

Chapter 8: Quadrilaterals

NCERT · Class 9 · Maths

Exercise 8.1 · Textbook page 146

Question 1

The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Solution

Let the common ratio between the angles be x .

We know that the sum of the interior angles of the quadrilateral = 360°

Now,

$$3x + 5x + 9x + 13x = 360^\circ$$

$$\Rightarrow 30x = 360^\circ$$

$$\Rightarrow x = 12^\circ$$

, Angles of the quadrilateral are:

$$3x = 3 \times 12^\circ = 36^\circ$$

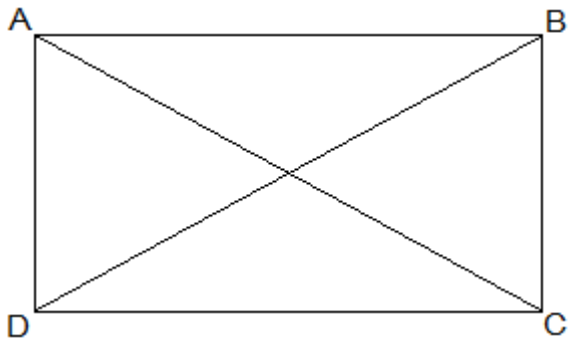
$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

Question 2

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution

Given that,

$$AC = BD$$

To show that ABCD is a rectangle if the diagonals of a parallelogram are equal

To show ABCD is a rectangle, we have to prove that one of its interior angles is right-angled.

Proof,

In $\triangle ABC$ and $\triangle BAD$,

$$AB = BA \text{ (Common)}$$

$$BC = AD \text{ (Opposite sides of a parallelogram are equal)}$$

$$AC = BD \text{ (Given)}$$

Therefore, $\triangle ABC \cong \triangle BAD$ [SSS congruency]

$$\angle A = \angle B \text{ [Corresponding parts of Congruent Triangles]}$$

also,

$$\angle A + \angle B = 180^\circ \text{ (Sum of the angles on the same side of the transversal)}$$

$$\Rightarrow 2\angle A = 180^\circ$$

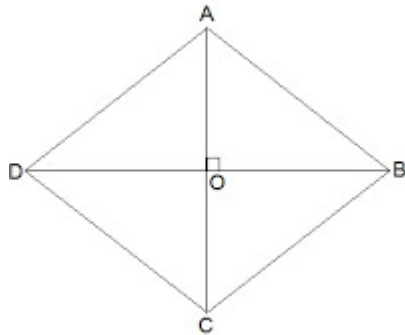
$$\Rightarrow \angle A = 90^\circ = \angle B$$

Therefore, ABCD is a rectangle.

Hence Proved.

Question 3

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution

Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given that,

$$OA = OC$$

$$OB = OD$$

$$\text{and } \angle AOB = \angle BOC = \angle OCD = \angle ODA = 90^\circ$$

To show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus, we have to prove that ABCD is a parallelogram and $AB = BC = CD = AD$

Proof,

In $\triangle AOB$ and $\triangle COB$,

$$OA = OC \text{ (Given)}$$

$$\angle AOB = \angle COB \text{ (Opposite sides of a parallelogram are equal)}$$

$$OB = OB \text{ (Common)}$$

Therefore, $\triangle AOB \cong \triangle COB$ [SAS congruency]

$$\text{Thus, } AB = BC \text{ [CPCT]}$$

Similarly, we can prove,

$$BC = CD$$

$$CD = AD$$

$$AD = AB$$

$$, AB = BC = CD = AD$$

Opposite sides of a quadrilateral are equal. Hence, it is a parallelogram.

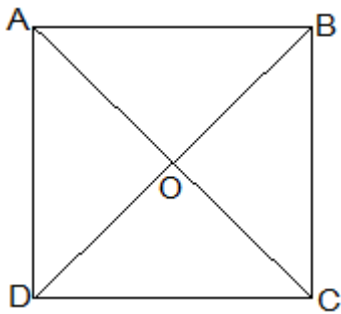
ABCD is rhombus as it is a parallelogram whose diagonals intersect at a right angle.

