

Chapter 9: Circles

NCERT Exemplar · Class 10 · Maths

Exercise 9.1 · Textbook page 102

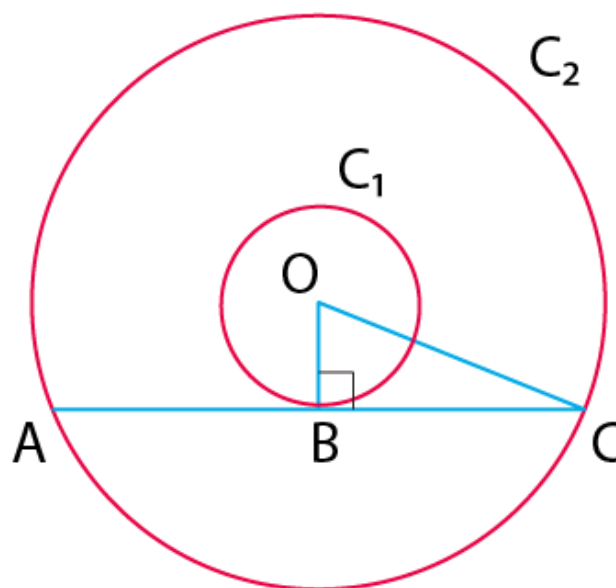
Question 1

If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle, which is tangent to the other circle, is

(A) 3 cm (B) 6 cm

(C) 9 cm (D) 1 cm

Solution



According to the question,

$OA = 4\text{ cm}$, $OB = 5\text{ cm}$

And, $OA \perp BC$

Therefore, $OB^2 = OA^2 + AB^2$

$$\Rightarrow 5^2 = 4^2 + AB^2$$

$$\Rightarrow AB = \sqrt{25 - 16} = 3\text{cm}$$

$$\Rightarrow BC = 2AB = 2 \times 3\text{cm} = 6\text{cm}$$

Question 2

In Fig. 9.3, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to

(A) 62.5° (B) 45°

(C) 35° (D) 55°

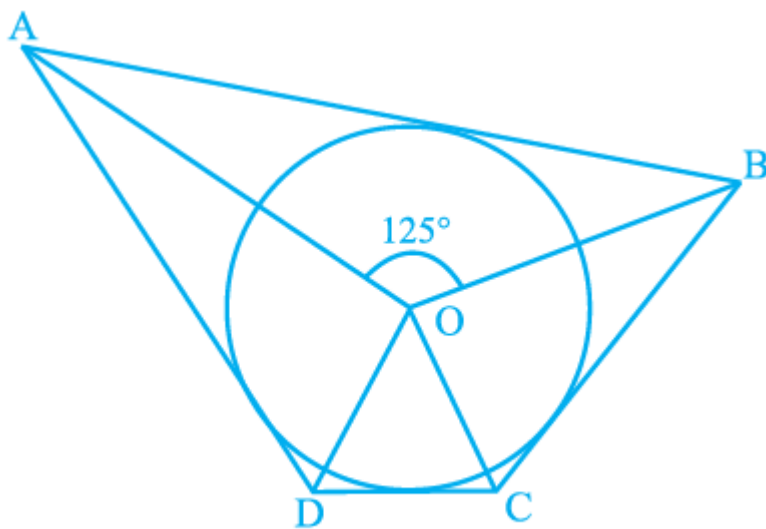


Fig. 9.3

Solution

ABCD is a quadrilateral circumscribing the circle

We know that, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.

So, we have

$$\angle AOB + \angle COD = 180^\circ$$

$$125^\circ + \angle COD = 180^\circ$$

$$\angle COD = 55^\circ$$

Question 3

In Fig. 9.4, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is tangent to the circle at point A, then $\angle BAT$ is equal to

(A) 65° (B) 60°

(C) 50° (D) 40°

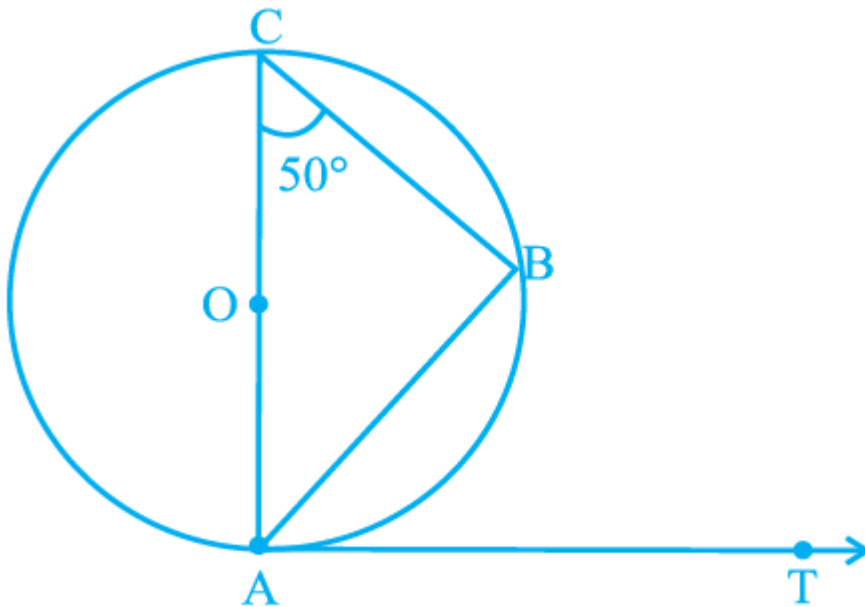


Fig. 9.4

Solution

According to the question,

A circle with centre O, diameter AC and $\angle ACB = 50^\circ$

AT is a tangent to the circle at point A

Since, angle in a semicircle is a right angle

$$\angle CBA = 90^\circ$$

By angle sum property of a triangle,

$$\angle ACB + \angle CAB + \angle CBA = 180^\circ$$

$$50^\circ + \angle CAB + 90^\circ = 180^\circ$$

$$\angle CAB = 40^\circ \dots (1)$$

Since tangent to at any point on the circle is perpendicular to the radius through point of contact,

