the entire item and the sum of the squares of the items.

Solution

Ans: Given:

(a) Mean

(b) The standard deviation of respectively.

With the help of the below formula,

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2$$

$$\bar{x} = 50, n = 100 \text{ and } \sigma = 4$$

$$\frac{\sum x_i}{100} = 50$$

$$\sum x_i = 5000$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2$$

$$\Rightarrow 4^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (50)^2$$

$$\Rightarrow 4^2 + (50)^2 = \frac{\sum f_i x_i^2}{\sum f_i}$$

$$\Rightarrow \frac{\sum f_i x_i^2}{\sum f_i} = 16 + 2500$$

$$\Rightarrow \frac{\sum f_i x_i^2}{100} = 2516$$

$$\Rightarrow \sum f_i x_i^2 = 251600$$

Question 14

If for distribution

$$\sum (x - 5) = 3 \text{ and } \sum (x - 5)^2 = 43$$

and the total number of items is

$$18$$

. Find the mean and standard deviation.

Solution

Ans: Given:

$$\sum (x - 5) = 3 \text{ and } \sum (x - 5)^2 = 43$$

The total number of items is

$$18$$

Mean

$$= A + \frac{\sum (x - 5)}{n}$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum (x - 5)^2}{n} - \left(\frac{\sum (x - 5)}{n} \right)^2}$$

Then,

$$\sum (x - 5) = 3 \text{ and } \sum (x - 5)^2 = 43$$

Mean

$$= A + \frac{\sum (x - 5)}{n}$$

$$= 5 + \frac{3}{18}$$

$$= 5 + 0.1666$$

$$= 5.17$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum (x - 5)^2}{n} - \left(\frac{\sum (x - 5)}{n} \right)^2}$$

$$= \sqrt{\frac{43}{18} - \left(\frac{3}{18} \right)^2}$$

$$= \sqrt{2.3889 - 0.0277} = 1.53$$

Question 15

Find the mean and variance of the frequency distribution given below:

x	$1 \leq x < 3$	$3 \leq x < 5$	$5 \leq x < 7$	$7 \leq x < 10$
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f	6	4	5	1
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Solution

Ans: By using the below basic formulas, we can find the mean and variance.

Mean

$$= \frac{\sum f_i y_i}{f_i}$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum f_i y_i^2}{n} - \left(\frac{\sum f_i y_i}{n}\right)^2}$$

x	f_i	x_i	$f_i x_i$	$f_i x_i^2$
1-3	6	2	12	24
3-5	4	4	16	64
5-7	5	6	30	180
7-10	1	8.5	8.5	72.25
Total	n=16		$\sum f_i x_i = 66.5$	$\sum f_i x_i^2 = 340.25$

Mean

$$= \frac{\sum f_i y_i}{f_i}$$

$$= \frac{66.5}{16}$$

$$= 4.16$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum f_i y_i^2}{n} - \left(\frac{\sum f_i y_i}{n}\right)^2}$$

$$= \sqrt{\frac{340.25}{16} - \left(\frac{66.5}{16}\right)^2}$$

$$= \sqrt{21.26 - 17.22} = 4.04$$

Long Answer Questions**Question 16**

Calculate the mean deviation about the mean for the following frequency distribution:

Class interval	0-4	4-8	8-12	12-16	16-20
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Frequency	4	6	8	5	2
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Solution

Ans: We know that,

$$X = A + h \left(\frac{1}{N} \sum f_i d_i \right)$$

Mean deviation about mean =

$$= \frac{1}{N} \left(\sum f_i |x_i - X| \right)$$

Let us assume

$$A = 10$$

and

$$h = 4$$

Class interval	Mid value (xi)	Frequency	$d_i = \frac{x_i - 10}{4}$	$f_i d_i$	$ x_i - X = x_i - 9.2 $	$f_i x_i - X $
0-4	2	4	-2	-8	-7.2	-28.8
4-8	6	6	-1	-6	-3.2	-19.2
8-12	10	8	0	0	0.8	6.4
12-16	14	5	1	5	4.8	24
16-20	18	2	2	4	8.8	17.6
		N=25		$\sum f_i d_i = -5$		$\sum f_i x_i - X = 96$

$$X = A + h \left(\frac{1}{N} \sum f_i d_i \right)$$

$$= 10 + 4 \left(\frac{1}{25} (-5) \right)$$

$$= 10 - \frac{4}{5} = 10 - 0.8$$

$$= 9.2$$

Mean deviation about mean =

$$= \frac{1}{N} \left(\sum f_i |x_i - X| \right) = \frac{96}{25} = 3.84$$

Question 17

Calculate the mean deviation from the median of the following data:

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	5	3	6	2

Solution

Ans: With the help of the basic formula,

Median

$$= l + \frac{\frac{N}{2} - C}{f} \times h$$

Class interval	Frequency	(xi)	cf	$d_i = x_i - \overline{m_d} $	fidi
0-6	4	3	4	11	44
6-12	5	9	9	5	25
12-18	3	15	12	1	3
18-24	6	21	18	7	42
24-30	2	27	20	13	26
Total	N=20		$\sum f_i x_i = 230$		$\sum f_i d_i = 140$

$$= 12 + \frac{(10 - 9)}{3} \times 6 = 14$$

Since, $\frac{N}{2} = 10$ median class is 12 – 18

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

$$\text{Median} = 12 + \frac{10 - 9}{3} \times 6 = 14$$

$$\text{MD} = \frac{\sum f_i d_i}{\sum f_i} = \frac{140}{20} = 7.$$

So the required mean deviation is 7.