Hence Proved.

Question 4

Show that the diagonals of a square are equal and bisect each other at right angles.

Solution



Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show that,

$$AC = BD$$

$$AO = OC$$

$$\text{and } \angle AOB = 90^\circ$$

Proof,

In $\triangle ABC$ and $\triangle BAD$,

$$AB = BA \text{ (Common)}$$

$$\angle ABC = \angle BAD = 90^\circ$$

$$BC = AD \text{ (Given)}$$

$$\triangle ABC \cong \triangle BAD \text{ [SAS congruency]}$$

Thus,

$$AC = BD \text{ [CPCT]}$$

diagonals are equal.

Now,

In $\triangle AOB$ and $\triangle COD$,

$$\angle BAO = \angle DCO \text{ (Alternate interior angles)}$$

$$\angle AOB = \angle COD \text{ (Vertically opposite)}$$

$$AB = CD \text{ (Given)}$$

$$\therefore \triangle AOB \cong \triangle COD \text{ [AAS congruency]}$$

Thus,

$$AO = CO \text{ [CPCT]}.$$

, Diagonal bisect each other.

Now,

In $\triangle AOB$ and $\triangle COB$,

$$OB = OB \text{ (Given)}$$

$$AO = CO \text{ (diagonals are bisected)}$$

$$AB = CB \text{ (Sides of the square)}$$

$$\therefore \triangle AOB \cong \triangle COB \text{ [SSS congruency]}$$

$$\text{also, } \angle AOB = \angle COB$$

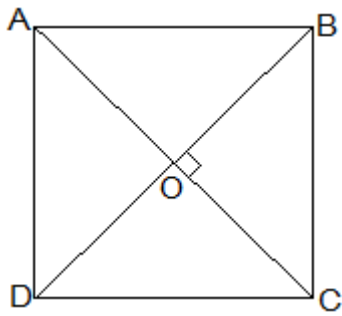
$$\angle AOB + \angle COB = 180^\circ \text{ (Linear pair)}$$

$$\text{Thus, } \angle AOB = \angle COB = 90^\circ$$

, Diagonals bisect each other at right angles

Question 5

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution

Given that,

Let ABCD be a quadrilateral and its diagonals AC and BD bisect each other at a right angle at O.

To prove that,

The Quadrilateral ABCD is a square.

Proof,

In $\triangle AOB$ and $\triangle COD$,

$AO = CO$ (Diagonals bisect each other)

$\angle AOB = \angle COD$ (Vertically opposite)

$OB = OD$ (Diagonals bisect each other)

, $\triangle AOB \cong \triangle COD$ [SAS congruency]

Thus,

$AB = CD$ [CPCT] — (i)

also,

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\Rightarrow AB \parallel CD$

Now,

In $\triangle AOD$ and $\triangle COB$,

$AO = CO$ (Diagonals bisect each other)

$\angle AOD = \angle COB$ (Vertically opposite)

$OD = OD$ (Common)

, $\triangle AOD \cong \triangle COD$ [SAS congruency]

Thus,

$AD = CD$ [CPCT] — (ii)

also,

$AD = BC$ and $AD = CD$

$\Rightarrow AD = BC = CD = AB$ — (ii)

also, $\angle ADC = \angle BCD$ [CPCT]

and $\angle ADC + \angle BCD = 180^\circ$ (co-interior angles)

$\Rightarrow 2\angle ADC = 180^\circ$

$\Rightarrow \angle ADC = 90^\circ$ — (iii)

One of the interior angles is a right angle.

Thus, from (i), (ii) and (iii), given quadrilateral ABCD is a square.

Hence Proved.

Question 6

Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see Fig. 8.19). Show that

(i) it bisects $\angle C$ also,

(ii) ABCD is a rhombus.