We get,

$$OA \perp AT$$

$$\angle OAT = 90^\circ$$

$$\angle OAT + \angle BAT = 90^\circ$$

$$\angle CAT + \angle BAT = 90^\circ$$

$$40^\circ + \angle BAT = 90^\circ \text{ [from equation (1)]}$$

$$\angle BAT = 50^\circ$$

Question 4

From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(A) 60 cm^2 (B) 65 cm^2

(C) 30 cm^2 (D) 32.5 cm^2

Solution

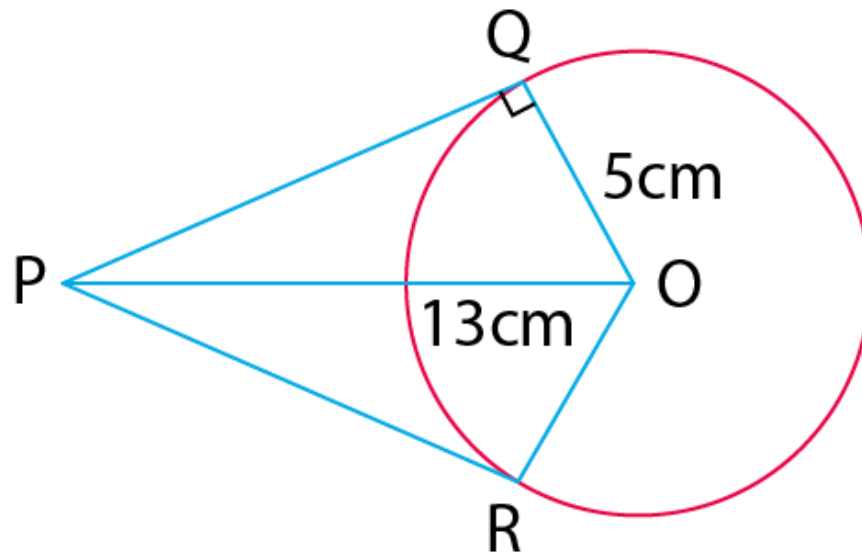
Construction: Draw a circle of radius 5 cm with centre O.

Let P be a point at a distance of 13 cm from O.

Draw a pair of tangents, PQ and PR.

$$OQ = OR = \text{radius} = 5 \text{ cm} \dots \text{equation (1)}$$

$$\text{And } OP = 13 \text{ cm}$$



We know that, tangent to at any point on the circle is perpendicular to the radius through point of contact,

Hence, we get,

$$OQ \perp PQ \text{ and } OR \perp PR$$

$\triangle POQ$ and $\triangle POR$ are right-angled triangles.

Using Pythagoras Theorem in $\triangle PQO$,

$$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$$

$$(PQ)^2 + (OQ)^2 = (OP)^2$$

$$(PQ)^2 + (5)^2 = (13)^2$$

$$(PQ)^2 + 25 = 169$$

$$(PQ)^2 = 144$$

$$PQ = 12 \text{ cm}$$

Tangents through an external point to a circle are equal.

So,

$$PQ = PR = 12 \text{ cm} \dots (2)$$

Therefore, Area of quadrilateral PQRS, $A = \text{area of } \triangle POQ + \text{area of } \triangle POR$

Area of right angled triangle = $\frac{1}{2} \times \text{base} \times \text{perpendicular}$

$$A = (\frac{1}{2} \times OQ \times PQ) + (\frac{1}{2} \times OR \times PR)$$

$$A = (\frac{1}{2} \times 5 \times 12) + (\frac{1}{2} \times 5 \times 12)$$

$$A = 30 + 30 = 60 \text{ cm}^2$$

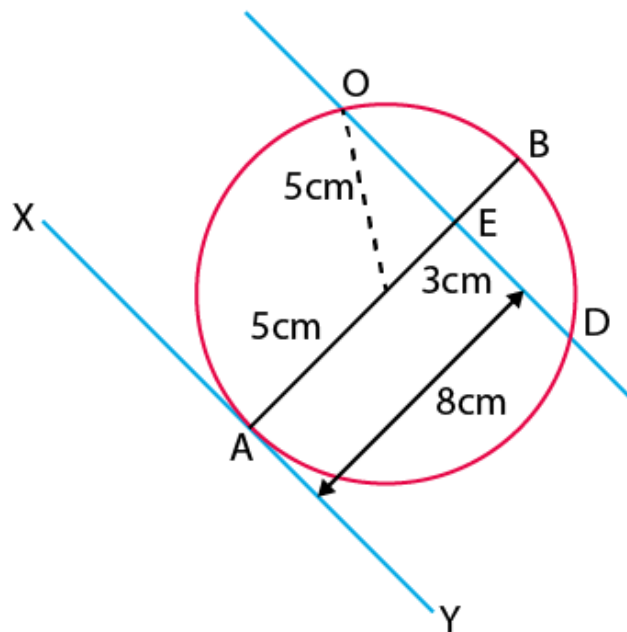
Question 5

At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

(A) 4 cm (B) 5 cm

(C) 6 cm (D) 8 cm

Solution



According to the question,

Radius of circle, $AO = OC = 5 \text{ cm}$

$AM = 8 \text{ cm}$

$AM = OM + AO$

$OM = AM - AO$

Substituting these values in the equation,

$$OM = (8-5) = 3\text{CM}$$

OM is perpendicular to the chord CD.

$$\text{In } \triangle OCM \angle OMC = 90^\circ$$

By Pythagoras theorem,

$$OC^2 = OM^2 + MC^2$$

Therefore,

$$CD = 2 \times CM = 8 \text{ cm}$$

Exercise 9.2 · Textbook page 105

Question 1

If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60° .

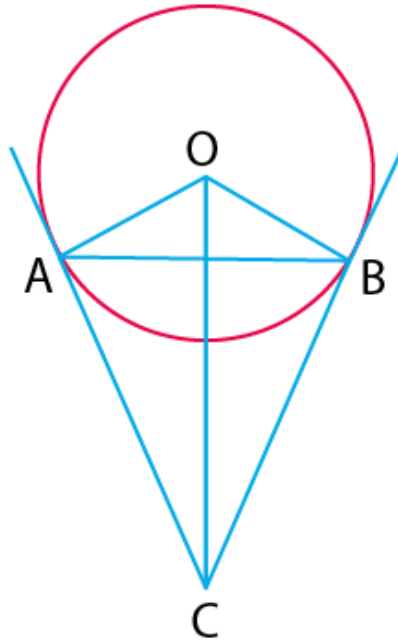
Solution

False

Justification:

For example,

Consider the given figure. In which we have a circle with centre O and AB a chord with $\angle AOB = 60^\circ$



Since, tangent to any point on the circle is perpendicular to the radius through point of contact,

We get,

$$OA \perp AC \text{ and } OB \perp CB$$

$$\angle OBC = \angle OAC = 90^\circ \dots \text{eq(1)}$$

Using angle sum property of quadrilateral in Quadrilateral AOBC,

We get,

$$\angle OBC + \angle OAC + \angle AOB + \angle ACB = 360^\circ$$

$$90^\circ + 90^\circ + 60^\circ + \angle ACB = 360^\circ$$

$$\angle ACB = 120^\circ$$

Hence, the angle between two tangents is 120° .