Question 18

Determine the mean and standard deviation of the following distribution:

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

Solution

$$\text{Ans: Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i} \right)^2}$$

Marks	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
2	1	2	-4	-4	16
3	6	18	-3	-18	54
4	6	24	-2	-12	24
5	8	40	-1	-8	8
6	8	48	0	0	0
7	2	14	1	2	2
8	2	16	2	4	8
9	3	27	3	9	27
10	0	0	4	0	0
11	2	22	5	10	50
12	1	12	6	6	36
13	0	0	7	0	0
14	0	0	8	0	0
15	0	0	9	0	0
16	1	16	10	10	100
Total	$\sum f_i = 40$	$\sum f_i x_i = 239$		$\sum f_i d_i = -1$	$\sum f_i d_i^2 = 325$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{239}{40} = 5.975 = 6$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2} \\ &= \sqrt{\frac{325}{40} - \left(\frac{-1}{40}\right)^2} \\ &= \sqrt{8.124} = 2.85\end{aligned}$$

Question 19

The weight of coffee in

70

jars is shown in the following table:

Weight Frequency (in grams)	Frequency
-----------------------------	-----------

200-201	13
201-202	27
202-203	18
203-204	10
204-205	1
205-206	1

Determine variance and standard deviation of the above distribution.

Solution

Ans: We know that,

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2$$

Class interval	f_i	x_i	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
200-201	13	200.5	-2	-26	52
201-202	27	201.5	-1	-27	27
202-203	18	202.5	0	0	0
203-204	10	203.5	1	10	10
204-205	1	204.5	2	2	4
205-206	1	205.5	3	3	9
Total	$\sum f_i = 70$			$\sum f_i d_i = -38$	$\sum f_i d_i^2 = 102$

$$\text{The mean } \bar{x} = \frac{200.5 \times 13 + 201.5 \times 27 + 202.5 \times 18 + 203.5 \times 10 + 204.5 \times 1 + 205.5 \times 1}{70} = \frac{14137}{70} = 202.5$$

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2$$

$$= \frac{102}{70} - \left(\frac{-38}{70} \right)^2$$

$$= 1.4571 - 0.2946 = 1.1655$$

$$\sigma = \sqrt{1.1655} = 1.08gm$$

Question 20

Determine the mean and standard deviation of first n terms of an A.P. whose first term is

$$a$$

and the common difference is

$$d.$$

Solution

Ans: Given: First-term of an A.P is

$$a$$

.

In addition, the common difference is

$$d.$$

Mean

$$= \frac{\sum x_i}{n} = \frac{1}{n} \left[\frac{n}{2} [2a + (n-1)d] \right] = a + \frac{n-1}{2}d$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n} \right)^2}$$

x_i	$x_i - a$	$(x_i - a)^2$
a	0	0
$a + d$	d	d^2
$a + 2d$	$2d$	$4d^2$
.....		$9d^2$
.....		
.....		
$a + (n-1)d$	$(n-1)d$	$(n-1)^2d^2$

Mean

$$= \frac{\sum x_i}{n} = \frac{1}{n} \left[\frac{n}{2} [2a + (n-1)d] \right] = a + \frac{n-1}{2}d$$

$$\sum (x_i - a) = d[1 + 2 + 3 + \dots + (n-1)] = \frac{d(n-1)n}{2}$$

$$\sum (x_i - a)^2 = d^2 [1^2 + 2^2 + 3^2 + \dots + (n-1)^2] = d^2 \frac{n(n-1)(2n-1)}{6}$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n} \right)^2}$$

$$= \sqrt{\frac{d^2 (n-1)(2n-1)}{6} - \frac{d^2 (n-1)^2}{4}}$$

$$= d \sqrt{\frac{n-1}{2} \left[\frac{(2n-1)}{3} - \frac{(n-1)}{2} \right]}$$

$$= d\sqrt{\frac{n-1}{2} \left[\frac{2(2n-1) - 3(n-1)}{6} \right]}$$

$$= d\sqrt{\frac{n-1}{2} \left[\frac{(n+1)}{6} \right]} = d\sqrt{\frac{n^2-1}{12}}$$

Question 21

Following are the marks obtained, out of 100, by two students Ravi and Hashina in

10

tests.