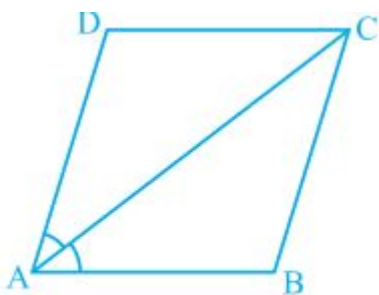


Fig. 8.19

Solution

(i) In $\triangle ADC$ and $\triangle CBA$,

$AD = CB$ (Opposite sides of a parallelogram)

$DC = BA$ (Opposite sides of a parallelogram)

$AC = CA$ (Common Side)

, $\triangle ADC \cong \triangle CBA$ [SSS congruency]

Thus,

$\angle ACD = \angle CAB$ by CPCT

and $\angle CAB = \angle CAD$ (Given)

$\Rightarrow \angle ACD = \angle BCA$

Thus,

AC bisects $\angle C$ also.

(ii) $\angle ACD = \angle CAD$ (Proved above)

$\Rightarrow AD = CD$ (Opposite sides of equal angles of a triangle are equal)

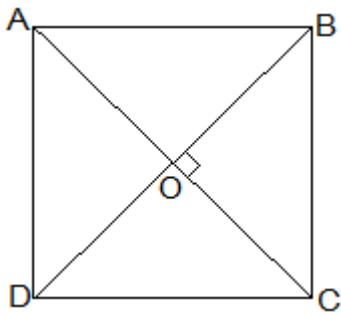
Also, $AB = BC = CD = DA$ (Opposite sides of a parallelogram)

Thus,

$ABCD$ is a rhombus.

Question 7

$ABCD$ is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution

Given that,

ABCD is a rhombus.

AC and BD are its diagonals.

Proof,

$AD = CD$ (Sides of a rhombus)

$\angle DAC = \angle DCA$ (Angles opposite of equal sides of a triangle are equal.)

also, $AB \parallel CD$

$\Rightarrow \angle DAC = \angle BCA$ (Alternate interior angles)

$\Rightarrow \angle DCA = \angle BCA$

, AC bisects $\angle C$.

Similarly,

We can prove that diagonal AC bisects $\angle A$.

Following the same method,

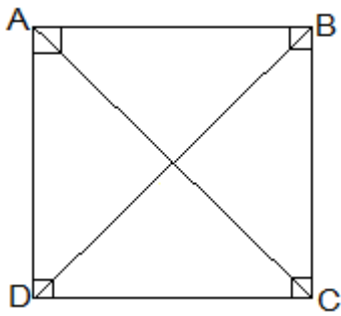
We can prove that the diagonal BD bisects $\angle B$ and $\angle D$.

Question 8

ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

(i) ABCD is a square

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.

Solution

(i) $\angle DAC = \angle DCA$ (AC bisects $\angle A$ as well as $\angle C$)

$\Rightarrow AD = CD$ (Sides opposite to equal angles of a triangle are equal)

also, $CD = AB$ (Opposite sides of a rectangle)

, $AB = BC = CD = AD$

Thus, ABCD is a square.

(ii) In $\triangle BCD$,

$BC = CD$

$\Rightarrow \angle CDB = \angle CBD$ (Angles opposite to equal sides are equal)

also, $\angle CDB = \angle ABD$ (Alternate interior angles)

$\Rightarrow \angle CBD = \angle ABD$

Thus, BD bisects $\angle B$

Now,

$\angle CBD = \angle ADB$

$\Rightarrow \angle CDB = \angle ADB$

Thus, BD bisects $\angle B$ as well as $\angle D$.

