

Chapter 8: Introduction to Trigonometry & Its Equations

NCERT Exemplar · Class 10 · Maths

Exercise 8.1

Question 1

If $\cos A = 4/5$, then the value of $\tan A$ is

(A) $3/5$ (B) $3/4$ (C) $4/3$ (D) $5/3$

Solution

(B) $3/4$

According to the question,

$$\cos A = 4/5 \dots(1)$$

We know,

$$\tan A = \sin A / \cos A$$

To find the value of $\sin A$,

We have the equation,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{So, } \sin \theta = \sqrt{1 - \cos^2 \theta}$$

Then,

$$\sin A = \sqrt{1 - \cos^2 A} \dots(2)$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \sqrt{1 - \cos^2 A}$$

Substituting equation (1) in (2),

We get,

$$\sin A = \sqrt{1 - (4/5)^2}$$

$$= \sqrt{1 - (16/25)}$$

$$= \sqrt{9/25}$$

$$= \frac{3}{5}$$

Therefore,

$$\tan A = \frac{3}{4} \times \frac{5}{3} = \frac{5}{4}$$

Question 2

If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(A) $\sqrt{3}$ (B) $1/\sqrt{3}$ (C) $\sqrt{3}/2$ (D) 1

Solution

(A) $\sqrt{3}$

According to the question,

$$\sin A = \frac{1}{2} \dots (1)$$

We know that,

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \dots (2)$$

To find the value of $\cos A$.

We have the equation,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{So, } \cos \theta = \sqrt{1 - \sin^2 \theta}$$

Then,

$$\cos A = \sqrt{1 - \sin^2 A} \dots (3)$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos A = \sqrt{1 - \sin^2 A}$$

Substituting equation 1 in 3, we get,

$$\cos A = \sqrt{1-1/4} = \sqrt{3/4} = \sqrt{3}/2$$

Substituting values of $\sin A$ and $\cos A$ in equation 2, we get

$$\cot A = (\sqrt{3}/2) \times 2 = \sqrt{3}$$

Question 3

The value of the expression $[\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)]$ is

(A) -1 (B) 0 (C) 1 (D) 3 2

Solution

(B) 0

According to the question,

We have to find the value of the equation,

$$\begin{aligned} & \operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta) \\ &= \operatorname{cosec}[90^\circ - (15^\circ - \theta)] - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot[90^\circ - (55^\circ + \theta)] \end{aligned}$$

Since, $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

And, $\cot(90^\circ - \theta) = \tan \theta$

We get,

$$\begin{aligned} &= \sec(15^\circ - \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \tan(55^\circ + \theta) \\ &= 0 \end{aligned}$$

Question 4

Given that $\sin \theta = a/b$, then $\cos \theta$ is equal to

(A) $b/\sqrt{b^2 - a^2}$ (B) b/a (C) $\sqrt{b^2 - a^2}/b$ (D) $a/\sqrt{b^2 - a^2}$

Solution

(C) $\sqrt{b^2 - a^2}/b$

According to the question,

$$\sin \theta = a/b$$

We know, $\sin^2 \theta + \cos^2 \theta = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$\text{So, } \cos \theta = \sqrt{1 - a^2/b^2} = \sqrt{(b^2 - a^2)/b^2} = \sqrt{b^2 - a^2}/b$$

$$\text{Hence, } \cos \theta = \sqrt{b^2 - a^2}/b$$

Question 5

If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(A) $\cos \beta$ (B) $\cos 2\beta$ (C) $\sin \alpha$ (D) $\sin 2\alpha$

Solution

(B) $\cos 2\beta$

According to the question,

$$\cos(\alpha + \beta) = 0$$

$$\text{Since, } \cos 90^\circ = 0$$

We can write,

$$\cos(\alpha + \beta) = \cos 90^\circ$$

By comparing cosine equation on L.H.S and R.H.S,

We get,

$$(\alpha + \beta) = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

Now we need to reduce $\sin(\alpha - \beta)$,

So, we take,

$$\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta) = \sin(90^\circ - 2\beta)$$

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\text{So, } \sin(90^\circ - 2\beta) = \cos 2\beta$$

$$\text{Therefore, } \sin(\alpha - \beta) = \cos 2\beta$$

Question 6

The value of $(\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ)$ is

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

Solution

(B) 1

$$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot \tan 45^\circ \cdot \tan 46^\circ \cdot \tan 47^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$$

$$\text{Since, } \tan 45^\circ = 1,$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \tan 46^\circ \cdot \tan 47^\circ \dots \tan 87^\circ \cdot \tan 88^\circ \cdot \tan 89^\circ$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \tan(90^\circ - 44^\circ) \cdot \tan(90^\circ - 43^\circ) \dots \tan(90^\circ - 3^\circ) \cdot \tan(90^\circ - 2^\circ) \cdot \tan(90^\circ - 1^\circ)$$

