

Chapter 12: Introduction to Three Dimensional Geometry

NCERT Exemplar · Class 11 · Maths

Short Answer Questions

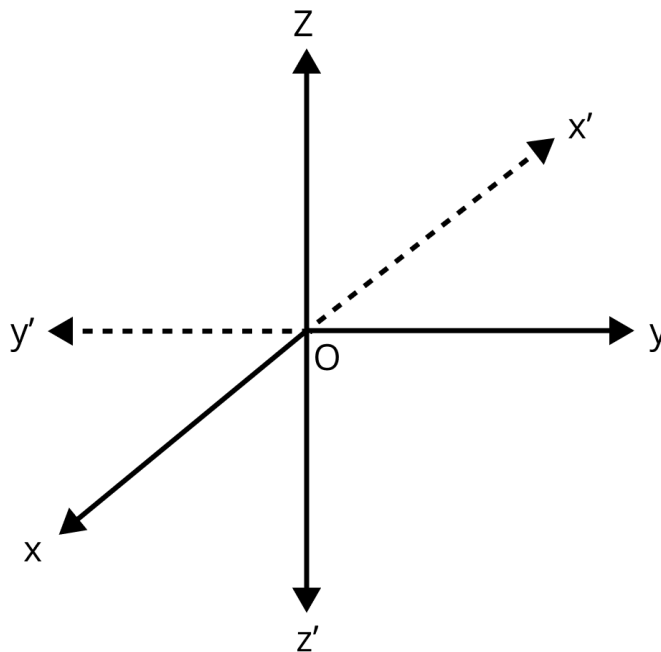
Question 1

Locate the following points:

- (i) $(1, -1, 3)$
- (ii) $(-1, 2, 4)$
- (iii) $(-2, -4, -7)$
- (iv) $(-4, 2, -5)$

Solution

Ans:

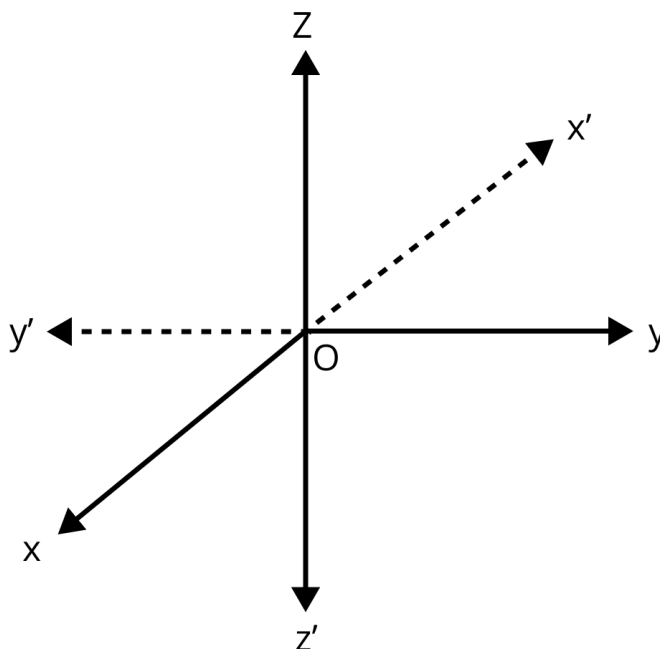


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P.

The point $P(1, -1, 3)$ is located in the IV octant.

Ans:

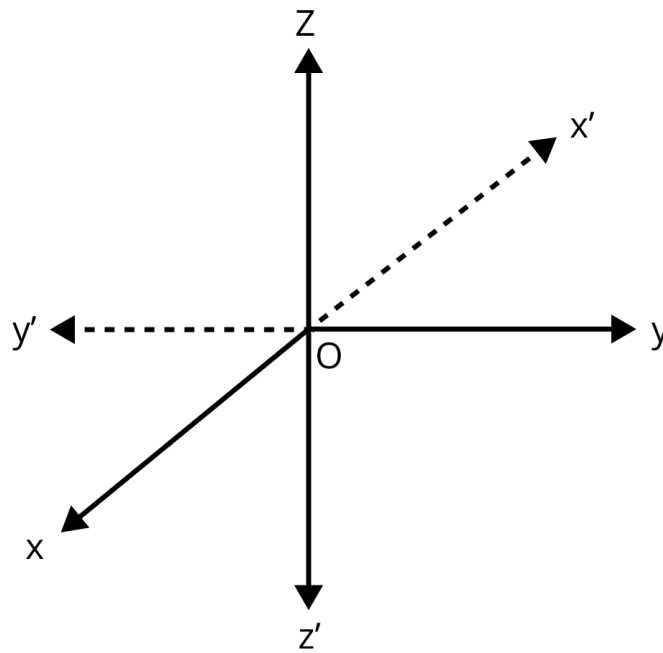


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(-1, 2, 4)$ is located in the II octant.

Ans:

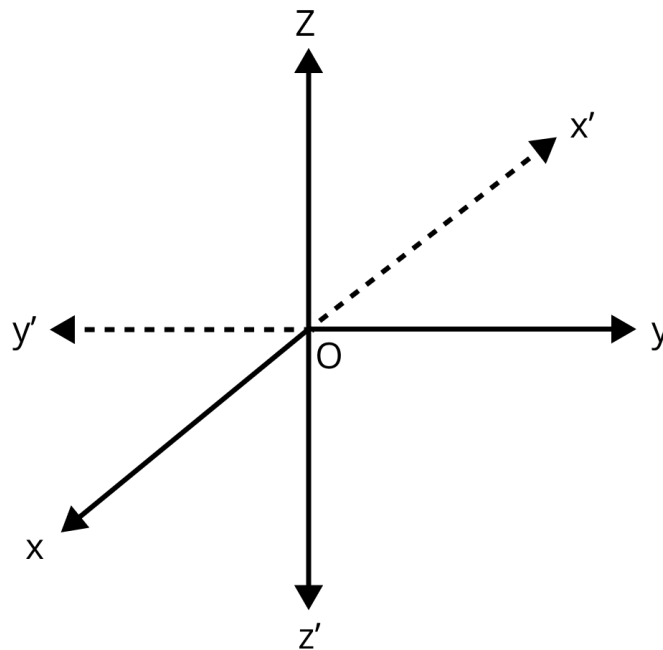


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(-2, -4, -7)$ is located in the *VII* octant.

Ans:



Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(-4, 2, -5)$ is located in the VI octant.

Question 2

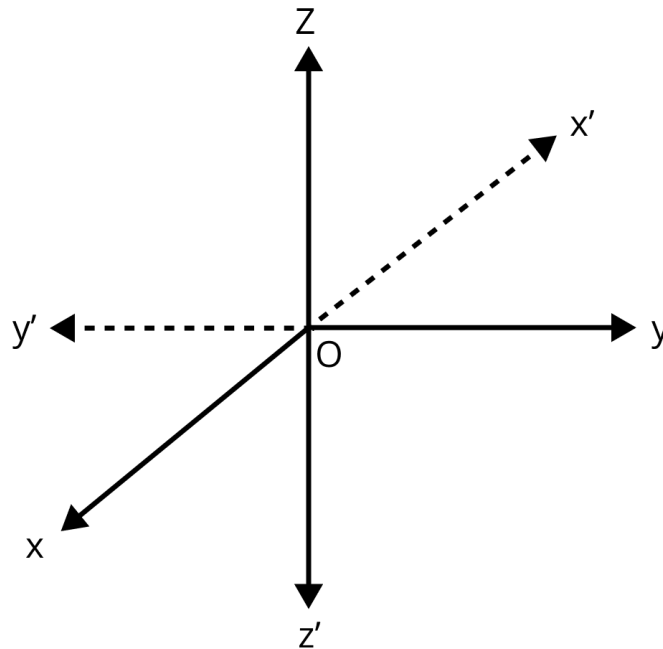
Name the octant in which each of the following points lies.

- (i) $(1, 2, 3)$
- (ii) $(4, -2, 3)$
- (iii) $(4, -2, -5)$
- (iv) $(4, 2, -5)$
- (v) $(-4, 2, 5)$
- (vi) $(-3, -1, 6)$
- (vii) $P(2, -4, -7)$

(viii) $(-4, 2, -5)$

Solution

Ans:

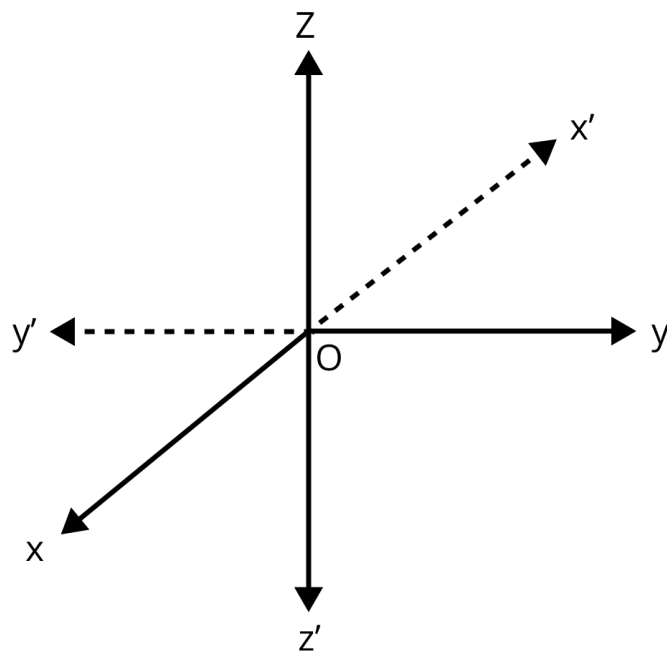


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point P $(1, 2, 3)$ is located in the I octant.

Ans:

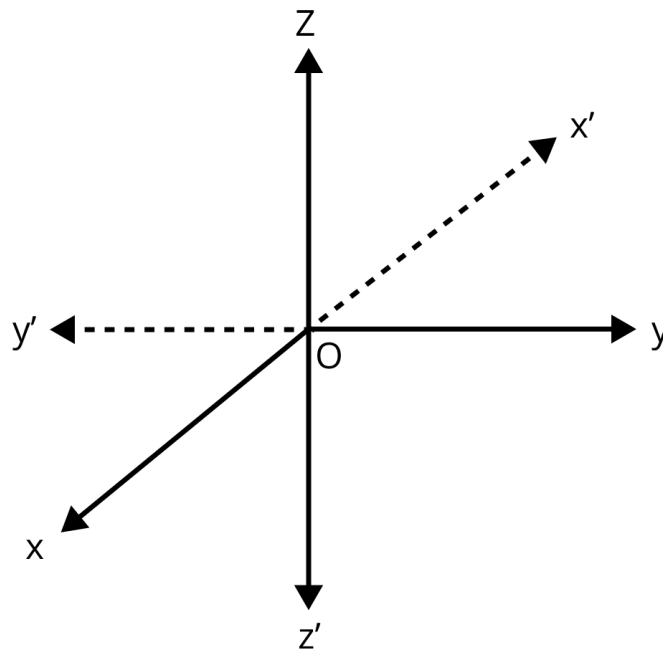


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(4, -2, 3)$ is located in the IV octant.

