

Chapter 10: Straight Lines

NCERT Exemplar · Class 11 · Maths

Short Answer Questions

Question 1

Find the equation of a line which passes through the point $(1, -2)$ and cuts off equal intercepts from axes.

Solution

Ans:

Given: Line passes through the point $(1, -2)$.

Let the intercept made by the line be a on x -axis and the intercept on y -axis be b .

So, equation of line is $\frac{x}{a} + \frac{y}{b} = 1$.

Given that $a = b$.

$$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1$$

It passes through $(1, -2)$, so $\frac{1}{a} - \frac{2}{a} = 1$

$$\Rightarrow -\frac{1}{a} = 1$$

$$\Rightarrow a = -1$$

So, equation of line will be $\frac{x}{-1} + \frac{y}{-1} = 1$

$$x + y = -1$$

Therefore, the equation is $x + y + 1 = 0$.

Question 2

Find the equation of a line which passes through the point $(5, 2)$ and perpendicular to the line joining the points $(2, 3)$ and $(3, -1)$.

Solution

Ans:

Given: $(2, 3)$ and $(3, -1)$.

Slope of line joining $(2, 3)$ and $(3, -1)$ is $\frac{-1-3}{3-2} = -4$.

Slope of the line perpendicular to this line is $\frac{-1}{-4} = \frac{1}{4}$.

The equation of line that passes through $(5, 2)$ is $y - 2 = \frac{1}{4}(x - 5)$.

$$\Rightarrow 4y - 8 = x - 5$$

$$\Rightarrow x - 4y + 3 = 0$$

Therefore, the equation is $x - 4y + 3 = 0$.

Question 3

Find the angle between the lines $y = (2 - \sqrt{3})(x + 5)$ and $y = (2 + \sqrt{3})(x - 7)$.

Solution

Ans:

Given: $y = (2 - \sqrt{3})(x + 5)$ and $y = (2 + \sqrt{3})(x - 7)$.

Slope of line $y = (2 - \sqrt{3})(x + 5)$ is $2 - \sqrt{3}$.

Slope of the line $y = (2 + \sqrt{3})(x - 7)$ is $2 + \sqrt{3}$.

θ be the angle between two lines.

$$\text{Using } \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

$$\Rightarrow \tan \theta = \left| \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{-2\sqrt{3}}{1 + 4 - 3} \right|$$

$$\Rightarrow \tan \theta = |-\sqrt{3}|$$

$$\Rightarrow \tan \theta = \sqrt{3} \text{ or } -\sqrt{3}$$

$$\Rightarrow \tan \theta = \tan 60^\circ \text{ or } \tan 120^\circ$$

Therefore, $\theta = 60^\circ$ or $\theta = 120^\circ$

Question 4

Find the equation of lines which passes through the point $(3, 4)$ and cuts off intercepts from the coordinate axes such that their sum is 14 .

Solution

Ans:

Given: Line passes through the point $(3, 4)$.

Let the intercept made by the line be a on x - axis and the intercept on y - axis be b .

So, equation of line is $\frac{x}{a} + \frac{y}{b} = 1$.

Given that $a + b = 14$.

$$\Rightarrow b = 14 - a$$

$$\text{So, } \frac{x}{a} + \frac{y}{14-a} = 1$$

This equation passes through $(3, 4)$.

$$\Rightarrow \frac{3}{a} + \frac{4}{14-a} = 1$$

$$\Rightarrow \frac{3(14-a)+4a}{a(14-a)} = 1$$

$$\Rightarrow 42 + a = 14a - a^2$$

$$\Rightarrow a^2 - 13a + 42 = 0$$

$$\Rightarrow (a - 6)(a - 7) = 0$$

$$\Rightarrow a = 6, 7$$

$$\Rightarrow b = 14 - 6, 14 - 7$$

$$\Rightarrow b = 8, 7$$

So, the equation of line when

$$a = 6$$

and

$$b = 8$$

is $\frac{x}{6} + \frac{y}{8} = 1$ i.e.

$$4x + 3y = 24$$

.

The equation of line when

$$a = 7$$

and

$$b = 7$$

is $\frac{x}{7} + \frac{y}{7} = 1$ i.e.

$$x + y = 7$$

.

Therefore, the equations are $x + y = 7$ and

$$4x + 3y = 24$$

.

Question 5

Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

Solution

Ans:

Given: $x + y = 4$ and $4x + 3y = 10$.

Let the point (x_1, y_1) be the point lying on $x + y = 4$ i.e. $x_1 + y_1 = 4$ (1)

The distance of (x_1, y_1) from $4x + 3y = 10$:

$$\Rightarrow \left| \frac{4x_1 + 3y_1 - 10}{\sqrt{4^2 + 3^2}} \right| = 1$$

$$\Rightarrow \left| \frac{4x_1 + 3y_1 - 10}{5} \right| = 1$$

$$\Rightarrow 4x_1 + 3y_1 - 10 = \pm 5$$

Take the + sign,

$$\Rightarrow 4x_1 + 3y_1 - 10 = 5$$

$$\Rightarrow 4x_1 + 3y_1 = 15 \dots (2)$$

From

$$(1)$$

,

$$\Rightarrow y_1 = 4 - x_1$$

Put this value in

$$(2)$$

,

$$\Rightarrow 4x_1 + 3(4 - x_1) = 15$$

$$\Rightarrow x_1 = 3$$

and

$$y_1 = 4 - 3 = 1$$

.

So, the point is $(3, 1)$.

Take the - sign,

$$\Rightarrow 4x_1 + 3y_1 - 10 = -5$$

$$\Rightarrow 4x_1 + 3y_1 = 5 \dots (3)$$

From

$$(1)$$

,

$$\Rightarrow y_1 = 4 - x_1$$

Put this value in

$$(3)$$

,

$$\Rightarrow 4x_1 + 3(4 - x_1) = 5$$

$$\Rightarrow x_1 = -7$$

and

$$y_1 = 4 - (-7) = 11$$

.

So, the point is $(-7, 11)$.

Therefore, the points are $(3, 1)$ and $(-7, 11)$.

Question 6

Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

Solution

Ans:

Given: The tangent of an angle between $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

Given that $\frac{x}{a} + \frac{y}{b} = 1 \dots (1)$

$$\frac{x}{a} - \frac{y}{b} = 1 \dots (2)$$

Slope of the line $\frac{x}{a} + \frac{y}{b} = 1$ is $-\frac{b}{a}$.

Slope of the line $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{b}{a}$.

Using $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$,

$$\Rightarrow \tan \theta = \left| \frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{-\frac{2b}{a}}{1 - \left(\frac{b}{a}\right)^2} \right|$$

$$\Rightarrow \tan \theta = \left| \frac{-2ab}{a^2 - b^2} \right|$$

$$\Rightarrow \tan \theta = \frac{2ab}{a^2 - b^2}$$

Hence verified.

Question 7

Find the equation of lines passing through $(1, 2)$ and making angle 30° with y -axis.

Solution

Ans:

Given: lines passing through $(1, 2)$.

The line makes 30° with y -axis.

So, the angle made with x -axis is 60° .

Slope of line $= \tan 60^\circ = \sqrt{3}$.

The equation of line that passes through $(1, 2)$ and slope $\sqrt{3}$ is:

$$\Rightarrow y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 2 = \sqrt{3}(x - 1)$$

$$\Rightarrow y - \sqrt{3}x + \sqrt{3} - 2 = 0$$

Therefore, the equation is $y - \sqrt{3}x + \sqrt{3} - 2 = 0$.

Question 8

Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.

Solution

Ans:

Given: Line passes through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to $3x + 4y = 7$.

The equation of line that passes through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ is $(2x + y - 5) + \lambda(x + 3y + 8) = 0$.

$$\Rightarrow (2 + \lambda)x + (1 + 3\lambda)y - 5 + 8\lambda = 0$$

Slope of this line is $\frac{-(2+\lambda)}{1+3\lambda}$.

Slope of $3x + 4y = 7$ is $-\frac{3}{4}$.

As the two lines are parallel, $\frac{-(2+\lambda)}{1+3\lambda} = -\frac{3}{4}$

$$\Rightarrow \frac{2+\lambda}{1+3\lambda} = \frac{3}{4}$$

$$\Rightarrow 8 + 4\lambda = 3 + 9\lambda$$

$$\Rightarrow 5\lambda = 5$$

$$\Rightarrow \lambda = 1$$

The equation $(2x + y - 5) + \lambda(x + 3y + 8) = 0$ becomes $3x + 4y + 3 = 0$.

Therefore, the equation is $3x + 4y + 3 = 0$.

Question 9

For what values of a and b the intercepts cut off on the coordinate axes by the line

$$ax + by + 8 = 0$$

are equal in length but opposite in signs to those cut off by the line

$$2x - -3y + 6 = 0$$

on the axes.

Solution

Ans:

Given:

$$ax + by + 8 = 0$$

and

$$2x - -3y + 6 = 0$$

.

We can write

$$ax + by + 8 = 0$$

as:

$$\Rightarrow ax + by = -8$$

$$\Rightarrow \frac{a}{-8}x + \frac{b}{-8}y = 1$$

$$\Rightarrow \frac{x}{\frac{-8}{a}} + \frac{y}{\frac{-8}{b}} = 1$$

The intercepts are

$$\frac{-8}{a}, \frac{-8}{b}$$

.