Ravi	25	50	45	30	70	42	36	48	35	60
Hashina	10	70	50	20	95	55	42	60	48	80

Who is more intelligent and who is more consistent?

Solution

Ans: We know that,

$$\sigma = \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n} \right)^2}$$

For Ravi

Let assumed mean = 45

x_i	$d_i = x_i - 45$	d_i^2
25	-20	400
50	5	25
45	0	0
30	-15	225
70	25	625
42	-3	9
36	-9	81
48	3	9
35	-10	100
60	15	225
Total	$\sum d_i = -9$	$\sum d_i^2 = 1699$

$$\begin{aligned}
 \sigma &= \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} \\
 &= \sqrt{\frac{1699}{10} - \left(\frac{-9}{10}\right)^2} \\
 &= \sqrt{169.9 - 0.81} \\
 &= \sqrt{169.09} = 13.003 \\
 \bar{x} &= A + \frac{\sum d_i}{\sum f_i} = 45 - \frac{9}{10} = 44.1
 \end{aligned}$$

for Hashina,

x_i	$d_i = x_i - 55$	d_i^2
10	-45	2025
70	15	225
50	-5	25
20	-35	1225
95	40	1600
55	0	0
42	-13	169
60	5	25
48	-7	49
80	25	625
Total	$\sum d_i = -20$	$\sum d_i^2 = 5968$

$$\begin{aligned}
 \sigma &= \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} \\
 &= \sqrt{\frac{5698}{10} - \left(\frac{20}{10}\right)^2} \\
 &= \sqrt{569.8 - 2} \\
 &= \sqrt{594.8} = 24.38 \\
 \text{And, } \bar{x} &= A + \frac{\sum d_i}{\sum f_i} = 55 - \frac{20}{10} = 53
 \end{aligned}$$

For Ravi,

$$\frac{\sigma}{\bar{x}} \times 100 = \frac{13.003}{44.1} \times 100 = 29.48$$

For hashina,

$$\frac{\sigma}{\bar{x}} \times 100 = \frac{24.38}{53} \times 100 = 46.01$$

Hence hashina is more intelligent and consistent.

Question 22

Mean and standard deviation

100

observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Solution

Ans: Given: $n = 100, \bar{x} = 40, \sigma = 10$

Two Observations were wrongly recorded as

30 and 70

in the place of

3 and 27

$$\text{We know that, } \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$n = 100, \bar{x} = 40, \sigma = 10$$

$$\frac{\sum x_i}{n} = 40 \Rightarrow \sum x_i = 40 \times 100$$

$$\sum x_i = 4000$$

$$\text{Corrected } \sum x_i = 4000 - 30 - 70 + 3 + 27 = 3930$$

$$\text{Corrected mean} = \frac{3930}{100} = 39.3$$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$100 = \frac{\sum x_i^2}{100} - 1600$$

$$\sum x_i^2 = 170000$$

$$\begin{aligned} \text{Corrected } \sum x_i^2 &= 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2 \\ &= 164938 \end{aligned}$$

$$\text{corrected } \sigma = \sqrt{\frac{164938}{100} - (39.3)^2} = \sqrt{(1649.38 - 1544.49)}$$

$$= \sqrt{104.9} = 10.24$$

Question 23

While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Solution

Ans: Given: Mistaken mean of the 10 observations = 45

Mistaken variance of the 10 observations = 16

The correct reading is 25 instead of 52.

Use the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$ to find the sum of all the observations. Subtract the incorrect value and add the correct value to find the correct total. Here $\bar{x}$ is the mean.

Use the formula $\sigma^2 = \frac{\sum_{i=1}^n (x_i)^2}{n} - (\bar{x})^2$, where σ^2 is the variance, to calculate $\sum_{i=1}^n (x_i)^2$. Now, subtract $(52)^2$ from $\sum_{i=1}^n (x_i)^2$ and add $(25)^2$.

The incorrect mean of the given 10 observations is 45. Assuming this mean as $\bar{x}$ and using the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$,

$$\Rightarrow 45 = \frac{\left(\sum_{i=1}^{10} x_i\right)}{10}$$

$$\Rightarrow \left(\sum_{i=1}^{10} x_i\right) = 450$$

That means the sum of all the observations (including the incorrect one) is 450. Since one reading is to be 25 instead of 52, so the correct sum must be,

$$\Rightarrow \left(\sum_{i=1}^{10} x_i\right)_{co} = 450 - 52 + 25$$

$$\Rightarrow \left(\sum_{i=1}^{10} x_i\right)_{co} = 423$$

Therefore, the correct mean is $\bar{x}_{co} = \frac{\left(\sum_{i=1}^{10} x_i\right)_{co}}{10} = 42.3$.