Ans:

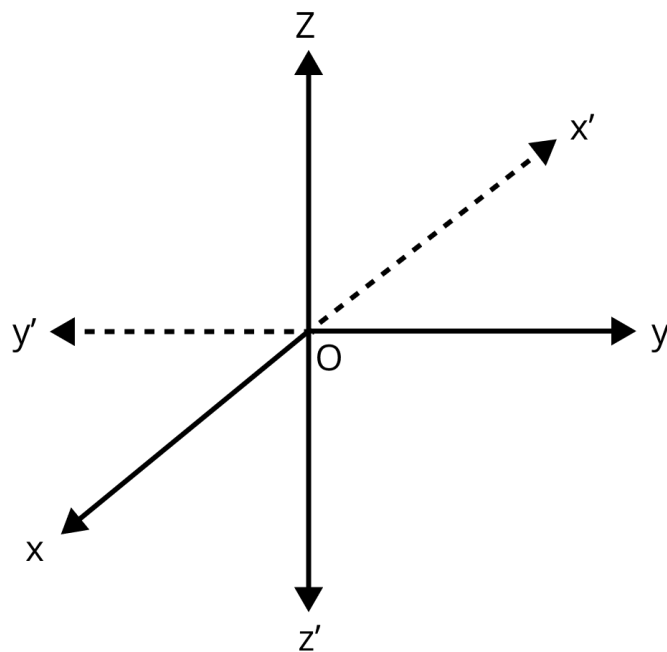


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(4, -2, -5)$ is located in the *VIII* octant.

Ans:

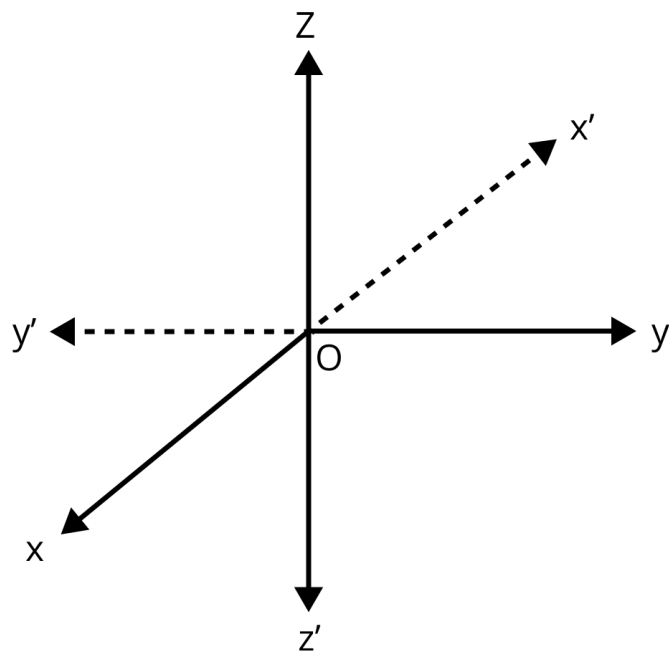


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(4, 2, -5)$ is located in the V octant.

Ans:

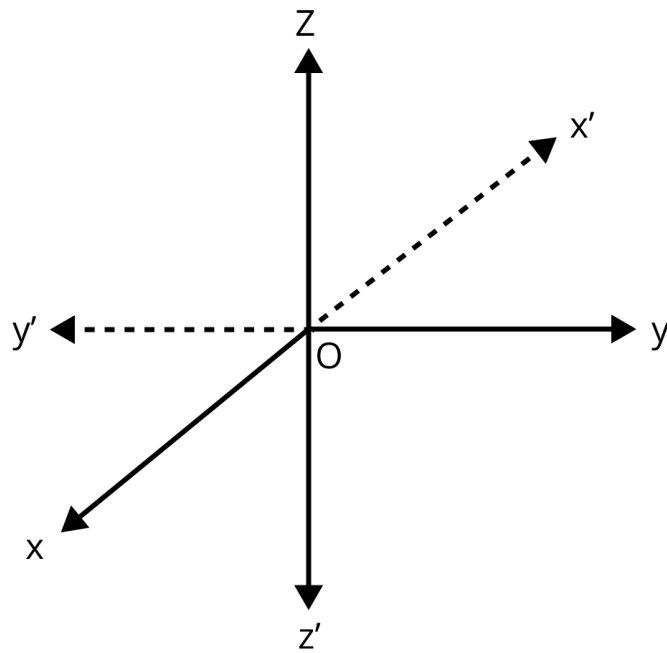


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P.]

The point $P(-4, 2, 5)$ is located in the II octant.

Ans:

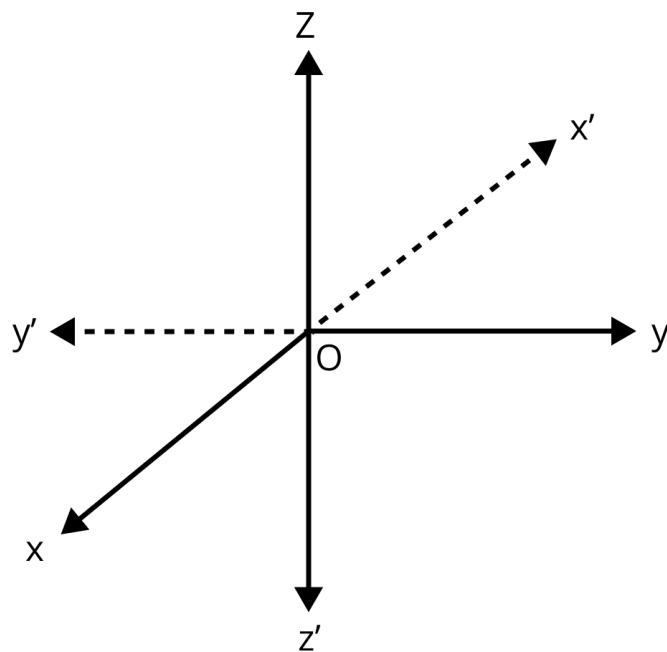


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(-3, -1, 6)$ is located in the *III* octant.

Ans:

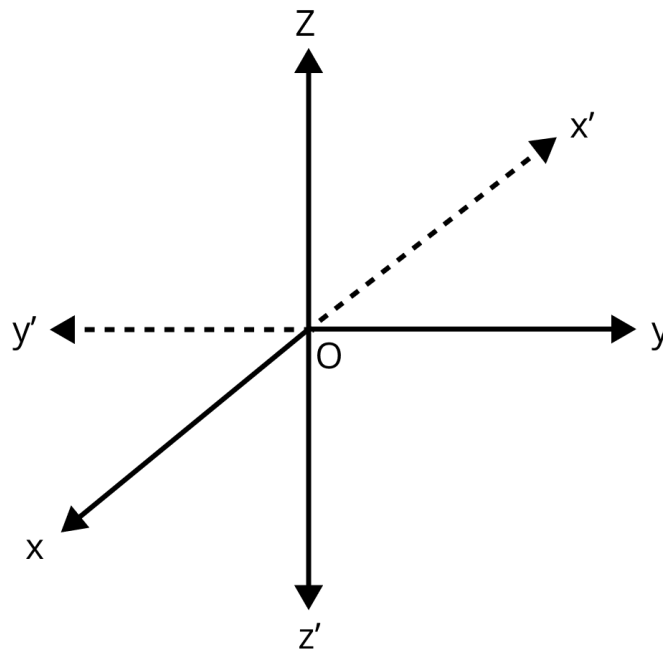


Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P .

The point $P(2, -4, -7)$ is located in the $VIII$ octant.

Ans:



Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the point P

The point $P(-4, 2, -5)$ is located in the VI octant.

Question 3

Let A , B , C be the feet of perpendiculars from a point P on the x , y , z - axis respectively. Find the coordinates of A , B and C in each of the following where the point P is:

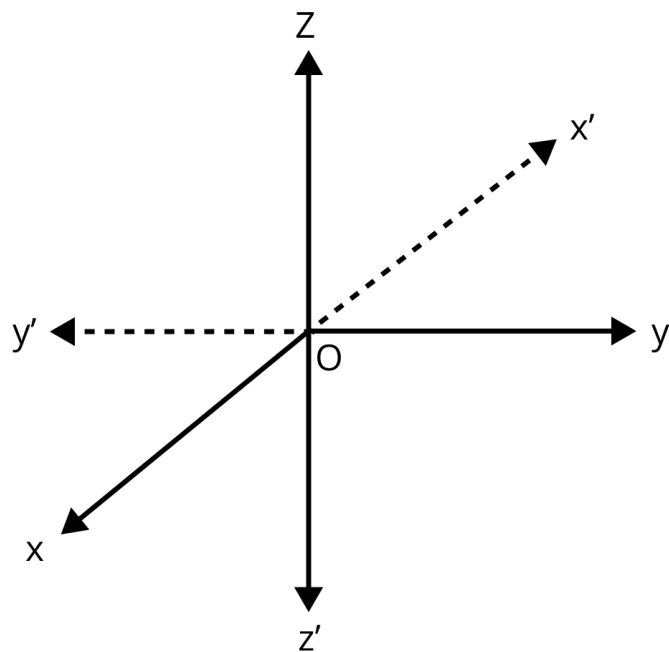
(i) $(3, 4, 2)$

(ii) $(-5, 3, 7)$

(iii) $(4, -3, -5)$

Solution

Ans:



Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

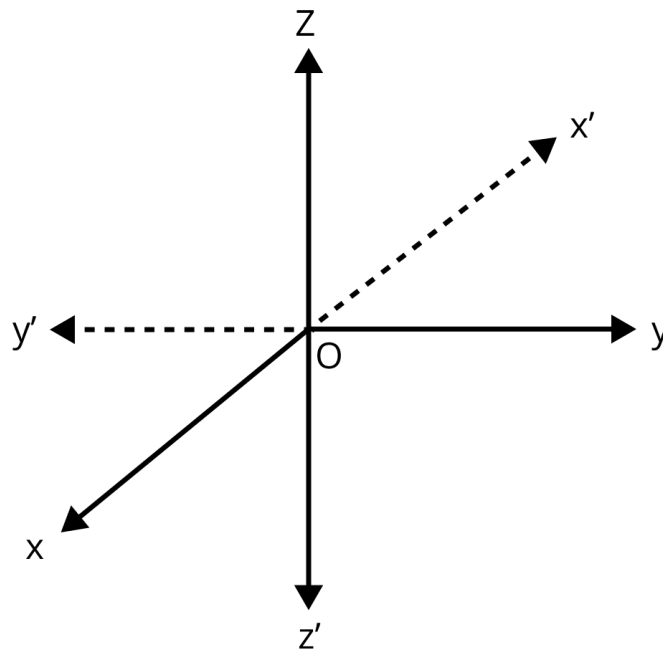
Locate the points.