We can write

$$2x - -3y + 6 = 0$$

as:

$$\Rightarrow 2x - -3y = -6$$

$$\Rightarrow \frac{2x}{-6} - \frac{3y}{-6} = 1$$

$$\Rightarrow \frac{x}{-3} + \frac{y}{2} = 1$$

The intercepts are

$$-3, 2$$

.

From the question,

$$\frac{-8}{a} = 3, \frac{-8}{b} = -2$$

i.e.

$$a = \frac{-8}{3}, b = 4$$

Therefore, $a = -\frac{8}{3}$ and $b = 4$.

Question 10

If the intercept of a line between the coordinate axes is divided by the point

$$(-5, 4)$$

in the ratio

$$1 : 2$$

, then find the equation of the line.

Solution

Ans:

Given:

$$(-5, 4)$$

Assume

$$a$$

and

$$b$$

are intercepts on given line.

Then, the coordinates are

$$A(a, 0)$$

and

$$B(0, b)$$

.

Using

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

and

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

,

$$\Rightarrow -5 = \frac{1 \times 0 + 2 \times a}{1 + 2}$$

$$\Rightarrow 2a = -15$$

$$\Rightarrow a = \frac{-15}{2}$$

So,

$$A = \left(\frac{-15}{2}, 0 \right)$$

.

And

$$4 = \frac{1 \times b + 0 \times 2}{1 + 2}$$

$$\Rightarrow 4 = \frac{b}{3}$$

$$\Rightarrow b = 12$$

So,

$$B = (0, 12)$$

.

The equation of line

$$AB$$

is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

.

$$\Rightarrow y - 0 = \left(\frac{12 - 0}{0 + \frac{15}{2}} \right) \left(x + \frac{15}{2} \right)$$

$$\Rightarrow y = \frac{8}{5} \left(x + \frac{15}{2} \right)$$

$$\Rightarrow 5y = 8x + 60$$

Therefore,

$$8x - 5y + 60 = 0$$

.

Question 11

Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x -axis.

Solution

Ans:

Given: Line makes 120° with the positive direction of x -axis.

The line makes 120° with the positive direction of x -axis.

So, slope of the line is $\tan 120^\circ = -\sqrt{3}$.

Equation of the required line is:

$$y = -\sqrt{3}x + c$$

$$\Rightarrow \sqrt{3}x + y - c = 0.$$

This line is 4 units away from $(0, 0)$.

$$\Rightarrow \frac{|\sqrt{3}(0) + 0 - c|}{\sqrt{3+1}} = 4$$

$$\Rightarrow |c| = 8$$

$$\Rightarrow c = \pm 8$$

Therefore, the equation of line is $\sqrt{3}x + y \pm 8 = 0$.

Question 12

Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y + 4 = 0$ and the opposite vertex of the hypotenuse is $(2, 2)$.

Solution

Ans:

Given: $3x + 4y + 4 = 0$

The hypotenuse is along the line $3x + 4y + 4 = 0$

Slope of the hypotenuse is $-\frac{3}{4}$.

As the triangle is isosceles right angled triangle, two angles will be equal to 45° .

Let the slope of the line making 45° with the hypotenuse be m .

$$\Rightarrow \tan 45^\circ = \left| \frac{m - \left(-\frac{3}{4}\right)}{1 + m\left(-\frac{3}{4}\right)} \right|$$

$$\Rightarrow \tan 45^\circ = \left| \frac{4m+3}{4-3m} \right|$$

$$\Rightarrow \frac{4m+3}{4-3m} = \pm 1$$

$$\Rightarrow 4m + 3 = 4 - 3m \text{ or } 4m + 3 = 3m - 4$$

$$\Rightarrow m = \frac{1}{7} \text{ or } m = -7$$

Slope of this line is $\frac{1}{7}$, so the equation of this line is $y - 2 = (1/7)(x - 2)$ i.e. $x - 7y + 12 = 0$.

The slope of line perpendicular to it is -7 .

The equation of this line is: $y - 2 = -7(x - 2)$ i.e. $7x + y - 16 = 0$.

Therefore, the equation of required line is $7x + y - 16 = 0$.

Question 13

If the equation of the base of an equilateral triangle is $x + y - 2$ and the vertex is $(2, -1)$, then find the length of the side of the triangle.

Solution

Ans:

Given: $x + y - 2$ and $(2, -1)$.

The altitude from the vertex $(2, -1)$ on the base $x + y - 2$ meets at the midpoint of base.

The length of altitude is the distance of $(2, -1)$ from $x + y - 2 = 0$ $= \left| \frac{2+(-1)-2}{\sqrt{1^2+1^2}} \right| = \frac{1}{\sqrt{2}}$.

The triangle is an equilateral triangle. So, $\sin 60^\circ = \frac{\text{Altitude}}{\text{side of triangle}}$.

$$\Rightarrow \text{length of side of triangle} = \frac{\text{Altitude}}{\sin 60^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \sqrt{\frac{2}{3}}.$$

Therefore, the length of side of the triangle is $\sqrt{\frac{2}{3}}$.

Question 14

A variable line passes through a fixed point P . The algebraic sum of the perpendiculars drawn from the points $(2, 0)$, $(0, 2)$ and $(1, 1)$ on the line is zero. Find the coordinates of the point P .

Solution

Ans:

Given: $(2, 0)$, $(0, 2)$ and $(1, 1)$

Assume that the variable line passing through the fixed point P is $ax + by + c = 0$.

$$\text{Perpendicular distance from the point } A(2, 0) = \frac{2a+0+c}{\sqrt{a^2+b^2}}$$

$$\text{Perpendicular distance from the point } B(0, 2) = \frac{0+2b+c}{\sqrt{a^2+b^2}}$$

$$\text{Perpendicular distance from the point } C(1, 1) = \frac{a+b+c}{\sqrt{a^2+b^2}}$$

$$\text{Given that, } \frac{2a+0+c}{\sqrt{a^2+b^2}} + \frac{0+2b+c}{\sqrt{a^2+b^2}} + \frac{a+b+c}{\sqrt{a^2+b^2}} = 0$$

$$\Rightarrow 3a + 3b + 3c = 0 \text{ or } a + b + c = 0$$

Therefore, the variable line passes through the fixed point $(1, 1)$.

Question 15

In what direction should a line be drawn through the point $(1, 2)$ so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

Solution

Ans:

Given: $x + y = 4$ and $(1, 2)$.

Let the slope of line be m .

It passes through $A(1, 2)$

$$\text{So, the equation of line is } y - 2 = m(x - 1) \text{ or } mx - y + 2 - m = 0 \dots (1)$$

$$\text{The equation of the given line is } x + y - 4 = 0 \dots (2)$$

Assume that these lines meet at point B .

Solving (1) and (2),

$$B \equiv \left(\frac{m+2}{m+1}, \frac{3m+2}{m+1} \right)$$

$$\text{Given that } AB = \frac{\sqrt{6}}{3}$$

$$\Rightarrow AB^2 = \frac{6}{9}$$

$$\Rightarrow \left(\frac{m+2}{m+1} - 1 \right)^2 + \left(\frac{3m+2}{m+1} - 2 \right)^2 = \frac{6}{9}$$

$$\Rightarrow \left(\frac{1}{m+1} \right)^2 + \left(\frac{m}{m+1} \right)^2 = \frac{2}{3}$$

$$\Rightarrow \frac{1+m^2}{(1+m)^2} = \frac{2}{3}$$

$$\Rightarrow 3 + 3m^2 = 2 + 2m^2 + 4m$$

$$\Rightarrow m^2 - 4m + 1 = 0$$

$$\Rightarrow m = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3} = 2 + \sqrt{3} \text{ or } 2 - \sqrt{3}$$

$$\Rightarrow \tan \theta = 2 + \sqrt{3} \text{ or } 2 - \sqrt{3}$$

$$\Rightarrow \theta = 75^\circ \text{ or } \theta = 15^\circ.$$

Therefore, $\theta = 75^\circ$ or $\theta = 15^\circ$

Question 16

A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

Solution

Ans:

$$\text{Given: } \frac{1}{a} + \frac{1}{b} = \text{constant}$$

The equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$.

$$\text{As } \frac{1}{a} + \frac{1}{b} = \text{constant},$$

Let $\frac{1}{a} + \frac{1}{b} = \frac{1}{k}$.

So, $\frac{k}{a} + \frac{k}{b} = 1$

Thus, (k, k) lies on $\frac{x}{a} + \frac{y}{b} = 1$.