Now, using the formula $\sigma^2 = \frac{\sum_{i=1}^n (x_i)^2}{n} - (\bar{x})^2$ for the variance of the incorrect mean, where $\sigma^2 = 16$ is provided,

$$\Rightarrow \frac{\left(\sum_{i=1}^{10} (x_i)^2\right)}{10} - (45)^2 = 16$$

$$\Rightarrow \sum_{i=1}^{10} (x_i)^2 = 20410$$

That means the incorrect sum of squares of all the observations is 20410. Since 52 is to be replaced with 25, so the correct value of $\sum_{i=1}^{10} (x_i)^2$ is,

$$\Rightarrow \left(\sum_{i=1}^{10} (x_i)^2\right)_{co} = 20410 - (52)^2 + (25)^2$$

$$\Rightarrow \left(\sum_{i=1}^{10} (x_i)^2\right)_{co} = 18331$$

Hence, the variance for the correct mean is,

$$\Rightarrow (\sigma^2)_{co} = \frac{\left(\sum_{i=1}^{10} (x_i)^2\right)_{co}}{10} - (\bar{x})_{co}^2$$

$$\Rightarrow (\sigma^2)_{co} = \frac{18331}{10} - (42.3)_{co}^2$$

$$\Rightarrow (\sigma^2)_{co} = 43.81$$

Question 24

The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

- (A) 2
(B) 2.57
(C) 3
(D) 3.75

Solution

Ans: The correct option is (B).

Given: The data 3, 10, 10, 4, 7, 10, 5.

Use the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$ to calculate the mean of the given data. Here $\bar{x}$ is the mean.

Now, use the formula $M.D. = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$ to calculate the mean deviation.

The total number of observations is $n = 7$. Using the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$, where $\bar{x}$ is the mean,

$$\Rightarrow \bar{x} = \frac{\left(\sum_{i=1}^7 x_i\right)}{7}$$

$$\Rightarrow \bar{x} = \frac{3 + 10 + 10 + 4 + 7 + 10 + 5}{7}$$

$$\Rightarrow \bar{x} = 7$$

Now, the mean deviation is given as $M.D. = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$. So for the given data,

$$\Rightarrow M.D. = \frac{\sum_{i=1}^7 |x_i - 7|}{7}$$

$$\Rightarrow M.D. = \frac{|3 - 7| + |10 - 7| + |10 - 7| + |4 - 7| + |7 - 7| + |10 - 7| + |5 - 7|}{7}$$

$$\Rightarrow M.D. = 2.57$$

Correct Answer: B

Question 25

Mean deviation for n observations $x_1, x_2, \dots, x_n$ from their mean $\bar{x}$ is given by

(A) $\sum_{i=1}^n (x_i - \bar{x})$

(B) $\frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$

(C) $\sum_{i=1}^n (x_i - \bar{x})^2$

$$(D) \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Solution

Ans: The correct option is (B).

Given: n observations $x_1, x_2, \dots, x_n$.

The mean value of these observations is $\bar{x}$.

Use the definition of the mean deviation which is 'the ratio of the summation of absolute deviation of each data from the mean and the total number of observations given'.

According to the definition of the mean deviation it is the ratio of the summation of absolute deviation of each data from the mean and the total number of observations given.

Now, the absolute deviation of n observations $x_1, x_2, \dots, x_n$ from their mean $\bar{x}$ is given as $|x_1 - \bar{x}|, |x_2 - \bar{x}|, \dots, |x_n - \bar{x}|$ respectively. Taking the sum of these deviations and dividing by the number of observations gives,

$$\Rightarrow M.D. = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$$

Correct Answer: B

Question 26

When tested the lives (in hours) of 5 bulbs were noted as follows:

1357, 1090, 1666, 1494, 1623

The mean deviation (in hours) from their mean is

(A) 178

(B) 179

(C) 220

(D) 356

Solution

Ans: The correct option is (A).

Given: The lives in hours of 5 bulbs are 1357, 1090, 1666, 1494, 1623.

Use the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$ to calculate the mean of the given data. Here $\bar{x}$ is the mean.

Now, use the formula $M.D. = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$ to calculate the mean deviation.

The total number of observations is $n = 5$. Using the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$, where $\bar{x}$ is the mean,

$$\Rightarrow \bar{x} = \frac{\left(\sum_{i=1}^5 x_i\right)}{5}$$

$$\Rightarrow \bar{x} = \frac{1357 + 1090 + 1666 + 1494 + 1623}{5}$$

$$\Rightarrow \bar{x} = 1446 \text{ hours}$$

Now, the mean deviation is given as $M.D. = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$. So for the given data,

$$\Rightarrow M.D. = \frac{\sum_{i=1}^5 |x_i - 1446|}{5}$$

$$\Rightarrow M.D. = \frac{|1357 - 1446| + |1090 - 1446| + |1666 - 1446| + |1494 - 1446| + |1623 - 1446|}{5}$$

$$\Rightarrow M.D. = 178 \text{ hours}$$

Correct Answer: A

Question 27

Following are the marks obtained by 9 students in a mathematics test:

50, 69, 20, 33, 53, 39, 40, 65, 59

The mean deviation from the median is:

- (A) 9
- (B) 10.5
- (C) 12.67
- (D) 14.76

Solution

Ans: The correct option is (C).