Therefore, we can conclude that,

the given statement is false.

Question 2

The length of tangent from an external point on a circle is always greater than the radius of the circle.

Solution

False

Justification:

Length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

Question 3

The length of tangent from an external point P on a circle with centre O is always less than OP.

Solution

True

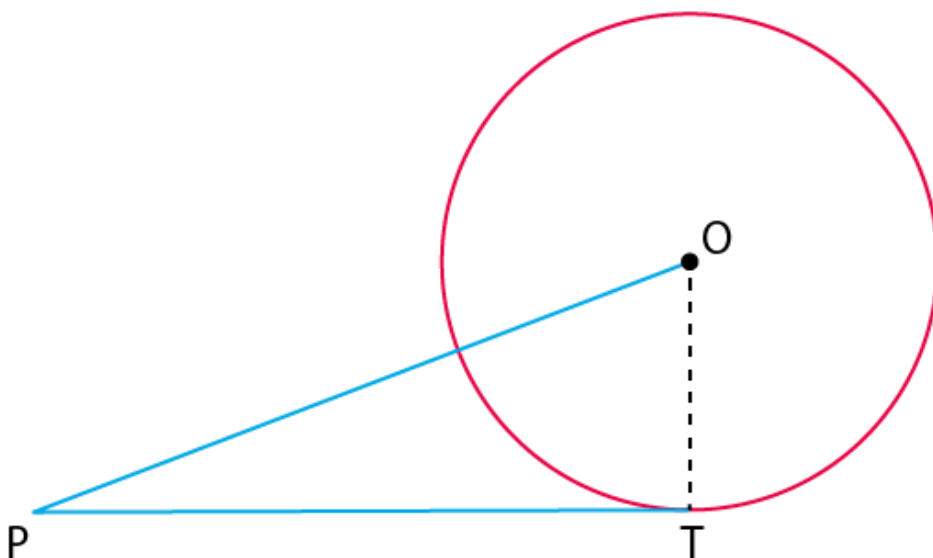
Justification:

Consider the figure of a circle with centre O.

Let PT be a tangent drawn from external point P.

Now, Join OT.

$OT \perp PT$



We know that,

Tangent at any point on the circle is perpendicular to the radius through point of contact

Hence, OPT is a right-angled triangle formed.

We also know that,

In a right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

Hence,

$$OP > PT \text{ or } PT < OP$$

Hence, length of tangent from an external point P on a circle with centre O is always less than OP.

Question 4

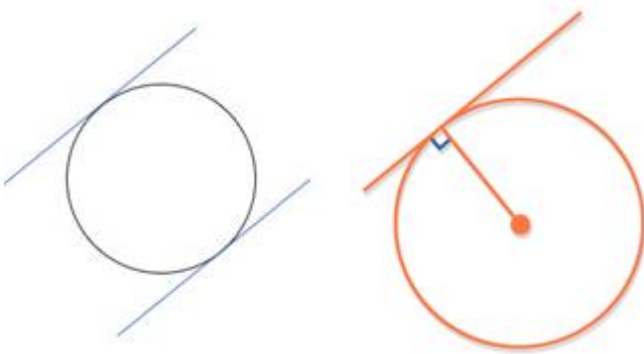
The angle between two tangents to a circle may be 0° .

Solution

True

Justification:

The angle between two tangents to a circle may be 0° only when both tangent lines coincide or are parallel to each other.



Question 5

If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90° , then $OP = a\sqrt{2}$.

Solution

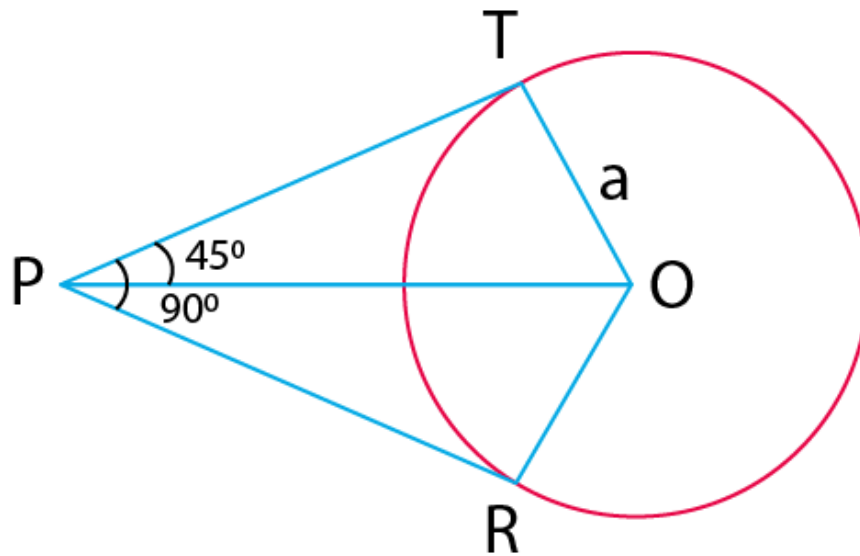
True.

Tangent is always perpendicular to the radius at the point of contact.

Hence, $\angle RPT = 90^\circ$

If 2 tangents are drawn from an external point, then they are equally inclined to the line segment joining the centre to that point.

Consider the following figure,



From point P, two tangents are drawn.

It is given that, $OT = a$

And line OP bisects $\angle RPT$.