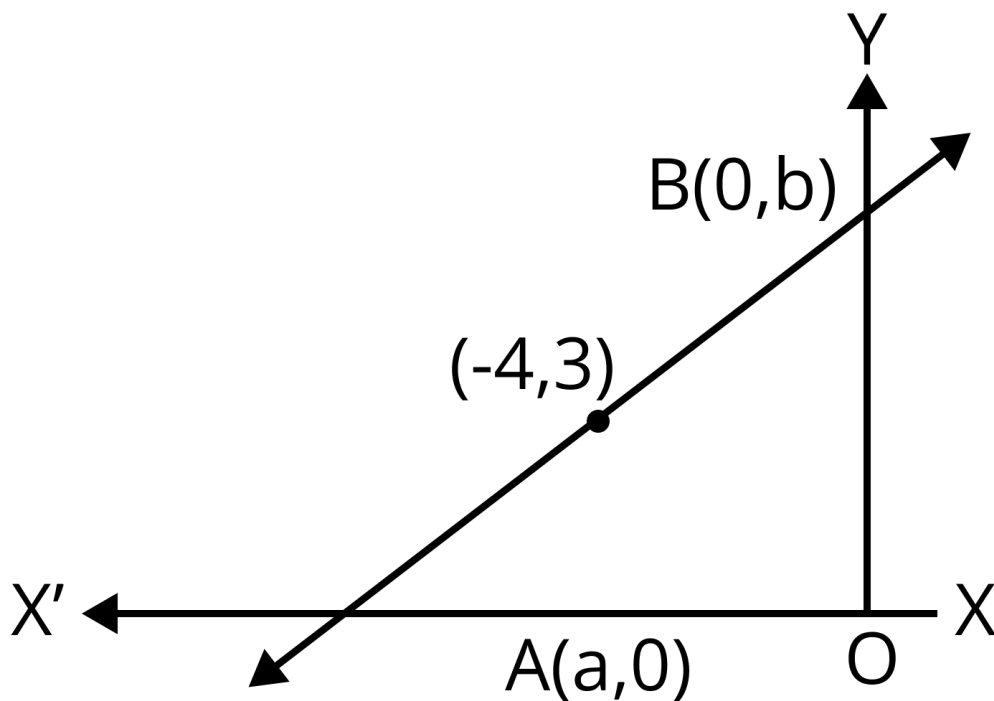
Therefore, the line passes through the fixed point (k, k) .

Question 17

Find the equation of the line which passes through the fixed point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in the ratio 5:3 by this point.

Solution

Ans: Let AB be a line passing through a point $(-4, 3)$ and meets x-axis at $A(a, 0)$ and y-axis at $B(0, b)$.



$$\Rightarrow \therefore -4 = \frac{5 \times 0 + 3a}{5+3}$$

$$\Rightarrow -4 = \frac{3a}{8} \left[\because X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right]$$

$$\Rightarrow 3a = -32$$

$$\Rightarrow a = \frac{-32}{3} \text{ and } 3 = \frac{5 \times b + 3 \times 0}{5+3}$$

$$\Rightarrow 3 = \frac{5 \times b}{8} \Rightarrow 5b = 24 \Rightarrow b = \frac{24}{5}$$

$$\text{Intercept form of the line is } \frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1$$

$$\Rightarrow \frac{-3x}{32} + \frac{5y}{24} = 1$$

$$\Rightarrow -9x + 20y = 96 \Rightarrow 9x - 20y + 96 = 0$$

Hence, The required equation is $9x - 20y + 96 = 0$

Question 18

Find the equation of the lines through the point of intersection of the lines $x - y + 1 = 0$ and

$$2x - 3y + 5 = 0$$

and whose distance from the point $(3, 2)$ is $\frac{7}{5}$.

Solution

Ans: Given equation are

$$\mathbf{x - y + 1 = 0 \dots (i)} \text{ And } \mathbf{2x - 3y + 5 = 0 \dots (ii)} \text{ Solving eq. (i) and eq. (ii)}$$

$$\text{we get } 2x - 2y + 2 = 0 \quad 2x - 3y + 5 = 0$$

$$y - 3 = 0, y = 3$$

From eq. (i) we have

$$\mathbf{x - 3 + 1 = 0 \Rightarrow x = 2}$$

So, $(2, 3)$ is the point of intersection of eq.(i) and eq. (ii).

Let m be the slope of the required line

$\therefore$ Equation of the line is

$$y - 3 = m(x - 2)$$

$$\Rightarrow y - 3 = mx - 2m$$

$$\Rightarrow mx - y + 3 - 2m = 0$$

Since, the perpendicular distance from $(3, 2)$

to the line is $\frac{7}{5}$ then,

$$\frac{7}{5} = \left| \frac{m(3) - 2 + 3 - 2m}{\sqrt{m^2 + 1}} \right|$$

$$\Rightarrow \frac{49}{25} = \frac{(3m - 2 + 3 - 2m)^2}{m^2 + 1}$$

$$\Rightarrow \frac{49}{25} = \frac{(m+1)^2}{m^2+1}$$

$$\Rightarrow 49m^2 + 49 = 25m^2 + 50m + 25$$

$$\Rightarrow 49m^2 - 25m^2 - 50m + 49 - 25 = 0$$

$$\Rightarrow 24m^2 - 50m + 24 = 0$$

$$\Rightarrow 12m^2 - 25m + 12 = 0$$

$$\Rightarrow 12m^2 - 16m - 9m + 12 = 0$$

$$\Rightarrow 4m(3m - 4) - 3(3m - 4) = 0$$

$$\Rightarrow (3m - 4)(4m - 3) = 0$$

$$\Rightarrow 3m - 4 = 0 \text{ and } 4m - 3 = 0$$

$$\Rightarrow 3m - 4 = 0 \text{ and } 4m - 3 = 0$$

$$\therefore m = \frac{4}{3}, \frac{3}{4}$$

Equation of the line taking $m = \frac{4}{3}$ is

$$y - 3 = \frac{4}{3}(x - 2)$$

$$\Rightarrow 3y - 9 = 4x - 8 \Rightarrow 4x - 3y + 1 = 0$$

Equation of the line taking $m = \frac{3}{4}$ is

$$y - 3 = \frac{3}{4}(x - 2)$$

$$\Rightarrow 4y - 12 = 3x - 6 \Rightarrow 3x - 4y + 6 = 0$$

Hence, the required equation are

$$4x - 3y + 1 = 0 \text{ and } 3x - 4y + 6 = 0$$

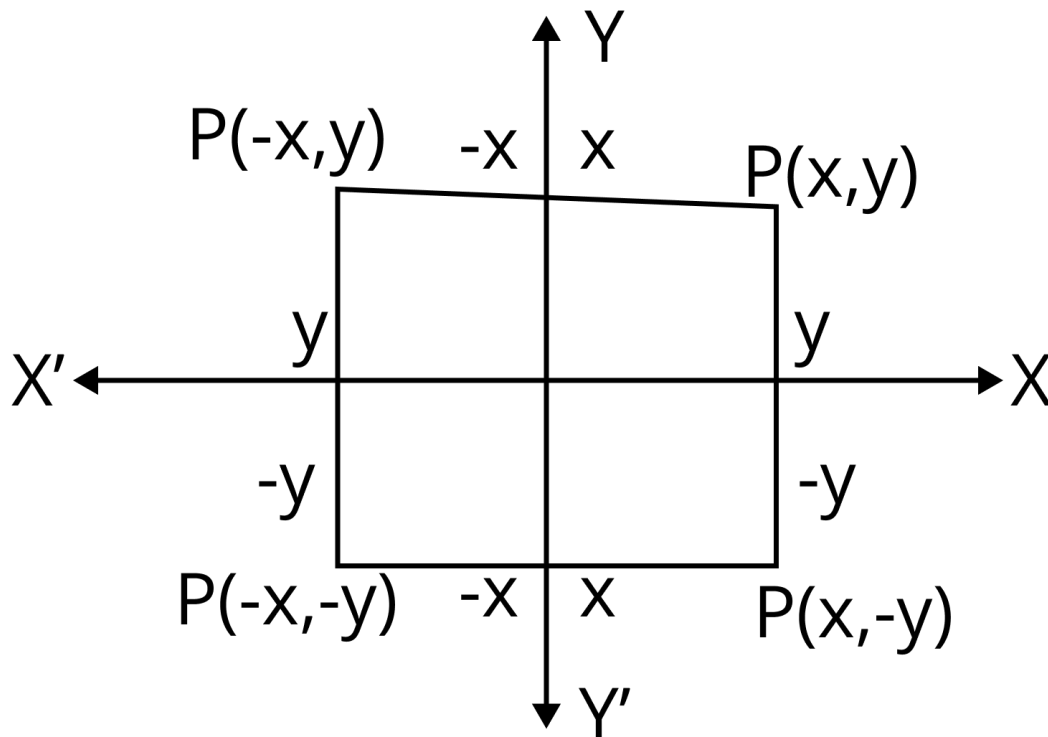
Question 19

If the sum of the distance of a moving point in a plane from the axes is 1, then find the locus of the point.

Solution

Ans: Let coordinates of a moving point P be (x, y) .