The coordinates of A are $(3, 0, 0)$

The coordinates of B are $(0, 4, 0)$

The coordinates of C are $(0, 0, 2)$

Ans:



Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

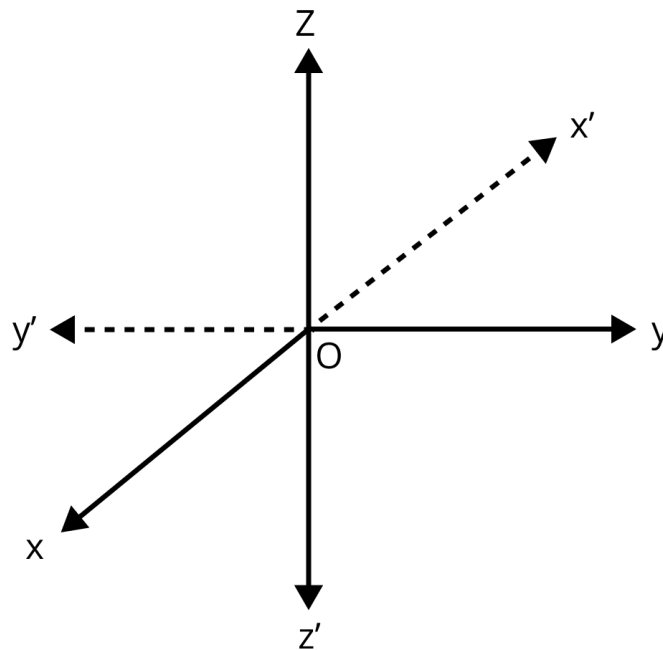
Locate the points.

The coordinates of A are $(-5, 0, 0)$

The coordinates of B are $(0, 3, 0,)$

The coordinates of C are $(0, 0, 7)$

Ans:



Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.

Locate the points.

The coordinates of A are $(4, 0, 0)$

The coordinates of B are $(0, -3, 0)$

The coordinates of C are $(0, 0, -5)$

Question 4

Let A , B , C be the feet of perpendiculars from a point P on the xy , yz and zx planes respectively. Find the coordinates of A , B , C in each of the following where the point P is

(i) $(3, 4, 5)$

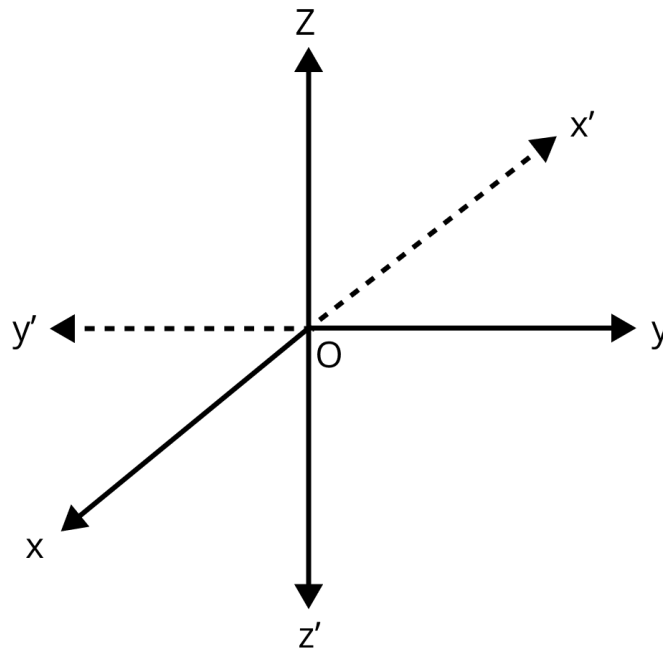
(ii) $(-5, 3, 7)$

(iii) $(4, -3, -5)$.

Solution

Ans:

Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.



Locate the points.

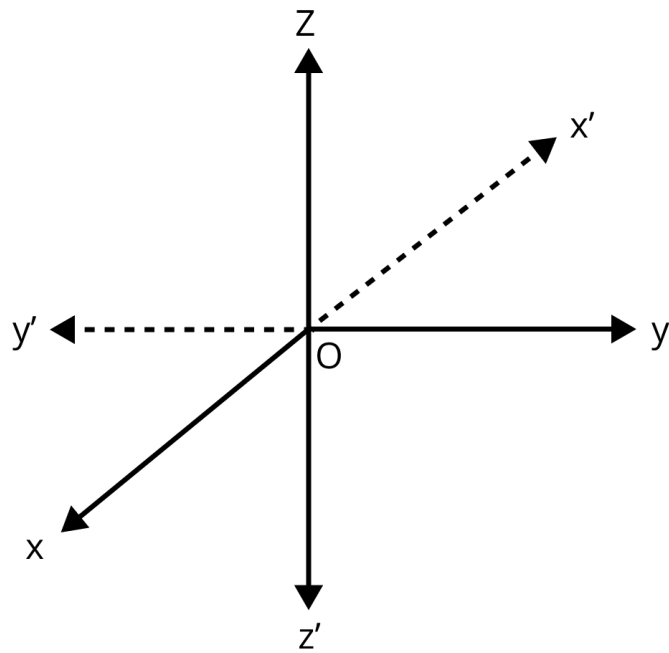
The coordinates of A are $(3, 4, 0)$

The coordinates of B are $(0, 4, 5)$

The coordinates of C are $(3, 0, 5)$

Ans:

Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.



Locate the points.

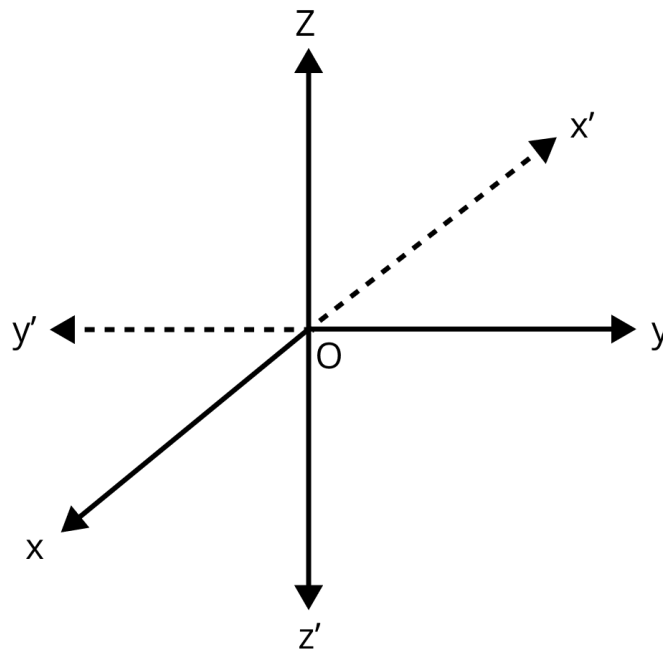
The coordinates of A are $(-5, 3, 0)$

The coordinates of B are $(0, 3, 7)$

The coordinates of C are $(-5, 0, 7)$

Ans:

Each plane divides this space in two parts and these three coordinate planes together divide the space in eight regions called octants.



Locate the points.

The coordinates of A are $(4, -3, 0)$

The coordinates of B are $(0, -3, -5)$

The coordinates of C are $C(4, 0, -5)$

Question 5

How far apart are the points $(2, 0, 0)$ and $(-3, 0, 0)$

Solution

Ans: Given: The points $(2, 0, 0)$ and $(-3, 0, 0)$

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Given points are $(2, 0, 0)$ and $(-3, 0, 0)$

Distance between the given points

$$\begin{aligned} &= \sqrt{(2+3)^2 + (0-0)^2 + (0-0)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Hence, the required distance is 5.

Question 6

Find the distance from the origin to $(6, 6, 7)$.

Solution

Ans: Given: The origin to $(6, 6, 7)$.

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Coordinates of the origin are $(0, 0, 0)$

Distance from $(0, 0, 0)$ to $(6, 6, 7)$

$$\begin{aligned} D &= \sqrt{(6-0)^2 + (6-0)^2 + (7-0)^2} \\ &= \sqrt{36 + 36 + 49} \\ &= \sqrt{121} \\ &= 11 \text{ units.} \end{aligned}$$

Hence, the required distance is 11 units.

Question 7

Show that if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a distance 1 unit from the origin.

Solution

Ans: Given: Point $(x, y, \sqrt{1 - x^2 - y^2})$

Given point is $(x, y, \sqrt{1 - x^2 - y^2})$

Distance between the origin and the point is

$$\begin{aligned} D &= \sqrt{(x - 0)^2 + (y - 0)^2 + (\sqrt{1 - x^2 - y^2} - 0)^2} \\ &= \sqrt{1} = 1 \end{aligned}$$

Hence, proved.

Question 8

Show that the point $A(1, -1, 3)$, $B(2, -4, 5)$ and $(5, -13, 11)$ are collinear.