So,

$$\angle TPO = \angle RPO = 45^\circ$$

We know that, $OT \perp PT$

In right-angled triangle OTP,

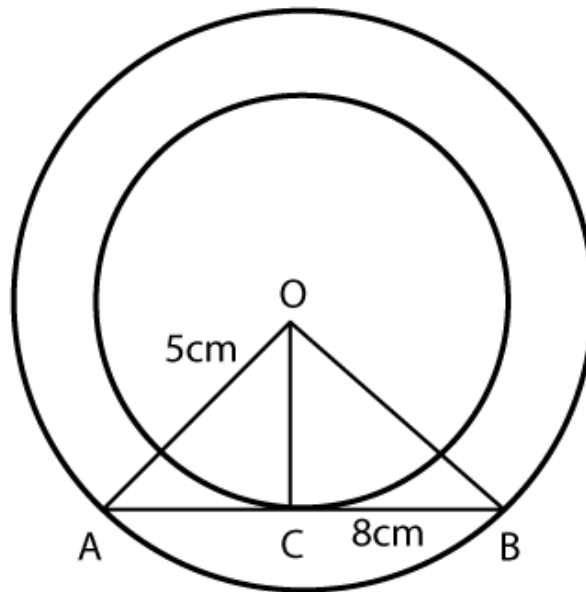
$$\sin 45^\circ = OT/OP$$

$$= 1/\sqrt{2} = a/OP$$

$$\text{Hence, } OP = a\sqrt{2}$$

Exercise 9.3 · Textbook page 107**Question 1**

Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Solution

From the figure,

Chord AB = 8 cm

OC is perpendicular to the chord AB

AC = CB = 4 cm

In right triangle OCA

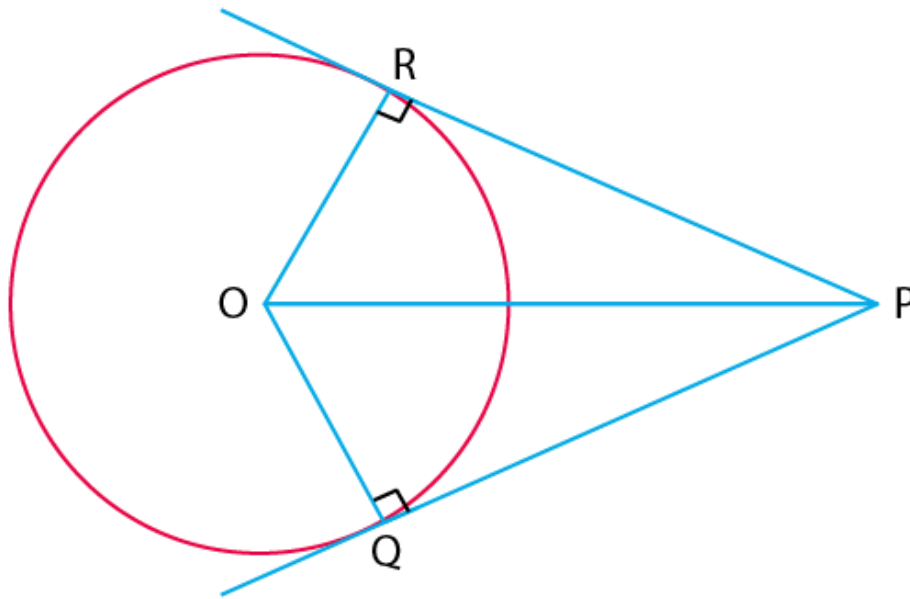
$$OC^2 + CA^2 = OA^2$$

$$OC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$OC = 3 \text{ cm}$$

Question 2

Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Solution

We know that,

Radius $\perp$ Tangent = $OR \perp PR$

i.e., $\angle ORP = 90^\circ$

Likewise,

Radius $\perp$ Tangent = $OQ \perp PQ$

$\angle OQP = 90^\circ$

In quadrilateral ORPQ,

Sum of all interior angles = 360°

$\angle ORP + \angle RPQ + \angle PQO + \angle QOR = 360^\circ$

$90^\circ + \angle RPQ + 90^\circ + \angle QOR = 360^\circ$

Hence, $\angle O + \angle P = 180^\circ$

PROQ is a cyclic quadrilateral.

Question 3

If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that angle $DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.

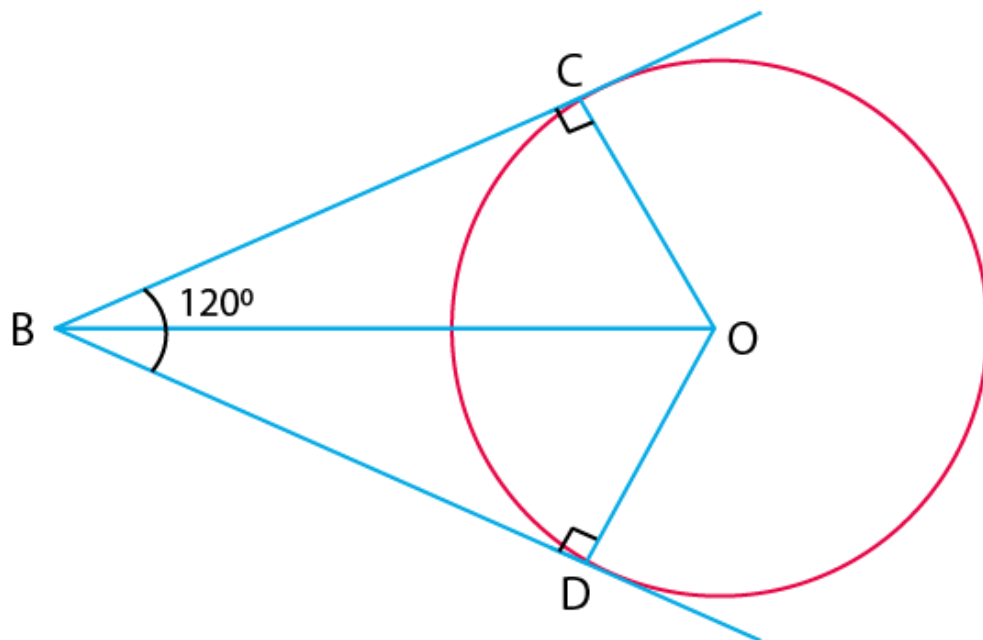
Solution

According to the question,

By RHS rule,

$\triangle OBC$ and $\triangle OBD$ are congruent

By CPCT



$\angle OBC$ and $\angle OBD$ are equal

Therefore,

$$\angle OBC = \angle OBD = 60^\circ$$

In triangle OBC,

$$\cos 60^\circ = BC/OB$$

$$\frac{1}{2} = BC/OB$$

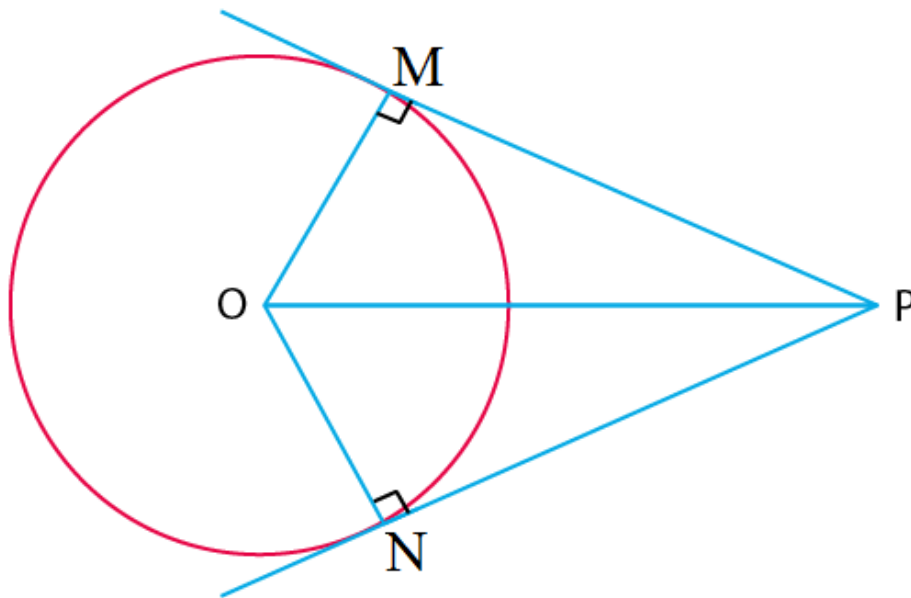
$$OB = 2BC$$

Hence proved

Question 4

Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Solution



Let the lines be l_1 and l_2 .