Given that the sum of the distance from the axes to the point is always 1



$$\therefore |x| + |y| = 1$$

$$\Rightarrow x + y = 1$$

$$\Rightarrow -x - y = 1$$

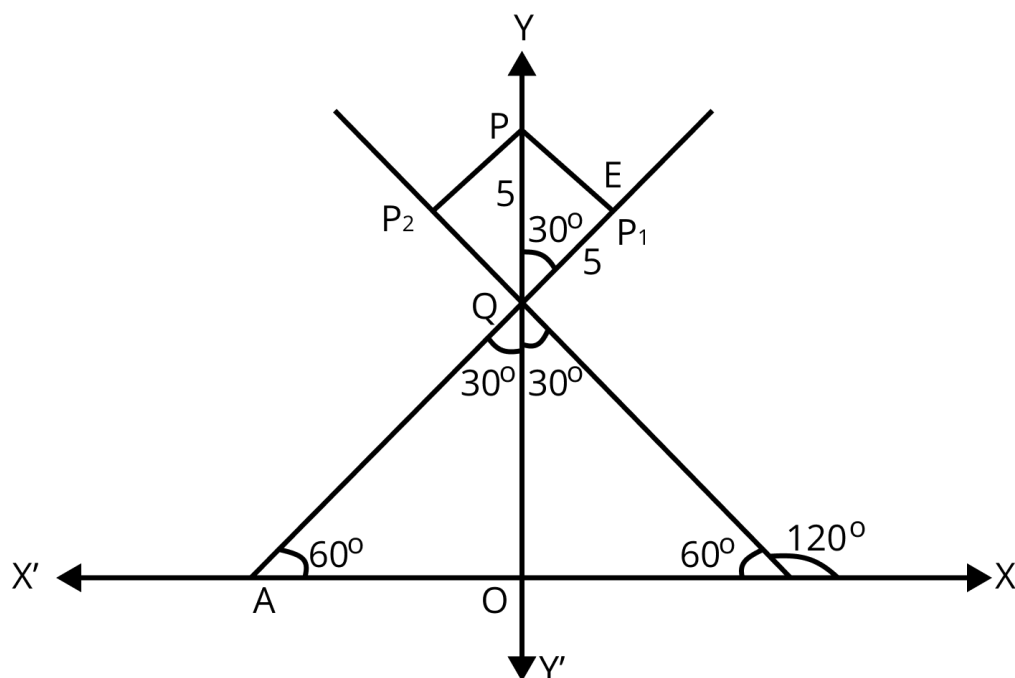
$$\Rightarrow -x + y = 1$$

$$\Rightarrow x - y = 1$$

Hence, these equations gives us the locus of the point P which is a square.

Question 20

P₁, P₂ are points on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P₁, P₂ on the bisector of the angle between the given lines.



Solution

Ans: Given lines are $y - \sqrt{3}|x| = 2$

$$\Rightarrow y - \sqrt{3}x = 2 \quad \text{if } x \geq 0 \dots(i)$$

$$\text{And } y + \sqrt{3}x = 2 \quad \text{if } x < 0 \dots(ii)$$

Slope of eq. (i) is $\tan \theta = \sqrt{3} \therefore \theta = 60^\circ$

Slope of eq. (ii) is $\tan \theta = -\sqrt{3} \therefore \theta = 120^\circ$

120° Solving eq. (i) and eq. (ii) we get

$$y - \sqrt{3}x = 2$$

$$y + \sqrt{3}x = 2$$

$$2y = 4 \Rightarrow y = 2$$

Putting the value of y in eq. (i) we get

$$x = 0$$

Point of intersection of line (i) and (ii) is $Q(0, 2)$

$$\therefore QO = 2$$

In $\triangle PEQ$,

$$\cos 30^\circ = \frac{PQ}{QE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{PQ}{5}$$

$$\Rightarrow \therefore PQ = \frac{5\sqrt{3}}{2}$$

$$\therefore OP = OQ + PQ$$

$$= 2 + \frac{5\sqrt{3}}{2}$$

Hence, the coordinates of the foot of perpendicular = $(0, 2 + \frac{5\sqrt{3}}{2})$

Question 21

If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P. then show that $a^4 + b^4 = 0$.

Solution

Ans: Given equation is $\frac{x}{a} + \frac{y}{b} = 1$

Since, p is the length of perpendicular drawn from the origin to the given line

$$\therefore p = \left| \frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right|$$

Squaring both sides, we have

$$p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} \dots\dots (i) \text{ Since } a^2, p^2, b^2 \text{ are in A.P. } \therefore 2p^2 = a^2 + b^2$$

$$\Rightarrow p^2 = \frac{a^2 + b^2}{2} \Rightarrow \frac{1}{p^2} = \frac{2}{a^2 + b^2}$$

Putting the value of $\frac{1}{p^2}$ is eq. (i) we get,

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{a^2 + b^2}$$

$$\Rightarrow \frac{a^2 + b^2}{a^2 b^2} = \frac{2}{a^2 + b^2}$$

$$\Rightarrow (a^2 + b^2)^2 = 2a^2 b^2$$

$$\Rightarrow a^4 + b^4 + 2a^2 b^2 = 2a^2 b^2$$

$$\Rightarrow a^4 + b^4 = 0$$

Hence proved.

Question 22

A line cutting off intercept -3 from the y -axis and the tangent at angle to the x -axis is $\frac{3}{5}$, its equation is

(A) $5y - 3x + 15 = 0$

(B) $3y - 5x + 15 = 0$

(C) $5y - 3x - 15 = 0$

(D) None of these

Solution

Ans:

Given: A line cuts off an intercept -3 from the y -axis.

Let the equation of required line be $y = mx + c$.

As $c = -3$ and $m = \frac{3}{5}$,

The equation of line is: $y = \frac{3}{5}x - 3$

$$\Rightarrow 5y - 3x + 15 = 0$$

Hence, the equation is $5y - 3x + 15 = 0$.

Correct Option : A

Question 23

Slope of a line which cut off intercepts of equal lengths on the axes is

(A) -1

(B) 0

(C) 2

(D) $\sqrt{3}$

Solution

Ans:

Given: A line cuts off intercepts of equal lengths on the axes.

The equation of required line is $\frac{x}{a} + \frac{y}{a} = 1$.

$$\Rightarrow x + y = a$$

$$\Rightarrow y = -x + a$$

$$\text{Slope} = -1$$

Hence, the slope is -1 .

Correct Option: A

Question 24

The equation of the straight line passing through the point $(3, 2)$ and perpendicular to the line $y = x$ is

(A) $x - y = 5$

(B) $x + y = 5$

(C) $x + y = 1$

(D) $x - y = 1$

Solution

Ans:

Given: $(3, 2)$ and $y = x$.

Slope of $y = x$ is 1 .

So, the slope of line perpendicular to it is -1 .

The line passes through $(3, 2)$.

So, equation of required line is $y - 2 = -1(x - 3)$ i.e. $x + y = 5$.

Hence, equation is $x + y = 5$.

Correct Option: B

Question 25

The equation of the line passing through the point $(1, 2)$ and perpendicular to the line $x + y + 1 = 0$ is

(A) $y - x + 1 = 0$

(B) $y - x - 1 = 0$

(C) $y - x + 2 = 0$

(D) $y - x - 2 = 0$

Solution

Ans:

Given: $(1, 2)$ and $x + y + 1 = 0$.

Slope of $x + y + 1 = 0$ is -1 .

So, the slope of line perpendicular to it is 1 .

The line passes through $(1, 2)$.

So, equation of required line is $y - 2 = 1(x - 1)$ i.e. $y - x - 1 = 0$.

Hence, equation is $y - x - 1 = 0$.

Correct Option: B

Question 26

The tangent of angle between the lines whose intercepts on the axes are $a, -b$ and $b, -a$, respectively, is

(A) $\frac{a^2 - b^2}{ab}$

(B) $\frac{b^2 - a^2}{2}$

(C) $\frac{b^2 - a^2}{2ab}$

(D) None of these

Solution

Ans:

Given: The intercepts on the axes are $a, -b$ and $b, -a$.

The intercepts of line are a and $-b$.

So, the line passes through the points $(a, 0)$ and $(0, -b)$.

Thus, the slope of line is $m_1 = \frac{-b-0}{0-a} = \frac{b}{a}$.

The intercepts of line are b and $-a$.

So, the line passes through the points $(b, 0)$ and $(0, -a)$.

Thus, the slope of line is $m_2 = \frac{-a-0}{0-b} = \frac{a}{b}$.

Let θ be the angle between the lines.

$$\Rightarrow \tan \theta = \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{a}{b} \cdot \frac{b}{a}}$$

$$\Rightarrow \tan \theta = \frac{\frac{b^2 - a^2}{ab}}{2}$$

$$\Rightarrow \tan \theta = \frac{b^2 - a^2}{2ab}$$

$$\text{Hence, } \tan \theta = \frac{b^2 - a^2}{2ab} .$$

Correct Option: C

Question 27

If the line $\frac{x}{a} + \frac{y}{b}$ passes through the points $(2, -3)$ and $(4, -5)$, then (a, b) is

- (A) $(1, 1)$
- (B) $(-1, 1)$
- (C) $(1, -1)$
- (D) $(-1, -1)$

Solution

Ans:

Given: $(2, -3)$ and $(4, -5)$.

The points $(2, -3)$ and $(4, -5)$ lie on $\frac{x}{a} + \frac{y}{b} = 1$.

$$\Rightarrow \frac{2}{a} - \frac{3}{b} = 1 \dots (1)$$

$$\text{And } \frac{4}{a} - \frac{5}{b} = 1 \dots (2)$$

Multiply (1) by 2 and subtract (2) from it.

$$\Rightarrow -\frac{6}{b} + \frac{5}{b} = 1$$

$$\Rightarrow \frac{-1}{b} = 1$$

$$\Rightarrow b = -1$$

Put $b = -1$ in (1) ,

$$\Rightarrow \frac{2}{a} + 3 = 1$$

$$\Rightarrow a = -1$$

So, $(a, b) = (-1, -1)$

Hence, $(a, b) = (-1, -1)$.