$$\text{Since, } \tan(90^\circ - \theta) = \cot \theta,$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot \cot 44^\circ \cdot \cot 43^\circ \dots \cot 3^\circ \cdot \cot 2^\circ \cdot \cot 1^\circ$$

$$\text{Since, } \tan \theta = (1/\cot \theta)$$

$$= \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 43^\circ \cdot \tan 44^\circ \cdot 1 \cdot (1/\tan 44^\circ) \cdot (1/\tan 43^\circ) \dots (1/\tan 3^\circ) \cdot (1/\tan 2^\circ) \cdot (1/\tan 1^\circ)$$

$$= \left(\tan 1^\circ \times \frac{1}{\tan 1^\circ}\right) \cdot \left(\tan 2^\circ \times \frac{1}{\tan 2^\circ}\right) \dots \left(\tan 44^\circ \times \frac{1}{\tan 44^\circ}\right)$$

$$= 1$$

$$\text{Hence, } \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 89^\circ = 1$$

Question 7

If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(A) $1/\sqrt{3}$ (B) $\sqrt{3}$ (C) 1 (D) 0

Solution

(C) 1

According to the question,

$$\cos 9\alpha = \sin \alpha \text{ and } 9\alpha < 90^\circ$$

i.e. 9α is an acute angle

We know that,

$$\sin(90^\circ - \theta) = \cos \theta$$

So,

$$\cos 9\alpha = \sin (90^\circ - \alpha)$$

$$\text{Since, } \cos 9\alpha = \sin(90^\circ - 9\alpha) \text{ and } \sin(90^\circ - \alpha) = \sin \alpha$$

$$\text{Thus, } \sin (90^\circ - 9\alpha) = \sin \alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ$$

$$\alpha = 9^\circ$$

Substituting $\alpha = 9^\circ$ in $\tan 5\alpha$, we get,

$$\tan 5\alpha = \tan (5 \times 9) = \tan 45^\circ = 1$$

$$\therefore, \tan 5\alpha = 1$$

Exercise 8.2

Question 1

$$\tan 47^\circ / \cot 43^\circ = 1$$

Solution

True

Justification:

Since, $\tan (90^\circ - \theta) = \cot \theta$

$$\begin{aligned}\frac{\tan 47^\circ}{\cot 43^\circ} &= \frac{\tan(90^\circ - 43^\circ)}{\cot 43^\circ} \\ \frac{\tan 47^\circ}{\cot 43^\circ} &= \frac{\cot 43^\circ}{\cot 43^\circ} = 1 \\ \frac{\tan 47^\circ}{\cot 43^\circ} &= 1\end{aligned}$$

Question 2

The value of the expression $(\cos^2 23^\circ - \sin^2 67^\circ)$ is positive.

Solution

False

Justification:

Since, $(a^2 - b^2) = (a+b)(a-b)$

$$\cos^2 23^\circ - \sin^2 67^\circ = (\cos 23^\circ + \sin 67^\circ)(\cos 23^\circ - \sin 67^\circ)$$

$$= [\cos 23^\circ + \sin(90^\circ - 23^\circ)] [\cos 23^\circ - \sin(90^\circ - 23^\circ)]$$

$$= (\cos 23^\circ + \cos 23^\circ)(\cos 23^\circ - \cos 23^\circ) (\because \sin(90^\circ - \theta) = \cos \theta)$$

$$= (\cos 23^\circ + \cos 23^\circ) \cdot 0$$

$$= 0, \text{ which is neither positive nor negative}$$

Question 3

The value of the expression $(\sin 80^\circ - \cos 80^\circ)$ is negative.

Solution

False

Justification:

We know that,

$\sin \theta$ increases when $0^\circ \leq \theta \leq 90^\circ$

$\cos \theta$ decreases when $0^\circ \leq \theta \leq 90^\circ$

And $(\sin 80^\circ - \cos 80^\circ) = (\text{increasing value} - \text{decreasing value})$

= a positive value.

Therefore, $(\sin 80^\circ - \cos 80^\circ) > 0$.

Question 4

$$\sqrt{(1 - \cos^2 \theta) \sec^2 \theta} = \tan \theta$$

Solution

True

Justification:

$$\begin{aligned} \text{LHS: } & \sqrt{(1 - \cos^2 \theta) \sec^2 \theta} \\ &= \sqrt{\sin^2 \theta \sec^2 \theta} \\ (\because \sin^2 \theta + \cos^2 \theta &= 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta) \\ &= \sqrt{\frac{\sin^2 \theta}{\cos^2 \theta}} \left(\text{Since, } \sec^2 \theta = \frac{1}{\cos^2 \theta} \right) \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned}$$

Question 5

If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.