Assume that O touches l_1 and l_2 at M and N,

We get,

$OM = ON$ (Radius of the circle)

Therefore,

From the centre "O" of the circle, it has equal distance from l_1 & l_2 .

In $\triangle OPM$ & OPN ,

$OM = ON$ (Radius of the circle)

$\angle OMP = \angle ONP$ (As, Radius is perpendicular to its tangent)

$OP = OP$ (Common sides)

Therefore,

$\triangle OPM = \triangle OPN$ (SSS congruence rule)

By C.P.C.T,

$\angle MPO = \angle NPO$

So, l bisects $\angle MPN$.

Therefore, O lies on the bisector of the angle between l_1 & l_2 .

Hence, we prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Question 5

In Fig. 9.13, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.

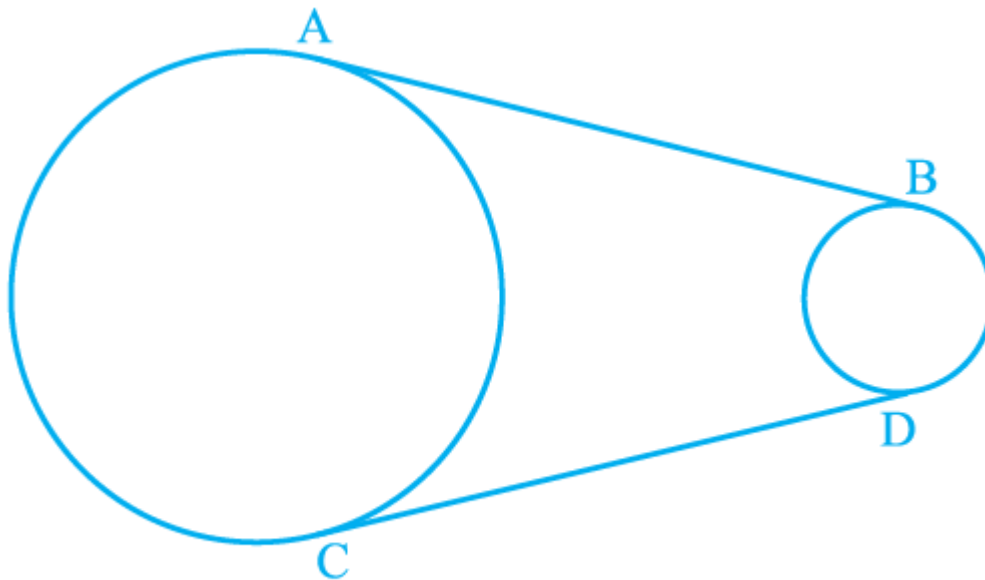
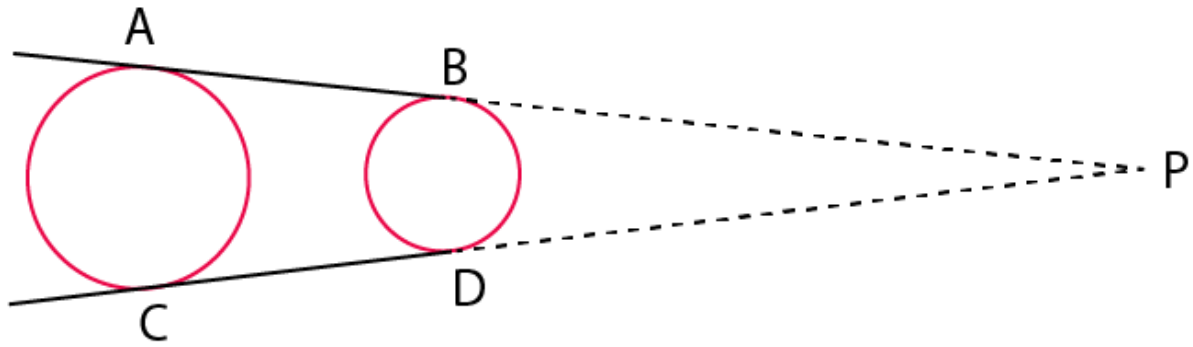


Fig. 9.13

Solution

According to the question,

$AB = CD$



Construction: Produce AB and CD, to intersect at P.

Proof:

Consider the circle with greater radius.

Tangents drawn from an external point to a circle are equal

$$AP = CP \dots(1)$$

Also,

Consider the circle with smaller radius.

