

Chapter 13: Limits and Derivatives

NCERT Exemplar · Class 11 · Maths

Short Answer Questions

Question 1

Evaluate: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$\because [a^2 - b^2 = (a + b)(a - b)]$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} \text{ (given)}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - (3)^2}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x + 3)(x - 3)}{(x - 3)}$$

$$= \lim_{x \rightarrow 3} (x + 3)$$

$$= 3 + 3 = 6$$

Question 2

Evaluate: $\lim_{x \rightarrow 1/2} \frac{4x^2 - 1}{2x - 1}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 1/2} \frac{4x^2 - 1}{2x - 1}$$

Then,

$$= \lim_{x \rightarrow 1/2} \frac{(2x)^2 - (1)^2}{2x - 1}$$

$$= \lim_{x \rightarrow 1/2} \frac{(2x + 1)(2x - 1)}{(2x - 1)}$$

$$= \lim_{x \rightarrow 1/2} (2x + 1)$$

$$= 2 \times \frac{1}{2} + 1$$

$$= 1 + 1$$

$$= 2$$

Question 3

Evaluate: $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

Solution

Ans: Given:

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\because h \rightarrow 0 \Rightarrow x + h \rightarrow x$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \quad (\text{given})$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^{1/2} - (x)^{1/2}}{x+h-x}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^{1/2} - (x)^{1/2}}{(x+h) - x} \quad \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= \frac{1}{2} x^{-1/2} \quad [\because h \rightarrow 0 \Rightarrow x + h \rightarrow x]$$

$$= \frac{1}{2\sqrt{x}}$$

Question 4

Evaluate: $\lim_{x \rightarrow 0} \frac{(x+2)^{1/3} - 2^{1/3}}{x}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{(x+2)^{1/3} - 2^{1/3}}{x}$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{(x+2)^{1/3} - 2^{1/3}}{(x+2) - 2} \\
&= \frac{1}{3} \times 2^{\frac{1}{3}-1} = \frac{1}{3} \times (2)^{-2/3} \quad \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\
&= \frac{1}{3(2)^{2/3}} \quad [\because x \rightarrow 0 \Rightarrow x+2 \rightarrow 2]
\end{aligned}$$

Question 5

Evaluate: $\lim_{x \rightarrow 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$$

Divide numerator and denominator by

$$x$$

$$= \lim_{x \rightarrow 0} \frac{\frac{(1+x)^6 - 1}{x}}{\frac{(1+x)^2 - 1}{x}}$$

$$[\because x \rightarrow 0 \Rightarrow 1+x \rightarrow 1]$$

$$= \lim_{x \rightarrow 0} \frac{\frac{(1+x)^6 - 1}{(1+x) - 1}}{\frac{(1+x)^2 - 1}{(1+x) - 1}}$$

$$= \frac{\lim_{x \rightarrow 0} \frac{(1+x)^6 - (1)^6}{(1+x) - 1}}{\lim_{x \rightarrow 0} \frac{(1+x)^2 - (1)^2}{(1+x) - 1}} \left[\because \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \right]$$

$$= \frac{6(1)^{6-1}}{2(1)^{2-1}}$$

$$\left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= \frac{6 \times 1}{2 \times 1}$$

$$= \frac{6}{2}$$

$$= 3$$

Question 6

Evaluate: $\lim_{x \rightarrow a} \frac{(2+x)^{5/2} - (a+2)^{5/2}}{x-a}.$

Solution

Ans: Given:

$$\lim_{x \rightarrow a} \frac{(2+x)^{5/2} - (a+2)^{5/2}}{x-a}$$

Then,

$$\begin{aligned} &= \lim_{x \rightarrow a} \frac{(2+x)^{5/2} - (a+2)^{5/2}}{(2+x) - (a+2)} \\ &= \frac{5}{2}(a+2)^{\frac{5}{2}-1} \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = na^{n-1} \right] \\ &= \frac{5}{2}(a+2)^{\frac{3}{2}} \quad [\because x \rightarrow a \Rightarrow x+2 \rightarrow a+2] \end{aligned}$$

Question 7

Evaluate: $\lim_{x \rightarrow 1} \frac{x - \sqrt{x}}{\sqrt{x} - 1}.$

Solution

Ans: Given:

$$\lim_{x \rightarrow 1} \frac{x - \sqrt{x}}{\sqrt{x} - 1}$$

Then,

Then,

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{\sqrt{x} \left[(x)^{\frac{7}{2}} - 1 \right]}{\sqrt{x} - 1} \\ &= \lim_{x \rightarrow 1} \frac{(x)^{7/2} - 1}{\sqrt{x} - 1} \cdot \lim_{x \rightarrow 1} \sqrt{x} \\ &\left[\because \lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) \right] \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 1} \frac{\frac{x^{7/2} - 1}{x - 1}}{\frac{(x)^{1/2} - 1}{x - 1}} \cdot 1 \\
&= \frac{\lim_{x \rightarrow 1} \frac{x^{7/2} - 1}{x - 1}}{\lim_{x \rightarrow 1} \frac{(x)^{1/2} - 1}{x - 1}} \left[\because \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \right] \\
&= \frac{\frac{7}{2}(1)^{\frac{7}{2}-1}}{\frac{1}{2}(1)^{\frac{1}{2}-1}} \\
&= \frac{\frac{7}{2}}{\frac{1}{2}} \\
&= 7
\end{aligned}$$

Question 8

Evaluate: $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x - 2} - \sqrt{x + 2}}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x - 2} - \sqrt{x + 2}}$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow 2} \frac{(x^2 - 4) (\sqrt{3x - 2} + \sqrt{x + 2})}{(\sqrt{3x - 2} - \sqrt{x + 2})(\sqrt{3x - 2} + \sqrt{x + 2})} \\
&= \lim_{x \rightarrow 2} \frac{(x^2 - 4) (\sqrt{3x - 2} + \sqrt{x + 2})}{(\sqrt{3x - 2})^2 - (\sqrt{x + 2})^2} \quad [\because (a + b)(a - b) = a^2 - b^2] \\
&= \lim_{x \rightarrow 2} \frac{(x^2 - 4) (\sqrt{3x - 2} + \sqrt{x + 2})}{(3x - 2) - (x + 2)} \\
&= \lim_{x \rightarrow 2} \frac{(x^2 - 4) (\sqrt{3x - 2} + \sqrt{x + 2})}{3x - 2 - x - 2} \\
&= \lim_{x \rightarrow 2} \frac{(x^2 - 4) (\sqrt{3x - 2} + \sqrt{x + 2})}{2x - 4} \\
&= \lim_{x \rightarrow 2} \frac{(x + 2)(x - 2)(\sqrt{3x - 2} + \sqrt{x + 2})}{2(x - 2)}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2} \\
&= \frac{(2+2)(\sqrt{6-2} + \sqrt{2+2})}{2} \\
&= \frac{4(2+2)}{2} = 8
\end{aligned}$$

Question 9

Evaluate: $\lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$

Solution

Ans: Given:

$$\lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$$

Then,

$$\lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x^2)^2 - (2)^2}{x^2 + 3\sqrt{2}x - 8}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x^2 - 2)(x^2 + 2)}{x^2 + 4\sqrt{2}x - \sqrt{2}x - 8}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})(x^2 + 2)}{x(x + 4\sqrt{2}) - \sqrt{2}(x + 4\sqrt{2})}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})(x^2 + 2)}{(x - \sqrt{2})(x + 4\sqrt{2})}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x^2 + 2)}{(x + 4\sqrt{2})}$$

$$\begin{aligned}
&= \frac{(\sqrt{2} + \sqrt{2}) \left[(\sqrt{2})^2 + 2 \right]}{(\sqrt{2} + 4\sqrt{2})} \\
&= \frac{2\sqrt{2}(2+2)}{5\sqrt{2}} \\
&= \frac{8}{5}
\end{aligned}$$

Question 10

Evaluate: $\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$$

Then,

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{x^7 - x^5 - x^5 + 1}{x^3 - x^2 - 2x^2 + 2} \\ &= \lim_{x \rightarrow 1} \frac{x^5(x^2 - 1) - 1(x^5 - 1)}{x^2(x - 1) - 2(x^2 - 1)} \end{aligned}$$

On dividing numerator and denominator by $(x - 1)$, then

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{\frac{x^5(x^2 - 1)}{(x - 1)} - \frac{1(x^5 - 1)}{(x - 1)}}{\frac{x^2(x - 1)}{(x - 1)} - \frac{2(x^2 - 1)}{(x - 1)}} \\ &= \frac{\lim_{x \rightarrow 1} x^5(x + 1) - \lim_{x \rightarrow 1} \left(\frac{x^5 - 1}{x - 1} \right)}{\lim_{x \rightarrow 1} x^2 - \lim_{x \rightarrow 1} (x + 1)} \\ &= \frac{1 \times 2 - 5 \times (1)^4}{1 - 2 \times 2} \\ &= \frac{2 - 5}{1 - 4} \\ &= \frac{-3}{-3} = 1 \end{aligned}$$

Question 11

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2}$$

Then,

$$= \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} \quad (\text{given})$$

Rationalise with $\sqrt{1+x^3} + \sqrt{1-x^3}$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{x^2} \cdot \frac{\sqrt{1+x^3} + \sqrt{1-x^3}}{\sqrt{1+x^3} + \sqrt{1-x^3}} \\
&= \lim_{x \rightarrow 0} \frac{(1+x^3) - (1-x^3)}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})} \\
&= \lim_{x \rightarrow 0} \frac{1+x^3 - 1+x^3}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})} \\
&= \lim_{x \rightarrow 0} \frac{2x^3}{x^2 (\sqrt{1+x^3} + \sqrt{1-x^3})} \\
&= \lim_{x \rightarrow 0} \frac{2x}{(\sqrt{1+x^3} + \sqrt{1-x^3})} \\
&= 0
\end{aligned}$$

Question 12**Evaluate:**

$$\lim_{x \rightarrow -3} \frac{x^3 + 27}{x^5 + 243}$$

Solution

Ans: Given:

$$\lim_{x \rightarrow -3} \frac{x^3 + 27}{x^5 + 243}$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow -3} \frac{x^3 + 27}{x^5 + 243} \text{ (given)} \\
&= \lim_{x \rightarrow -3} \frac{\frac{x^3 + 27}{x + 3}}{\frac{x^5 + 243}{x + 3}} \\
&= \lim_{x \rightarrow -3} \frac{\frac{x^3 - (-3)^3}{x - (-3)}}{\frac{x^5 - (-3)^5}{x - (-3)}} \\
&= \frac{\lim_{x \rightarrow -3} \frac{x^3 - (-3)^3}{x - (-3)}}{\lim_{x \rightarrow -3} \frac{x^5 - (-3)^5}{x - (-3)}} \left[\because \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{3(-3)^{3-1}}{5(-3)^{5-1}} \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\
&= \frac{3}{5} \frac{(-3)^2}{(-3)^4} \\
&= \frac{3}{5(-3)^2} \\
&= \frac{3}{45} \\
&= \frac{1}{15}
\end{aligned}$$