Solution

True

Justification:

According to the question,

$$\cos A + \cos^2 A = 1$$

$$\text{i.e., } \cos A = 1 - \cos^2 A$$

Since,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

We get,

$$\cos A = \sin^2 A \dots (1)$$

Squaring L.H.S and R.H.S,

$$\cos^2 A = \sin^4 A \dots (2)$$

To find $\sin^2 A + \sin^4 A = 1$

Adding equations (1) and (2),

We get

$$\sin^2 A + \sin^4 A = \cos A + \cos^2 A$$

Therefore, $\sin^2 A + \sin^4 A = 1$

Question 6

$$(\tan \theta + 2)(2 \tan \theta + 1) = 5 \tan \theta + \sec^2 \theta.$$

Solution

False

Justification:

$$\text{L.H.S} = (\tan \theta + 2)(2 \tan \theta + 1)$$

$$= 2 \tan^2 \theta + \tan \theta + 4 \tan \theta + 2$$

$$= 2 \tan^2 \theta + 5 \tan \theta + 2$$

Since, $\sec^2 \theta - \tan^2 \theta = 1$, we get, $\tan^2 \theta = \sec^2 \theta - 1$

$$= 2(\sec^2 \theta - 1) + 5 \tan \theta + 2$$

$$= 2 \sec^2 \theta - 2 + 5 \tan \theta + 2$$

$$= 5 \tan \theta + 2 \sec^2 \theta \neq \text{R.H.S}$$

∴, L.H.S ≠ R.H.S

Exercise 8.3

Question 1

$$\sin \theta / (1 + \cos \theta) + (1 + \cos \theta) / \sin \theta = 2 \operatorname{cosec} \theta$$

Solution

L.H.S=

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$$

Taking the L.C.M of the denominators,

We get,

$$= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{(1 + \cos \theta) \cdot \sin \theta}$$

$$= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{(1 + \cos \theta) \cdot \sin \theta}$$

Since, $\sin^2 \theta + \cos^2 \theta = 1$

$$= \frac{1 + 1 + 2 \cos \theta}{(1 + \cos \theta) \cdot \sin \theta}$$

$$= \frac{2 + 2 \cos \theta}{(1 + \cos \theta) \cdot \sin \theta}$$

$$= \frac{2(1 + \cos \theta)}{(1 + \cos \theta) \cdot \sin \theta}$$

Since, $1 / \sin \theta = \operatorname{cosec} \theta$

$$= \frac{2}{\sin \theta} = 2 \operatorname{cosec} \theta$$

R.H.S

Hence proved.

Question 2

$$\tan A / (1 + \sec A) - \tan A / (1 - \sec A) = 2 \operatorname{cosec} A$$

Solution

L.H.S:

$$\begin{aligned}
& \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} \\
& \text{Taking LCM of the denominators,} \\
& = \frac{\tan A(1-\sec A) - \tan A(1+\sec A)}{(1+\sec A)(1-\sec A)} \\
& \text{Since, } (1 + \sec A)(1 - \sec A) = 1 - \sec^2 A \\
& = \frac{\tan A(1-\sec A-1-\sec A)}{1-\sec^2 A} \\
& = \frac{\tan A(-2 \sec A)}{1-\sec^2 A} \\
& = \frac{2 \tan A \sec A}{\sec^2 A - 1}
\end{aligned}$$

Since,

$$\sec^2 A - \tan^2 A = 1$$

$$\sec^2 A - 1 = \tan^2 A$$

$$\begin{aligned}
& = \frac{2 \tan A \sec A}{\tan^2 A} \\
& \text{Since, } \sec A = (1/\cos A) \text{ and } \tan A = (\sin A / \cos A) \\
& = \frac{2 \sec A}{\tan A} = \frac{2 \cos A}{\cos A \sin A} \\
& = \frac{2}{\sin A} \\
& = 2 \operatorname{cosec} A \left(\because \frac{1}{\sin A} = \operatorname{cosec} A \right)
\end{aligned}$$

= R.H.S

Hence proved.

Question 3

If $\tan A = \frac{3}{4}$, then $\sin A \cos A = 12/25$

Solution

According to the question,

$$\tan A = \frac{3}{4}$$

We know,

$$\tan A = \text{perpendicular} / \text{base}$$

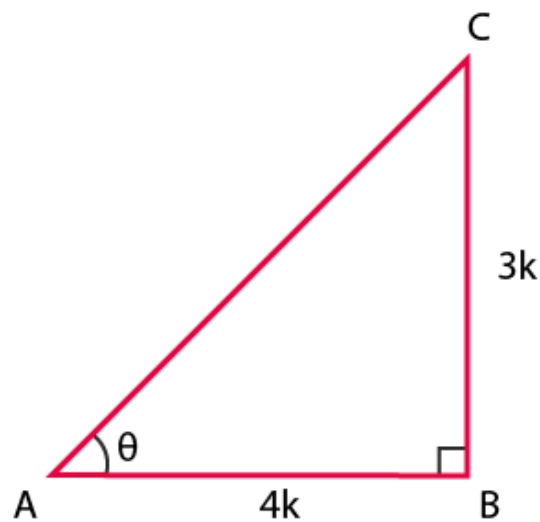
So,

$$\tan A = 3k/4k$$

Where,

$$\text{Perpendicular} = 3k$$

$$\text{Base} = 4k$$



Using Pythagoras Theorem,

$$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$$

$$(\text{hypotenuse})^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$$

$$\text{hypotenuse} = 5k$$

To find $\sin A$ and $\cos A$,

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4k}{5k} = \frac{4}{5}$$

Multiplying $\sin A$ and $\cos A$,

$$\sin A \cos A = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$

Hence, proved.