Given: The marks obtained by 9 students in the test are 50, 69, 20, 33, 53, 39, 40, 65, 59.

First arrange the marks in ascending order as 20, 33, 39, 40, 50, 53, 59, 65, 69 and find the median (M) which will be $\left(\frac{n+1}{2}\right)^{th}$ term after the arrangement.

Now, use the formula $M.D. = \frac{\sum_{i=1}^n |x_i - M|}{n}$ to calculate the mean deviation from the median.

To find the median the data must be arranged in either ascending or descending order of their value. So the arrangement in ascending order is 20, 33, 39, 40, 50, 53, 59, 65, 69.

The number of observations (n) is odd that means the median (M) is given as $\left(\frac{n+1}{2}\right)^{th}$ term.

$$\Rightarrow M = \left(\frac{9+1}{2}\right)^{th} \text{ term}$$

$$\Rightarrow M = (5)^{th} \text{ term}$$

$$\Rightarrow M = 50$$

Now, the mean deviation from the median is given as $M.D. = \frac{\sum_{i=1}^n |x_i - M|}{n}$. So for the given data,

$$\Rightarrow M.D. = \frac{\sum_{i=1}^9 |x_i - 50|}{9}$$

$$\Rightarrow M.D. = \frac{|20 - 50| + |33 - 50| + |39 - 50| + |40 - 50| + |50 - 50| + |53 - 50| + |59 - 50| + |65 - 50| + |69 - 50|}{9}$$

$$\Rightarrow M. D. = 12.67$$

Correct Answer: C

Question 28

The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(A) $\sqrt{\frac{52}{7}}$

(B) $\frac{52}{7}$

(C) $\sqrt{6}$

(D) 6

Solution

Ans: The correct option is (A).

Given: The data 6, 5, 9, 13, 12, 8, 10.

Use the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$ to calculate the mean of the given data. Here $\bar{x}$ is the mean.

Now, use the formula $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$, where σ is the standard deviation.

The total number of observations is $n = 7$. Using the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$, where $\bar{x}$ is the mean,

$$\Rightarrow \bar{x} = \frac{\left(\sum_{i=1}^7 x_i\right)}{7}$$

$$\Rightarrow \bar{x} = \frac{6 + 5 + 9 + 13 + 12 + 8 + 10}{7}$$

$$\Rightarrow \bar{x} = 9$$

Now, the standard deviation is given as $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$, where σ is the standard deviation.

$$\Rightarrow \sigma = \sqrt{\frac{\sum_{i=1}^7 (x_i - 9)^2}{7}}$$

$$\Rightarrow \sigma = \sqrt{\frac{(6-9)^2 + (5-9)^2 + (9-9)^2 + (13-9)^2 + (12-9)^2 + (8-9)^2 + (10-9)^2}{7}}$$

$$\Rightarrow \sigma = \sqrt{\frac{52}{7}}$$

Correct Answer: A

Question 29

Let $x_1, x_2, \dots, x_n$ be n observations and

$$\bar{x}$$

be their arithmetic mean. The formula for the standard deviation is given by

(A) $\sum_{i=1}^n (x_i - \bar{x})^2$

(B) $\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$

(C) $\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$

(D) $\sqrt{\frac{\sum_{i=1}^n x_i^2}{n}} + (\bar{x})^2$

Solution

Ans: The correct option is (C).

Given: n observations $x_1, x_2, \dots, x_n$ and their arithmetic mean as

$$\bar{x}$$

Use the definition of the variance which is 'the ratio of the summation of the squares of the deviations of each observation from their mean and the total number of observations'.

Take the square root of this mathematical expression of the variance to get the standard deviation.

The arithmetic means is generally referred to as the mean $\bar{x}$ of the data.

According to the definition of variance it is the ratio of the summation of the squares of the deviations of each observation from their mean and the total number of observations. Mathematically it is given as,

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

Now, the standard deviation is the square root of the variance, so it is given by the relation,

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

Correct Answer: C

Question 30

The mean of 100 observations is 50 and their standard deviation is 5. The sum of all squares of all the observations is

(A) 50000

(B) 250000

(C) 252500

(D) 255000

Solution

Ans: The correct option is (C).

Given: Total number of observations = 100

Mean of the observations = 50

Standard deviation = 5

Use the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$ to calculate the sum of all the observations, i.e. $\left(\sum_{i=1}^n x_i\right)$. Here $\bar{x}$ is the mean.

Now, use the formula $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$, where σ is the standard deviation, and calculate the variance. Break the summation into several parts and find the value of $\left(\sum_{i=1}^n (x_i)^2\right)$.