Question 13

Evaluate: $\lim_{x \rightarrow 1/2} \left(\frac{8x - 3}{2x - 1} - \frac{4x^2 + 1}{4x^2 - 1} \right).$

Solution

Ans: Given:

$$\lim_{x \rightarrow 1/2} \left(\frac{8x - 3}{2x - 1} - \frac{4x^2 + 1}{4x^2 - 1} \right)$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow 1/2} \left(\frac{8x - 3}{2x - 1} - \frac{4x^2 + 1}{4x^2 - 1} \right) \text{ (given)} \\
&= \lim_{x \rightarrow 1/2} \left[\frac{(8x - 3)(2x + 1) - (4x^2 + 1)}{(4x^2 - 1)} \right] \\
&= \lim_{x \rightarrow 1/2} \left[\frac{16x^2 + 8x - 6x - 3 - 4x^2 - 1}{4x^2 - 1} \right] \\
&= \lim_{x \rightarrow 1/2} \left[\frac{12x^2 + 2x - 4}{4x^2 - 1} \right] \\
&= \lim_{x \rightarrow 1/2} \frac{2(6x^2 + x - 2)}{4x^2 - 1} \\
&= \lim_{x \rightarrow 1/2} \frac{2(6x^2 + 4x - 3x - 2)}{4x^2 - 1} \\
&= \lim_{x \rightarrow 1/2} \frac{2[2x(3x + 2) - 1(3x + 2)]}{4x^2 - 1} \\
&= \lim_{x \rightarrow 1/2} \frac{2[(3x + 2)(2x - 1)]}{(2x)^2 - (1)^2} \\
&= \lim_{x \rightarrow 1/2} \frac{2(3x + 2)(2x - 1)}{(2x - 1)(2x + 1)}
\end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 1/2} \frac{2(3x+2)}{2x-1} \\
 &= \frac{2\left(3 \times \frac{1}{2} + 2\right)}{2 \times \frac{1}{2} + 1} \\
 &= \frac{3}{2} + 2 \\
 &= \frac{7}{2}
 \end{aligned}$$

Question 14

Find 'n', if

$$\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80, \quad n \in N$$

Solution

Ans: Given:

$$\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80, \quad n \in N$$

Then,

$$\begin{aligned}
 \Rightarrow n(2)^{n-1} &= 80 \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\
 \Rightarrow n \times 2^{n-1} &= 5 \times 16 \\
 \Rightarrow n \times 2^{n-1} &= 5 \times (2)^4 \\
 \Rightarrow n \times 2^{n-1} &= 5 \times (2)^{5-1} \\
 \therefore n &= 5
 \end{aligned}$$

Question 15Evaluate: $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x}$.**Solution**

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x}$$

We know that,

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$\therefore x \rightarrow 0 \Rightarrow (kx \rightarrow 0)$, here k is real number

$$= \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x} \text{ (given)}$$

Multiply and divide $3x$ in numerator and denominator

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{3x} \cdot 3x}{\frac{\sin 7x}{7x} \cdot 7x} \\ &= \frac{\lim_{x \rightarrow 0} \frac{\sin 3x}{3x}}{\lim_{x \rightarrow 0} \frac{\sin 7x}{7x}} \cdot \frac{3x}{7x} \\ &= \frac{3}{7} \cdot \frac{\lim_{x \rightarrow 0} \frac{\sin 3x}{3x}}{\lim_{x \rightarrow 0} \frac{\sin 7x}{7x}} \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= \frac{3}{7} [\because x \rightarrow 0 \Rightarrow (kx \rightarrow 0), \text{ here } k \text{ is real number}] \end{aligned}$$

Question 16

Evaluate: $\lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 4x}$$

We know that,

$$\therefore [\sin 2\theta = 2 \sin \theta \cos \theta]$$

$$\therefore \cos 0 = 1$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 2x}{[\sin 2(2x)]^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 2x}{(2 \sin 2x \cos 2x)^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 2x}{4 \sin^2 2x \cos^2 2x} [\because \sin 2\theta = 2 \sin \theta \cos \theta]$$

$$= \lim_{x \rightarrow 0} \frac{1}{4 \cos^2 2x} [\because \cos 0 = 1]$$

$$= \frac{1}{4}$$

Question 17

Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

We know that, $\cos 2x = 1 - 2\sin^2 x$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Then,

$$= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \text{ (given)}$$

$$= \lim_{x \rightarrow 0} \frac{1 - 1 + 2\sin^2 x}{x^2} \left[\because \cos 2x = 1 - 2\sin^2 x \right]$$

$$= \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2}$$

$$= 2 \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$$

$$= 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 2 \times 1 = 2$$

Question 18

Evaluate: $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$$

We know that,

$$\because \sin 2x = 2 \sin x \cos x$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3} \text{ (given)}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{2 \sin x - 2 \sin x \cos x}{x^3} \quad [\because \sin 2x = 2 \sin x \cos x] \\
&= \lim_{x \rightarrow 0} \frac{2 \sin x (1 - \cos x)}{x^3} \\
&= 2 \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \\
&= 2 \cdot 1 \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= 2 \lim_{x \rightarrow 0} \frac{1 - 1 + 2 \sin^2 \frac{x}{2}}{x^2} \\
&= 2 \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{4 \times \frac{x^2}{4}} \\
&= \frac{2 \cdot 2}{4} \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \\
&= \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 = 1
\end{aligned}$$

Question 19

Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$$

We know that,

$$\because \cos mx = 1 - 2 \sin^2 \frac{mx}{2} \text{ and } \sin nx = 1 - 2 \sin^2 \frac{nx}{2}$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Then,

$$= \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} \quad (\text{given})$$

$$= \lim_{x \rightarrow 0} \frac{1 - 1 + 2\sin^2 \frac{mx}{2}}{1 - 1 + 2\sin^2 \frac{nx}{2}}$$

$$\because \cos mx = 1 - 2\sin^2 \frac{mx}{2}$$

$$\text{and } \sin nx = 1 - 2\sin^2 \frac{nx}{2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 \frac{mx}{2}}{\sin^2 \frac{nx}{2}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin^2 \frac{mx}{2}}{\left(\frac{mx}{2}\right)^2} \cdot \left(\frac{mx}{2}\right)^2}{\frac{\sin^2 \frac{nx}{2}}{\left(\frac{nx}{2}\right)^2} \cdot \left(\frac{nx}{2}\right)^2}$$

$$= \lim_{x \rightarrow 0} \frac{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{mx}{2}}{\frac{mx}{2}} \right)^2 \cdot \frac{m^2 x^2}{4}}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{nx}{2}}{\frac{nx}{2}} \right)^2 \cdot \frac{n^2 x^2}{4}}$$

$$= \frac{m^2}{n^2} \cdot \frac{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{mx}{2}}{\frac{mx}{2}} \right)^2}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{nx}{2}}{\frac{nx}{2}} \right)^2} \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{m^2}{n^2} \left[\because x \rightarrow 0 \Rightarrow kx \rightarrow 0 \right]$$

Question 20

Evaluate: $\lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)}$$

We know that,

$$\therefore \cos 2x = 1 - 2\sin^2 x$$

$$\therefore \sin(\pi - \theta) = \sin \theta$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\therefore x \rightarrow \frac{\pi}{3} \Rightarrow \left(x - \frac{\pi}{3}\right) \rightarrow 0$$

Then,

$$= \lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - \cos 6x}}{\sqrt{2} \left(\frac{\pi}{3} - x\right)} \quad (\text{given})$$

$$= \lim_{x \rightarrow \pi/3} \frac{\sqrt{1 - 1 + 2\sin^2 3x}}{\sqrt{2} \left(\frac{\pi}{3} - x\right)} \quad [\because \cos 2x = 1 - 2\sin^2 x]$$

$$= \lim_{x \rightarrow \pi/3} \frac{\sqrt{2} \sin 3x}{\sqrt{2} \left(\frac{\pi}{3} - x\right)}$$

$$= \lim_{x \rightarrow \pi/3} \frac{\sin 3x}{\frac{\pi}{3} - x}$$

$$= \lim_{x \rightarrow \pi/3} \frac{\sin(\pi - 3x)}{\frac{\pi - 3x}{3}} \quad [\because \sin(\pi - \theta) = \sin \theta]$$

$$= 3 \lim_{x \rightarrow \pi/3} \frac{\sin(\pi - 3x)}{(\pi - 3x)} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 3 \times 1$$

$$= 3 \quad \left[\because x \rightarrow \frac{\pi}{3} \Rightarrow \left(x - \frac{\pi}{3}\right) \rightarrow 0 \right]$$

Question 21

Evaluate: $\lim_{x \rightarrow \pi/4} \frac{(\sin x - \cos x)}{\left(x - \frac{\pi}{4}\right)}$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/4} \frac{(\sin x - \cos x)}{\left(x - \frac{\pi}{4}\right)}$$

We know that,

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\therefore x \rightarrow \frac{\pi}{4} \Rightarrow \left(x - \frac{\pi}{4}\right) \rightarrow 0$$

Then,

$$\begin{aligned}
 & \lim_{x \rightarrow \pi/4} \frac{(\sin x - \cos x)}{\left(x - \frac{\pi}{4}\right)} \quad (\text{given}) \\
 &= \lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \left(\sin x \cdot \frac{1}{\sqrt{2}} - \cos x \cdot \frac{1}{\sqrt{2}} \right)}{\left(x - \frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \left(\sin x \cos \frac{\pi}{4} - \cos x \cdot \sin \frac{\pi}{4} \right)}{\left(x - \frac{\pi}{4}\right)} \\
 &= \lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \left\{ \sin \left(x - \frac{\pi}{4} \right) \right\}}{\left(x - \frac{\pi}{4}\right)} \\
 &= \sqrt{2} \lim_{x \rightarrow \pi/4} \frac{\sin \left(x - \frac{\pi}{4} \right)}{\left(x - \frac{\pi}{4}\right)} \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
 &= \sqrt{2} \quad \left[\because x \rightarrow \frac{\pi}{4} \Rightarrow \left(x - \frac{\pi}{4} \right) \rightarrow 0 \right]
 \end{aligned}$$