Question 4

$$(\sin \alpha + \cos \alpha) (\tan \alpha + \cot \alpha) = \sec \alpha + \operatorname{cosec} \alpha$$

Solution

L.H.S:

$$(\sin \alpha + \cos \alpha) (\tan \alpha + \cot \alpha)$$

As we know,

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

$$= (\sin \alpha + \cos \alpha) \left(\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} \right)$$

$$= (\sin \alpha + \cos \alpha) \left(\frac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cos \alpha} \right)$$

$$[\because \sin^2 \alpha + \cos^2 \alpha = 1]$$

$$= (\sin \alpha + \cos \alpha) \left(\frac{1}{\sin \alpha \cos \alpha} \right)$$

$$= \frac{\sin \alpha}{\sin \alpha \cos \alpha} + \frac{\cos \alpha}{\sin \alpha \cos \alpha}$$

$$= \frac{1}{\cos \alpha} + \frac{1}{\sin \alpha}$$

$$= \sec \alpha + \operatorname{cosec} \alpha \left[\because \frac{1}{\cos \alpha} = \sec \alpha \text{ and } \frac{1}{\sin \alpha} = \operatorname{cosec} \alpha \right]$$

= R.H.S

Hence, proved.

Question 5

$$(\sqrt{3}+1) (3 - \cot 30^\circ) = \tan^3 60^\circ - 2 \sin 60^\circ$$

Solution

$$\text{L.H.S: } (\sqrt{3} + 1) (3 - \cot 30^\circ)$$

$$= (\sqrt{3} + 1) (3 - \sqrt{3}) [\because \cot 30^\circ = \sqrt{3}]$$

$$= (\sqrt{3} + 1) \sqrt{3} (\sqrt{3} - 1) [\because (3 - \sqrt{3}) = \sqrt{3} (\sqrt{3} - 1)]$$

$$= ((\sqrt{3})^2 - 1) \sqrt{3} [\because (\sqrt{3}+1)(\sqrt{3}-1) = ((\sqrt{3})^2 - 1)]$$

$$= (3-1) \sqrt{3}$$

$$= 2\sqrt{3}$$

Similarly solving R.H.S: $\tan^3 60^\circ - 2 \sin 60^\circ$

Since, $\tan 60^\circ = \sqrt{3}$ and $\sin 60^\circ = \sqrt{3}/2$,

We get,

$$(\sqrt{3})^3 - 2 \cdot (\sqrt{3}/2) = 3\sqrt{3} - \sqrt{3}$$

$$= 2\sqrt{3}$$

Therefore, L.H.S = R.H.S

Hence, proved.

Question 6

$$1 + (\cot^2 \alpha / 1 + \operatorname{cosec} \alpha) = \operatorname{cosec} \alpha$$

Solution

L.H.S:

Since,

$$\cot^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} \text{ and } \operatorname{cosec} \alpha = \frac{1}{\sin \alpha}]$$

We get,

$$1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = 1 + \frac{\cos^2 \alpha / \sin^2 \alpha}{1 + 1/\sin \alpha}$$

$$= 1 + \frac{\cos^2 \alpha / \sin^2 \alpha}{\frac{\sin \alpha + 1}{\sin \alpha}}$$

$$= 1 + \frac{\cos^2 \alpha}{\sin \alpha (1 + \sin \alpha)}$$

$$= \frac{\sin \alpha + \sin^2 \alpha + \cos^2 \alpha}{\sin \alpha + \sin^2 \alpha}$$

And, we know that,

$$\begin{aligned}
 \sin^2 \alpha + \cos^2 \alpha &= 1 \\
 &= \frac{1 + \sin \alpha}{\sin \alpha (1 + \sin \alpha)} \\
 \text{Since,} \\
 \frac{1}{\sin \alpha} &= \operatorname{cosec} \alpha \\
 &= \frac{1}{\sin \alpha} = \operatorname{cosec} \alpha \\
 &= \text{R.H.S}
 \end{aligned}$$

Question 7

$$\tan \theta + \tan (90^\circ - \theta) = \sec \theta \sec (90^\circ - \theta)$$

Solution

$$\text{L.H.S} =$$

$$\text{Since, } \tan (90^\circ - \theta) = \cot \theta$$

$$\tan \theta + \tan (90^\circ - \theta) = \tan \theta + \cot \theta$$

$$\begin{aligned}
 &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{Since,} \\
 \sin^2 \theta + \cos^2 \theta &= 1 \\
 &= \frac{1}{\sin \theta \cos \theta} \\
 &= \frac{1}{\cos \theta} \times \frac{1}{\sin \theta} \\
 &= \sec \theta \operatorname{cosec} \theta
 \end{aligned}$$

$$\begin{aligned}
 \text{Since,} \\
 \operatorname{cosec} \theta &= \sec (90^\circ - \theta) \\
 &= \sec \theta \sec (90^\circ - \theta) \\
 &= \text{R.H.S}
 \end{aligned}$$

Hence, proved.