Since there are 100 observations and the mean is 50, so using the formula $\bar{x} = \frac{\left(\sum_{i=1}^n x_i\right)}{n}$,

$$\Rightarrow 50 = \frac{\left(\sum_{i=1}^{100} x_i\right)}{100}$$

$$\Rightarrow \left(\sum_{i=1}^{100} x_i\right) = 5000 \quad \dots\dots (1)$$

Now, the standard deviation is given as $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$, where σ is the standard deviation. On squaring both the sides the variance is obtained. The provided value of σ is 5.

$$\Rightarrow \sqrt{\frac{\sum_{i=1}^{100} (x_i - \bar{x})^2}{100}} = 5$$

$$\Rightarrow \frac{\sum_{i=1}^{100} (x_i - \bar{x})^2}{100} = 25$$

$$\Rightarrow \sum_{i=1}^{100} (x_i - \bar{x})^2 = 2500$$

Breaking the summation into several parts,

$$\Rightarrow \sum_{i=1}^{100} (x_i)^2 + \sum_{i=1}^{100} (\bar{x})^2 - \sum_{i=1}^{100} 2\bar{x}x_i = 2500$$

Using the value from equation (1),

$$\Rightarrow \sum_{i=1}^{100} (x_i)^2 + \sum_{i=1}^{100} (50)^2 - \sum_{i=1}^{100} 2 \times 50 \times x_i = 2500$$

$$\Rightarrow \sum_{i=1}^{100} (x_i)^2 + 100 \times (50)^2 - 100 \sum_{i=1}^{100} x_i = 2500$$

$$\Rightarrow \sum_{i=1}^{100} (x_i)^2 + 250000 - 500000 = 2500$$

$$\Rightarrow \sum_{i=1}^{100} (x_i)^2 = 252500$$

Correct Answer: C

Question 31

Let

$$a, b, c, d, e$$

be the observations with mean

$$m$$

and standard deviation

$$s.$$

the standard deviation of the observations

$$a + k, b + k, c + k, d + k, e + k$$

is

(A)

$$s$$

(B)

$$s \cdot k$$

(C)

$$s + k$$

(D)

$$\frac{s}{k}$$

Solution

Ans: The correct option is (A).

Given:

$$a, b, c, d, e$$

be the observations of mean m and standard deviations.

We know that,

$$Var(ax + b) = a^2 \cdot Var(x)$$

or

$$S.D(ax + b) = a \cdot S.D(x)$$

Then,

$$Var(ax + b) = a^2 \cdot Var(x)$$

$$S.D(ax + b) = a \cdot S.D(x)$$

$$a = 1, b = k$$

$$\therefore S.D(x + k) = S.D(a + k, b + k, c + k, d + k, e + k)$$

$$= 1 \cdot S.D(a, b, c, d, e) = s$$

Correct Answer: A

Question 32

Let

$$x_1, x_2, x_3, x_4, x_5$$

be the observations with mean

$$m$$

and standard deviation

$$s$$

. The standard deviation of the observations

$$kx_1, kx_2, kx_3, kx_4, kx_5$$

is

(A)

$$k + s$$

(B)

$$\frac{s}{k}$$

(C)

$$k \cdot s$$

(D)

$$s$$

Solution

Ans: The correct option is (C).

Given:

$$x_1, x_2, x_3, x_4, x_5$$

be the observations of mean

$$m$$

and standard deviation S.

We know that,

$$S.D = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sum x_i^2 = (kx_1)^2 + \dots + (kx_5)^2$$

$$= k^2 \sum x_i^2$$

$$\sum x_i = (kx_1) + \dots + (kx_5)$$

$$= k \sum x_i$$

Then,

$$S.D = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sum x_i^2 = (kx_1)^2 + \dots + (kx_5)^2$$

$$= k^2 \sum x_i^2$$

$$\sum x_i = (kx_1) + \dots + (kx_5)$$

$$= k \sum x_i$$

$$\begin{aligned} \text{S.D} &= \sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2} \quad \text{Since, } \bar{x} = \frac{\sum x_i}{n}; \sigma = S = \text{S.D} \\ &= |k| \sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i^2}{5}\right)^2} \\ &= |k| s \end{aligned}$$

Correct Answer: C

Question 33

Let

$$x_1, x_2, \dots, x_n$$

be

n

observations. Let

$$w_i = lx_i + k$$

for

$$l = 1, 2, \dots, n,$$

where

$$l \text{ and } k$$

are constants. If the mean of x_i 's is 48 and their standard deviation is

$$12$$

, the mean of w_i 's is 55 and standard deviation of w_i 's is 15, the mean of the values of

$$l$$

and

$$k$$

should be

(A)

$$l = 1.25, k = -5$$

(B)

$$l = -1.25, k = 5$$

(C)

$$l = 2.5, k = -5$$

(D)

$$l = 2.5, k = 5$$

Solution

Ans: The correct option is (A).