Question 22

Evaluate: $\lim_{x \rightarrow \pi/6} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}.$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/6} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$$

We know that,

$$\because \sin A \cos B - \cos A \sin B = \sin(A - B)$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } x \rightarrow \frac{\pi}{6} \Rightarrow \left(x - \frac{\pi}{6} \right) \rightarrow 0$$

Then,

$$\begin{aligned}
 &= \lim_{x \rightarrow \pi/6} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} \quad (\text{given}) \\
 &= \lim_{x \rightarrow \pi/6} \frac{2 \left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right)}{\left(x - \frac{\pi}{6}\right)}
 \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow \pi/6} \frac{2 \left(\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6} \right)}{\left(x - \frac{\pi}{6} \right)} \left[\because \sin A \cos B - \cos A \sin B = \sin(A - B) \right] \\
&= 2 \lim_{x \rightarrow \pi/6} \frac{\sin \left(x - \frac{\pi}{6} \right)}{\left(x - \frac{\pi}{6} \right)} \\
&\left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&\left[\because x \rightarrow \frac{\pi}{6} \Rightarrow \left(x - \frac{\pi}{6} \right) \rightarrow 0 \right] \\
&= 2
\end{aligned}$$

Question 23

Evaluate: $\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x}$$

We know that,

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\sin 2x + 3x}{2x + \tan 3x} \text{ (given)} \\
&= \lim_{x \rightarrow 0} \frac{\frac{\sin 2x + 3x}{2x} \cdot 2x}{\frac{2x + \tan 3x}{3x} \cdot 3x} \\
&= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin 2x}{2x} + \frac{3x}{2x} \right) 2x}{\left(\frac{2x}{3x} + \frac{\tan 3x}{3x} \right) 3x} \\
&= \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{2x} + \frac{3}{2}}{\frac{2}{3} + \frac{\tan 3x}{3x}} \cdot \frac{2}{3} \\
&= \frac{2}{3} \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{2x} + \frac{3}{2}}{\frac{2}{3} + \lim_{x \rightarrow 0} \frac{\tan 3x}{3x}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2}{3} \left(\frac{1 + \frac{3}{2}}{\frac{2}{3} + 1} \right) \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right] \\
&= \frac{2}{3} \times \frac{\frac{5}{2}}{\frac{5}{3}} = \frac{2}{3} \times \frac{5}{2} \times \frac{3}{5} \\
&= 1
\end{aligned}$$

Question 24

Evaluate: $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$.

Solution

Ans: Given:

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$$

We know that,

$$\because \sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \quad (\text{given}) \\
&= \lim_{x \rightarrow 0} \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right)}{\sqrt{x} - \sqrt{a}} \left[\because \sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \right] \\
&= \lim_{x \rightarrow a} \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right) (\sqrt{x} + \sqrt{a})}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})} \\
&= \lim_{x \rightarrow 0} \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right) (\sqrt{x} + \sqrt{a})}{x - a} \\
&= 2 \lim_{x \rightarrow a} \cos \left(\frac{x+a}{2} \right) (\sqrt{x} + \sqrt{a}) \lim_{x \rightarrow 0} \frac{\sin \left(\frac{x-a}{2} \right)}{2 \left(\frac{x-a}{2} \right)}
\end{aligned}$$

$$\begin{aligned}
&= 2 \lim_{x \rightarrow 0} \cos\left(\frac{x+a}{2}\right) (\sqrt{x} + \sqrt{a}) \cdot \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin\left(\frac{x-a}{2}\right)}{\left(\frac{x-a}{2}\right)} \\
&= 2 \cdot \cos \frac{a}{2} \cdot \sqrt{a} \cdot \frac{1}{2} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= \sqrt{a} \cos \frac{a}{2}
\end{aligned}$$

Question 25

Evaluate: $\lim_{x \rightarrow \pi/6} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}.$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/6} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

We know that,

$$\because \operatorname{cosec}^2 x = 1 + \cot^2 x$$

Then,

$$\begin{aligned}
&= \lim_{x \rightarrow \pi/6} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} \quad (\text{given}) \\
&= \lim_{x \rightarrow \pi/6} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2} \quad [\because \operatorname{cosec}^2 x = 1 + \cot^2 x] \\
&= \lim_{x \rightarrow \pi/6} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} \\
&= \lim_{x \rightarrow \pi/6} \frac{(\operatorname{cosec} x)^2 - (2)^2}{\operatorname{cosec} x - 2} \\
&= \lim_{x \rightarrow \pi/6} \frac{(\operatorname{cosec} x + 2)(\operatorname{cosec} x - 2)}{(\operatorname{cosec} x - 2)} \\
&= \lim_{x \rightarrow \pi/6} (\operatorname{cosec} x + 2) \\
&= \operatorname{cosec} \frac{\pi}{6} + 2 = 2 + 2 \\
&= 4
\end{aligned}$$

Question 26

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

We know that,

$$\because \cos x = 2\cos^2 \frac{x}{2} - 1$$

$$\because \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

Then,

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + 2\cos^2 \frac{x}{2} - 1}}{\sin^2 x} \quad \left[\because \cos x = 2\cos^2 \frac{x}{2} - 1 \right]$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{2\cos^2 \frac{x}{2}}}{\sin^2 x} \quad \left[\because \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2} \right]$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} \left(1 - \cos \frac{x}{2} \right)}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} \left(1 - 1 + 2\sin^2 \frac{x}{4} \right)}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} \left(2\sin^2 \frac{x}{4} \right)}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0} 2\sqrt{2} \frac{\sin^2 \frac{x}{4}}{\left(\frac{x}{4} \right)^2} \cdot \frac{\left(\frac{x}{4} \right)^2}{\sin^2 x}$$

$$= 2\sqrt{2} \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{4}}{\frac{x}{4}} \right)^2 \cdot \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right)^2 \cdot \frac{1}{16}$$

$$= 2\sqrt{2} \cdot 1 \cdot 1 \cdot \frac{1}{16}$$

$$= \frac{1}{4\sqrt{2}}$$

Question 27

Evaluate: $\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$$

We know that,

$$\therefore \sin x + \sin y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Then,

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin 5x + \sin x - 2 \sin 3x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin 3x \cos 2x - 2 \sin 3x}{x} \left[\therefore \sin x + \sin y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \right]$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin 3x (\cos 2x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin 3x}{\frac{1}{3} \times 3x} (\cos 2x - 1)$$

$$= 6 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} (\cos 2x - 1)$$

$$= 6 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot \lim_{x \rightarrow 0} (\cos 2x - 1) \left[\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 6 \times 1 \times 0 = 0$$

Question 28

Evaluate: If $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$, then find the value of

k

.

Solution

Ans: Given:

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$$

We know that,

$$\therefore \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\therefore \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

Then,

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2} \text{ (given)}$$

$$\Rightarrow 4(1)^{4-1} = \lim_{x \rightarrow k} \frac{\frac{x^3 - k^3}{x - k}}{\frac{x^2 - k^2}{x - k}} \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$\Rightarrow 4 = \frac{\lim_{x \rightarrow k} \frac{x^3 - k^3}{x - k}}{\lim_{x \rightarrow k} \frac{x^2 - k^2}{x - k}} \cdot 1 \left[\because \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \right]$$

$$\Rightarrow 4 = \frac{3k^2}{2k}$$

$$\Rightarrow 4 = \frac{3}{2}k$$

$$\therefore k = \frac{4 \times 2}{3} = \frac{8}{3}$$

Differentiate each of the functions w. r. t x in Exercises 29 to 42:

Question 29

$$\frac{x^4 + x^3 + x^2 + 1}{x}$$

Solution

Ans: Given:

$$\frac{x^4 + x^3 + x^2 + 1}{x}$$

Then

$$\begin{aligned} &= \frac{d}{dx} \left(\frac{x^4 + x^3 + x^2 + 1}{x} \right) \\ &= \frac{d}{dx} \left(x^3 + x^2 + x + \frac{1}{x} \right) \\ &= \frac{d}{dx} x^3 + \frac{d}{dx} x^2 + \frac{d}{dx} x + \frac{d}{dx} \left(\frac{1}{x} \right) \end{aligned}$$

(by chain rule)

$$\begin{aligned} &= 3x^2 + 2x + 1 + \left(-\frac{1}{x^2} \right) \\ &= 3x^2 + 2x + 1 - \frac{1}{x^2} \\ &= \frac{3x^4 + 2x^3 + x^2 - 1}{x^2} \end{aligned}$$

Question 30

$$\left(x + \frac{1}{x}\right)^3$$

Solution

Ans: Given:

$$\left(x + \frac{1}{x}\right)^3$$

Let $y = \left(x + \frac{1}{x}\right)^3$

Differentiate on both side w.r.t

$$x$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{d}{dx} \left(x + \frac{1}{x}\right)^3 \\ &= 3 \left(x + \frac{1}{x}\right)^{3-1} \frac{d}{dx} \left(x + \frac{1}{x}\right) \text{ (by chain rule)} \\ &= 3 \left(x + \frac{1}{x}\right)^2 \left(1 - \frac{1}{x^2}\right) \\ &= 3x^2 - \frac{3}{x^2} - \frac{3}{x^4} + 3 \end{aligned}$$

Question 31

$$(3x + 5)(1 + \tan x)$$

Solution

Ans: Given:

$$(3x + 5)(1 + \tan x)$$

We know that,

Differentiate the given equation w.r.t

$$x$$

using product rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

Then, $y = (3x + 5)(1 + \tan x)$

Differentiate on both side w.r.t

$$x$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= \frac{d}{dx} [(3x+5)(1+\tan x)] \\
 &= (3x+5) \frac{d}{dx} (1+\tan x) + (1+\tan x) \frac{d}{dx} (3x+5) \quad [\text{product rule}] \\
 &= (3x+5) (\sec^2 x) + (1+\tan x) \cdot 3 \\
 &= (3x+5) \sec^2 x + 3(1+\tan x) \\
 &= 3x \sec^2 x + 5 \sec^2 x + 3 \tan x + 3
 \end{aligned}$$