Question 9

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see Fig. 8.20). Show that:

(i) $\triangle APD \cong \triangle CQB$

(ii) $AP = CQ$

(iii) $\triangle AQB \cong \triangle CPD$

(iv) $AQ = CP$

(v) $APCQ$ is a parallelogram

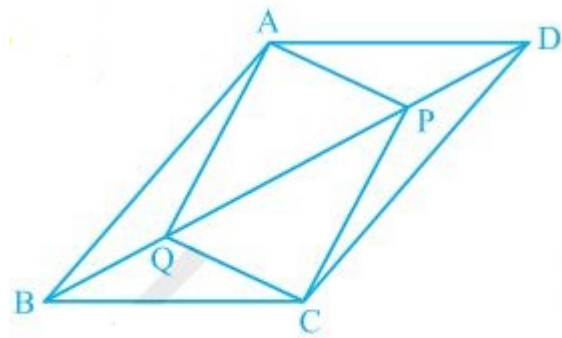


Fig. 8.20

Solution

(i) In $\triangle APD$ and $\triangle CQB$,

$DP = BQ$ (Given)

$\angle ADP = \angle CBQ$ (Alternate interior angles)

$AD = BC$ (Opposite sides of a parallelogram)

Thus, $\triangle APD \cong \triangle CQB$ [SAS congruency]

(ii) $AP = CQ$ by CPCT as $\triangle APD \cong \triangle CQB$.

(iii) In $\triangle AQB$ and $\triangle CPD$,

$BQ = DP$ (Given)

$\angle ABQ = \angle CDP$ (Alternate interior angles)

$AB = CD$ (Opposite sides of a parallelogram)

Thus, $\triangle AQB \cong \triangle CPD$ [SAS congruency]

(iv) As $\triangle AQB \cong \triangle CPD$

$AQ = CP$ [CPCT]

(v) From the questions (ii) and (iv), it is clear that APCQ has equal opposite sides and also has equal and opposite angles. , APCQ is a parallelogram.

Question 10

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that

(i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$

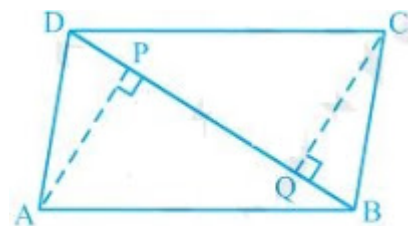


Fig. 8.21

Solution

(i) In $\triangle APB$ and $\triangle CQD$,

$\angle ABP = \angle CDQ$ (Alternate interior angles)

$\angle APB = \angle CQD$ ($= 90^\circ$ as AP and CQ are perpendiculars)

$AB = CD$ (ABCD is a parallelogram)

, $\triangle APB \cong \triangle CQD$ [AAS congruency]

(ii) As $\triangle APB \cong \triangle CQD$.

, $AP = CQ$ [CPCT]

Question 11

In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F, respectively (see Fig. 8.22).

Show that

(i) quadrilateral ABED is a parallelogram

(ii) quadrilateral BEFC is a parallelogram

(iii) $AD \parallel CF$ and $AD = CF$

(iv) quadrilateral $ACFD$ is a parallelogram

(v) $AC = DF$

(vi) $\triangle ABC \cong \triangle DEF$.

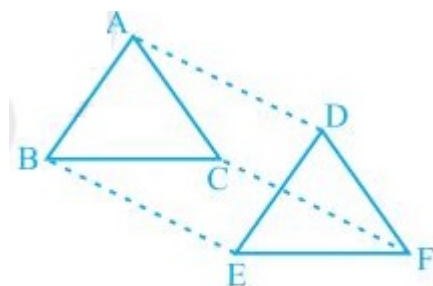


Fig. 8.22

Solution

(i) $AB = DE$ and $AB \parallel DE$ (Given)

Two opposite sides of a quadrilateral are equal and parallel to each other.

Thus, quadrilateral $ABED$ is a parallelogram

(ii) Again $BC = EF$ and $BC \parallel EF$.

Thus, quadrilateral $BEFC$ is a parallelogram.

(iii) Since $ABED$ and $BEFC$ are parallelograms.