Tangents drawn from an external point to a circle are equal

$$BP = BD \dots(2)$$

Subtract Equation (2) from (1). We Get

$$AP - BP = CP - BD$$

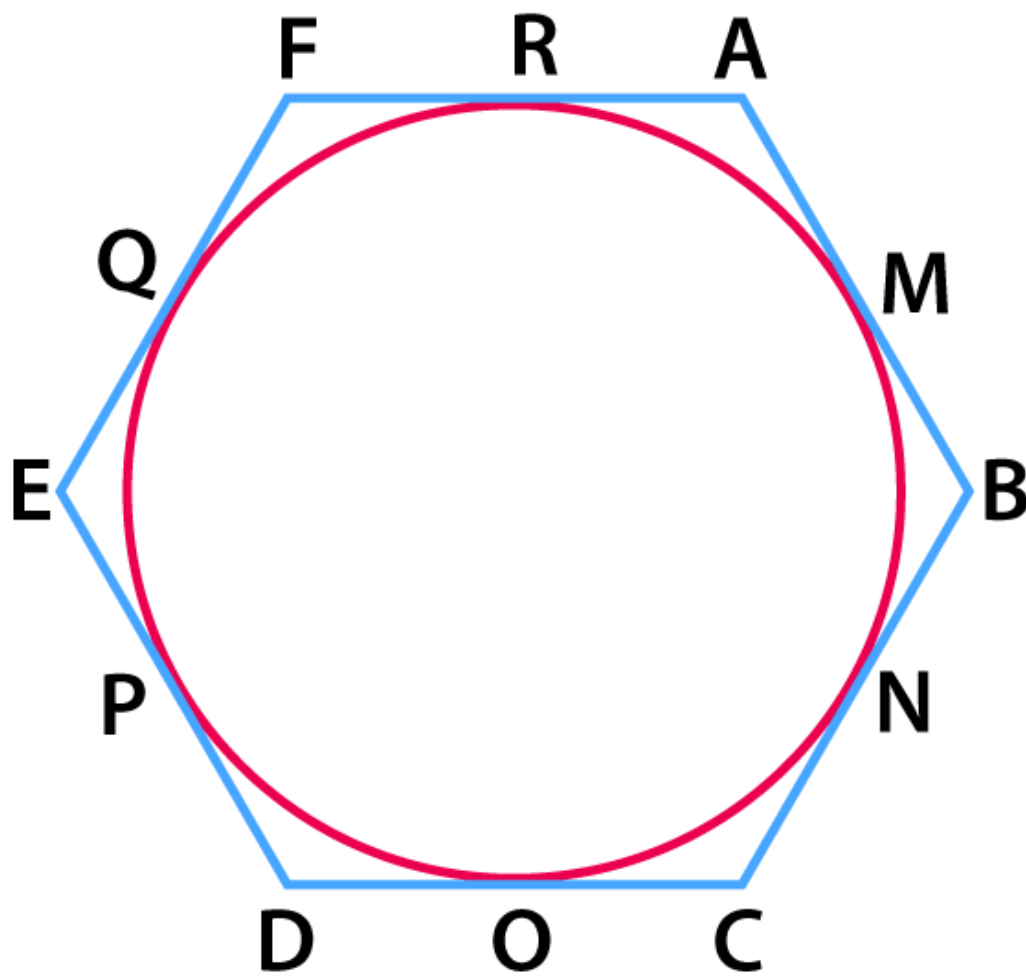
$$AB = CD$$

Hence Proved.

Exercise 9.4 · Textbook page 110

Question 1

If a hexagon ABCDEF circumscribes a circle, prove that $AB + CD + EF = BC + DE + FA$.

Solution

According to the question,

A Hexagon ABCDEF circumscribes a circle.

To prove:

$$AB + CD + EF = BC + DE + FA$$

Proof:

Tangents drawn from an external point to a circle are equal.

Hence, we have

$$AM = RA \dots \text{eq 1 [tangents from point A]}$$

$$BM = BN \dots \text{eq 2 [tangents from point B]}$$

$$CO = NC \dots \text{eq 3 [tangents from point C]}$$

$$OD = DP \dots \text{eq 4 [tangents from point D]}$$

$$EQ = PE \dots \text{eq 5 [tangents from point E]}$$

$$QF = FR \dots \text{eq 6 [tangents from point F] [eq 1]+[eq 2]+[eq 3]+[eq 4]+[eq 5]+[eq 6]}$$

$$AM + BM + CO + OD + EQ + QF = RA + BN + NC + DP + PE + FR$$

On rearranging, we get,

$$(AM + BM) + (CO + OD) + (EQ + QF) = (BN + NC) + (DP + PE) + (FR + RA)$$

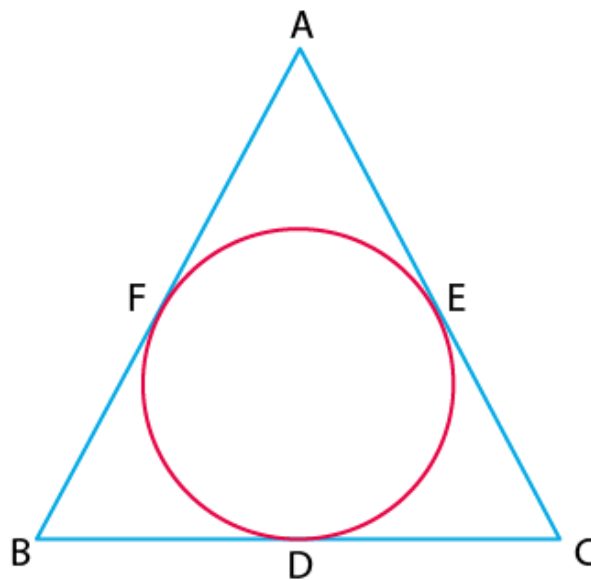
$$AB + CD + EF = BC + DE + FA$$

Hence Proved!

Question 2

Let s denote the semi-perimeter of a triangle ABC in which $BC = a$, $CA = b$, $AB = c$. If a circle touches the sides BC , CA , AB at D , E , F , respectively, prove that $BD = s - b$.

Solution



According to the question,

A triangle ABC with $BC = a$, $CA = b$ and $AB = c$. Also, a circle is inscribed, which touches the sides BC , CA and AB at D , E and F , respectively and s is semi-perimeter of the triangle

To Prove: $BD = s - b$

Proof:

According to the question,

We have,

Semi Perimeter = s

Perimeter = $2s$

$$2s = AB + BC + AC \text{ [1]}$$

As we know,

Tangents drawn from an external point to a circle are equal

So we have

$$AF = AE \text{ [2] [Tangents from point A]}$$

$$BF = BD \text{ [3] [Tangents From point B]}$$

$$CD = CE \text{ [4] [Tangents From point C]}$$

Adding [2] [3] and [4]

$$AF + BF + CD = AE + BD + CE$$

$$AB + CD = AC + BD$$

Adding BD both side

$$AB + CD + BD = AC + BD + BD$$

$$AB + BC - AC = 2BD$$

$$AB + BC + AC - AC - AC = 2BD$$

$$2s - 2AC = 2BD \text{ [From 1]}$$

$$2BD = 2s - 2b \text{ [as } AC = b]$$

$$BD = s - b$$

Hence Proved.

Question 3

From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn, which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

Solution

According to the question,

From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle tangent is drawn, which intersects PA and PB at C and D, respectively. And PA = 10 cm

To Find: Perimeter of $\triangle PCD$

As we know that, Tangents drawn from an external point to a circle are equal.