Question 32

$$(\sec x - 1)(\sec x + 1)$$

Solution

Ans: Given:

$$(\sec x - 1)(\sec x + 1)$$

We know that, Differentiate the given equation w.r.t

$$x$$

using chain rule

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\therefore (a+b)(a-b) = a^2 - b^2$$

$$\text{Let, } y = (\sec x - 1)(\sec x + 1)$$

$$y = (\sec^2 - 1) \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \tan^2 x$$

$$\therefore \frac{dy}{dx} = 2 \tan x \cdot \frac{d}{dx} \tan x$$

$$= 2 \tan x \cdot \sec^2 x \quad (\text{By chain rule})$$

Question 33

$$\frac{3x+4}{5x^2-7x+9}$$

Solution

Ans: Given:

$$\frac{3x+4}{5x^2-7x+9}$$

We know that,

Differentiate the given equation w.r.t

$$x$$

using quotient rule

$$\therefore \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$\text{Let, } y = \frac{3x + 4}{5x^2 - 7x + 9}$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{(5x^2 - 7x + 9) \frac{d}{dx} (3x + 4) - (3x + 4) \frac{d}{dx} (5x^2 - 7x + 9)}{(5x^2 - 7x + 9)^2} \quad [\text{by quotient rule}] \\ &= \frac{(5x^2 - 7x + 9) \cdot 3 - (3x + 4)(10x - 7)}{(5x^2 - 7x + 9)^2} \\ &= \frac{15x^2 - 21x + 27 - 30x^2 + 21x - 40x + 28}{(5x^2 - 7x + 9)^2} \\ &= \frac{-15x^2 - 40x + 55}{(5x^2 - 7x + 9)^2} \\ &= \frac{55 - 15x^2 - 40x}{(5x^2 - 7x + 9)^2} \end{aligned}$$

Question 34

$$\frac{x^5 - \cos x}{\sin x}$$

Solution

Ans: Given:

$$\frac{x^5 - \cos x}{\sin x}$$

We know that, Differentiate the given equation w.r.t

$$x$$

using quotient rule

$$\therefore \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

$$\text{Then, Let } y = \frac{x^5 - \cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx} (x^5 - \cos x) - (x^5 - \cos x) \frac{dy}{dx} \sin x}{(\sin x)^2} \quad (\text{by quotient rule})$$

$$\begin{aligned}
&= \frac{\sin x (5x^4 + \sin x) - (x^5 - \cos x) \cos x}{\sin^2 x} \\
&= \frac{5x^4 \sin x + \sin^2 x - x^5 \cos x + \cos^2 x}{\sin^2 x} \\
&= \frac{5x^4 \sin x - x^5 \cos x + \sin^2 x + \cos^2 x}{\sin^2 x} \\
&= \frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}
\end{aligned}$$

Question 35

$$\frac{x^2 \cos \frac{\pi}{4}}{\sin x}$$

Solution

Ans: Given:

$$\frac{x^2 \cos \frac{\pi}{4}}{\sin x}$$

We know that, Differentiate the given equation w.r.t

$$x$$

using quotient rule

$$\therefore \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

Let

$$y = \frac{x^2 \cos \frac{\pi}{4}}{\sin x}$$

$$y = \frac{1}{\sqrt{2}} \cdot \frac{x^2}{\sin x}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}} \left[\frac{\sin x \cdot \frac{d}{dx} x^2 - x^2 \frac{d}{dx} \sin x}{\sin^2 x} \right] \quad (\text{by quotient rule})$$

$$= \frac{1}{\sqrt{2}} \left[\frac{\sin x \cdot 2x - x^2 \cdot \cos x}{\sin^2 x} \right]$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{2x \sin x - x^2 \cos x}{\sin^2 x}$$

$$= \frac{x}{\sqrt{2}} [2 \operatorname{cosec} x - x \cot x \operatorname{cosec} x]$$

$$= \frac{x}{\sqrt{2}} \operatorname{cosec}[2 - x \cot x]$$

Question 36

$$(ax^2 + \cot x)(p + q \cos x)$$

Solution

Ans: Given:

$$(ax^2 + \cot x)(p + q \cos x)$$

We know that, Differentiate the given equation w.r.t

$$x$$

using product rule

$$\therefore \frac{d}{dx}(uv) = u \frac{d}{dx}v + v \frac{d}{dx}u$$

Let

$$y = (ax^2 + \cot x)(p + q \cos x)$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= (ax^2 + \cot x) \frac{d}{dx}(p + q \cos x) + (p + q \cos x) \frac{d}{dx}(ax^2 + \cot x) \quad [\text{by product rule}] \\ &= (ax^2 + \cot x)(-q \sin x) + (p + q \cos x)(2ax - \operatorname{cosec}^2 x) \\ &= -q \sin x (ax^2 + \cot x) + (p + q \cos x)(2ax - \operatorname{cosec}^2 x) \end{aligned}$$

Question 37

$$\frac{a + b \sin x}{c + d \cos x}$$

Solution

Ans: Given:

$$\frac{a + b \sin x}{c + d \cos x}$$

We know that, Differentiate the given equation w.r.t

$$x$$

using quotient rule

$$\therefore \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx}u - u \frac{d}{dx}v}{v^2}$$

$$\text{Let, } y = \frac{a + b \sin x}{c + d \cos x}$$

$$\begin{aligned}
\therefore \frac{dy}{dx} &= \frac{(c + d \cos x) \frac{d}{dx}(a + b \sin x) - (a + b \sin x) \frac{d}{dx}(c + d \cos x)}{(c + d \cos x)^2} \quad [\text{by quotient rule}] \\
&= \frac{(c + d \cos x)(b \cos x) - (a + b \sin x)(-d \sin x)}{(c + d \cos x)^2} \\
&= \frac{bc \cos x + bd \cos^2 x + ad \sin x + bd \sin^2 x}{(c + d \cos x)^2} \\
&= \frac{bc \cos x + ad \sin x + bd (\cos^2 x + \sin^2 x)}{(c + d \cos x)^2} \\
&= \frac{bc \cos x + ad \sin x + bd}{(c + d \cos x)^2}
\end{aligned}$$

Question 38

$$(\sin x + \cos x)^2$$

Solution

Ans: Given:

$$(\sin x + \cos x)^2$$

We know that, Differentiate the given equation w.r.t

$$x$$

using quotient rule

$$\begin{aligned}
\therefore \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
\therefore \cos 2x &= \cos^2 x - \sin^2 x \\
\text{Let } y &= (\sin x + \cos x)^2 \\
\therefore \frac{dy}{dx} &= 2(\sin x + \cos x)(\sin x - \cos x)
\end{aligned}$$

(By chain rule)

$$\begin{aligned}
&= 2(\cos^2 x - \sin^2 x) \quad [\because \cos 2x = \cos^2 x - \sin^2 x] \\
&= 2 \cos 2x
\end{aligned}$$

Question 39

$$(2x - 7)^2(3x + 5)^3$$

Solution

Ans: Given:

$$(2x - 7)^2(3x + 5)^3$$

We know that, Differentiate the given equation w.r.t

$$x$$

using product rule and chain rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

and

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\text{Let } y = (2x - 7)^2(3x + 5)^3$$

$$\therefore \frac{dy}{dx} = (2x - 7)^2 \frac{d}{dx}(3x + 5)^3 + (3x + 5)^3 \frac{d}{dx}(2x - 7)^2 \text{ [by product rule]}$$

$$= (2x - 7)^2(3)(3x + 5)^2(3) + (3x + 5)^3 2(2x - 7)(2) \text{ [by chain rule]}$$

$$= 9(2x - 7)^2(3x + 5)^2 + 4(3x + 5)^3(2x - 7)$$

$$= (2x - 7)(3x + 5)^2[9(2x - 7) + 4(3x + 5)]$$

$$= (2x - 7)(3x + 5)^2(18x - 63 + 12x + 20)$$

$$= (2x - 7)(3x + 5)^2(30x - 43)$$

Question 40

$$x^2 \sin x + \cos 2x$$

Solution

Ans: Given:

$$x^2 \sin x + \cos 2x$$

We know that, Differentiate the given equation w.r.t

$$x$$

using product rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

$$\text{Let } y = x^2 \sin x + \cos 2x$$

$$\therefore \frac{dy}{dx} = \frac{d(x^2 \sin x)}{dx} + \frac{d(\cos 2x)}{dx}$$

$$= (2x \sin x + x^2 \cdot \cos x) + (-\sin 2x \cdot 2) \text{ [By product rule]}$$

$$= 2x \sin x + x^2 \cdot \cos x - 2 \sin 2x \text{ [By chain rule]}$$

Question 41

$$\sin^3 x \cos^3 x$$

Solution

Ans: Given:

$$\sin^3 x \cos^3 x$$

We know that, Differentiate the given equation w.r.t

$$x$$

using product rule and chain rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\therefore \sin 2x = 2 \sin x \cos x$$

Let

$$y = \sin^3 x \cos^3 x$$

$$\frac{dy}{dx} = \sin^3 x \cdot \frac{d}{dx} \cos^3 x + \cos^3 x \frac{d}{dx} \sin^3 x$$

(by product rule)

$$= \sin^3 x \cdot 3\cos^2 x(-\sin x) + \cos^3 x \cdot 3\sin^2 x \cos x \quad (\text{by chain rule})$$

$$= -3\cos^2 x \sin^4 x + 3\sin^2 x \cos^4 x$$

$$= 3\sin^2 x \cos^2 x (\cos^2 x - \sin^2 x)$$

$$= 3\sin^2 x \cos^2 x \cos 2x$$

$$= \frac{3}{4} (2 \sin x \cos x)^2 \cos 2x \quad [\therefore \sin 2x = 2 \sin x \cos x]$$

$$= \frac{3}{4} \sin^2 2x \cos 2x$$

Question 42

$$\frac{1}{ax^2 + bx + c}$$

Solution

Ans: Given:

$$\frac{1}{ax^2 + bx + c}$$

We know that, Differentiate the given equation w.r.t

$$x$$

using product rule and chain rule

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\text{Let } y = \frac{1}{ax^2 + bx + c} = (ax^2 + bx + c)^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= -(ax^2 + bx + c)^{-2} (2ax + b) \quad [\text{By chain rule}] \\ &= \frac{-(2ax + b)}{(ax^2 + bx + c)^2} \end{aligned}$$