Correct Option: D

Question 28

The distance of the point of intersection of the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$ from the line $5x - 2y = 0$ is

(A) $\frac{130}{17\sqrt{29}}$

(B) $\frac{13}{7\sqrt{29}}$

(C) $\frac{130}{7}$

(D) None of these

Solution

Ans:

Given: $2x - 3y + 5 = 0$ and $3x + 4y = 0$.

Solve the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$.

The point of intersection is $\left(\frac{-20}{17}, \frac{15}{17}\right)$.

The distance of $\left(\frac{-20}{17}, \frac{15}{17}\right)$ from $5x - 2y = 0$ is $\frac{\left|5 \times \left(\frac{-20}{17}\right) - 2 \times \left(\frac{15}{17}\right)\right|}{\sqrt{25+4}}$

The distance of $\left(\frac{-20}{17}, \frac{15}{17}\right)$ from $5x - 2y = 0$ is $\frac{\left|\frac{-100}{17} - \frac{30}{17}\right|}{\sqrt{29}}$

The distance of $\left(\frac{-20}{17}, \frac{15}{17}\right)$ from $5x - 2y = 0$ is $\frac{130}{17\sqrt{29}}$

Hence, the distance is $\frac{130}{17\sqrt{29}}$.

Correct Option: A

Question 29

The equations of the lines which pass through the point $(3, -2)$ and are inclined at 60° to the line $\sqrt{3}x + y = 1$ is

(A) $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(B) $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(D) None of these

Solution

Ans:

Given: $(3, -2)$ and $\sqrt{3}x + y = 1$.

The slope of $\sqrt{3}x + y = 1$ is $m_1 = -\sqrt{3}$.

Let the slope of required line which makes 60° with $\sqrt{3}x + y = 1$ be m .

$$\text{Then, } \tan 60^\circ = \left| \frac{-\sqrt{3}-m}{1-\sqrt{3}m} \right|$$

$$\Rightarrow \left| \frac{-\sqrt{3}-m}{1-\sqrt{3}m} \right| = \sqrt{3}$$

$$\Rightarrow -\sqrt{3} - m = \sqrt{3} - 3m \text{ or}$$

$$-\sqrt{3} - m = -\sqrt{3} + 3m$$

$$\Rightarrow m = \sqrt{3} \text{ or } m = 0$$

Line passes through $(3, -2)$.

So, the equation of line is: $y + 2 = \sqrt{3}(x - 3)$ or $y + 2 = 0$

$$\Rightarrow \sqrt{3}x - y - 2 - 3\sqrt{3} = 0 \text{ and } y + 2 = 0$$

Hence, the equations are $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$.

Correct Option: A

Question 30

The equations of the lines passing through the point $(1, 0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin, are

(A) $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$

(B) $\sqrt{3}x + y + \sqrt{3} = 0, \sqrt{3}x - y + \sqrt{3} = 0$

(C) $x + \sqrt{3}y - 3 = 0, x - \sqrt{3}y - \sqrt{3} = 0$

(D) None of these.

Solution

Ans:

Given: Line passes through $(1, 0)$.

Let the slope of the required line be m .

The equation of line passing through $(1, 0)$ is: $y - 0 = m(x - 1)$ i.e. $y - mx + m = 0$.

The distance of this line from $(0, 0)$ is $\frac{\sqrt{3}}{2}$.

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{|0-0+m|}{\sqrt{1+m^2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{|m|}{\sqrt{1+m^2}}$$

$$\Rightarrow 3 + 3m^2 = 4m^2$$

$$\Rightarrow m^2 = 3$$

$$\Rightarrow m = \pm\sqrt{3}$$

Hence, equations of lines are: $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$.

Correct Option: A

Question 31

The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is

(A) $\frac{c_1 - c_2}{\sqrt{m^2 + 1}}$

(B) $\frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$

(C) $\frac{c_2 - c_1}{\sqrt{1 + m^2}}$

(D) 0

Solution

Ans:

Given: $y = mx + c_1$ and $y = mx + c_2$.

Let $P(x_1, y_1)$ be a point on $y = mx + c_1$.

The other line is: $y = mx + c_2$ i.e. $mx - y + c_2 = 0$.

Distance of $P(x_1, y_1)$ from this line is $d = \frac{|mx_1 - y_1 + c_2|}{\sqrt{m^2 + 1}}$.

$P(x_1, y_1)$ lies on $y = mx + c_1$,

$$\Rightarrow y_1 = mx_1 + c_1$$

$$\Rightarrow mx_1 - y_1 = -c_1$$

$$\Rightarrow d = \frac{|c_1 - c_2|}{\sqrt{m^2 + 1}}$$

Hence, distance is $\frac{|c_1 - c_2|}{\sqrt{m^2 + 1}}$.

Correct Option: B

Question 32

The coordinates of the foot of perpendicular from the point $(2, 3)$ on the line $y = 3x + 4$ is given by

(A) $\left(\frac{37}{10}, \frac{-1}{10}\right)$

(B) $\left(-\frac{1}{10}, \frac{37}{10}\right)$

(C) $\left(\frac{10}{37}, -10\right)$

(D) $\left(\frac{2}{3}, -\frac{1}{3}\right)$

Solution

Ans:

Given: $y = 3x + 4$ and $(2, 3)$.

Let the foot of perpendicular from $P(2, 3)$ on $3x - y + 4 = 0$ be $M(h, k)$.

$M(h, k)$ lies on $y = 3x + 4$.

$$\Rightarrow 3h - k + 4 = 0 \dots (1)$$

The slope of the given line is 3.

Slope of the line joining $P(2, 3)$ and $M(h, k)$ is $-\frac{1}{3} = \frac{k-3}{h-2}$ or $h + 3k - 11 = 0 \dots (2)$

Solve (1) and (2),

$$\Rightarrow (h, k) = \left(-\frac{1}{10}, \frac{37}{10}\right)$$

Hence, the coordinates of foot of perpendicular is $\left(-\frac{1}{10}, \frac{37}{10}\right)$.

Correct Option: B

Question 33

If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is $(3, 2)$, then the equation of the line will be

(A) $2x + 3y = 12$

(B) $3x + 2y = 12$

(C) $4x - 3y = 6$

(D) $5x - 2y = 10$

Solution

Ans:

Given: $(3, 2)$

As the middle point is $P(3, 2)$, the line meets the axes at $A(6, 0)$ and $B(0, 4)$.

So, the equation of the line using intercept form is: $\frac{x}{6} + \frac{y}{4} = 1$ i.e. $2x + 3y = 12$.

Hence, the equation is $2x + 3y = 12$.

Correct Option: A

Question 34

Equation of the line passing through $(1, 2)$ and parallel to the line $y = 3x - 1$ is

(A) $y + 2 = x + 1$

(B) $y + 2 = 3(x + 1)$

(C) $y - 2 = 3(x - 1)$

(D) $y - 2 = x - 1$

Solution

Ans:

Given: $(1, 2)$ and $y = 3x - 1$.

The required line is parallel to $y = 3x - 1$.

Thus, slope of the line is 3.

The line also passes through $(1, 2)$.

So, the equation of line is: $y - 2 = 3(x - 1)$

Hence, the equation is $y - 2 = 3(x - 1)$.

Correct Option: C

Question 35

Equations of diagonals of the square formed by the lines $x = 0, y = 0, x = 1$ and $y = 1$ are

(A) $y = x, y + x = 1$

(B) $y = x, x + y = 2$

(C) $2y = x, y + x = \frac{1}{3}$

(D) $y = 2x, y + 2x = 1$

Solution

Ans:

Given: $x = 0, y = 0, x = 1$ and $y = 1$.

Using the equations $x = 0, y = 0, x = 1$ and $y = 1$, the vertices of square are $A(0, 0), B(1, 0), C(1, 1), D(0, 1)$.

The equation of AC is: $y - 0 = \frac{1-0}{1-0}(x - 0)$ i.e. $x - y = 0$.

The equation of BD is: $y - 0 = \frac{1-0}{0-1}(x - 1)$ i.e. $x + y - 1 = 0$.

Hence, the diagonals are $y = x, y + x = 1$.

Correct Option: A

Question 36

For specifying a straight line, many geometrical parameters should be known?

(A) 1

(B) 2

(C) 4

(D) 3

Solution

Ans:

Given: A straight line is specified.

The general equation of a linear equation in two variables is $ax + by + c = 0$

At least one of a and b should be non-zero.

Assume $a \neq 0$.

Then, the equation of the line is: $x + \frac{b}{a}y + \frac{c}{a} = 0$

Hence, two parameters should be known.

Correct Option: B

Question 37

The point $(4, 1)$ undergoes the following two successive transformations:

(i) Reflection about the line $y = x$

(ii) Translation through a distance 2 units along the positive x -axis. Then the final coordinates of the point are

(A) $(4, 3)$

(B) $(3, 4)$

(C) $(1, 4)$

(D) $\left(\frac{7}{2}, \frac{7}{2}\right)$

Solution

Ans:

Given: $(4, 1)$ undergoes two successive transformations.

The reflection of $A(4, 1)$ in $y = x$ is $B(1, 4)$.

Then, translation of B through a distance 2 units along the positive x -axis shifts B to $C(1 + 2, 4)$ i.e. $C(3, 4)$.

Hence, the final coordinates of point is $(3, 4)$.