Solution

Ans: Given: $A(1, -1, 3)$, $B(2, -4, 5)$ and $(5, -13, 11)$

Examine the points.

$$\begin{aligned} \text{The distance between points } A \text{ and } B &\text{ is } \sqrt{(2 - 1)^2 + (-4 + 1)^2 + (5 - 3)^2} \\ &= \sqrt{(1)^2 + (-3)^2 + (2)^2} \\ &= \sqrt{1 + 9 + 4} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} \text{The distance between points } B \text{ and } C &\text{ is } \sqrt{(5 - 2)^2 + (-13 + 4)^2 + (11 - 5)^2} \\ &= \sqrt{(3)^2 + (-9)^2 + (6)^2} \\ \text{Solve further, } &\sqrt{9 + 81 + 36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

$$\begin{aligned} \text{The distance between points } A \text{ and } C &\text{ is } \sqrt{(5 - 1)^2 + (-13 + 1)^2 + (11 - 3)^2} \\ &= \sqrt{16 + 144 + 64} \\ &= \sqrt{224} \end{aligned}$$

i.e. $4\sqrt{14}$

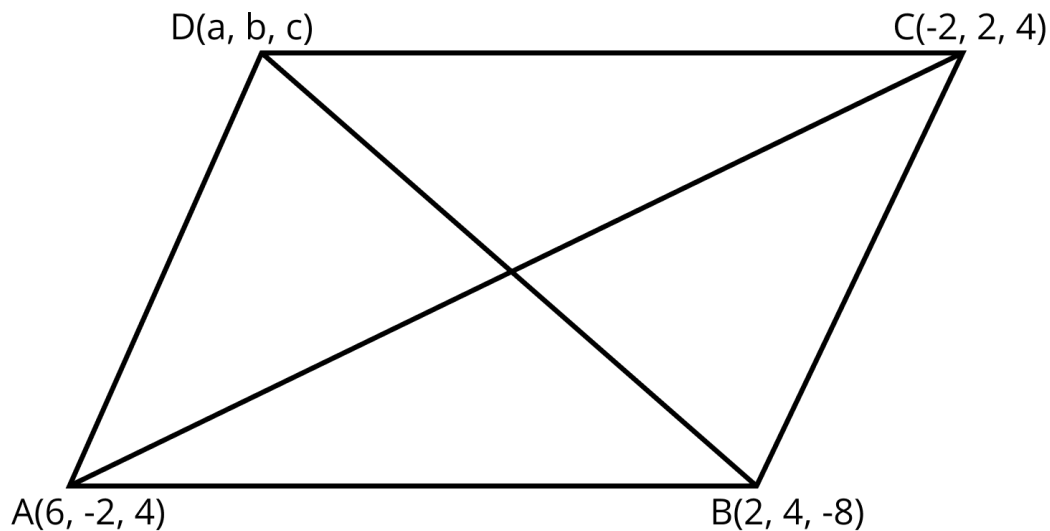
Therefore,

$$AB + BC = AC$$

The points A, B, and C are collinear.

Question 9

Three consecutive vertices of a parallelogram $ABCD$ are $A(6, -2, 4)$, $B(2, 4, -8)$, $C(-2, 2, 4)$. Find the coordinates of the fourth vertex.



$$= (2, 0, 4)$$

Solution

Ans: Given: $A(6, -2, 4)$, $B(2, 4, -8)$, $C(-2, 2, 4)$

To find: Coordinates of the fourth vertex.

The diagonals of a parallelogram bisect each other.

Midpoint of diagonal

$$AC = \left(\frac{6-2}{2}, \frac{-2+2}{2}, \frac{4+4}{2} \right)$$

And the midpoint of diagonal BD

$$= \left(\frac{a+2}{2}, \frac{b+4}{2}, \frac{c-8}{2} \right)$$

$$\frac{a+2}{2} = 2$$

$$\Rightarrow a = 2$$

And

$$\frac{b+4}{2} = 0$$

$$\Rightarrow b = -4$$

$$\frac{c-8}{2} = 4$$

$$\Rightarrow c = 16$$

Hence, the required coordinates are $(2, -4, 16)$.

Question 10

Show that the triangle ABC with vertices $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$ is right angled.

Solution

Ans: Given:

$$A(0, 4, 1), B(2, 3, -1)$$

$$C(4, 5, 0)$$

Given vertices are $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$

$$\text{So, } AB = \sqrt{(2-0)^2 + (3-4)^2 + (-1-1)^2}$$

$$= \sqrt{4+1+4}$$

$$= \sqrt{9}$$

$$= 3$$

Also,

$$\begin{aligned} BC &= \sqrt{(4-2)^2 + (5-3)^2 + (0+1)^2} \\ &= \sqrt{4+4+1} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

Also,

$$\begin{aligned} AC &= \sqrt{(4-0)^2 + (5-4)^2 + (0-1)^2} \\ &= \sqrt{16+1+1} \\ &= \sqrt{18} \end{aligned}$$

For right angled triangle $(AB)^2 + (BC)^2 = (AC)^2$

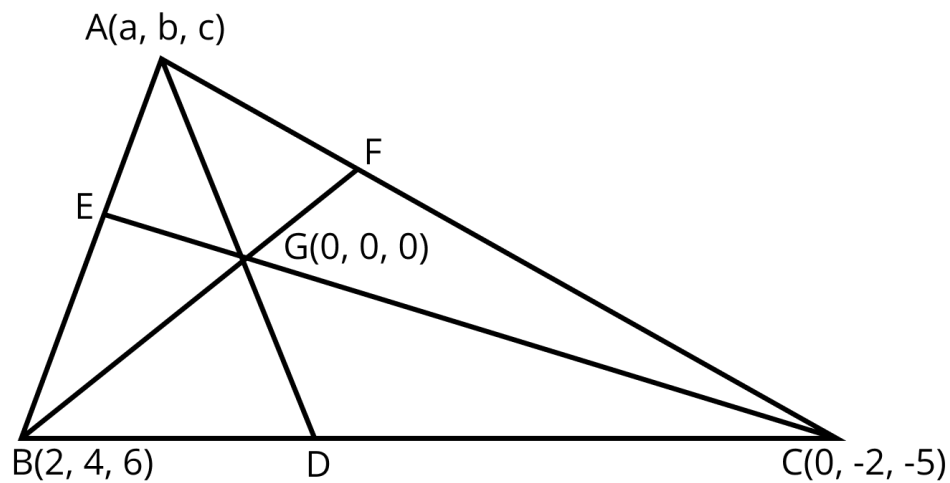
$$(3)^2 + (3)^2 = (\sqrt{18})^2$$

$$\text{So } AB^2 + BC^2 = AC^2$$

Hence, $\triangle ABC$ is a right angled triangle.

Question 11

Find the third vertex of triangle whose centroid is origin and two vertices are $(2, 4, 6)$ and $(0, -2, -5)$.



Solution

Ans: Given: Triangle ABC

Two vertices $B = (2, 4, 6)$ and $C = (0, -2, -5)$.

Let the coordinates of the third vertex A be (a, b, c) .

Since the centroid is at origin $(0, 0, 0)$

Hence, the x coordinate is $\frac{a + 2 + 0}{3} = 0$

$$a + 2 = 0$$

$$\Rightarrow a = -2$$

Hence, the y coordinate is $\frac{b + 4 - 2}{3} = 0$

$$b + 2 = 0$$

$$\Rightarrow b = -2$$

Hence, the z coordinate is $\frac{c + 6 - 5}{3} = 0$

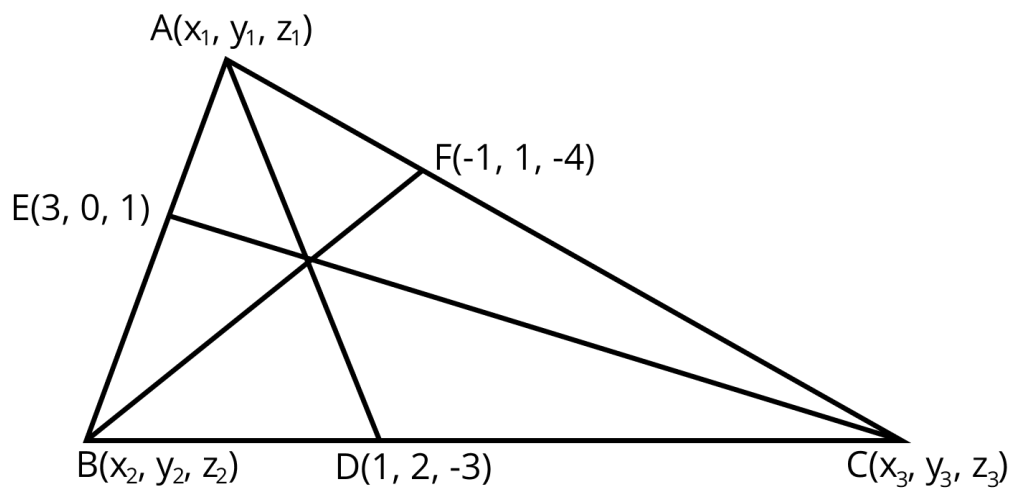
$$c + 1 = 0$$

$$\Rightarrow c = -1$$

The third vertex of the triangle is $(-2, -2, -1)$

Question 12

Find the centroid of a triangle, the mid-point of whose sides are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.



Solution

Ans: Given: In Triangle ABC

Midpoint $D(1, 2, -3)$

$E(3, 0, 1)$

$F(-1, 1, -4)$.

Let the coordinates of the vertices of $\triangle ABC$ be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$

Midpoint of $BC = (1, 2, -3)$

$$1 = \frac{x_2 + x_3}{2}$$

$$x_2 + x_3 = 2 \dots (1)$$

$$2 = \frac{y_2 + y_3}{2}$$

$$y_2 + y_3 = 4 \dots (2)$$

Similarly,

$$-3 = \frac{z_2 + z_3}{2}$$

$$z_2 + z_3 = -6 \dots (3)$$

Midpoint of $AB = (3, 0, 1)$

$$3 = \frac{x_1 + x_2}{2}$$

$$x_1 + x_2 = 6 \dots (4)$$

So,

$$0 = \frac{y_1 + y_2}{2}$$

$$y_1 + y_2 = 0 \dots (5)$$

Similarly,

$$1 = \frac{z_1 + z_2}{2}$$

$$z_1 + z_2 = 2 \dots (6)$$

Similarly, midpoint of $AC = (-1, 1, -4)$

$$-1 = \frac{x_1 + x_3}{2}$$

$$x_1 + x_3 = -2 \dots (7)$$

Similarly,

$$1 = \frac{y_1 + y_3}{2}$$

$$y_1 + y_3 = 2 \dots (8)$$

So,

$$-4 = \frac{z_1 + z_3}{2}$$

$$z_1 + z_3 = -8. \dots (9)$$

Adding the equation of (1), (4) and (7) get

$$2x_1 + 2x_2 + 2x_3 = 2 + 6 - 2 = 6$$

$$x_1 + x_2 + x_3 = 3$$

$$6 + x_3 = 3$$

$$\Rightarrow x_3 = -3$$

Again,

$$x_1 + 2 = 3$$

$$x_1 = 1$$

Let,

$$x_2 - 2 = 3$$

$$x_2 = 5$$

So, $x_1 = 1$, $x_2 = 5$ and $x_3 = -3$.

Similarly,

Adding the equation (2), (5), and (8) get

$$2(y_1 + y_2 + y_3) = 4 + 0 + 2 = 6$$

$$y_1 + y_2 + y_3 = 3$$

$$y_1 + 4 = 3$$

$$y_1 = -1$$

$$0 + y_3 = 3$$

$$y_3 = 3$$

$$y_2 + 2 = 3$$

$$\Rightarrow y_2 = 1$$

So, $y_1 = -1$, $y_2 = 1$, $y_3 = 3$

Adding the equation (3), (6) and (8) get

$$2(z_1 + z_2 + z_3) = -6 + 2 - 8 = -12$$

$$z_1 + z_2 + z_3 = -6$$

$$z_1 - 6 = -6$$

$$z_1 = 0$$

$$2 + z_3 = -6$$

$$z_3 = -8$$

$$z_3 = -8$$

$$z_2 - 8 = -6$$

$$\Rightarrow z_2 = 2$$

And

$$z_1 = 1, z_2 = 1, z_3 = -8.$$

So, the points are A(1, -1, 0), B(5, 1, 2) and C(-3, 3, -8).