Long Answer Questions

Question 43

$$\cos(x^2 + 1)$$

Solution

Ans: Given:

$$\cos(x^2 + 1)$$

We know that, To differentiate the given equation

$$\therefore \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$$

$$\therefore x \rightarrow 0 \Rightarrow kx \rightarrow 0$$

Let,

$$\therefore \frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\cos\{(x+h)^2 + 1\} - \cos(x^2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left\{\frac{(x+h)^2 + 1 + x^2 + 1}{2}\right\} \sin\left\{\frac{(x+h)^2 + 1 - x^2 - 1}{2}\right\}}{h} \left[\therefore \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left\{\frac{(x+h)^2 + x^2 + 2}{2}\right\} \sin\left\{\frac{(x+h)^2 - x^2}{2}\right\} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left\{\frac{(x+h)^2 + x^2 + 2}{2}\right\} \sin\left\{\frac{x^2 + h^2 + 2xh - x^2}{2}\right\} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[-2 \sin\left\{\frac{(x+h)^2 + x^2 + 2}{2}\right\} \sin\left\{\frac{h^2 + 2hx}{2}\right\} \right] \end{aligned}$$

$$\begin{aligned}
&= -2 \lim_{h \rightarrow 0} \sin \left\{ \frac{(x+h)^2 + x^2 + 2}{2} \right\} \lim_{h \rightarrow 0} \left\{ \frac{\sin h \left(\frac{h+2x}{2} \right)}{h \left(\frac{h+2x}{2} \right)} \right\} \times \left(\frac{h+2x}{2} \right) \\
&= -2 \lim_{h \rightarrow 0} \sin \left\{ \frac{(x+h)^2 + x^2 + 2}{2} \right\} \lim_{h \rightarrow 0} \left(\frac{h+2x}{2} \right) \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= -2x \sin(x^2 + 1) \quad [\because x \rightarrow 0 \Rightarrow hx \rightarrow 0]
\end{aligned}$$

Question 44

$$\frac{ax+b}{cx+d}$$

Solution

Ans: Given, $\frac{ax+b}{cx+d}$

Let,

$$f(x) = \frac{ax+b}{cx+d}$$

$$f(x+h) = \frac{a(x+h)+b}{c(x+h)+d}$$

$$\therefore \frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{1}{h} [f(x+h) - f(x)]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{a(x+h)+b}{c(x+h)+d} - \frac{ax+b}{cx+d} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b+ah}{c(x+h)+d} - \frac{ax+b}{cx+d} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{(ax+ah+b)(cx+d) - (ax+b)\{c(x+h)+d\}}{\{c(x+h)+d\}(cx+d)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{(ax+ah+b)(cx+d) - (ax+b)(cx+ch+d)}{\{c(x+h)+d\}(cx+d)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{acx^2 + achx + bcx + adx + adh + bd - acx^2 - achx - adx - bch - bd}{\{c(x+h)+d\}(cx+d)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{adh - bch}{\{c(x+h)+d\}(cx+d)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{ad - bc}{\{c(x+h)+d\}(cx+d)}$$

$$= \frac{ad - bc}{(cx+d)^2}$$

Question 45

$$\frac{2}{x^3}$$

Solution

Ans: Given:

$$\frac{2}{x^3}$$

We know that,

$$\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

Let,

$$f(x) = x^{2/3}$$

$$f(x+h) = (x+h)^{2/3}$$

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{(h \rightarrow 0)} \left[\frac{(x+h)^{2/3} - x^{2/3}}{h} \right]$$

$$= \lim_{(x+h) \rightarrow x} \left[\frac{(x+h)^{2/3} - x^{2/3}}{(x+h) - x} \right] \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= \frac{2}{3} (x)^{2/3-1}$$

$$= \frac{2}{3} x^{-1/3}$$

Question 46

$$x \cos x$$

Solution

Ans: Given:

$$x \cos x$$

We know that,

$$\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$$

Then,

$$= \lim_{h \rightarrow 0} \frac{1}{h} [(x+h) \cos(x+h) - x \cos x]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} [x \cos(x+h) + h \cos(x+h) - x \cos x]$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{1}{h} [x \{ \cos(x+h) - \cos x \} + h \cos(x+h)] \\
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[x \left\{ -2 \sin \left(\frac{2x+h}{2} \right) \sin \frac{h}{2} \right\} + h \cos(x+h) \right] \\
&= \lim_{h \rightarrow 0} \left[-2x \sin \left(x + \frac{h}{2} \right) \frac{\sin \frac{h}{2}}{h} + \cos(x+h) \right] \left[\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \right] \\
&= -2 \lim_{h \rightarrow 0} x \sin \left(x + \frac{h}{2} \right) \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2} + \lim_{h \rightarrow 0} \cos(x+h) \\
&= -2 \cdot \frac{1}{2} x \sin x + \cos x \\
&= \cos x - x \sin x
\end{aligned}$$

Evaluate each of the following limits in Exercises 47 to 53:

Question 47

$$\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$$

Solution

Ans: Given:

$$\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$$

Let,

$$\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\frac{x+y}{\cos(x+y)} - \frac{x}{\cos x}}{y}$$

$$= \lim_{y \rightarrow 0} \frac{(x+y) \cos x - x \cos(x+y)}{y \cos x \cos(x+y)}$$

$$= \lim_{y \rightarrow 0} \left[\frac{x \cos x + y \cos x - x \cos(x+y)}{y \cos x \cos(x+y)} \right]$$

$$= \lim_{y \rightarrow 0} \left[\frac{x \cos x - x \cos(x+y) + y \cos x}{y \cos x \cos(x+y)} \right]$$

$$= \lim_{y \rightarrow 0} \frac{x \{ \cos x - \cos(x+y) \} + y \cos x}{y \cos x \cos(x+y)}$$

$$= \lim_{y \rightarrow 0} \frac{x \left[-2 \sin \left(x + \frac{y}{2} \right) \sin \left(\frac{-y}{2} \right) \right] + y \cos x}{y \cos x \cos(x+y)} \left[\because \cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \right]$$

$$\begin{aligned}
&= \lim_{y \rightarrow 0} \left[\frac{x \left\{ 2 \sin \left(x + \frac{y}{2} \right) \sin \frac{y}{2} \right\} + y \cos x}{y \cos x \cos(x+y)} \right] \\
&= \lim_{y \rightarrow 0} \frac{2x \sin \left(x + \frac{y}{2} \right)}{\cos x \cos(x+y)} \cdot \lim_{y \rightarrow 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}} \cdot \frac{1}{2} + \lim_{y \rightarrow 0} \sec(x+y) \\
&\quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } x \rightarrow 0 \Rightarrow kx \rightarrow 0 \right] \\
&= \lim_{y \rightarrow 0} \frac{2x \sin \left(x + \frac{y}{2} \right)}{\cos x \cos(x+y)} \cdot \frac{1}{2} + \lim_{y \rightarrow 0} \sec(x+y) \\
&= \frac{2x \sin x}{\cos x \cos x} \cdot \frac{1}{2} + \sec x \\
&= x \tan x \sec x + \sec x \\
&= \sec x (x \tan x + 1)
\end{aligned}$$

Question 48

$$\lim_{x \rightarrow 0} \frac{(\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin 2\alpha x)}{\cos 2\beta x - \cos 2\alpha x} \cdot x$$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{[\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin 2\alpha x]}{\cos 2\beta x - \cos 2\alpha x} \cdot x$$

We know that,

$$\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\because \cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } x \rightarrow 0 \Rightarrow kx \rightarrow 0$$

Let,

$$\lim_{x \rightarrow 0} \frac{[\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin 2\alpha x]}{\cos 2\beta x - \cos 2\alpha x} \cdot x$$

$$= \lim_{x \rightarrow 0} \frac{[\sin(\alpha + \beta)x + \sin(\alpha - \beta)x + \sin 2\alpha x]}{\cos 2\beta x - \cos 2\alpha x} \cdot x$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2} \right]$$

$$= \lim_{x \rightarrow 0} \frac{[2 \sin \alpha x \cdot \cos \beta x + \sin 2\alpha x]}{\cos 2\beta x - \cos 2\alpha x} \cdot x$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{[2 \sin \alpha x \cos \beta x + \sin 2\alpha x]x}{2 \sin(\alpha + \beta)x \sin(\alpha - \beta)x} \\
&= \lim_{x \rightarrow 0} \frac{[2 \sin \alpha x \cos \beta x + 2 \sin \alpha x \cos \alpha x]x}{2 \sin(\alpha + \beta)x \sin(\alpha - \beta)x} \\
&= \lim_{x \rightarrow 0} \frac{2 \sin \alpha x [\cos \beta x + \cos \alpha x]x}{2 \sin(\alpha + \beta)x \sin(\alpha - \beta)x} \\
&\left[\because \cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2} \right] \\
&= \lim_{x \rightarrow 0} \frac{\sin \alpha x \left[2 \cos \left(\frac{\alpha + \beta}{2} \right) x \cos \left(\frac{\alpha - \beta}{2} \right) x \right] x}{2 \sin \left(\frac{\alpha + \beta}{2} \right) x \cos \left(\frac{\alpha + \beta}{2} \right) x \cdot 2 \sin \left(\frac{\alpha - \beta}{2} \right) x \cos \left(\frac{\alpha - \beta}{2} \right) x} \\
&\left[\because \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \text{ and } \sin 2\theta = 2 \sin \theta \cos \theta \right] \\
&= \lim_{x \rightarrow 0} \frac{\sin \alpha x \cdot x}{2 \sin \left(\frac{\alpha + \beta}{2} \right) x \sin \left(\frac{\alpha - \beta}{2} \right) x} \\
&= \frac{1}{2} \lim_{x \rightarrow 0} \frac{\frac{\sin \alpha x}{\alpha x} \cdot x \cdot (\alpha x)}{2 \sin \frac{\left(\frac{\alpha + \beta}{2} \right) x}{\left(\frac{\alpha + \beta}{2} \right) x} \cdot \sin \frac{\left(\frac{\alpha - \beta}{2} \right) x}{\left(\frac{\alpha - \beta}{2} \right) x} \cdot \left(\frac{\alpha + \beta}{2} \right) x \cdot \left(\frac{\alpha - \beta}{2} \right) x} \\
&= \frac{\frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin \alpha x}{\alpha x} \cdot \alpha x^2}{\lim_{x \rightarrow 0} \sin \frac{\left(\frac{\alpha + \beta}{2} \right) x}{\left(\frac{\alpha + \beta}{2} \right) x} \lim_{x \rightarrow 0} \frac{\left(\frac{\alpha - \beta}{2} \right) x}{\left(\frac{\alpha - \beta}{2} \right) x} \cdot \left(\frac{\alpha^2 - \beta^2}{4} \right) x^2} \\
&= \frac{1}{2} \cdot \frac{\alpha \cdot 4}{\alpha^2 - \beta^2} \left[\frac{\lim_{x \rightarrow 0} \frac{\sin \alpha x}{\alpha x}}{\lim_{x \rightarrow 0} \sin \frac{\left(\frac{\alpha + \beta}{2} \right) x}{\left(\frac{\alpha + \beta}{2} \right) x} \lim_{x \rightarrow 0} \frac{\left(\frac{\alpha - \beta}{2} \right) x}{\left(\frac{\alpha - \beta}{2} \right) x}} \right] \\
&\left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } x \rightarrow 0 \Rightarrow kx \rightarrow 0 \right] \\
&= \frac{1}{2} \cdot \frac{4\alpha}{\alpha^2 - \beta^2} \\
&= \frac{2\alpha}{\alpha^2 - \beta^2}
\end{aligned}$$