Correct Option: B

Question 38

A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is

- (A) $(1, -1)$
(B) $(1, 1)$
(C) $(0, 0)$
(D) $(0, 1)$

Solution

Ans: Given: $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$.

The distance of $(0, 0)$ from $4x + 3y + 10 = 0$ is $\frac{|0+0+10|}{\sqrt{4^2+3^2}} = 2$.

The distance of $(0, 0)$ from $5x - 12y + 26 = 0$ is $\frac{|0+0+26|}{\sqrt{5^2+12^2}} = 2$.

The distance of $(0, 0)$ from $7x + 24y - 50 = 0$ is $\frac{|0+0-50|}{\sqrt{7^2+24^2}} = 2$.

The distance of all the three lines from $(0, 0)$ is 2 units.

Hence, $(0, 0)$ is the required point.

Correct Option: C

Question 39

A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y -intercept is

- (A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{4}{3}$

Solution

Ans:

Given: Line passes through $(2, 2)$.

The slope of $3x + y = 3$ is -3 .

So, slope of a line perpendicular to it = $\frac{1}{3}$.

The equation of the required line is: $y - 2 = \frac{1}{3}(x - 2)$ i.e. $x - 3y + 4 = 0$.

Put $x = 0$ to find y - intercept.

$$\Rightarrow 0 - 3y + 4 = 0$$

$$\Rightarrow y = \frac{4}{3} \text{ which is the } y\text{- intercept.}$$

Hence, the y - intercept is $\frac{4}{3}$.

Correct Option: D

Question 40

The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ is

(A) 1 : 2

(B) 3 : 7

(C) 2 : 3

(D) 2 : 5

Solution

Ans: Given: $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$.

Distance between $3x + 4y + 5 = 0$ and $3x + 4y + 2 = 0$ is $\frac{|5-2|}{\sqrt{9+16}} = \frac{3}{5}$

Distance between $3x + 4y - 5 = 0$ and $3x + 4y + 2 = 0$ is $\frac{|-5-2|}{\sqrt{9+16}} = \frac{7}{5}$

Hence, the ratio is $\frac{3}{5} : \frac{7}{5}$ i.e. 3 : 7.

Correct Option: B

Question 41

One vertex of the equilateral triangle with centroid at the origin and one side as $x + y - 2 = 0$ is

(A) $(-1, -1)$

(B) $(2, 2)$

(C) $(-2, -2)$

(D) $(2, -2)$

Solution

Ans: Given: $x + y - 2 = 0$

Let ABC be the equilateral triangle with vertex $A(h, k)$.

The centroid is $G(0, 0)$.

Slope of line BC i.e. $x + y - 2 = 0$ is -1 .

As $AG \perp BC$,

The slope of AG is $\frac{k}{h} = 1$ or $h = k$.

The distance of $(0, 0)$ from $BC = \frac{|0+0-2|}{\sqrt{1^2+1^2}} = \sqrt{2}$.

Distance of A from $BC = 3\sqrt{2} = \frac{|h+k-2|}{\sqrt{1^2+1^2}}$

$$\Rightarrow |h + k - 2| = 6$$

$$\Rightarrow h + k - 8 = 0 \text{ or } h + k + 4 = 0$$

$$\Rightarrow h + h - 8 = 0 \text{ or } h + h + 4 = 0$$

$$\Rightarrow h = 4 \text{ or } h = -2$$

Hence, the vertex is $(-2, -2)$

Correct Option: C

Question 42

If a, b, c are in A.P., then the straight line $ax + by + c = 0$ will always pass through.

Solution

Ans: Given: a, b, c are in A.P.

a, b, c are in A.P.

So, $b = \frac{a+c}{2}$ i.e. $a - 2b + c = 0$

Compare $a - 2b + c = 0$ and line $ax + by + c = 0$,

$$\Rightarrow x = 1, y = -2$$

Hence, the point $(1, -2)$ lies on the line.

Question 43

The line which cuts off equal intercept from the axes and pass through the point $(1, -2)$ is

Solution

Ans: Given: $(1, -2)$

The line which cuts equal intercepts from the axes is $\frac{x}{a} + \frac{y}{a} = 1$.

As this line passes through $(1, -2)$,

$$\Rightarrow 1 - 2 = a \text{ or } a = -1.$$

Thus, the required equation of line is: $x + y = -1$

$$\Rightarrow x + y + 1 = 0$$

Hence, the equation is $x + y + 1 = 0$.

Question 44

Equations of the lines through the point $(3, 2)$ and making an angle of 45° with the line $x - 2y = 3$ are

Solution

Ans: Given: Line passes through $(3, 2)$.

Slope of $x - 2y = 3$ is $\frac{1}{2}$.

Let the slope of the required line be m .

$$\Rightarrow \tan 45^\circ = \left| \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} \right|$$

$$\Rightarrow 1 = \pm \frac{2m-1}{2+m}$$

$$\Rightarrow 2m - 1 = 2 + m \text{ or } 1 - 2m = 2 + m$$

$$\Rightarrow m = 3 \text{ or } m = -\frac{1}{3}$$

The line also passes through $(3, 2)$.

The equation of the line is: $y - 2 = 3(x - 3)$ or $y - 2 = -\frac{1}{3}(x - 3)$

Hence, equations of lines are: $3x - y - 7 = 0$, $x + 3y - 9 = 0$.

Question 45

The points $(3, 4)$ and $(2, -6)$ are situated on the _____ of the line $3x - 4y - 8 = 0$.

Solution

Ans: Given: $(3, 4)$ and $(2, -6)$.

Given line is $3x - 4y - 8 = 0$.

For the point $(3, 4)$: $3(3) - 4(4) - 8 = -15 < 0$

For the point $(2, -6)$: $3(2) - 4(-6) - 8 = 22 > 0$

Hence, $(3, 4)$ and $(2, -6)$ lie on opposite side of $3x - 4y - 8 = 0$.

Question 46

A point moves so that square of its distance from the point $(3, -2)$ is numerically equal to its distance from the line $5x - 12y = 3$. The equation of its locus is

Solution

Ans: Given: $(3, -2)$.

Let the moving point be $P(h, k)$.

Given point is $A(3, -2)$.

$$\Rightarrow AP^2 = (h - 3)^2 + (k + 2)^2 = d_1^2$$

The distance of (h, k) from $5x - 12y - 3 = 0$ is:

$$\Rightarrow d_2 = \left| \frac{5h - 12k - 3}{\sqrt{25 + 144}} \right|$$

$$\Rightarrow d_2 = \left| \frac{5h - 12k - 3}{13} \right|$$

Given that, $d_1^2 = d_2^2$

$$\Rightarrow (h - 3)^2 + (k + 2)^2 = \frac{5h - 12k - 3}{13}$$

$$\Rightarrow h^2 - 6h + 9 + k^2 + 4k + 4 = \frac{5h - 12k - 3}{13}$$

$$\Rightarrow 13h^2 + 13k^2 - 78h + 52k + 169 = 5h - 12k - 3$$

$$\Rightarrow 13h^2 + 13k^2 - 83h + 64k + 172 = 0$$

Hence, locus is: $13x^2 + 13y^2 - 83x + 64y + 172 = 0$

Question 47

Locus of the mid-points of the portion of the line $x \sin \theta + y \cos \theta = p$ intercepted between the axes is

Solution

Ans: Given: $x \sin \theta + y \cos \theta = p$.

$x \sin \theta + y \cos \theta = p$ meets axes at points $A \left(\frac{p}{\sin \theta}, 0 \right)$ and $B \left(0, \frac{p}{\cos \theta} \right)$.

Let $P(h, k)$ be the mid-point of AB .

$$\Rightarrow h = \frac{p}{2 \sin \theta} \text{ and } k = \frac{p}{2 \cos \theta}$$

$$\Rightarrow \sin \theta = \frac{p}{2h} \text{ and } \cos \theta = \frac{p}{2k}$$

By squaring and adding,

$$\Rightarrow \sin^2 \theta + \cos^2 \theta = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow 1 = \frac{p^2}{4x^2} + \frac{p^2}{4y^2}$$

$$\Rightarrow 4x^2y^2 = p^2 (x^2 + y^2)$$

Hence, the locus is $4x^2y^2 = p^2 (x^2 + y^2)$.

State whether the Statements in Exercises 48 to 56 True or False? Justify.

Question 48

If the vertices of a triangle have integral coordinates, then the triangle cannot be equilateral.

Solution

Ans: The given statement is true.

Let us assume an equilateral triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. All the angles of an equilateral triangle are 60° . Assuming the slope of line AB as m_1 and the slope of line BC as m_2 .

$$\Rightarrow \tan 60^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \sqrt{3} = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \dots\dots (1)$$

Here $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$ and $m_2 = \frac{y_3 - y_2}{x_3 - x_2}$. Since all the coordinates are integers so m_1 and m_2 are rational numbers.

Therefore, in expression (1), the R.H.S is a rational number and the L.H.S is an irrational number. This results in rational number = irrational numbers which are not possible.

Question 49

The points $A(-2, 1)$, $B(0, 5)$ and $C(-1, 2)$ are collinear.

Solution

Ans: The given statement is false.