Centroid of the triangle

$$G = \left(\frac{1+5-3}{3}, \frac{-1+1+3}{3}, \frac{0+2-8}{3} \right)$$
$$= (1, 1, -2)$$

Hence, the required coordinates (1, 1, -2).

Question 13

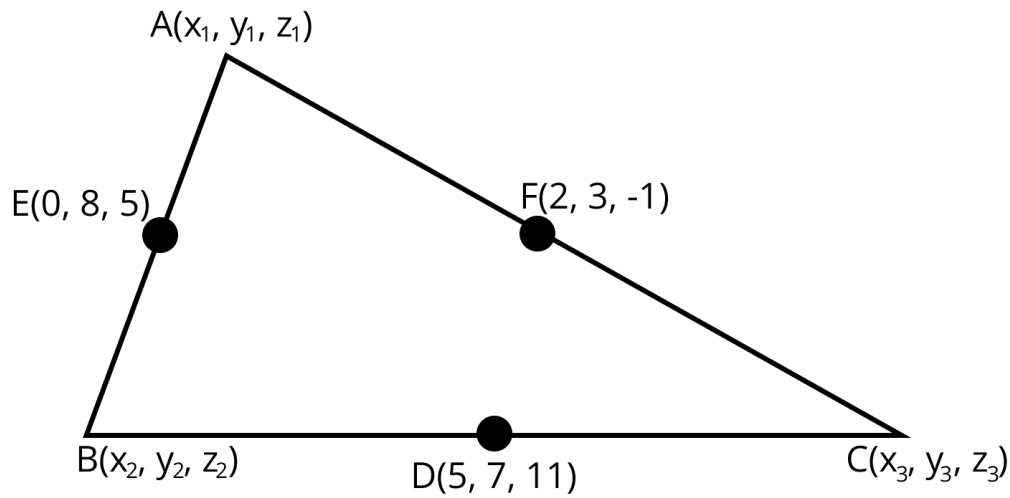
The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, -1). Find its vertices.

Solution

Ans: Given: In Triangle ABC

Midpoints are (5, 7, 11), (0, 8, 5) and (2, 3, -1).

Let the coordinates of the vertices be A (x_1, y_1, z_1), B (x_2, y_2, z_2) and C (x_3, y_3, z_3) respectively.



Since D(5, 7, 11) is the midpoint of BC

$$5 = \frac{x_2 + x_3}{2}$$

$$x_2 + x_3 = 10 \dots (1)$$

$$7 = \frac{y_2 + y_3}{2}$$

$$y_2 + y_3 = 14 \dots (2)$$

$$11 = \frac{z_2 + z_3}{2}$$

$$z_2 + z_3 = 22 \dots (3)$$

E(0, 8, 5) is the midpoint of AB

$$0 = \frac{x_1 + x_2}{2}$$

$$x_1 + x_2 = 0 \dots (4)$$

$$8 = \frac{y_1 + y_2}{2}$$

$$y_1 + y_2 = 16 \dots (5)$$

$$5 = \frac{z_1 + z_2}{2}$$

$$z_1 + z_2 = 10 \dots (6)$$

Similarly, $F(2, 3, -1)$ is the midpoint of AC

$$2 = \frac{x_1 + x_3}{2}$$

$$x_1 + x_3 = 4 \dots (7)$$

$$3 = \frac{y_1 + y_3}{2}$$

$$y_1 + y_3 = 6 \dots (8)$$

$$\text{and } -1 = \frac{z_1 + z_3}{2}$$

$$z_1 + z_3 = -2 \dots (9)$$

Adding the equation (1), (4) and (7)

$$2(x_1 + x_2 + x_3) = 10 + 0 + 4$$

$$x_1 + x_2 + x_3 = 7 \dots (10)$$

Subtracting (1) from (10)

$$x_1 = 7 - 10 = -3$$

Subtracting (4) from (10)

$$x_3 = 7 - 0 = 7$$

Subtracting (7) from (10)

$$x_2 = 7 - 4 = 3$$

Adding the equation (2), (5) and (8)

$$2(y_1 + y_2 + y_3) = (14 + 16 + 6)$$

$$y_1 + y_2 + y_3 = 18 \dots (11)$$

Subtracting (2) from (11)

$$y_1 = 18 - 14 = 4$$

Subtracting (5) from (11)

$$y_3 = 18 - 16 = 2$$

Subtracting (8) from (11)

$$y_2 = 18 - 6 = 12$$

Now adding the equation (3), (6) and (9)

$$2(z_1 + z_2 + z_3) = 22 + 10 - 2$$

$$z_1 + z_2 + z_3 = 15 \dots (12)$$

Subtracting (3) from (12)

$$z_1 = 15 - 22 = -7$$

Subtracting (6) from (12)

$$z_3 = 15 - 10 = 5$$

Subtracting (9) from (12)

$$z_2 = 15 + 2 = 17$$

So, the required coordinates are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.

That is $A(-3, 4, -7)$, $B(3, 12, 17)$ and $C(7, 2, 5)$.

Question 14

Three vertices of a Parallelogram $ABCD$ are $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$. Find the fourth vertex D

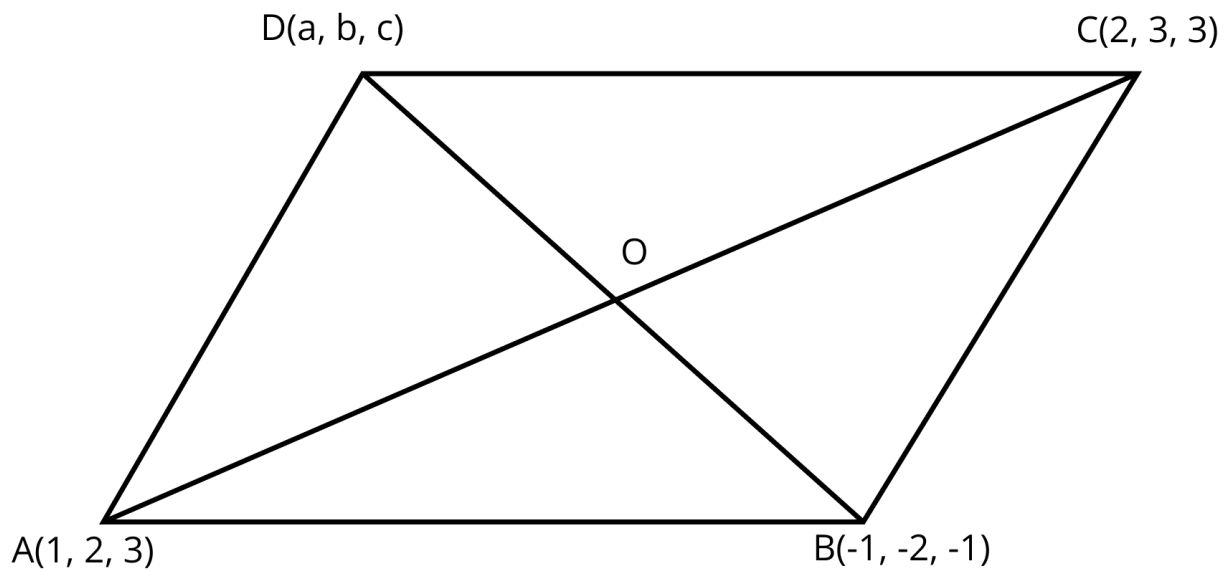
Solution

Ans: Given: In parallelogram $ABCD$

$A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$.

Let the coordinates of D be (a, b, c) .

The diagonals of a parallelogram bisect each other.



Midpoint of AC

$$\begin{aligned} O &= \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2} \right) \\ &= \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2} \right) \end{aligned}$$

Midpoint of BD

$$O = \left(\frac{a-1}{2}, \frac{b-2}{2}, \frac{c-1}{2} \right)$$

Equating the corresponding coordinate,

$$\frac{a-1}{2} = \frac{3}{2}$$

$$\Rightarrow a = 4$$

$$\frac{b-2}{2} = \frac{5}{2}$$

$$\Rightarrow b = 7$$

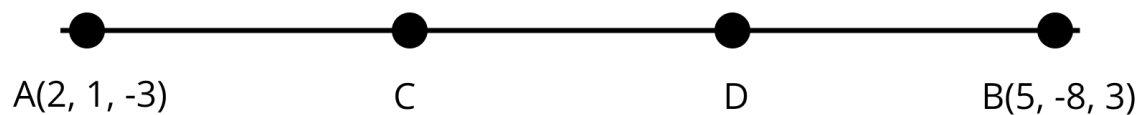
$$\frac{c-1}{2} = \frac{5}{2}$$

$$\Rightarrow c = 6$$

Hence, the coordinates of D = (4, 7, 6).

Question 15

Find the coordinate of the points which trisect the line segment joining the points A (2, 1, -3) and B (5, -8, 3).

**Solution**

Ans: Given: The points A (2, 1, -3) and B (5, -8, 3).

Let C and D be the points which divide the given line AB into three equal parts.

Here AC : CB = 1 : 2

Let (x_1, y_1, z_1) be the coordinates of C

$$x_1 = \frac{1 \times 5 + 2 \times 2}{1 + 2} = 3$$

$$y_1 = \frac{1 \times -8 + 2 \times 1}{1 + 2} = -2$$

$$z_1 = \frac{1 \times 3 + 2 \times -3}{1 + 2} = -1$$

So $C = (3, -2, -1)$.

Now D is the midpoint = CB

Let the coordinates of D be (x_2, y_2, z_2)

$$x_2 = \frac{3 + 5}{2} = 4$$

$$y_2 = \frac{-8 - 2}{2} = -5$$

$$z_2 = \frac{3 - 1}{2} = 1$$

So, $D = (4, -5, 1)$

Hence, the required coordinates are $C(3, -2, -1)$ and $D(4, -5, 1)$.

Question 16

If the origin is the centroid of a triangle ABC having vertices are $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, find the values of a , b and c .

Solution

Ans: Given: A triangle ABC whose centroid is the origin.

The vertices of the triangle are $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$.

Use the formula for the coordinates of a centroid of a triangle given as

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$. Here (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) are the vertices of the triangle.