Question 49

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos x + \frac{\pi}{4}}$$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

We know that,

$$\frac{0}{0} \text{ form}$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$\because \cos A \cdot \cos B - \sin A \sin B = \cos(A + B)$$

Let,

$$\lim_{x \rightarrow \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)} \left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow \pi/4} \frac{\tan x (\tan^2 x - 1)}{\cos\left(x + \frac{\pi}{4}\right)} = \lim_{x \rightarrow \pi/4} \tan x \cdot \lim_{x \rightarrow \pi/4} \left(\frac{1 - \tan^2 x}{\cos\left(x + \frac{\pi}{4}\right)} \right)$$

$$[\because a^2 - b^2 = (a + b)(a - b)]$$

$$= -1 \times \lim_{x \rightarrow \pi/4} \frac{(1 + \tan x)(1 - \tan x)}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$= - \lim_{x \rightarrow \pi/4} (1 + \tan x) \lim_{x \rightarrow \pi/4} \frac{\cos x - \sin x}{\cos x \cdot \cos\left(x + \frac{\pi}{4}\right)}$$

$$= -(1 + 1) \times \lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \left[\frac{1}{\sqrt{2}} \cdot \cos x - \frac{1}{\sqrt{2}} \cdot \sin x \right]}{\cos x \cdot \cos\left(x + \frac{\pi}{4}\right)}$$

$$= -2\sqrt{2} \lim_{x \rightarrow \pi/4} \left[\frac{\cos \frac{\pi}{4} \cdot \cos x - \sin \frac{\pi}{4} \cdot \sin x}{\cos x \cdot \cos\left(x + \frac{\pi}{4}\right)} \right]$$

$$\because \cos A \cdot \cos B - \sin A \sin B = \cos(A + B)$$

$$= -2\sqrt{2} \lim_{x \rightarrow \pi/4} \frac{\cos\left(x + \frac{\pi}{4}\right)}{\cos x \cdot \cos\left(x + \frac{\pi}{4}\right)}$$

$$\begin{aligned}
 &= -2\sqrt{2} \times \frac{1}{\frac{1}{\sqrt{2}}} \\
 &= -2\sqrt{2} \times \sqrt{2} \\
 &= -4
 \end{aligned}$$

Question 50

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \cos \frac{x}{4} - \sin \frac{x}{4}}$$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$$

We know that,

$$\because \sin^2 \theta + \cos^2 \theta = \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\because \cos^2 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

Let,

$$\begin{aligned}
 &\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} \\
 &= \lim_{x \rightarrow \pi} \frac{\cos^2 \frac{x}{4} + \sin^2 \frac{x}{4} - 2 \cdot \sin \frac{x}{4} \cdot \cos \frac{x}{4}}{\cos \frac{x}{2} \cdot \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} \quad [\because \sin^2 \theta + \cos^2 \theta = 1 \quad \sin 2\theta = 2 \sin \theta \cos \theta] \\
 &= \lim_{x \rightarrow \pi} \frac{\left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)^2}{\left(\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} \quad [\because \cos^2 2\theta = \cos^2 \theta - \sin^2 \theta] \\
 &= \lim_{x \rightarrow \pi} \frac{\left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}{\left(\cos \frac{x}{4} + \sin \frac{x}{4} \right) \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)} \quad [\because a^2 - b^2 = (a + b)(a - b)] \\
 &= \lim_{x \rightarrow \pi} \frac{1}{\cos \frac{x}{4} + \sin \frac{x}{4}} \\
 &= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}
 \end{aligned}$$

$$= \frac{\sqrt{2}}{2}$$

$$= \frac{1}{\sqrt{2}}$$

Question 51

Show that $\lim_{x \rightarrow 4} \frac{|x - 4|}{x - 4}$ does not exist

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/4} \frac{|x - 4|}{x - 4}$$

Let,

$$\lim_{x \rightarrow \pi/4} \frac{|x - 4|}{x - 4}$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{4}^-} \frac{-(x - 4)}{x - 4} \quad [\because |x - 4| = -(x - 4), x < 4]$$

$$= -1$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{4}^+} \frac{(x - 4)}{x - 4} \quad [\because |x - 4| = (x - 4), x > 4]$$

$$= 1$$

$$\therefore \text{LHL} \neq \text{RHL}$$

So, limit does not exist

Question 52

Let $f(x) = \frac{k \cos x}{\pi - 2x}$, when $x \neq \frac{\pi}{2}$ and $x = \frac{\pi}{2}$ and if $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, find the value of k .

Solution

Ans:

$$\therefore \text{LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x}$$

$$= \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{k \sin h}{\pi - \pi + 2h}$$

$$= \lim_{h \rightarrow 0} \frac{k \sin h}{2h}$$

$$= \frac{k}{2} \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

$$= \frac{k}{2} \cdot 1$$

$$= \frac{k}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{k \cos x}{\pi - 2x}$$

$$= \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-k \sin h}{\pi - \pi - 2h}$$

$$= \lim_{h \rightarrow 0} \frac{k \sin h}{2h}$$

$$= \frac{k}{2} \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

$$= \frac{k}{2}$$

$$f\left(\frac{\pi}{2}\right) = 3 \text{ (given)}$$

It is given that,

$$\lim_{x \rightarrow \pi/2} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\therefore k = 6$$

Question 53

Let $f(x) = \begin{cases} x + 2, & x \leq -1 \\ cx^2, & x > -1 \end{cases}$, find 'c' if $\lim_{x \rightarrow -1} f(x)$ exists.

Solution

$$\text{Ans: } f(x) = \begin{cases} x + 2, & x \leq -1 \\ cx^2, & x > -1 \end{cases}$$

$$LHL = \lim_{x \rightarrow -1^-} f(x)$$

$$= \lim_{x \rightarrow -1^-} (x + 2)$$

$$= \lim_{x \rightarrow -1^-} (x + 2)$$

$$= \lim_{h \rightarrow 0} (1 - h) = 1$$

$$RHL = \lim_{x \rightarrow -1^+} f(x)$$

$$= \lim_{x \rightarrow -1^+} cx^2$$

$$= \lim_{h \rightarrow 0} c(-1 + h)^2$$

$$= c$$

If $\lim_{x \rightarrow -1} f(x)$ exist, then $LHL = RHL$

$$\therefore c = 1$$

Objective Type Questions:

Choose the correct answer out of 4 options given against each Exercise 54 to 76 (M.C.Q).

Question 54

$$\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} \text{ is}$$

(A) 1

(B) 2

(C) -1

(D) -2

Solution

Ans:

Given:

$$\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$$

We know that,

$$\therefore \sin(\pi - \theta) = \sin \theta$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \pi - x \rightarrow 0 \Rightarrow x \rightarrow \pi$$

Let,

$$= \lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$$

$$\begin{aligned}
&= \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{-(\pi - x)} \quad [\because \sin(\pi - \theta) = \sin \theta] \\
&= -\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{(\pi - x)} \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \pi - x \rightarrow 0 \Rightarrow x \rightarrow \pi \right] \\
&= -1
\end{aligned}$$

Correct Option: C

Question 55

$$\lim_{x \rightarrow 0} \frac{x^2 \cos x}{1 - \cos x} \text{ is}$$

(A) 2

(B) $\frac{3}{2}$

(C) $\frac{-3}{2}$

(D) 1

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{x^2 \cos x}{1 - \cos x}$$

We know that,

$$\because 1 - \cos x = 2\sin^2 \frac{x}{2}$$

Let,

$$\begin{aligned}
&\lim_{x \rightarrow 0} \frac{x^2 \cos x}{1 - \cos x} \\
&= \lim_{x \rightarrow 0} \frac{x^2 \cos x}{2\sin^2 \frac{x}{2}} \quad \left[\because 1 - \cos x = 2\sin^2 \frac{x}{2} \right] \\
&= 2 \lim_{x \rightarrow 0} \frac{\left(\frac{x}{2}\right)^2}{\sin^2 \frac{x}{2}} \cdot \lim_{x \rightarrow 0} \cos x \\
&= 2 \cdot 1 = 2
\end{aligned}$$

Correct Option: A

Question 56

$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} \text{ is}$$

(A) n

(B) 1

(C) $-n$

(D) 0

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x}$$

We know that,

$$\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

Then,

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} \\ &= \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{(1+x) - 1} \\ &= \lim_{x \rightarrow 0} \frac{(1+x)^n - 1^n}{(1+x) - 1} \\ &= \lim_{(1+x) \rightarrow 1} \frac{(1+x)^n - 1^n}{(1+x) - 1} \\ &= n \cdot (1)^{n-1} \end{aligned}$$

$$\left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= n$$

Correct Option: A

Question 57

$$\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1} \text{ is}$$

(A) 1

(B) $\frac{m}{n}$

(C) $-\frac{m}{n}$

(D) $\frac{m^2}{n^2}$

Solution

Ans: Given:

$$\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}$$

We know that,

$$\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

Then,

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1} \\ &= \lim_{x \rightarrow 1} \frac{\frac{x^m - 1}{x - 1}}{\frac{x^n - 1}{x - 1}} \\ &= \frac{\lim_{x \rightarrow 1} \frac{x^m - 1^m}{x - 1}}{\lim_{x \rightarrow 1} \frac{x^n - 1^n}{x - 1}} \\ &= \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1} \\ &= \lim_{x \rightarrow 1} \frac{\frac{x^m - 1}{x - 1}}{\frac{x^n - 1}{x - 1}} \\ &= \frac{\lim_{x \rightarrow 1} \frac{x^m - 1^m}{x - 1}}{\lim_{x \rightarrow 1} \frac{x^n - 1^n}{x - 1}} \\ &= \frac{m(1)^{m-1}}{n(1)^{n-1}} \\ &= \frac{m}{n} \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \end{aligned}$$

Correct Option: B

Question 58

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta} \text{ is}$$

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

(C) $\frac{-1}{2}$

(D) -1

Solution

Ans: Given:

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$$

We know that,

$$\therefore 1 - \cos 2\theta = 2\sin^2\theta$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } x \rightarrow 0 \Rightarrow kx \rightarrow 0$$