Exercise 8.4

Question 1

If $\operatorname{cosec} \theta + \cot \theta = p$, then prove that $\cos \theta = (p^2 - 1) / (p^2 + 1)$.

Solution

According to the question,

$$\operatorname{cosec} \theta + \cot \theta = p$$

Since,

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \text{ \& \; } \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = p$$

$$\frac{1+\cos \theta}{\sin \theta} = p$$

Squaring on L.H.S and R.H.S,

$$\left(\frac{1+\cos \theta}{\sin \theta} \right)^2 = p^2$$

$$\frac{1+\cos^2 \theta+2 \cos \theta}{\sin^2 \theta} = p^2$$

Applying component and dividend rule,

$$\frac{(1+\cos^2 \theta+2 \cos \theta)-\sin^2 \theta}{(1+\cos^2 \theta+2 \cos \theta)+\sin^2 \theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{(1-\sin^2 \theta)+\cos^2 \theta+2 \cos \theta}{\sin^2 \theta+\cos^2 \theta+1+2 \cos \theta} = \frac{p^2-1}{p^2+1}$$

Since,

$$1-\sin^2 \theta = \cos^2 \theta \text{ \& \; } \sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{\cos^2 \theta+\cos^2 \theta+2 \cos \theta}{1+1+2 \cos \theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{2 \cos^2 \theta+2 \cos \theta}{2+2 \cos \theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{2 \cos \theta (\cos \theta+1)}{2 (\cos \theta+1)} = \frac{p^2-1}{p^2+1}$$

$$\cos \theta = \frac{p^2-1}{p^2+1}$$

Hence, proved.

Question 2

Prove that $\sqrt{(\sec^2 \theta + \operatorname{cosec}^2 \theta)} = \tan \theta + \cot \theta$

Solution

L.H.S=

$$\sqrt{(\sec^2 \theta + \operatorname{cosec}^2 \theta)}$$

Since,

$$\sqrt{(\sec^2 \theta + \operatorname{cosec}^2 \theta)}$$

Since,

$$\sec^2 \theta = \frac{1}{\cos^2 \theta} \text{ \& } \operatorname{cosec}^2 \theta = \frac{1}{\sin^2 \theta}$$

$$= \sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}$$

$$= \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}}$$

Since,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$= \sqrt{\frac{1}{\cos^2 \theta \sin^2 \theta}}$$

$$= \frac{1}{\cos \theta \sin \theta}$$

Since,

$$1 = \sin^2 \theta + \cos^2 \theta$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta \sin \theta} + \frac{\cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

Since,

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \text{ \& } \frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$= \tan \theta + \cot \theta$$

= R.H.S

Hence, proved.

Question 3

The angle of elevation of the top of a tower from certain point is 30° . If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15° . Find the height of the tower.

Solution

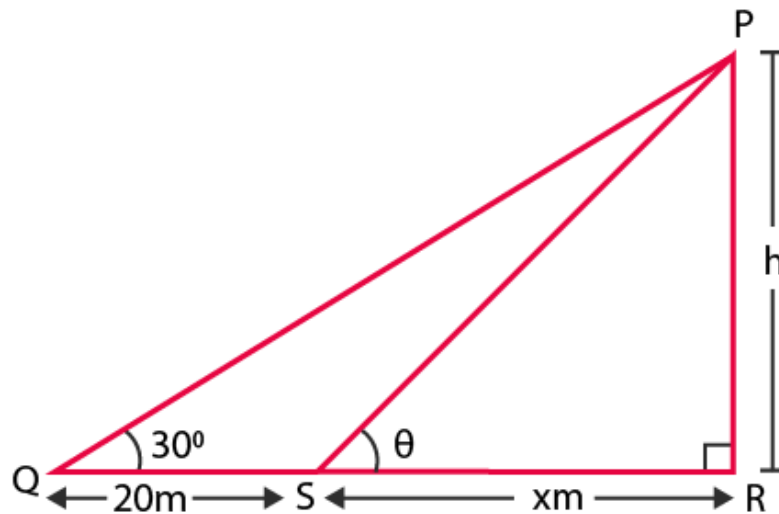
Let PR = h meter, be the height of the tower.