Since the centroid of the triangle is the origin, so its coordinates is $(0, 0, 0)$. If a triangle has its vertices (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) then its centroid is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right).$$

$$\Rightarrow \left(\frac{a + (-2) + 4}{3}, \frac{1 + b + 7}{3}, \frac{3 + (-5) + c}{3} \right) = (0, 0, 0)$$

$$\Rightarrow \left(\frac{a+2}{3}, \frac{b+8}{3}, \frac{c-2}{3} \right) = (0, 0, 0)$$

Equating the respective coordinates and solving,

$$a = -2, b = -8 \text{ and } c = 2$$

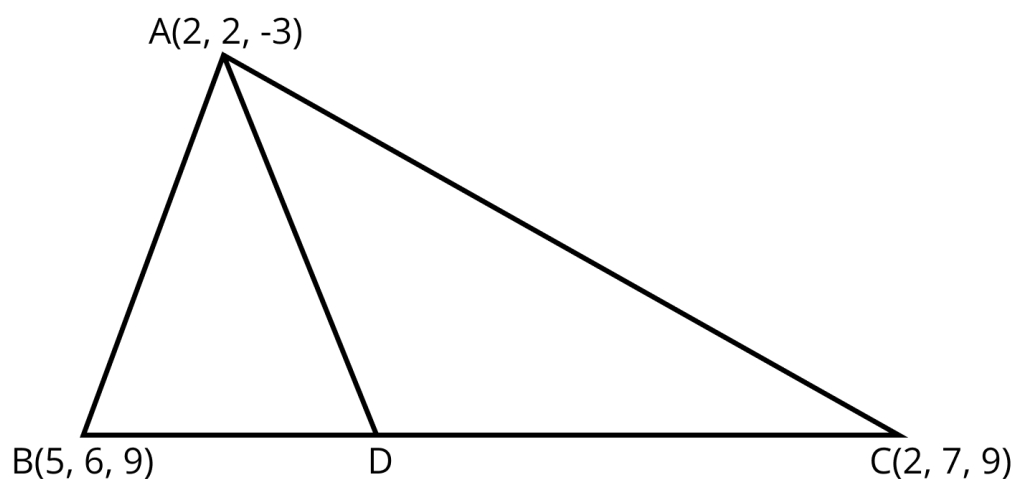
Question 17

Let $A(2, 2, -3)$, $B(5, 6, 9)$ and $C(2, 7, 9)$ be the vertices of a triangle. The internal bisector of the angle A meets BC at point D . Find the coordinates of point D .

Solution

Ans: Given: The vertices of the triangle are $A(2, 2, -3)$, $B(5, 6, 9)$ and $C(2, 7, 9)$. AD is the internal bisector of angle A and meets the side BC at D . Assume the coordinates of D as (x, y, z) . Use the angle bisector theorem given as $\frac{AB}{AC} = \frac{BD}{DC}$ and find the use the distance formula to find AB and AC .

Now, use the section formula for the points B , D and C . If AD is the angle bisector of the angle A and it meets BC at D then by the angle bisector theorem $\frac{AB}{AC} = \frac{BD}{DC}$.



Using the distance formula the length of AB

$$= \sqrt{(5-2)^2 + (6-2)^2 + (9-(-3))^2}$$

$$= \sqrt{9+16+144}$$

$$= 13 \text{ units}$$

Similarly, the length of AC

$$= \sqrt{(2-2)^2 + (7-2)^2 + (9-(-3))^2}$$

$$= \sqrt{25+144}$$

$$= 13 \text{ units}$$

Therefore, the ratio $\frac{BD}{DC} = 1$ and so $BD = DC$. Clearly the point D is the midpoint of BC . Using the midpoint formula for its coordinates,

$$\Rightarrow (x, y, z) = \left(\frac{5+2}{2}, \frac{7+6}{2}, \frac{9+9}{2} \right)$$

$$\Rightarrow (x, y, z) = \left(\frac{7}{2}, \frac{13}{2}, 9 \right)$$

Question 18

Show that the three points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$ are collinear and find the ratio in which C divides AB .

Solution

Ans: Given: The points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$.

Find the lengths AB , BC and AC . Check if the sum of two distances is equal to the third distance. If yes then the points are collinear.

To find the required ratio use the section formula for the points A , C and B . Using the distance formula, the length AB is,

$$\Rightarrow AB = \sqrt{(-1-2)^2 + (2-3)^2 + (-3-4)^2}$$

$$\Rightarrow AB = \sqrt{9 + 1 + 49}$$

$$\Rightarrow AB = \sqrt{59} \text{ units}$$

Similarly, the length BC is,

$$\Rightarrow BC = \sqrt{(-4 - (-1))^2 + (1 - 2)^2 + (-10 - (-3))^2}$$

$$\Rightarrow BC = \sqrt{9 + 1 + 49}$$

$$\Rightarrow BC = \sqrt{59} \text{ units}$$

Also, the length AC is,

$$\Rightarrow AC = \sqrt{(-4 - 2)^2 + (1 - 3)^2 + (-10 - 4)^2}$$

$$\Rightarrow AC = \sqrt{36 + 4 + 196}$$

$$\Rightarrow AC = 2\sqrt{59} \text{ units}$$

Clearly $AB + BC = AC$, so the three points are collinear.

Now, let us assume the ratio in which C divides AB is $m : n$. Using the section formula for the x coordinates of the points A , C and B ,

$$\Rightarrow x \text{ coordinate of } C = \frac{m \times (-1) + n \times (2)}{m + n}$$

$$\Rightarrow -4 = \frac{-m + 2n}{m + n}$$

$$\Rightarrow -4m - 4n = -m + 2n$$

$$\Rightarrow -3m = 6n$$

$$\Rightarrow \frac{m}{n} = \frac{-2}{1}$$

Hence, C divides AB externally in the ratio $2 : 1$.

Question 19

The mid-points of the sides of a triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$. Find its vertices. Also find the centroid of the triangle.

Solution

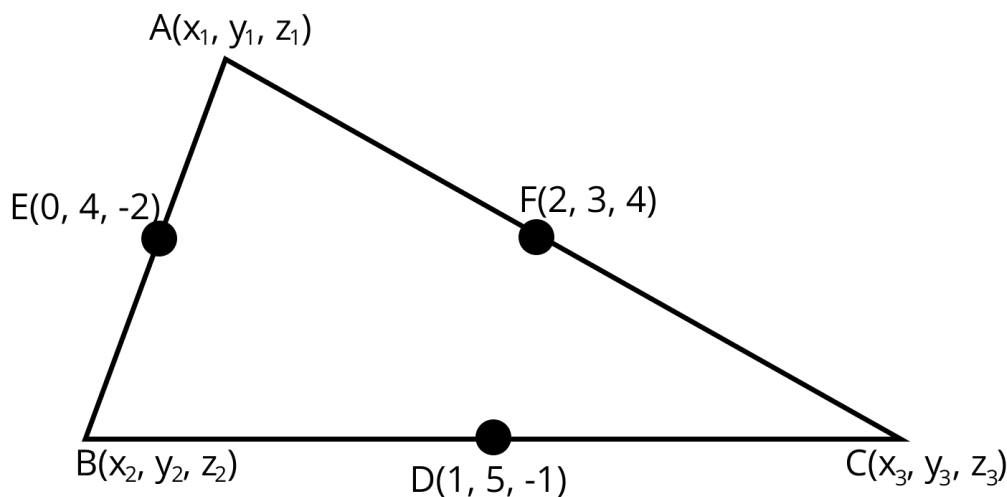
Ans: Given: The mid-points of the sides of a triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$

Assume (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) as the vertices of the triangle.

Use the midpoint formula to form the relations between the coordinates of the mid-points and the vertices.

To find the centroid use the formula $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$.

Let us assume the vertices of the triangle are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ whereas the mid-points of the sides AB , BC and CA are $D(1, 5, -1)$, $E(0, 4, -2)$ and $F(2, 3, 4)$ respectively.



Using the midpoint formula for the x coordinates,

$$x \text{ coordinate of } D = \frac{x_1 + x_2}{2}$$

$$\Rightarrow 1 = \frac{x_1 + x_2}{2} \dots (i)$$

$$x \text{ coordinate of } E = \frac{x_2 + x_3}{2}$$

$$\Rightarrow 0 = \frac{x_2 + x_3}{2} \dots (ii)$$

$$x \text{ coordinate of } F = \frac{x_3 + x_1}{2}$$

$$\Rightarrow 2 = \frac{x_3 + x_1}{2} \dots \text{(iii)}$$

Solving the three equations gives $x_1 = 3$, $x_2 = -1$ and $x_3 = 1$.

Similarly for the y coordinates,

$$y \text{ coordinate of } D = \frac{y_1 + y_2}{2}$$

$$\Rightarrow 5 = \frac{y_1 + y_2}{2} \dots \text{(i)}$$

$$y \text{ coordinate of } E = \frac{y_2 + y_3}{2}$$

$$\Rightarrow 4 = \frac{y_2 + y_3}{2} \dots \text{(ii)}$$

$$y \text{ coordinate of } F = \frac{y_3 + y_1}{2}$$

$$\Rightarrow 3 = \frac{y_3 + y_1}{2} \dots \text{(iii)}$$

Solving the three equations gives $y_1 = 4$, $y_2 = 6$ and $y_3 = 2$.

Finally, for the z coordinates,

$$z \text{ coordinate of } D = \frac{z_1 + z_2}{2}$$

$$\Rightarrow -1 = \frac{z_1 + z_2}{2} \dots \text{(i)}$$

$$z \text{ coordinate of } E = \frac{z_2 + z_3}{2}$$

$$\Rightarrow -2 = \frac{z_2 + z_3}{2} \dots \text{(ii)}$$

$$z \text{ coordinate of } F = \frac{z_3 + z_1}{2}$$

$$\Rightarrow 4 = \frac{z_3 + z_1}{2} \dots \text{(iii)}$$

Solving the three equations gives $z_1 = 5$, $z_2 = -7$ and $z_3 = 3$.

Therefore, the vertices of the triangle are $A(3, 4, 5)$, $B(-1, 6, -7)$ and $C(1, 2, 3)$.

Now, the centroid of the triangle is

$$\begin{aligned}
&= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right) \\
&= \left(\frac{3 + (-1) + 1}{3}, \frac{4 + 6 + 2}{3}, \frac{5 + (-7) + 3}{3} \right) \\
&= \left(1, 4, \frac{1}{3} \right)
\end{aligned}$$

Question 20

Prove that the points $(0, -1, -7)$, $(2, 1, -9)$ and $(6, 5, -13)$ are collinear. Find the ratio in which the first point divides the join of the two points.

Solution

Ans: Given: The points $(0, -1, -7)$, $(2, 1, -9)$ and $(6, 5, -13)$.