Given:

$$x_1, x_2, \dots, x_n$$

Be

n

observations

$$w_i = lx_i + k$$

for $l = 1, 2, \dots, n$,

$$l \text{ and } k$$

are constants. The mean of x_i is 48 and their Standard deviation is

$$12$$

The mean of w_i 's is 55 and standard deviation of w_i 's is 15.

We know that,

$$\sigma(ax + b) = |a| \sigma(x)$$

$$\sigma(w_i) = \sigma(lx_i + k)$$

Then,

$$\bar{x} = 48$$

$$\sigma(x) = 12$$

$$w_i = lx_i + k \dots (1)$$

for equation (1)

$$\sigma(ax + b) = |a| \sigma(x)$$

$$\Rightarrow \sigma(w_i) = \sigma(lx_i + k)$$

$$\Rightarrow \sigma(w) = (l)(12) \Rightarrow l = \frac{15}{12}$$

$$\Rightarrow l = 1.25$$

$$\text{Now, } \bar{w} = l\bar{x} + k$$

$$\Rightarrow 55 = (1.25 \times 48) + k$$

$$\Rightarrow k = -5$$

Correct Answer: A

Question 34

Standard deviations for the first

$$10$$

natural numbers is

(A)

$$5.5$$

(B)

$$3.87$$

(C)

$$2.97$$

(D)

$$2.87$$

Solution

Ans: The correct option is (D).

Given: First 10 natural numbers,

$$n = 10$$

To find: Standard deviations

We know that,

$$\sigma = \sqrt{\frac{n^2 - 1}{12}}$$

$$\Rightarrow \sigma = \sqrt{\frac{n^2 - 1}{12}}$$

$$n = 10$$

$$\Rightarrow \sigma = \sqrt{\frac{10^2 - 1}{12}}$$

$$= \sqrt{\frac{99}{12}}$$

$$= 2.87$$

Correct Answer: D

Question 35

Consider the numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

If

$$1$$

is added to each number, the variance of the numbers so obtained is

(A)

$$6.5$$

(B)

$$2.87$$

(C)

$$3.87$$

(D)

$$8.25$$

Solution

Ans: The correct option is (D).

Given: Let us consider the numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

$$1$$

is added to each number.

We know that,

$$\sigma^2 = \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n} \right)^2$$

$$\sum_{i=1}^{10} X_i^2 = 2^2 + 3^2 + \dots + 11^2$$

$$= \frac{11 \times 12 \times 23}{6} - 1 = 505$$

$$X_1, \dots, X_n = \sum_{i=1}^n X_i = 2 + 3 + \dots + 11 = 65$$

Then,

$$\begin{aligned}\sigma^2 &= \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n} \right)^2 \\ &= \frac{505}{10} - (6.5)^2 \\ &= 8.25\end{aligned}$$

Correct Answer: D

Question 36

Consider the first

10

positive integers. If we multiply each number by

−1

and then add 1 to each number, the variance of the numbers so obtained is

(A)

8.25

(B)

6.5

(C)

3.87

(D)

2.87

Solution

Ans: The correct option is (A).

Given: Let's consider the first 10 positive numbers

Multiply by

−1

and add

1

to each number

For Example,

$$1 \times (-1) = -1 + 1 = 0$$

We know that,

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

Since the first 10 positive integers are

1, 2, 3, 4, 5, 6, 7, 8, 9 and 10

On multiplying each number by

$$-1,$$

We get

$$-1, -2, -3, -4, -5, -6, -7, -8, -9, -10$$

On adding

$$1$$

in each number,

We get

$$-1, -2, -3, -4, -5, -6, -7, -8, -9, -10$$

$$\sigma^2 = \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n} \right)^2$$

$$\sum x_i = \frac{-9 \times 10}{2} = -45$$

$$\sum x_i^2 = 0^2 + (-1)^2 + (-2)^2 + (-3)^2 + \dots + (-9)^2 = \frac{9 \times 10 \times 19}{6} = 285$$

$$\sigma^2 = \frac{285}{10} - \left(\frac{-45}{10} \right)^2 = \frac{285}{10} - \left(\frac{2025}{100} \right)$$

$$= \sqrt{\frac{2850 - 2025}{100}} = (\sqrt{8.25})^2 = 8.25$$

Correct Answer: A

Question 37

The following information relates to sample size 60: $\sum x^2 = 18000$, $\sum x = 960$ the variance is

(A)

$$6.63$$

(B)

$$16$$

(C)

$$22$$

(D)

$$44$$

Solution

Ans: The correct option is (D).

Given: $\sum x^2 = 18000$, $\sum x = 960$

We know that,

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{n} \right)^2$$

Then,

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{n} \right)^2$$

$$N = 60$$

$$\sigma^2 = \frac{18000}{60} - \left(\frac{960}{60} \right)^2$$

$$= 300 - (16)^2$$

$$= 44$$

Correct Answer: D

Question 38

Coefficient of variation of two distributions is 50 and 60, and their arithmetic means are 30 and 25 respectively. The difference in their standard deviation is

(A) 0

(B) 1

(C)

1.5

(D)

2.5

Solution

Ans: The correct option is (A).