The observer is standing at point Q such that, the distance between the observer and tower is QR = (20+x) m, where

$$QR = QS + SR = 20 + x$$

$$\angle PQR = 30^\circ$$

$$\angle PSR = \theta$$



In $\triangle PQR$,

$$\tan 30^\circ = \frac{h}{20+x} \quad [\because \tan \theta = \frac{\text{perpendicular}}{\text{base}}]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x} \quad [\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

Rearranging the terms,

$$\text{We get } 20 + x = \sqrt{3}h$$

$$\Rightarrow x = \sqrt{3}h - 20 \quad \dots \text{eq.1}$$

In $\triangle PSR$,

$$\tan \theta = h/x$$

Since, angle of elevation increases by 15° when the observer moves 20 m towards the tower.

We have,

$$\theta = 30^\circ + 15^\circ = 45^\circ$$

So,

$$\tan 45^\circ = h/x$$

$$\Rightarrow 1 = h/x$$

$$\Rightarrow h = x$$

Substituting $x=h$ in eq. 1, we get

$$h = \sqrt{3} h - 20$$

$$\Rightarrow \sqrt{3} h - h = 20$$

$$\Rightarrow h (\sqrt{3} - 1) = 20$$

$$\Rightarrow h = \frac{20}{\sqrt{3}-1}$$

Rationalizing the denominator, we have

$$\Rightarrow h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow h = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{20(\sqrt{3}+1)}{3-1}$$

$$= \frac{20(\sqrt{3}+1)}{2}$$

$$= 10(\sqrt{3} + 1)$$

Hence, the required height of the tower is $10(\sqrt{3} + 1)$ meter.

Question 4

If $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$, then prove that $\tan \theta = 1$ or $\frac{1}{2}$.

Solution

$$\text{Given: } 1 + \sin^2 \theta = 3 \sin \theta \cos \theta$$

Dividing L.H.S and R.H.S equations with $\sin^2 \theta$,

We get,

$$\frac{1 + \sin^2 \theta}{\sin^2 \theta} = \frac{3 \sin \theta \cos \theta}{\sin^2 \theta}$$

$$\Rightarrow \frac{1}{\sin^2 \theta} + 1 = \frac{3 \cos \theta}{\sin \theta}$$

$$\operatorname{cosec}^2 \theta + 1 = 3 \cot \theta$$

Since,

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \Rightarrow \operatorname{cosec}^2 \theta = \cot^2 \theta + 1$$

$$\Rightarrow \cot^2 \theta + 1 + 1 = 3 \cot \theta$$

$$\Rightarrow \cot^2 \theta + 2 = 3 \cot \theta$$

$$\Rightarrow \cot^2 \theta - 3 \cot \theta + 2 = 0$$

Splitting the middle term and then solving the equation,

$$\Rightarrow \cot^2 \theta - \cot \theta - 2 \cot \theta + 2 = 0$$

$$\Rightarrow \cot \theta (\cot \theta - 1) - 2(\cot \theta + 1) = 0$$

$$\Rightarrow (\cot \theta - 1)(\cot \theta - 2) = 0$$

$$\Rightarrow \cot \theta = 1, 2$$

Since,

$$\tan \theta = 1/\cot \theta$$

$$\tan \theta = 1, \frac{1}{2}$$

Hence, proved.

Question 5

Given that $\sin \theta + 2 \cos \theta = 1$, then prove that $2 \sin \theta - \cos \theta = 2$.

Solution

Given: $\sin \theta + 2 \cos \theta = 1$

Squaring on both sides,

$$(\sin \theta + 2 \cos \theta)^2 = 1$$

$$\Rightarrow \sin^2 \theta + 4 \cos^2 \theta + 4 \sin \theta \cos \theta = 1$$

Since, $\sin^2 \theta = 1 - \cos^2 \theta$ and $\cos^2 \theta = 1 - \sin^2 \theta$

$$\Rightarrow (1 - \cos^2 \theta) + 4(1 - \sin^2 \theta) + 4 \sin \theta \cos \theta = 1$$

$$\Rightarrow 1 - \cos^2 \theta + 4 - 4 \sin^2 \theta + 4 \sin \theta \cos \theta = 1$$

$$\Rightarrow -4 \sin^2 \theta - \cos^2 \theta + 4 \sin \theta \cos \theta = -4$$

$$\Rightarrow 4 \sin^2 \theta + \cos^2 \theta - 4 \sin \theta \cos \theta = 4$$

We know that,

$$a^2 + b^2 - 2ab = (a - b)^2$$

So, we get,

$$(2 \sin \theta - \cos \theta)^2 = 4$$

$$\Rightarrow 2 \sin \theta - \cos \theta = 2$$

Hence proved.

Question 6

The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is $\sqrt{st}$.