So we have

$$AC = CE \text{ [1] [Tangents from point C]}$$

$$ED = DB \text{ [2] [Tangents from point D]}$$

Now Perimeter of Triangle PCD

$$= PC + CD + DP$$

$$= PC + CE + ED + DP$$

$$= PC + AC + DB + DP \text{ [From 1 and 2]}$$

$$= PA + PB$$

Now,

$$PA = PB = 10 \text{ cm as tangents drawn from an external point to a circle are equal}$$

So we have

$$\text{Perimeter} = PA + PB = 10 + 10 = 20 \text{ cm}$$

Question 4

If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig. 9.17. Prove that $\angle BAT = \angle ACB$

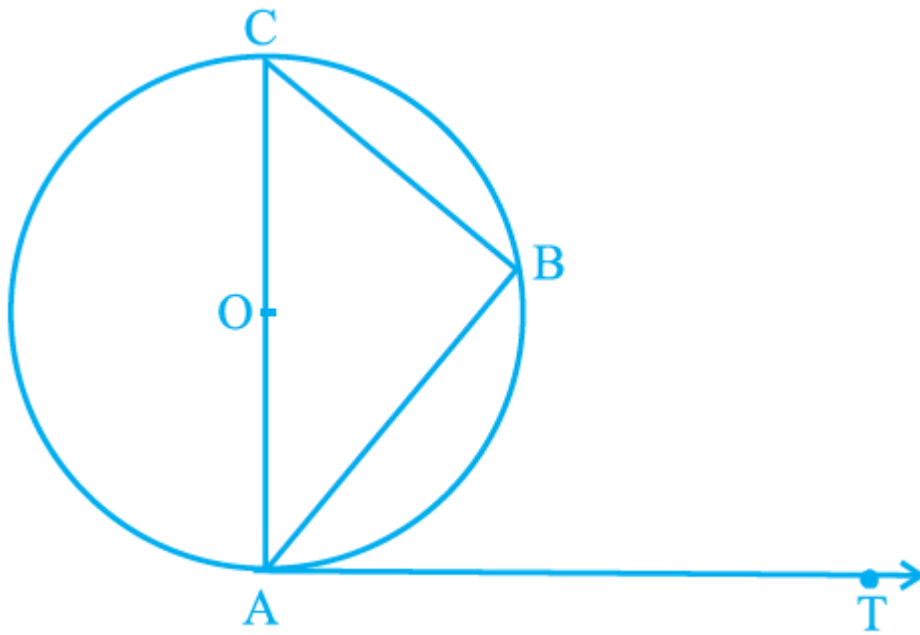


Fig. 9.17

Solution

According to the question,

A circle with centre O and AC as a diameter and AB and BC as two chords also, AT is a tangent at point A

To Prove: $\angle BAT = \angle ACB$

Proof:

$\angle ABC = 90^\circ$ [Angle in a semicircle is a right angle]

In $\triangle ABC$ By angle sum property of triangle

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\angle ACB + 90^\circ = 180^\circ - \angle BAC$$

$$\angle ACB = 90 - \angle BAC \quad [1]$$

Now,

$OA \perp AT$ [Tangent at a point on the circle is perpendicular to the radius through point of contact]

$$\angle OAT = \angle CAT = 90^\circ$$

$$\angle BAC + \angle BAT = 90^\circ$$

$$\angle BAT = 90^\circ - \angle BAC \text{ [2]}$$

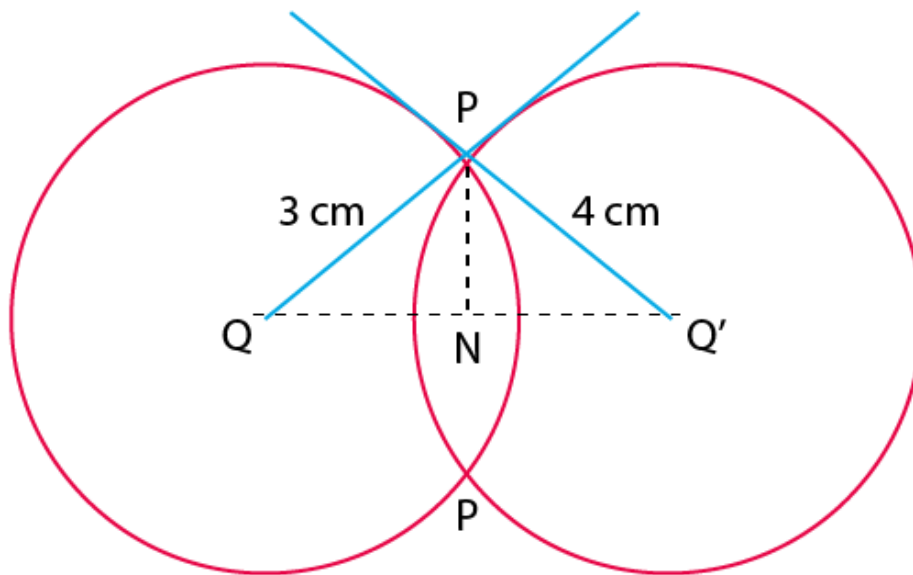
From [1] and [2]

$$\angle BAT = \angle ACB \text{ [Proved]}$$

Question 5

Two circles with centres O and O' of radii 3 cm and 4 cm, respectively, intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Solution



According to the question,

Two circles with centres O and O' of radii 3 cm and 4 cm, respectively, intersect at two points P and Q, such that OP and O'P are tangents to the two circles and PQ is a common chord.

To Find: Length of common chord PQ

$$\angle OPO' = 90^\circ \text{ [Tangent at a point on the circle is perpendicular to the radius through point of contact]}$$

So OPO is a right-angled triangle at P

Using Pythagoras in $\triangle OPO'$, we have

$$(OO')^2 = (O'P)^2 + (OP)^2$$

$$(OO')^2 = (4)^2 + (3)^2$$

$$(OO')^2 = 25$$

$$OO' = 5 \text{ cm}$$

$$\text{Let } ON = x \text{ cm and } NO' = 5 - x \text{ cm}$$

In right angled triangle ONP

$$(ON)^2 + (PN)^2 = (OP)^2$$

$$x^2 + (PN)^2 = (3)^2$$

$$(PN)^2 = 9 - x^2 \quad [1]$$

In right angled triangle O'NP

$$(O'N)^2 + (PN)^2 = (O'P)^2$$

$$(5 - x)^2 + (PN)^2 = (4)^2$$

$$25 - 10x + x^2 + (PN)^2 = 16$$

$$(PN)^2 = -x^2 + 10x - 9 \quad [2]$$

From [1] and [2]