Using the distance formula, the length AB

$$\begin{aligned} &= \sqrt{(0 - (-2))^2 + (5 - 1)^2} \\ &= \sqrt{4 + 16} \\ &= 2\sqrt{5} \text{ units} \end{aligned}$$

Similarly, the length BC

$$\begin{aligned} &= \sqrt{(-1 - 0)^2 + (2 - 5)^2} \\ &= \sqrt{1 + 9} \\ &= \sqrt{10} \text{ units} \end{aligned}$$

And, the length AC

$$\begin{aligned} &= \sqrt{(-2 - (-1))^2 + (1 - 2)^2} \\ &= \sqrt{1 + 1} \\ &= \sqrt{2} \text{ units} \end{aligned}$$

Clearly the sum of any two line segments is not equal to the third one in any case, so the points are not collinear.

Question 50

Equation of the line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta + y\csc\theta = a$ is $x\cos\theta - y\sin\theta = a\sin 2\theta$.

Solution

Ans: The given statement is false.

The slope of the given line $x\sec\theta + y\csc\theta = a$ is,

$$\Rightarrow m_1 = -\frac{\sec\theta}{\csc\theta}$$

$$\Rightarrow m_1 = - \frac{\left(\frac{1}{\cos \theta}\right)}{\left(\frac{1}{\sin \theta}\right)}$$

$$\Rightarrow m_1 = - \left(\frac{\sin \theta}{\cos \theta}\right)$$

Therefore, the slope of the required line which is perpendicular to the given line is,

$$\Rightarrow m_2 = \frac{-1}{m_1}$$

$$\Rightarrow m_2 = \frac{-1}{-\left(\frac{\sin \theta}{\cos \theta}\right)}$$

$$\Rightarrow m_2 = \frac{\cos \theta}{\sin \theta}$$

So the equation of the line passing through the point $(a \cos^3 \theta, a \sin^3 \theta)$ and having slope m_2 is,

$$\Rightarrow (y - a \sin^3 \theta) = m_2 (x - a \cos^3 \theta)$$

$$\Rightarrow (y - a \sin^3 \theta) = \frac{\cos \theta}{\sin \theta} (x - a \cos^3 \theta)$$

$$\Rightarrow \sin \theta (y - a \sin^3 \theta) = \cos \theta (x - a \cos^3 \theta)$$

$$\Rightarrow x \cos \theta - y \sin \theta = a \cos^4 \theta - a \sin^4 \theta$$

Using the algebraic identity $m^2 - n^2 = (m + n)(m - n)$ and the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$,

$$\Rightarrow x \cos \theta - y \sin \theta = a (\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow x \cos \theta - y \sin \theta = a (\cos^2 \theta - \sin^2 \theta)$$

Using the identity $\cos^2 \theta - \sin^2 \theta = \cos 2\theta$,

$$\Rightarrow x \cos \theta - y \sin \theta = a \cos 2\theta$$

Question 51

The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

Solution

Ans: The given statement is true.

The intersection of the lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is their solution coordinates. Solving the two equations algebraically gives,

$$x = \left(-\frac{20}{3}\right) \text{ and } y = \left(\frac{25}{3}\right), \text{ so the solution coordinate is } \left(-\frac{20}{3}, \frac{25}{3}\right).$$

Now, if the line $5x + 4y = 0$ passes through the above coordinates then it must satisfy the point. On substitution,

$$\Rightarrow 5 \times \left(-\frac{20}{3}\right) + 4 \times \left(\frac{25}{3}\right) = 0$$

$$\Rightarrow \left(-\frac{100}{3}\right) + \left(\frac{100}{3}\right) = 0$$

$$\Rightarrow 0 = 0$$

Hence, the point satisfies the line.

Question 52

The vertex of an equilateral triangle is $(2, 3)$ and the equation of the opposite side is $x + y = 2$.

Then the other two sides are $y - 3 = (2 \pm \sqrt{3})(x - 2)$.

Solution

Ans: The given statement is true.

Let us assume the slope of one of the remaining two sides as m . The slope of the given line $x + y = 2$ is -1 . If m_1 and m_2 are slopes of two line segments then the angle (θ) between them is given as $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$.

The angle between any two sides of an equilateral triangle is 60° ,

$$\Rightarrow \tan 60^\circ = \left| \frac{-1 - m}{1 + (-1) \times m} \right|$$

$$\Rightarrow \sqrt{3} = \left| \frac{-(1+m)}{1-m} \right|$$

$$\Rightarrow \sqrt{3} = \left| \frac{1+m}{1-m} \right|$$

Removing the modulus sign,

$$\Rightarrow \pm \sqrt{3} = \left(\frac{1+m}{1-m} \right)$$

1. Considering the positive sign,

$$\Rightarrow \sqrt{3} = \left(\frac{1+m}{1-m} \right)$$

$$\Rightarrow m = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right)$$

$$\Rightarrow m = (2 - \sqrt{3})$$

(By rationalization)

1. Considering the negative sign,

$$\Rightarrow -(\sqrt{3}) = \left(\frac{1+m}{1-m} \right)$$

$$\Rightarrow m = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right)$$

$$\Rightarrow m = (2 + \sqrt{3})$$

(By rationalization)

Hence, there are two values of m for the two remaining sides of the triangle. Both the sides will pass through the point $(2, 3)$. So, equation of the other two sides are $(y - 3) = (2 \pm \sqrt{3})(x - 2)$.

Question 53

The equation of the line joining the point $(3, 5)$ to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ is equidistant from the points $(0, 0)$ and $(8, 34)$.

Solution

Ans: The given statement is true.

Solving the equations $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ algebraically gives $x = 2$ and $y = -7$. So the point of intersection is $(2, -7)$.

The equation of the line passing through the points

$$(3, 5)$$

and $(2, -7)$ is,

$$\Rightarrow (y - 5) = \left(\frac{-7-5}{2-3} \right) (x - 3)$$

$$\Rightarrow (y - 5) = (12) (x - 3)$$

$$\Rightarrow 12x - y - 31 = 0$$

Now, the distance of the point $(0, 0)$ from the line $12x - y - 31 = 0$,

$$= \left| \frac{(0 \times 12) + (0 \times (-1)) - 31}{\sqrt{12^2 + (-1)^2}} \right|$$

$$= \frac{31}{\sqrt{145}}$$

Similarly, the distance of the point $(8, 34)$ from the line $12x - y - 31 = 0$,

$$= \left| \frac{(8 \times 12) + (34 \times (-1)) - 31}{\sqrt{12^2 + (-1)^2}} \right|$$

$$= \frac{31}{\sqrt{145}}$$

Hence, the points are equidistant from the required line.

Question 54

The lines $\frac{x}{a} + \frac{y}{b} = 1$ moves in such a ways that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$, where c is a constant. The locus of the foot of the perpendicular from the origin on the given line is $x^2 + y^2 = c^2$.

Solution

Ans: The given statement is true.

Let the coordinates of the foot of perpendicular drawn from the origin $(0, 0)$ is (x, y) .

The given line is $\frac{x}{a} + \frac{y}{b} = 1$ which can be written as $\frac{x}{a} + \frac{y}{b} - 1 = 0$.

The foot of perpendicular is given as,

$$\Rightarrow \frac{(x-0)}{\left(\frac{1}{a}\right)} = \frac{(y-0)}{\left(\frac{1}{b}\right)} = \frac{\left(\frac{1}{a} \times 0\right) + \left(\frac{1}{b} \times 0\right) - 1}{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\Rightarrow ax = by = \frac{-1}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)}$$

Using the given relation $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$,

$$\Rightarrow ax = by = \frac{-1}{\left(\frac{1}{c^2}\right)}$$

$$\Rightarrow ax = by = (-c^2)$$

Equating the relations $ax = (-c^2)$ and $by = (-c^2)$ gives

$$x = \left(\frac{-c^2}{a}\right)$$

and $y = \left(\frac{-c^2}{b}\right)$, respectively.

On squaring and adding,

$$\Rightarrow x^2 + y^2 = \left(\frac{c^4}{a^2}\right) + \left(\frac{c^4}{b^2}\right)$$

$$\Rightarrow x^2 + y^2 = c^4 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

$$\Rightarrow x^2 + y^2 = c^4 \left(\frac{1}{c^2}\right)$$

$$\Rightarrow x^2 + y^2 = c^2$$

Question 55

The lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent if a , b and c are in G.P.

Solution

Ans: The given statement is false.

The given lines can be written as $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$. For these to be concurrent it must satisfy the determinant relation,

$$\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$$

$$a(3 - 4c) + 2(c - b) + (4b - 3c) = 0$$

$$3a - 4ac + 2c - 2b + 4b - 3c = 0$$

$$c \quad \& \quad 4 \quad \& \quad 1$$

$$\end{array}\right|=0\$$$

Performing the row operations $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$,

$$\begin{array}{ccc} \end{array}\leftarrow\begin{array}{ccc}$$

$$a \quad \& \quad 2 \quad \& \quad 1 \quad \backslash \backslash$$

$$b-a \quad \& \quad 1 \quad \& \quad 0 \quad \backslash \backslash$$

$$c-a \quad \& \quad 2 \quad \& \quad 0$$

$$\end{array}\right|=0\$$$

Expanding the determinant along C_3 ,

$$\Rightarrow 2(b-a) - (c-a) = 0$$

$$\Rightarrow 2b - a - c = 0$$

$$\Rightarrow a + c = 2b$$

Therefore, for the lines to be concurrent a , b and c must be in A.P.