Given: Coefficients of variations are 50 and 60

Arithmetic means are 30 and 25

We know that,

$$CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$$

Then,

$$CV_1 = 50, CV_2 = 60$$

$$\bar{x}_1 = 30, \bar{x}_2 = 25$$

$$cv_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$$

$$\sigma_1 = \frac{50 \times 30}{100} = 15$$

$$CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$$

$$\sigma_2 = \frac{60 \times 25}{100} = 15$$

now

$$\sigma_1 = \sigma_2 = 15$$

The difference in standard deviation is

0.

Correct Answer: A

Question 39

The standard deviation of some temperature data in

$$^{\circ}C \text{ is } 5$$

. If the data were converted into $^{\circ}F$ then the variance would be

(A) 81

(B) 57

(C) 36

(D) 25

Solution

Ans: The correct option is (A).

Given: Some data are given in

$$^{\circ}C \text{ is } 5$$

We know that,

$$F = \frac{9}{5}C + 32$$

Then,

$$\sigma_c \text{ and } \sigma_f = ?$$

$$F = \frac{9}{5}c + 32$$

As We know that on adding and subtracting any number in data the standard deviation doesn't change but on multiplying and dividing the data by the same number the standard deviation changes according to the number.

$$x \rightarrow s$$

$$kx \rightarrow ks$$

$$x \rightarrow s$$

$$k + x \rightarrow s$$

$$\sigma_f = \frac{9}{5}\sigma_c$$

$$\Rightarrow \sigma_f = 9 \because \text{Given } \sigma_c = 5$$

$$\Rightarrow \text{variance} = (SD)^2 = 9^2 = 81$$

Correct Answer: A

Fill in the blanks in Exercises from 40 to 46.

Question 40

$$\text{Coefficient of variation} = \frac{\dots\dots\dots}{Mean} \times 100$$

Solution

Ans: The formula for computing variation is

$$\text{Coefficient of variation C.V} = \frac{\sigma}{\mu} \times 100$$

Where, σ = standard deviation

μ = Mean

Therefore,

$$\text{C.V.} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

Question 41

If x is the mean of n values of x , then $\sum_{i=1}^n (x_i - \bar{x})$ is always equal to _____.

If a has any value other than x , then $\sum_{i=1}^n (x_i - \bar{x})^2$ is _____ than $(x_i - a)^2$.

Solution

Ans: Mean of n observations $x_1, x_2, \dots, x_n = \bar{x}$

$$\text{Therefore, } \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\Rightarrow \sum_{i=1}^n x_i = n\bar{x} \text{ then,}$$

$$\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x}$$

$$= n\bar{x} - \bar{x} \sum_{i=1}^n 1 \left[\text{and } \sum_{i=1}^n 1 = n \right]$$

$$= n\bar{x} - \bar{x} \cdot n$$

$$= 0$$

Hence, the value of $\sum_{i=1}^n (x_i - \bar{x}) = 0$.

$$\text{Then, } \sum_{i=1}^n (x_i - \bar{x})^2 < (x_i - a)^2.$$

Question 42

If the variance of a data is

121

, then the standard deviation of the data is _____.

Solution

Ans: Square root of variance is standard deviation.

$$\text{i.e, standard deviation} = \sqrt{\text{variance}}$$

Therefore, given

$$\text{variance} = 121$$

$$\sigma^2 = 121$$

$$\Rightarrow \sigma = \sqrt{121}$$

$$\Rightarrow \sigma = 11$$

Hence, standard deviation is 11

Question 43

The standard deviation of a data is _____ of any change in origin, but is _____ on the change of scale.

Solution

Ans: Standard deviation is not affected by the addition or subtraction of any value from the observation therefore independent of change in origin. But change in scale affects the standard deviation i.e., any value multiplied or divided to the observation changes standard deviation too.

Question 44

The sum of the squares of the deviations of the values of the variable is _____ when taken about their arithmetic mean.

Solution

Ans: Mean of the number of observations converts the series of observations into a minimum value and mean deviation is the arithmetic average of deviations of the observation from an observed value. Sum of squares of deviation is minimum when deviations are taken from mean.

Question 45

The mean deviation of the data is _____ when measured from the median.

Solution

Ans: Sum of all the deviation of the data about the data mean is = mean deviation and the median lies between mean and mode. Therefore, mean deviation of the data is least when measured from the median.

Question 46

The standard deviation is _____ to the mean deviation taken from the arithmetic mean.

Solution

Ans: Difference between square of deviation of data about mean and square of mean is standard deviation. And mean deviation is the sum of all deviations of a series of data about its mean. Hence, its greater or equal to mean deviation.

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