$$9 - x^2 = -x^2 + 10x - 9$$

$$10x = 18$$

$$x = 1.8$$

From (1) we have

$$(PN)^2 = 9 - (1.8)^2$$

$$= 9 - 3.24 = 5.76$$

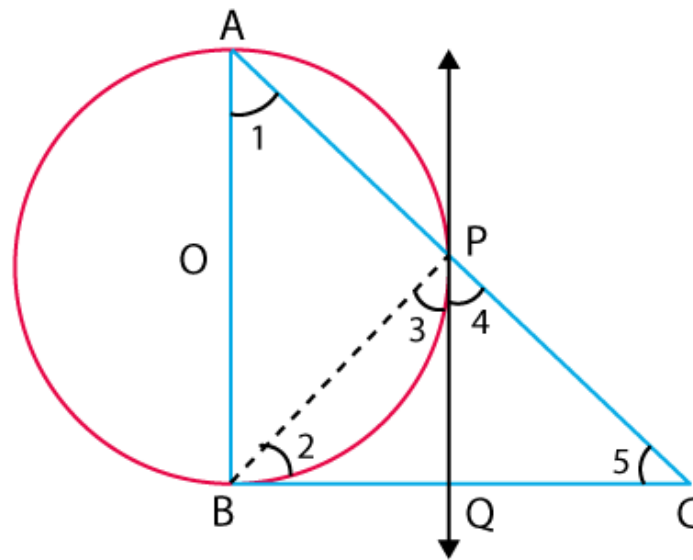
$$PN = 2.4 \text{ cm}$$

$$PQ = 2PN = 2(2.4) = 4.8 \text{ cm}$$

Question 6

In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

Solution



According to the question,

In a right angle $\triangle ABC$ in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Also PQ is a tangent at P

To Prove: PQ bisects BC i.e. $BQ = QC$

Proof:

$\angle APB = 90^\circ$ [Angle in a semicircle is a right-angle]

$\angle BPC = 90^\circ$ [Linear Pair]

$\angle 3 + \angle 4 = 90$ [1]

Now, $\angle ABC = 90^\circ$

So in $\triangle ABC$

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$90 + \angle 1 + \angle 5 = 180$

$\angle 1 + \angle 5 = 90$ [2]

Now,

$\angle 1 = \angle 3$ [angle between tangent and the chord equals angle made by the chord in alternate segment]

Using this in [2] we have

$$\angle 3 + \angle 5 = 90 \text{ [3]}$$

From [1] and [3] we have

$$\angle 3 + \angle 4 = \angle 3 + \angle 5$$

$$\angle 4 = \angle 5$$

$QC = PQ$ [Sides opposite to equal angles are equal]

But Also $PQ = BQ$ [Tangents drawn from an external point to a circle are equal]

So, $BQ = QC$

i.e. PQ bisects BC .

Question 7

In Fig. 9.18, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ . Find the $\angle RQS$.

[Hint: Draw a line through Q and perpendicular to QP .]

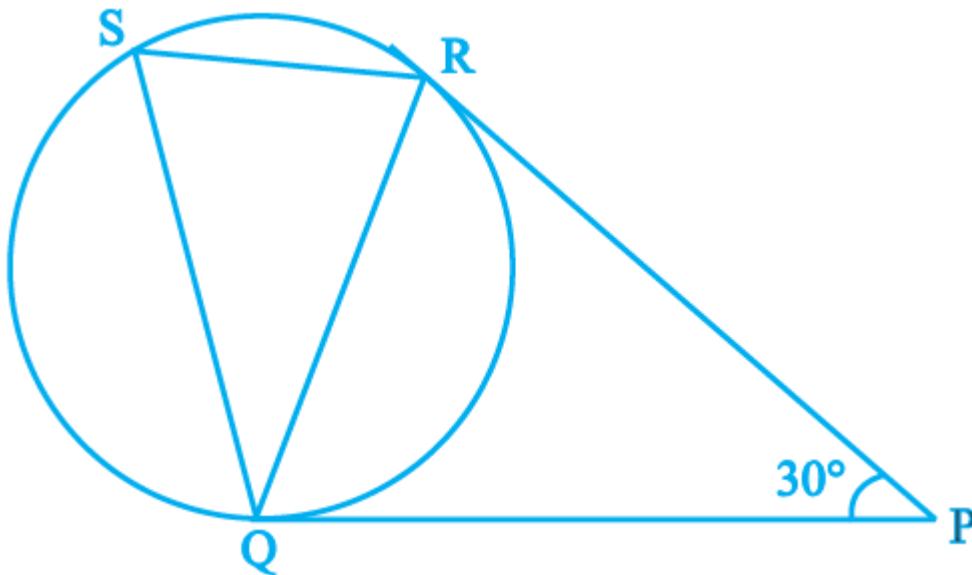
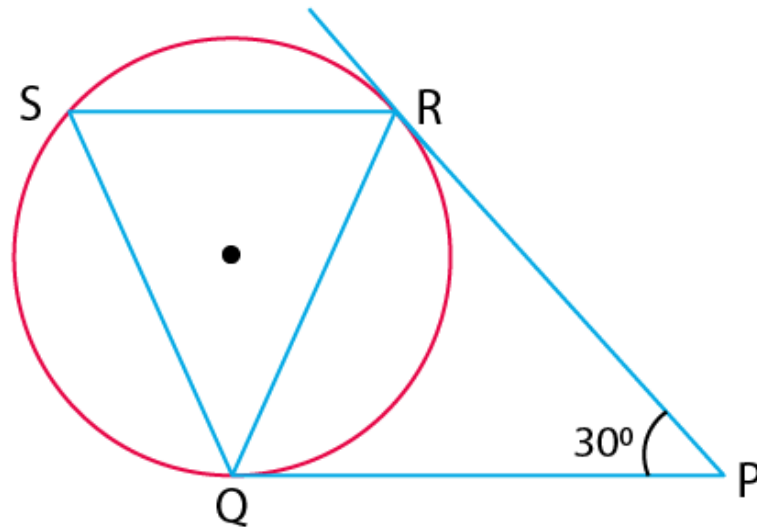


Fig. 9.18

Solution



According to the question,

Tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ.

To Find: $\angle RQS$

$PQ = PR$ [Tangents drawn from an external point to a circle are equal]

$\angle PRQ = \angle PQR$ [Angles opposite to equal sides are equal] [1]

In $\triangle PQR$

$$\angle PRQ + \angle PQR + \angle QPR = 180^\circ$$

$$\angle PQR + \angle PQR + \angle QPR = 180^\circ \text{ [Using 1]}$$

$$2\angle PQR + \angle RPQ = 180^\circ$$

$$2\angle PQR + 30 = 180$$

$$2\angle PQR = 150$$

$$\angle PQR = 75^\circ$$

$$\angle QRS = \angle PQR = 75^\circ \text{ [Alternate interior angles]}$$

$\angle QSR = \angle PQR = 75^\circ$ [angle between tangent and the chord equals angle made by the chord in alternate segment]

Now In $\triangle RQS$

$$\angle RQS + \angle QRS + \angle QSR = 180$$

$$\angle RQS + 75 + 75 = 180$$

$$\angle RQS = 30^\circ$$