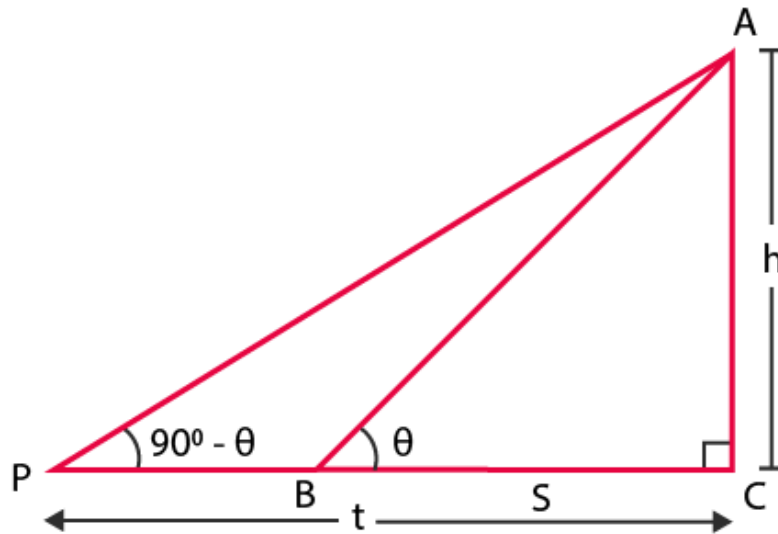
Solution

Let $BC = s$; $PC = t$

Let height of the tower be $AB = h$.

$$\angle ABC = \theta \text{ and } \angle APC = 90^\circ - \theta$$

($\because$ the angle of elevation of the top of the tower from two points P and B are complementary)



$$\text{In } \triangle ABC, \tan \theta = \frac{AC}{BC} = \frac{h}{s} \dots \text{eq. 1} \left[\because \tan \theta = \frac{\text{perpendicular}}{\text{base}} \right]$$

$$\text{In } \triangle APC, \tan(90^\circ - \theta) = \frac{AC}{PC} = \frac{h}{s}$$

$$\Rightarrow \cot \theta = \frac{h}{s} \dots \text{eq. 2}$$

Multiplying eq. 1 and eq. 2, we get

$$\tan \theta \times \cot \theta = \frac{h}{s} \times \frac{h}{s}$$

$$\Rightarrow 1 = \frac{h^2}{st} \left[\because \tan \theta \times \cot \theta = 1 \text{ as } \cot \theta = \frac{1}{\tan \theta} \right]$$

$$\Rightarrow h^2 = st$$

$$\Rightarrow h = \sqrt{st}$$

Hence the height of the tower is $\sqrt{st}$.

Question 7

The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60° . Find the height of the tower.

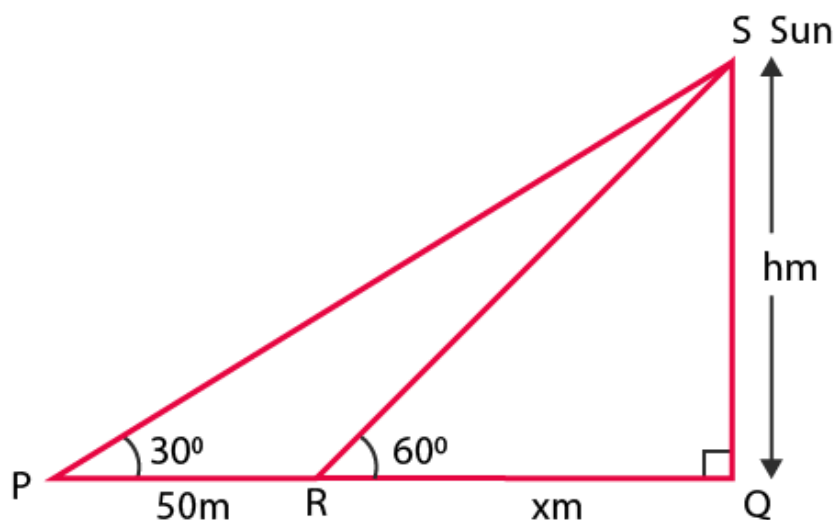
Solution

Let $SQ = h$ be the tower.

$$\angle SPQ = 30^\circ \text{ and } \angle SRQ = 60^\circ$$

According to the question, the length of shadow is 50 m long when angle of elevation of the sun is 30° than when it was 60° . So,

PR = 50 m and RQ = x m



So in ΔSRQ , we have

$$\begin{aligned}\tan 60^\circ &= \frac{h}{x} \\ [\because \tan \theta &= \frac{\text{perpendicular}}{\text{base}}] \\ \Rightarrow \tan 60^\circ &= \frac{SQ}{RQ} \\ \Rightarrow \sqrt{3} &= \frac{h}{x} [\because \tan 60^\circ = \sqrt{3}] \\ \Rightarrow x &= \frac{h}{\sqrt{3}}\end{aligned}$$

In ΔSPQ ,

$$\begin{aligned}\tan 30^\circ &= \frac{h}{50+x} \\ [\because \tan 30^\circ &= \frac{SQ}{PQ} = \frac{SQ}{PR+PQ}] \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{h}{50+x} [\because \tan 30^\circ = \frac{1}{\sqrt{3}}] \\ \Rightarrow 50 + x &= \sqrt{3}h\end{aligned}$$