Let,

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$$

$$= \lim_{\theta \rightarrow 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta} \quad [\because 1 - \cos 2\theta = 2\sin^2\theta]$$

$$= \frac{\lim_{\theta \rightarrow 0} \frac{\sin^2 2\theta}{(2\theta)^2} \cdot (2\theta)^2}{\lim_{\theta \rightarrow 0} \frac{\sin^2 3\theta}{(3\theta)^2} \cdot (3\theta)^2}$$

$$= \frac{4}{9} \cdot \frac{\lim_{\theta \rightarrow 0} \left(\frac{\sin 2\theta}{2\theta} \right)^2}{\lim_{\theta \rightarrow 0} \left(\frac{\sin 3\theta}{3\theta} \right)^2} \quad \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{4}{9} [x \rightarrow 0 \Rightarrow kx \rightarrow 0]$$

Correct Option: A

Question 59

$$\lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} \text{ is}$$

(A) $\frac{-1}{2}$

(B) 1

(C) $\frac{1}{2}$

(D) 1

Solution

Ans: Given:

$$\lim_{\theta \rightarrow 0} \frac{\operatorname{co sec} x - \cot x}{x}$$

We know that,

$$\therefore \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1$$

Let,

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\operatorname{cosec} x - \cot x}{x} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cdot \sin x} \\ &= \lim_{x \rightarrow 0} \frac{2\sin^2 \frac{x}{2}}{x \cdot 2 \sin \frac{x}{2} \cos \frac{x}{2}} \\ &= \lim_{x \rightarrow 0} \frac{\tan \frac{x}{2}}{x} \left[\therefore \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \right] \\ &= \lim_{x \rightarrow 0} \frac{\tan \frac{x}{2}}{\frac{x}{2}} \cdot \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

Correct Option: C

Question 60

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}} \text{ is}$$

- (A) 2
(B) 0
(C) 1
(D) -1

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{x-1}}$$

We know that,

$$\therefore \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Let,

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}} \\
&= \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}} \cdot \frac{\sqrt{x+1} + \sqrt{1-x}}{\sqrt{x+1} + \sqrt{1-x}} \\
&= \lim_{x \rightarrow 0} \frac{\sin x (\sqrt{x+1} + \sqrt{1-x})}{(x+1) - (1-x)} \\
&= \lim_{x \rightarrow 0} \frac{\sin x (\sqrt{x+1} + \sqrt{1-x})}{x+1-1+x} \\
&= \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} (\sqrt{x+1} + \sqrt{1-x}) \\
&= \frac{1}{2} \cdot 1 \cdot 2 \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= 1
\end{aligned}$$

Correct Option: C

Question 61

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} \text{ is}$$

- (A) 3
(B) 1
(C) 0
(D) $\sqrt{2}$

Solution

Ans: Given:

$$\lim_{x \rightarrow \pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$$

We know that,

$$\begin{aligned}
& \because \sec x = 1 + \tan x \\
& \therefore a^2 - b^2 = (a+b)(a-b)
\end{aligned}$$

Let,

$$\begin{aligned}
& \lim_{x \rightarrow \pi/4} \frac{\sec^2 x - 2}{\tan x - 1} \\
&= \lim_{x \rightarrow \pi/4} \frac{1 + \tan^2 x - 2}{\tan x - 1} \\
&= \lim_{x \rightarrow \pi/4} \frac{\tan^2 x - 1}{\tan x - 1}
\end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \pi/4} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)} \\
 &= \lim_{x \rightarrow \pi/4} (\tan x + 1) \\
 &= 2
 \end{aligned}$$

Correct Option: D

Question 62

$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(2x - 3)}{2x^2 + x - 3}$ is

(A) $\frac{1}{10}$

(B) $\frac{-1}{10}$

(C) 1

(D) None of these

Solution

Ans: Given:

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(2x - 3)}{(2x^2 + x - 3)}$$

We know that,

$$\because a^2 - b^2 = (a + b)(a - b)$$

Let,

$$\begin{aligned}
 &\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(2x - 3)}{2x^2 + x - 3} \\
 &= \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(2x - 3)}{(2x + 3)(x - 1)} \\
 &= \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(2x - 3)}{(2x + 3)(\sqrt{x} - 1)(\sqrt{x} + 1)} \\
 &= \lim_{x \rightarrow 1} \frac{2x - 3}{(2x + 3)(\sqrt{x} + 1)} \\
 &= \frac{-1}{5 \times 2} \\
 &= \frac{-1}{10}
 \end{aligned}$$

Correct Option: B

Question 63

If $f(x) = \frac{\sin[x]}{[x]}$, $[x] \neq 0$, $[x] = 0$, where $[.]$ denotes the greatest integer function, then $\lim_{x \rightarrow 0} f(x)$ is equal to

- (A) 1
 (B) 0
 (C) -1
 (D) None of these

Solution

Ans:

Let,

$$f(x) = \frac{\sin[x]}{[x]}, [x] \neq 0$$

$$0, [x] = 0$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{x \rightarrow 0^-} \frac{\sin[x]}{[x]} \\ &= \lim_{h \rightarrow 0} \frac{\sin[0 - h]}{[0 - h]} \\ &= \lim_{h \rightarrow 0} \frac{-\sin[-h]}{[-h]} = -1 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\ &= \lim_{x \rightarrow 0^+} \frac{\sin[x]}{[x]} \\ &= \lim_{x \rightarrow 0^+} \frac{\sin[0 + h]}{[0 + h]} \\ &= \lim_{h \rightarrow 0} \frac{\sin[h]}{[h]} = 1 \end{aligned}$$

$$\text{LHL} \neq \text{RHL}$$

Therefore, the limit does not exist.

Correct Option: D

Question 64

$$\lim_{x \rightarrow 0} \frac{|\sin x|}{x} \text{ is}$$

- (A) 1
 (B) -1

(C) does not exist

(D) None of these

Solution

Ans:

$$\begin{aligned}\therefore LHL &= \lim_{x \rightarrow 0^-} \left(\frac{-\sin x}{x} \right) \\ &= - \lim_{x \rightarrow 0^-} \frac{\sin x}{x} \\ &= -1 \\ RHL &= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \\ &= 1 \\ \therefore LHL &\neq RHL\end{aligned}$$

Thus, the limit does not exist.

Correct Option : C

Question 65

Let $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$, the quadratic equation whose roots are $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ is

(A) $x^2 - 6x + 9 = 0$

(B) $x^2 - 7x + 8 = 0$

(C) $x^2 - 14x + 49 = 0$

(D) $x^2 - 10x + 21 = 0$

Solution

$$\begin{aligned}\text{Ans: } \therefore \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} (x^2 - 1) \\ &= \lim_{h \rightarrow 0} [(2 - h)^2 - 1] \\ &= \lim_{h \rightarrow 0} (4 + h^2 - 4h - 1) \\ &= \lim_{h \rightarrow 0} (h^2 - 4h + 3) \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{and } \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (2x + 3) \\ &= \lim_{h \rightarrow 0} [2(2 + h) + 3] \\ &= \lim_{h \rightarrow 0} (4 + 2h + 3)\end{aligned}$$

$$= 7$$

Thus, quadratic equation whose roots are 3 and 7

$$x^2 - (3 + 7)x + 3 \times 7 = 0$$

$$i.e., x^2 - 10x + 21 = 0$$

Correct Option : D

Question 66

$$\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x} \text{ is}$$

(A) 2

(B) $\frac{1}{2}$

(C) $\frac{-1}{2}$

(D) $\frac{1}{4}$

Solution

Ans: Correct Option : B

Given:

$$\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$$

We know that,

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\because \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$x \rightarrow 0 \Rightarrow kx \rightarrow 0$$

Let,

$$\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{x \left[\frac{\tan 2x}{x} - 1 \right]}{x \left[3 - \frac{\sin x}{x} \right]}$$

$$= \frac{\lim_{x \rightarrow 0} 2 \times \frac{\tan 2x}{2x} - 1}{3 - \lim_{x \rightarrow 0} \frac{\sin x}{x}}$$

$$\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$[x \rightarrow 0 \Rightarrow kx \rightarrow 0]$$

$$\left[\because \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right]$$

$$= \frac{2-1}{3-1}$$

$$= \frac{1}{2}$$

Question 67

Let $f(x) = x - [x]; \in \mathbf{R}$, then $f' \frac{1}{2}$ is

(A) $\frac{3}{2}$

(B) 1

(C) 0

(D) -1

Solution

Ans: $f(x) = x - [x]$ (given)

$$f'(x) = 1 + 0$$

$$= 1$$

$$\left[\because \frac{d}{dx} x = 1 \text{ and } \frac{d}{dx} [x] = 0 \right]$$

Therefore,

$$f' \left(\frac{1}{2} \right) = 1$$

Correct Option: B

Question 68

If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at $x = 1$ is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Solution

Ans: Given:

$$y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

We know that,

We will differentiate the given equation using chain rule

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Then,

$$y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\text{now, } \frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$$

$$\therefore \left(\frac{dy}{dx} \right)_{x=1} = \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

Correct Option: D

Question 69

If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is

(A) $\frac{5}{4}$

(B) $\frac{4}{5}$

(C) 1

(D) 0

Solution

Ans: Given:

$$f(x) = \frac{x-4}{2\sqrt{x}}$$

We know that, Differentiate

$$f(x)$$

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

At last putting

$$1$$

in

$$f'(x)$$

to get the final answer

Then, $f(x) = \frac{x-4}{2\sqrt{x}}$

$$f'(x) = \frac{2\sqrt{x} - (x-4) \cdot 2 \cdot \frac{1}{2\sqrt{x}}}{4x} \quad (\text{by quotient rule})$$

$$= \frac{2x - (x-4)}{4x^{3/2}}$$

$$= \frac{2x - x + 4}{4x^{3/2}}$$

$$= \frac{x + 4}{4x^{3/2}}$$

$$f'(1) = \frac{1 + 4}{4 \times (1)^{3/2}}$$

$$= \frac{5}{4}$$

Correct Option : A

Question 70

If $y = \frac{1 + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$, then $\frac{dy}{dx}$ is

(A) $\frac{-4x}{(x^2 - 1)^2}$

(B) $\frac{-4x}{x^2 - 1}$

(C) $\frac{1 - x^2}{4x}$

(D) $\frac{4x}{x^2 - 1}$

Solution

Ans: Given:

$$y = \frac{1 + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$$

We know that, We will differentiate the given equation using quotient rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

Then,

$$\begin{aligned} y &= \frac{1 + \frac{1}{x^2}}{1 - \frac{1}{x^2}} \\ &= \frac{x^2 + 1}{x^2 - 1} \end{aligned}$$