Assume the points as A , B and C respectively. Find the lengths AB , BC and AC . Check if the sum of two distances is equal to the third distance. If yes then the points are collinear. To find the required ratio use the section formula for the points B , A and C .

Using the distance formula, the length AB is,

$$\Rightarrow AB = \sqrt{(2 - 0)^2 + (1 - (-1))^2 + (-9 - (-7))^2}$$

$$\Rightarrow AB = \sqrt{4 + 4 + 4}$$

$$\Rightarrow AB = 2\sqrt{3} \text{ units}$$

Similarly, the length BC is,

$$\Rightarrow BC = \sqrt{(6 - 2)^2 + (5 - 1)^2 + (-13 - (-9))^2}$$

$$\Rightarrow BC = \sqrt{16 + 16 + 16}$$

$$\Rightarrow BC = 4\sqrt{3} \text{ units}$$

Also, the length AC is,

$$\Rightarrow AC = \sqrt{(6 - 0)^2 + (5 - (-1))^2 + (-13 - (-7))^2}$$

$$\Rightarrow AC = \sqrt{36 + 36 + 36}$$

$$\Rightarrow AC = 6\sqrt{3} \text{ units}$$

Clearly $AB + BC = AC$, so the three points are collinear.

Now, let us assume the ratio in which the first point A divides BC is $m : n$. Using the section formula for the x coordinates of the points B , A and C ,

$$\Rightarrow x \text{ coordinate of } A = \frac{m \times 6 + n \times 2}{m + n}$$

$$\Rightarrow 0 = \frac{6m + 2n}{m + n}$$

$$\Rightarrow 0 = 6m + 2n$$

$$\Rightarrow 6m = -2n$$

$$\Rightarrow \frac{m}{n} = \frac{-1}{3}$$

Hence, A divides BC externally in the ratio $1 : 3$.

Question 21

What are the coordinates of the vertices of a cube whose edge is 2 units , one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Solution

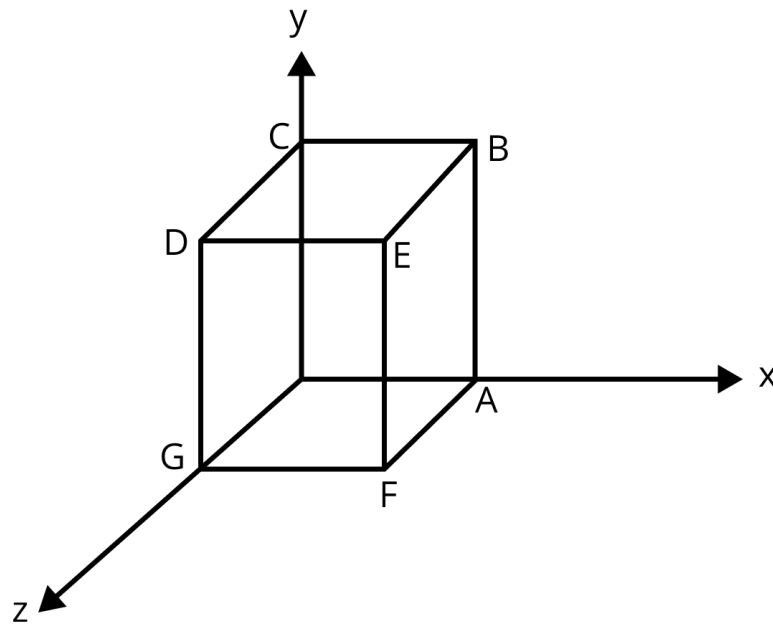
Ans: Given: Edge length of the cube = 2 units

One of the vertices coincides with the origin.

Three edges passing through the origin coincides with the positive direction of the axes.

Draw a diagram of the cube according to the conditions given. Start from the origin and use the sign convention of the coordinates in a specific quadrant to find the vertices.

The diagram of the cube according to the given conditions can be shown below: -



Here one of the vertices is the origin O , so its coordinate is $(0, 0, 0)$.

Since point A lies on the x axis so its coordinate is $(2, 0, 0)$.

Point C lies on the y axis so its coordinate is $(0, 2, 0)$.

Point G lies on the z axis so its coordinate is $(0, 0, 2)$.

Since point B lies in the xy plane so its coordinate is $(2, 2, 0)$.

Point D lies in the yz plane so its coordinate is $(0, 2, 2)$.

Point F lies in the xz plane so its coordinate is $(2, 0, 2)$.

Now, the point E lies in the 3 — D space so its coordinate is $(2, 2, 2)$.

Question 22

The distance of point $P(3, 4, 5)$ from the yz plane is

- (a) 3 units
- (b) 4 units
- (c) 5 units

(d) 550 units

Solution

Ans: Option (c)

Given: The point $P(3, 4, 5)$.

Use the fact that the distance of a point $P(x_1, y_1, z_1)$ from the yz plane is its x coordinate.

The distance of a point $P(x_1, y_1, z_1)$ from the yz plane is its x coordinate x_1 .

Therefore, the distance of the point $P(3, 4, 5)$ from the yz plane = 3 units.

Question 23

What is the length of the perpendicular foot drawn from the point $P(3, 4, 5)$ on y axis?

(a)

$$\sqrt{41}$$

(b)

$$\sqrt{34}$$

(c) 5

(d) None of these

Solution

Ans: Option (b)

Given: The point $P(3, 4, 5)$.

The length of foot of perpendicular from a point $P(x_1, y_1, z_1)$ on the y axis is $\sqrt{x_1^2 + z_1^2}$.

Therefore, the length of foot of perpendicular from the point $P(3, 4, 5)$ on the y axis

$$= \sqrt{3^2 + 5^2}$$

$$= \sqrt{34}$$

Question 24

Distance of the point $(3, 4, 5)$ from the origin is

(a)

$$\sqrt{50}$$

(b) 3

(c) 4

(d) 5

Solution

Ans: Option (a)

Given: The origin and the point $(3, 4, 5)$.

Use the formula of the distance between the two points (x_1, y_1, z_1) and (x_2, y_2, z_2) given as

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

The origin is given by the coordinates $(0, 0, 0)$. Let us assume the distance between the points $(0, 0, 0)$ and $(3, 4, 5)$ is d .

The distance between the two points having coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

$$\Rightarrow d = \sqrt{(3 - 0)^2 + (4 - 0)^2 + (5 - 0)^2}$$

$$\Rightarrow d = \sqrt{9 + 16 + 25}$$

$$\Rightarrow d = \sqrt{50}$$

Question 25

If the distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then the value of a is

(a) 5

(b)

$$\pm 5$$

(c) - 5

(d) None of these

Solution

Ans: Option (b)

Given: The points $(a, 0, 1)$ and $(0, 1, 2)$.

Distance between the two points $= \sqrt{27}$

The distance between the two points having coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Therefore, for the points $(a, 0, 1)$ and $(0, 1, 2)$,

$$\Rightarrow d = \sqrt{27}$$

$$\Rightarrow \sqrt{(0 - a)^2 + (1 - 0)^2 + (2 - 1)^2} = \sqrt{27}$$

$$\Rightarrow a^2 + 2 = 27$$

$$\Rightarrow a^2 = 25$$

Taking square root both the sides,

$$\Rightarrow a = \pm 5$$

Question 26

x — axis is the intersection of two planes

- (a) xy and xz
- (b) yz and zx
- (c) xy and yz
- (d) none of these

Solution

Ans: Option (a)

Given: x — axis is the intersection of two planes.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the xy and xz planes, the line of intersection is the x — axis.

Hence, the two planes are xy and xz .

Question 27

Equation of y - axis is considered as

- (a) $x = 0, y = 0$
- (b) $y = 0, z = 0$
- (c) $z = 0, x = 0$
- (d) none of these

Solution

Ans: Option (c)

Given: y — axis.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the y — axis: $z = 0, x = 0$

Hence $z = 0, x = 0$.

Question 28

The point $(-2, -3, -4)$ lies in the

- (a) First octant
- (b) Seventh octant
- (c) Second octant
- (d) Eighth octant

Solution

Ans: Option (b)

Given: $(-2, -3, -4)$

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

$(-2, -3, -4)$ lies in the seventh octant.

Hence seventh octant is the correct answer.

Question 29

A plane is parallel to yz — plane so it is perpendicular to

- (a) x - axis
- (b) y - axis
- (c) z - axis
- (d) none of these

Solution

Ans: Option (a)

Given: A plane is parallel to yz — plane.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

As the plane is parallel to yz — plane, it is perpendicular to x — axis.

Hence, the plane is perpendicular to x — axis.

Question 30

The locus of a point for which $y = 0, z = 0$ is

- (a) equation of x — axis
- (b) equation of y — axis
- (c) equation of z — axis
- (d) none of these

Solution

Ans: Option (a)

Given: $y = 0, z = 0$.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

$y = 0, z = 0$ is the equation of x — axis.

Hence, the locus is the equation of x — axis.

Question 31

The locus of a point for which $x = 0$ is

- (a) xy - plane
- (b) yz - plane
- (c) zx - plane
- (d) none of these

Solution

Ans: Option (b)

Given: $x = 0$.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

$x = 0$ on the yz — plane.

Hence, the locus is yz — plane.

Question 32

If a parallelepiped is formed by planes drawn through the points $(5, 8, 10)$ and $(3, 6, 8)$ parallel to the coordinate planes, then the length of diagonal of the parallelepiped is

(a)

$$2\sqrt{3}$$

(b)

$$3\sqrt{2}$$

(c)

$$\sqrt{2}$$

(d)

$$\sqrt{3}$$

Solution

Ans: Option (a)

Given: $(5, 8, 10)$ and $(3, 6, 8)$.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The length of diagonal is:

$$\Rightarrow AB = \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2}$$

$$\Rightarrow AB = \sqrt{4+4+4}$$

$$\Rightarrow AB = 2\sqrt{3}$$

Hence, the length of diagonal of the parallelepiped is $2\sqrt{3}$.

Question 33

L is the foot of the perpendicular drawn from a point $P(3, 4, 5)$ on the xy - plane. The coordinates of point L are

(a) $(3,0,0)$ (b) $(0,4,5)$ (c) $(3,0,5)$

(d) none of these

Solution

Ans: Option (d)

Given: $P(3, 4, 5)$.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the xy - plane, $z = 0$.

Hence, the coordinates of point L are $(3, 4, 0)$.

Question 34

L is the foot of the perpendicular drawn from a point $(3, 4, 5)$ on x - axis. The coordinates of L are

- (a) $(3,0,0)$
- (b) $(0,4,0)$
- (c) $(0,0,5)$
- (d) none of these

Solution

Ans: Option (a)

Given: $(3, 4, 5)$.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the x - axis, $y = 0$ and $z = 0$.