Question 56

Line joining the points $(3, -4)$ and $(-2, 6)$ is perpendicular to the line joining the points $(-3, 6)$ and $(9, -18)$.

Solution

Ans: The given statement is false.

The line joining the points $(3, -4)$ and $(-2, 6)$ is,

$$\Rightarrow (y - 6) = \left(\frac{6 - (-4)}{-2 - 3} \right) (x - (-2))$$

$$\Rightarrow (y - 6) = (-2)(x + 2)$$

$$\Rightarrow y = -2x + 2 \quad (9, -18)$$

The slope of this line is -2 .

Similarly, the line joining the points $(-3, 6)$ and is,

$$\Rightarrow (y - 6) = \left(\frac{6 - (-18)}{-3 - 9} \right) (x - (-3))$$

$$\Rightarrow (y - 6) = (-2)(x + 3)$$

$$\Rightarrow y = -2x$$

The slope of this line is also -2 .

For the two lines to be perpendicular the product of their slopes must be -1 , but here the product is $(-2) \times (-2) = 4$.

Match the questions given under Column C_1 with their appropriate answers given under the column C_2 in Exercises 57 to 59.

Question 57

Column C_1	Column C_2
(a) The coordinates of the points P and Q on the line $x + 5y = 13$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$ are	(i) $(3, 1)$, $(-7, 11)$
(b) The coordinates of the points on the line $x + y = 4$ which are at a unit distance from the line $4x + 3y - 10 = 0$ are	(ii) $\left(-\frac{1}{3}, \frac{11}{3}\right)$, $\left(\frac{4}{3}, \frac{7}{3}\right)$
(c) The coordinates of the points on the line joining $A(-2, 5)$ and $B(3, 1)$ such that $AP = PQ = QB$ are	(iii) $\left(1, \frac{12}{5}\right)$, $\left(-3, \frac{16}{5}\right)$

Solution

Ans:

(a) Let one of the required point is (a, b) . Since it lies on the line $x + 5y = 13$, so it will satisfy the line.

$$\Rightarrow a + 5b = 13 \quad \dots\dots (1)$$

The distance of the point (a, b) from the line $12x - 5y + 26 = 0$ is 2 units,

$$\Rightarrow \left| \frac{(12 \times a) + ((-5) \times b) + 26}{\sqrt{(12)^2 + (-5)^2}} \right| = 2$$

$$\Rightarrow |12a - 5b + 26| = 26$$

$$\Rightarrow (12a - 5b + 26) = \pm 26$$

Considering the positive sign and solving equation (1) and $(12a - 5b + 26) = 26$ gives the point $(a, b) = \left(1, \frac{12}{5}\right)$.

Considering the negative sign and solving equation (1) and $(12a - 5b + 26) = -26$ gives the point $(a, b) = \left(-3, \frac{16}{5}\right)$.

(b) Let one of the required point is (c, d) . Since it lies on the line $x + y = 4$, so it will satisfy the line.

$$\Rightarrow c + d = 4 \quad \dots\dots (1)$$

The distance of the point (c, d) from the line $4x + 3y - 10 = 0$ is 1 unit,

$$\Rightarrow \left| \frac{(4 \times c) + (3 \times d) - 10}{\sqrt{(4)^2 + (3)^2}} \right| = 1$$

$$\Rightarrow |4c + 3d - 10| = 5$$

$$\Rightarrow (4c + 3d - 10) = \pm 5$$

Considering the positive sign and solving equation (1) and $(4c + 3d - 10) = 5$ gives the point $(c, d) = (3, 1)$.

Considering the negative sign and solving equation (1) and $(4c + 3d - 10) = -5$ gives the point $(c, d) = (-7, 11)$.

(c) Here the points P and Q trisect the line AB . Let the points be $P(x_1, y_1)$ and $Q(x_2, y_2)$.

For point P , $AP : PB = m : n = 1 : 2$. The section formula states that if a point (x, y) divides a line segments joining the two points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ then,

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Using this formula for the points A , P and B ,

$$\Rightarrow (x_1, y_1) = \left(\frac{(3 \times 1) + (2 \times (-2))}{1+2}, \frac{(1 \times 1) + (5 \times 2)}{1+2} \right)$$

$$\Rightarrow (x_1, y_1) = \left(-\frac{1}{3}, \frac{11}{3} \right)$$

Now, P is the midpoint of AQ . The midpoint formula is a special case of the section formula with $m : n = 1 : 1$.

$$\text{Using the midpoint formula for } A, P \text{ and } Q, \Rightarrow \left(-\frac{1}{3}, \frac{11}{3} \right) = \left(\frac{-2+x_2}{2}, \frac{5+y_2}{2} \right)$$

$$\text{Solving for the coordinates of } Q \text{ by comparison gives } (x_2, y_2) = \left(\frac{4}{3}, \frac{7}{3} \right).$$

Hence, the correct match is (a) – (iii), (b) – (i), (c) – (ii).

Question 58

The value of the λ , if the lines $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$ are

Column C_1	Column C_2
(a) Parallel to y - axis is	(i) $\lambda = -\frac{3}{4}$
(b) Perpendicular to $7x + y - 4 = 0$ are	(ii) $\lambda = -\frac{1}{3}$

(c) Passes through $(1, 2)$ is	(iii) $\lambda = -\frac{17}{41}$
(d) Parallel to x - axis	(iv) $\lambda = 3$

Solution

Ans: Simplifying the line $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$

gives $(2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$.

(a) For the line $(2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$ to be parallel to the y - axis, the coefficient of y - coordinate must be 0.

$$\Rightarrow (3 - \lambda) = 0$$

$$\Rightarrow \lambda = 3$$

(b) The slope of the line $7x + y - 4 = 0$ is -7 , therefore the slope of the line $(2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$ must be $\frac{1}{7}$ so that the two lines are perpendicular.

$$\Rightarrow -\frac{(2+6\lambda)}{(3-\lambda)} = \frac{1}{7}$$

$$\Rightarrow -14 - 42\lambda = 3 - \lambda$$

$$\Rightarrow \lambda = -\frac{17}{41}$$

(c) If the point $(1, 2)$ passes through the line $(2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$ then it must satisfy the equation. On substitution,

$$\Rightarrow (2 + 6\lambda) + (6 - 2\lambda) + (4 + 12\lambda) = 0$$

$$\Rightarrow 16\lambda = -12$$

$$\Rightarrow \lambda = -\frac{3}{4}$$

(d) For the line $(2 + 6\lambda)x + (3 - \lambda)y + (4 + 12\lambda) = 0$ to be parallel to the x - axis, the coefficient of x - coordinate must be 0.

$$\Rightarrow (2 + 6\lambda) = 0$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

Hence, the correct match is (a) – (iv), (b) – (iii), (c) – (i), (d) – (ii).

Question 59

The equation of the line through the intersection of the lines $2x - 3y = 0$ and $4x - 5y = 2$ and

Column C_1	Column C_2
(a) Through the point $(2, 1)$ is	(i) $2x - y = 4$
(b) Perpendicular to the line $x + 2y + 1 = 0$ is	(ii) $x + y - 5 = 0$
(c) Parallel to the line $3x - 4y + 5 = 0$ is	(iii) $x - y - 1 = 0$
(d) Equally inclined to the axes is	(iv) $3x - 4y - 1 = 0$

Solution

Ans: The point of intersection of the lines $2x - 3y = 0$ and $4x - 5y = 2$ is obtained by solving the equations algebraically. So the intersection point is $(3, 2)$.

(a) The equation of the line passing through the points $(2, 1)$ and $(3, 2)$ is,

$$\Rightarrow (y - 1) = \left(\frac{2-1}{3-2} \right) (x - 2)$$

$$\Rightarrow (y - 1) = (x - 2)$$

$$\Rightarrow x - y - 1 = 0$$

(b) The slope of the line $x + 2y + 1 = 0$ is $\left(-\frac{1}{2}\right)$, so the slope of the line perpendicular to this line will be 2 . Now, the line passes through the point $(3, 2)$ so its equation is,

$$\Rightarrow (y - 2) = 2(x - 3)$$

$$\Rightarrow (y - 2) = (2x - 6)$$

$$\Rightarrow 2x - y = 4$$

(c) The slope of the line $3x - 4y + 5 = 0$ is $\left(\frac{3}{4}\right)$, so the slope of the line parallel to this line will also be $\left(\frac{3}{4}\right)$. Now, the line passes through the point $(3, 2)$ so its equation is,

$$\Rightarrow (y - 2) = \left(\frac{3}{4}\right)(x - 3)$$

$$\Rightarrow (4y - 8) = (3x - 9)$$

$$\Rightarrow 3x - 4y - 1 = 0$$

(d) The lines which are equally inclined to the axes have their slope values equal to $\tan 45^\circ = 1$ or $\tan 135^\circ = -1$. Now, the line passes through the point $(3, 2)$ so its equation is,

Considering the positive slope,

$$\Rightarrow (y - 2) = (1)(x - 3)$$

$$\Rightarrow x - y - 1 = 0$$

Considering the negative slope,

$$\Rightarrow (y - 2) = (-1)(x - 3)$$

$$\Rightarrow x + y - 5 = 0$$

Hence, the correct match is (a) – (iii), (b) – (i), (c) – (iv), (d) – (ii) or (iii).