Differentiating

$$y$$

w.r.t

$$x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x^2 - 1) 2x - (x^2 + 1) (2x)}{(x^2 - 1)^2} \quad [\text{By Quotient rule}] \\ &= \frac{2x (x^2 - 1 - x^2 - 1)}{(x^2 - 1)^2} \\ &= \frac{2x(-2)}{(x^2 - 1)^2} \\ &= \frac{-4x}{(x^2 - 1)^2} \end{aligned}$$

Correct Option: A

Question 71

If $y = \frac{\sin x + \cos x}{\sin x - \cos x}$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) -2

(B) 0

(C) $\frac{1}{2}$

(D) does not exist

Solution

Ans: Given,

$$y = \frac{\sin x + \cos x}{\sin x - \cos x}$$

Now, Differentiating

$$y$$

w.r.t

 x

$$\frac{dy}{dx} = \frac{\sin x + \cos x}{\sin x - \cos x}$$

$$\frac{dy}{dx} = \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2} \quad [\text{by quotient rule}]$$

$$= \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$$

$$= \frac{-[(\sin x - \cos x)^2 + (\sin x + \cos x)^2]}{(\sin x - \cos x)^2}$$

$$= \frac{-[\sin^2 x + \cos^2 x - 2 \sin x \cos x + \sin^2 x + \cos^2 x + 2 \sin x \cos x]}{(\sin x - \cos x)^2}$$

$$= \frac{-2}{(\sin x - \cos x)^2}$$

$$\therefore \left(\frac{dy}{dx} \right)_{x=0} = -2$$

Correct Option: A

Question 72

If $y = \frac{\sin(x+9)}{\cos x}$ then $\frac{dy}{dx}$ at $x = 0$ is

(A) $\cos 9$ **(B)** $\sin 9$ **(C)** 0**(D)** 1**Solution**

Ans: Given, $y = \frac{\sin(x+9)}{\cos x}$

Differentiating

 y

w.r.t

 x

$$\frac{dy}{dx} = \frac{\cos x \cos(x+9) - \sin(x+9)(-\sin x)}{(\cos x)^2} \quad [\text{by quotient rule}]$$

$$= \frac{\cos x \cos(x+9) + \sin x \sin(x+9)}{\cos^2 x} \quad [\because \cos x \cos y + \sin x \sin y = \cos(x-y)]$$

$$= \frac{\cos(x+9-x)}{\cos^2 x}$$

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{\cos 9}{1}$$

$$= \cos 9$$

Correct Option: A

Question 73

If $f(x) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^{100}}{100}$, then $f'(1)$ is equal to

(A) $\frac{1}{100}$

(B) 100

(C) does not exist

(D) 0

Solution

Ans: Given:

$$f(x) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^{100}}{100}$$

We know that, Differentiating the

$$f(x)$$

w.r.t

$$x$$

Then, putting

$$x = 1$$

in the solved equation

Then,

$$f(x) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^{100}}{100}$$

Differentiating the equation w.r.t

$$x$$

$$f(x)' = 0 + 1 + 2 \times \frac{x}{2} + \dots + \frac{x^{100}}{100}$$

$$\therefore f(x)' = 1 + x + x^2 + \dots + x^{99}$$

Now, putting

$$x = 1$$

in the solved equation

$$\begin{aligned}\text{Now, } f(x)' &= 1 + 1 + 1 + \dots + 1 \text{ (100 times)} \\ &= 100\end{aligned}$$

Correct Option : B

Question 74

If $f(x) = \frac{x^n - a^n}{x - a}$ for some constant 'a', then $f'(a)$ is

- (A) 1
- (B) 0
- (C) does not exist
- (D) $\frac{1}{2}$

Solution

Ans: Given:

$$f(x) = \frac{x^n - a^n}{x - a}$$

We know that, Differentiating the given equation by quotient rule

$$\therefore \frac{d}{dx}(uv) = v \frac{d}{dx}u + u \frac{d}{dx}v$$

Putting

$$a$$

in the solved equation to get the final answer

Then,

$$f(x) = \frac{x^n - a^n}{x - a}$$

Differentiating the given equation w.r.t

$$x$$

$$\therefore f'(x) = \frac{(x - a)nx^{n-1} - (x^n - a^n)(1)}{(x - a)^2} \text{ [by quotient rule]}$$

$$\Rightarrow f'(x) = \frac{nx^{n-1}(x - a) - x^n + a^n}{(x - a)^2}$$

$$\text{Now, } f'(a) = \frac{na^{n-1}(0) - a^n + a^n}{(x - a)^2}$$

$$\Rightarrow f'(a) = \frac{0}{0}$$

So,

$$f'(a)$$

does not exist,

Since,

$$f(x)$$

is not defined at

$$x = a$$

.

Hence,

$$f'(x)$$

at

$$x = a$$

does not exist.

Correct Option : D

Question 75

If $f(x) = x^{100} + x^{99} + \dots + x + 1$, then $f'(1)$ is equal to

(A) 5050

(B) 5049

(C) 5051

(D) 50051

Solution

Ans: Given:

$$f(x) = x^{100} + x^{99} + \dots + x + 1$$

We know that, Differentiating $f(x)$ w.r.t x

Then using this formula to find sum

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$\text{Then, } f(x) = x^{100} + x^{99} + \dots + x + 1$$

Differentiating w.r.t x

$$\therefore f'(x) = 100x^{99} + 99x^{98} + \dots + 1 + 0$$

$$= 100x^{99} + 99x^{98} + \dots + 1$$

$$\text{Now, } f'(1) = 100 + 99 + \dots + 1$$

$$\begin{aligned}
&= \frac{100}{2} [2 \times 100 + (100 - 1)(-1)] \left[\because S_n = \frac{n}{2} \{2a + (n - 1)d\} \right] \\
&= 50[200 - 99] \\
&= 50 \times 101 \\
&= 5050
\end{aligned}$$

Correct Option: A

Question 76

If $f(x) = 1 - x + x^2 - x^3 \dots - x^{99} + x^{100}$, then $f'(1)$ is equal to

(A) 150

(B) -50

(C) -150

(D) 50

Solution

Ans: Given:

$$f(x) = 1 - x + x^2 - x^3 + \dots - x^{99} + x^{100}$$

We know that, Differentiating $f(x)$ w.r.t x

Then using this formula to find sum

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

Then,

$$f(x) = 1 - x + x^2 - x^3 + \dots - x^{99} + x^{100}$$

Differentiating above equation w.r.t x

$$f'(x) = 0 - 1 + 2x - 3x^2 + \dots - 99x^{98} + 100x^{99}$$

$$= -1 + 2x - 3x^2 + \dots - 99x^{98} + 100x^{99} f'(1)$$

$$= -1 + 2 - 3 + \dots - 99 + 100$$

$$\therefore f'(x) = (-1 - 3 - 5 - \dots - 99) + (2 + 4 + \dots + 100)$$

$$= -\frac{50}{2} [2 \times 1 + (50 - 1)2] + \frac{50}{2} [2 \times 2 + (50 - 1)2] \left[S_n = \frac{n}{2} \{2a + (n - 1)d\} \right]$$

$$= -25[2 + 49 \times 2] + 25[4 + 49 \times 2]$$

$$= -25(2 + 98) + 25(4 + 98)$$

$$= -25 \times 100 + 25 \times 102$$

$$= -2500 + 2550$$

$$= 50$$

Correct Option : A

Fill in the blanks in Exercises 77 to 80:

Question 77

If $f(x) = \frac{\tan x}{x - \pi}$, then $\lim_{x \rightarrow \pi} f(x) =$

Solution

Ans: Given:

$$f(x) = \frac{\tan x}{x - \pi}$$

We know that,

$$\therefore \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

Then,

$$f(x) = \frac{\tan x}{x - \pi}$$

$$\Rightarrow \lim_{x \rightarrow \pi} f(x) = \lim_{x \rightarrow \pi} \frac{\tan x}{x - \pi}$$

$$= \lim_{\pi - x \rightarrow 0} \frac{-\tan(\pi - x)}{x - \pi}$$

$$\left[\therefore \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right]$$

$$= 1$$

Question 78

$\lim_{x \rightarrow 0} \sin mx \cot \frac{x}{\sqrt{3}} = 2$, then $m =$

Solution

Ans: Given:

$$\lim_{x \rightarrow 0} \left(\sin mx \cot \frac{x}{\sqrt{3}} \right) = 2$$

We know that,

$$\therefore \cot \frac{x}{\sqrt{3}} = \frac{1}{\tan \frac{x}{\sqrt{3}}}$$

Then,

$$\lim_{x \rightarrow 0} \left(\sin mx \cot \frac{x}{\sqrt{3}} \right) = 2$$

(given)

Multiplying $\frac{mx}{mx}$ in L.H.S $\left[\because \frac{mx}{mx} = 1 \right]$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot mx \cdot \frac{1}{\tan \frac{x}{\sqrt{3}}} = 2$$

$$\left[\because \cot \frac{x}{\sqrt{3}} = \frac{1}{\tan \frac{x}{\sqrt{3}}} \right]$$

Multiplying and dividing

$$\frac{x}{\sqrt{3}}$$

in numerator and denominator in L.H.S

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot mx \cdot \frac{\frac{x}{\sqrt{3}}}{\tan \frac{x}{\sqrt{3}}} \cdot \frac{1}{\frac{x}{\sqrt{3}}} = 2$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin mx}{mx} \cdot \lim_{x \rightarrow 0} \frac{\frac{x}{\sqrt{3}}}{\tan \frac{x}{\sqrt{3}}} \cdot \lim_{x \rightarrow 0} \frac{mx}{\frac{x}{\sqrt{3}}} = 2$$

$$\Rightarrow \sqrt{3}x = 2$$

$$\therefore m = \frac{2\sqrt{3}}{3}$$

Question 79

if $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, then $\frac{dy}{dx} =$

Solution

Ans: Given:

$$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Then,

$$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \text{ (given)}$$

$$\therefore \frac{dy}{dx} = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{4!}$$

$$\begin{aligned} &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \\ &= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\ &= y \end{aligned}$$

Question 80

$$\lim_{x \rightarrow 3^+} \frac{x}{[x]} =$$

Solution

$$\begin{aligned} \text{Ans: } &\lim_{x \rightarrow 3^+} \frac{x}{[x]} \\ &= \lim_{h \rightarrow 0} \frac{(3+h)}{[3+h]} \\ &= \lim_{h \rightarrow 0} \frac{(3+h)}{[3]} \\ &= 1 \end{aligned}$$

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