Substituting the value of x in the above equation, we get

$$\begin{aligned}\Rightarrow 50 + \frac{h}{\sqrt{3}} &= \sqrt{3}h \\ \Rightarrow \frac{50\sqrt{3}+h}{\sqrt{3}} &= \sqrt{3}h\end{aligned}$$

$$\Rightarrow 50\sqrt{3}+h = 3h$$

$$\Rightarrow 50\sqrt{3} = 3h - h$$

$$\Rightarrow 3h - h = 50\sqrt{3}$$

$$\Rightarrow 2h = 50\sqrt{3}$$

$$\Rightarrow h = (50\sqrt{3})/2$$

$$\Rightarrow h = 25\sqrt{3}$$

Hence, the required height is $25\sqrt{3}$ m.

Question 8

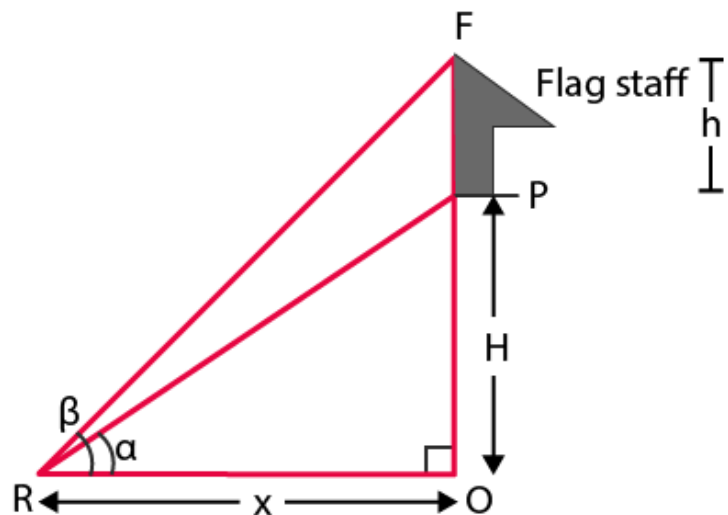
A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h . At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β , respectively. Prove that the height of the tower is $[h \tan \alpha / (\tan \beta - \tan \alpha)]$.

Solution

Given that a vertical flag staff of height h is surmounted on a vertical tower of height H (say),

such that $FP = h$ and $FO = H$.

The angle of elevation of the bottom and top of the flag staff on the plane is $\angle PRO = \alpha$ and $\angle FRO = \beta$ respectively



In $\triangle PRO$, we have

$$\tan \alpha = \frac{PO}{RO} = \frac{H}{x}$$

$$[\because \tan \theta = \frac{\text{perpendicular}}{\text{base}}]$$

$$\Rightarrow x = \frac{H}{\tan \alpha} \dots \text{eq. 1}$$

And in ΔFRO , we have

$$\tan \beta = \frac{FO}{RO} = \frac{FP+PO}{RO} = \frac{h+H}{x}$$

$$\Rightarrow x = \frac{h+H}{\tan \beta} \dots \text{eq. 2}$$

Comparing eq. 1 and eq. 2,

$$\Rightarrow \frac{H}{\tan \alpha} = \frac{h+H}{\tan \beta}$$

Solving for H,

$$\Rightarrow H \tan \beta = (h+H) \tan \alpha$$

$$\Rightarrow H \tan \beta - H \tan \alpha = h \tan \alpha$$

$$\Rightarrow H (\tan \beta - \tan \alpha) = h \tan \alpha$$

$$\Rightarrow H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

Hence, proved.

Question 9

If $\tan \theta + \sec \theta = I$, then prove that $\sec \theta = (I^2 + 1)/2I$.

Solution

Given: $\tan \theta + \sec \theta = I \dots \text{eq. 1}$

Multiplying and dividing by $(\sec \theta - \tan \theta)$ on numerator and denominator of L.H.S,

$$\frac{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{\sec \theta - \tan \theta} = I$$

$$\Rightarrow \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} = I$$

Since, $\sec^2 \theta - \tan^2 \theta = 1$

$$\Rightarrow \frac{1}{\sec \theta - \tan \theta} = I$$

So, $\sec \theta - \tan \theta = 1 \dots \text{eq. 2}$

Adding eq. 1 and eq. 2, we get

$$(\tan \theta + \sec \theta) + (\sec \theta - \tan \theta) = 1$$

$$\Rightarrow 2 \sec \theta = \frac{l^2+1}{1}$$
$$\Rightarrow \sec \theta = \frac{l^2+1}{21}$$

Hence, proved.

Class 10 students are advised to solve sample papers and previous years' question papers to have an idea of the types of questions asked from the chapter **Trigonometry**. Also, find our online reading materials, such as notes, exemplar books, 10th Class Maths NCERT Solutions and question papers in downloadable PDF format to have a quick revision before the board exam.

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