Hence, the coordinates of L are $(3, 0, 0)$.

Question 35

The three axes OX, OY, OZ determine.

Solution

Ans: Given: OX, OY, OZ

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The three axes OX, OY, OZ determine 3 coordinate planes.

Hence, OX, OY, OZ determine 3 coordinate planes.

Question 36

The three planes determine a rectangular parallelepiped which has _____ of rectangular faces.

Solution

Ans: Given: The three planes determine a rectangular parallelepiped.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The rectangular parallelepiped determined by three planes has three pairs of rectangular faces.

Hence, the rectangular parallelepiped has three pairs of rectangular faces.

Question 37

The coordinates of a point are the perpendicular distance from the _____ on the respective axes.

Solution

Ans: Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The coordinates of a point are the perpendicular distance from the given points on the respective axes.

Question 38

The three coordinate planes divide the space into _____ parts.

Solution

Ans: Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The three coordinate planes divide the space into 8 parts.

Hence, the space is divided into 8 parts.

Question 39

If a point P lies on yz - plane, then the coordinates of a point on yz - plane is of the form _____.

Solution

Ans: Given: P lies on yz - plane.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the yz - plane, $x = 0$.

Hence, the coordinates are of the form $(0, y, z)$.

Question 40

The equation of yz - plane is _____.

Solution

Ans: Given: yz - plane.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

For any point on the yz - plane, the x - coordinate is 0.

Hence, the equation of yz - plane is $x = 0$.

Question 41

If the point P lies on z – axis, then coordinates of P are of the form _____.

Solution

Ans: Given: P lies on z – axis.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the z – axis, $x = 0$ and $y = 0$.

Hence, the coordinates are of the form $(0, 0, z)$.

Question 42

The equations of z – axis are _____.

Solution

Ans: Given: z – axis

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

On the z – axis, the coordinates of any point are of the form $(0, 0, z)$.

Hence, the equation of z – axis is $x = 0$ and $y = 0$.

Question 43

A line is parallel to xy – plane if all the points on the line have equal _____.

Solution

Ans: Given: A line is parallel to xy – plane. Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

A line is parallel to xy – plane if every point on it is at the same distance from the xy – plane.

The distance of any point from xy – plane is z . Hence, all the points on the line have equal z – coordinate.

Question 44

A line is parallel to x — axis if all the points on the line have equal _____.

Solution

Ans: Given: A line is parallel to x — axis.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

A line is parallel to x — axis if every point on it maintains a constant distance from y — axis and z — axis.

Hence, all the points on the line have equal y and z — coordinates.

Question 45

$x = a$ represents a plane parallel to _____.

Solution

Ans: Given: $x = a$

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The locus of a point $P(x, y, z)$ is $x = a$.

Every point P has a constant x — coordinate.

x is the distance of P from the yz — plane. The plane $x = a$ is at constant distance a from the yz — plane.

Hence, $x = a$ represents a plane parallel to the yz — plane.

Question 46

The plane parallel to yz — plane is perpendicular to _____.

Solution

Ans: Given: yz — plane.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

The plane parallel to yz — plane is perpendicular to the x — axis.

Hence, the plane which is parallel to yz — plane is perpendicular to the x — axis.

Question 47

The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 10, 13 and 8 units are _____.

Solution

Ans: Given: Dimensions of rectangular rooms are 10, 13 and 8 units.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

Given dimensions are $a = 10, b = 13$ and $c = 8$.

The length of string required = $\sqrt{a^2 + b^2 + c^2}$

$$\text{Required length} = \sqrt{(10^2 + 13^2 + 8^2)}$$

$$\text{Required length} = \sqrt{(100 + 169 + 64)}$$

$$\text{Required length} = \sqrt{333}$$

Hence, the length of the longest piece of a string is $\sqrt{333}$.

Question 48

If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then a _____.

Solution

Ans: Given: The distance between $(a, 2, 1)$ and $(1, -1, 1)$ is 5.

Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

Given points are $(a, 2, 1)$ and $(1, -1, 1)$.

$$\text{Given } \sqrt{(a-1)^2 + (2+1)^2 + (1-1)^2} = 5$$

$$\Rightarrow (a-1)^2 + 9 + 0 = 25$$

$$\Rightarrow a^2 - 2a - 15 = 0$$

$$\Rightarrow (a-5)(a+3) = 0$$

$$\Rightarrow a = 5 \text{ or } -3$$

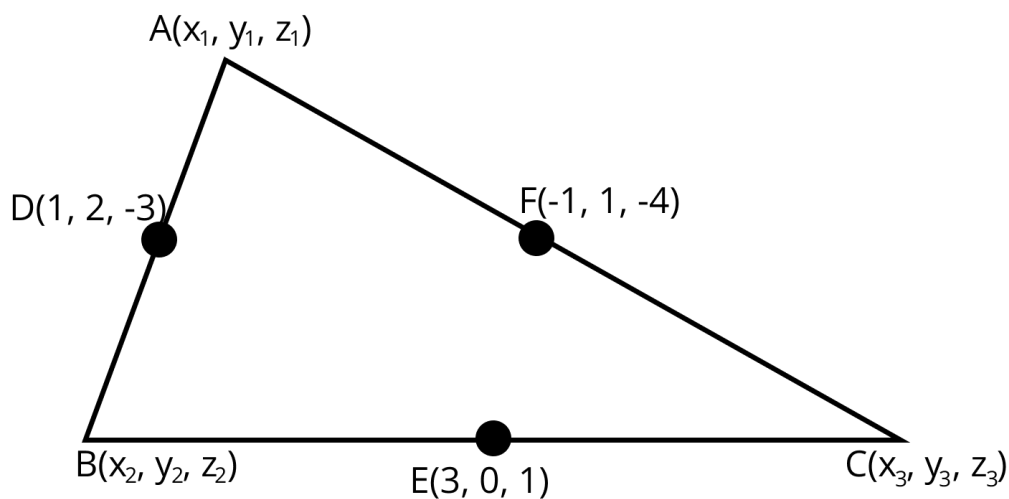
Hence, $a = 5$ or -3 .

Question 49

If the mid-points of the sides of a triangle $AB;BC;CA$; are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$, then the centroid of the triangle ABC is _____.

Solution

Ans: Given: The mid-points of the sides of a triangle $AB;BC;CA$; are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.



Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

Given that the mid-points of the sides of $\triangle ABC$ are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.

The centroid of the triangle formed by the midpoints of the sides of a triangle is the same as the centroid of the triangle.

So, the centroid of $\triangle DEF$ and the centroid of $\triangle ABC$ are the same.

$$\text{Centroid of } \triangle ABC = G \left(\frac{1+3-1}{3}, \frac{2+0+1}{3}, \frac{-3+1-4}{3} \right)$$

$$\text{Centroid of } \triangle ABC = G(1, 1, -2)$$

Hence, the centroid of the triangle is $(1, 1, -2)$.

Question 50

Match each item given under the column C_1 to its correct answer given under column C_2 .

Column C_1	Column C_2
(a) In xy - plane	(i) 1st octant
(b) Point $(2, 3, 4)$ lies in the	(ii) yz - plane
(c) Locus of the points having x coordinate 0 is	(iii) z - coordinate is zero
(d) A line is parallel to x - axis if and only	(iv) z - axis
(e) If $x = 0, y = 0$ taken together will represent the	(v) plane parallel to xy - plane
(f) $z = c$ represent the plane	(vi) if all the points on the line have equal y and z - coordinates.
(g) Planes $x = a, y = b$ represent the line	(vii) from the point on the respective
(h) Coordinates of a point are the distances from the origin to the feet of perpendiculars	(viii) parallel to z - axis.

(i) A ball is the solid region in the space enclosed by a	(ix) disc
(j) Region in the plane enclosed by a circle is known as a	(x) sphere

Solution

Ans: Three dimensional geometry involves the mathematics of shapes in $3D$ space and the three coordinates in XYZ plane.

(a) In xy — plane, z — coordinate is zero.

(b) The point $(2, 3, 4)$ lies in Ist octant.

(c) Locus of the points having x — coordinate zero is yz — plane.

(d) A line is parallel to x — axis if and only if all the points on the line have equal y and z — coordinates.

(e) $x = 0, y = 0$ represent z — axis.

(f) $z = c$ represents the plane parallel to xy — plane.

(g) The plane $x = a$ is parallel to yz — plane.

The plane $y = b$ is parallel to xz — plane.

The planes $x = a$ and $y = b$ is the line of intersection of these planes.

The line of intersection of yz — plane and xz — plane is z — axis.

Thus, the line of intersection of planes $x = a$ and $y = b$ is a line parallel to z — axis.

(h) The coordinates of a point are the distances from the origin to the feet of perpendicular from the point on the respective axis.

(i) A ball is the solid region in the space enclosed by a sphere.

(j) The region in the plane enclosed by a circle is known as a disc.

Hence,

(a) - (iii), (b) - (i), (c) - (ii), (d) - (vi), (e) - (iv), (f) - (v), (g) - (viii), (h) - (vii), (i) - (x), (j) - (ix)

.

About NCERT

NCERT Exemplar for Class 11 chapter 12 deals with the Introduction to Three Dimensional Geometry. It involves the vector-algebra approach towards three-dimensional geometry with minimal use of complicated three-dimensional diagrams like triangles, etc. This chapter also explains the standard properties of lines and planes as well as three-dimensional space.

Three Dimensional Geometry includes three coordinates- x , y and z . They are mainly used to find the exact location of a point on a plane. The three coordinates are together known as the three-dimensional space. The three coordinate axes are then paired together- xy , yz , and zx which form three coordinate planes. We also discuss here the coordinates of a point in three-dimensional space as arbitrary point P is marked as $(x_0y_0z_0)$. These space coordinates are called Cartesian coordinates of P or rectangular coordinates of P . Next in line, we talk about the signs of coordinates of a point in which the sign depends on the octant in which the point lies. For example, in the first octant, all coordinates are positive and by the seventh octant, all the coordinates turn negative. In the fourth octant, x and z are positive while y is negative and in the sixth octant, it is the opposite, i.e., x and z are negative and y is positive. There are two formulas included in this chapter:

- The Distance Formula between two points $(A, B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ which is also the length of the diagonal.
- The Section Formula where the coordinates of point R that divides a line segment joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m:n$ is calculated as $\left[\frac{mx_2+nx_1}{(m+n)}+\frac{my_2+ny_1}{(m+n)}+\frac{mz_2+nz_1}{(m+n)}\right]$, $\left[\frac{mx_2-nx_1}{(m-n)}+\frac{my_2-ny_1}{(m-n)}+\frac{mz_2-nz_1}{(m-n)}\right]$