$\Rightarrow AD = BE$ and $BE = CF$ (Opposite sides of a parallelogram are equal)

, $AD = CF$.

Also, $AD \parallel BE$ and $BE \parallel CF$ (Opposite sides of a parallelogram are parallel)

, $AD \parallel CF$

(iv) AD and CF are opposite sides of quadrilateral $ACFD$ which are equal and parallel to each other.

Thus, it is a parallelogram.

(v) Since $ACFD$ is a parallelogram

$AC \parallel DF$ and $AC = DF$

(vi) In $\triangle ABC$ and $\triangle DEF$,

$$AB = DE \text{ (Given)}$$

$$BC = EF \text{ (Given)}$$

$$AC = DF \text{ (Opposite sides of a parallelogram)}$$

$$\therefore \triangle ABC \cong \triangle DEF \text{ [SSS congruency]}$$

Question 12

ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see Fig. 8.23). Show that

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

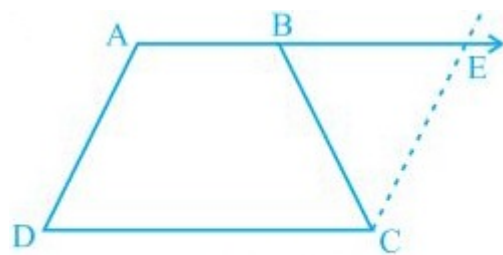


Fig. 8.23

A.

$$\angle A = \angle B$$

B.

$$\angle C = \angle D$$

C.

$$\triangle ABC \cong \triangle BAD$$

D.

$$\text{diagonal AC} = \text{diagonal BD}$$

Solution

To Construct: Draw a line through C parallel to DA intersecting AB produced at E.

(i) $CE = AD$ (Opposite sides of a parallelogram)

$$AD = BC \text{ (Given)}$$

$$, BC = CE$$

$$\Rightarrow \angle CBE = \angle CEB$$

also,

$$\angle A + \angle CBE = 180^\circ \text{ (Angles on the same side of transversal and } \angle CBE = \angle CEB)$$

$$\angle B + \angle CBE = 180^\circ \text{ (As Linear pair)}$$

$$\Rightarrow \angle A = \angle B$$

$$\text{(ii) } \angle A + \angle D = \angle B + \angle C = 180^\circ \text{ (Angles on the same side of transversal)}$$

$$\Rightarrow \angle A + \angle D = \angle A + \angle C \text{ (} \angle A = \angle B)$$

$$\Rightarrow \angle D = \angle C$$

(iii) In $\triangle ABC$ and $\triangle BAD$,

$$AB = AB \text{ (Common)}$$

$$\angle DBA = \angle CBA$$

$$AD = BC \text{ (Given)}$$

$$, \triangle ABC \cong \triangle BAD \text{ [SAS congruency]}$$

(iv) Diagonal $AC =$ diagonal BD by CPCT as $\triangle ABC \cong \triangle BAD$.

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Question 1

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that:

(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

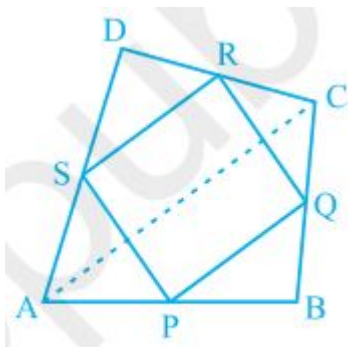


Fig. 8.29

Solution

(i) In $\triangle DAC$,

R is the mid point of DC and S is the mid point of DA.

Thus by mid point theorem, $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) In $\triangle BAC$,

P is the mid point of AB and Q is the mid point of BC.

Thus by mid point theorem, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

also, $SR = \frac{1}{2} AC$

, $PQ = SR$

(iii) $SR \parallel AC$ ————— from question (i)

and, $PQ \parallel AC$ ————— from question (ii)

$\Rightarrow SR \parallel PQ$ – from (i) and (ii)

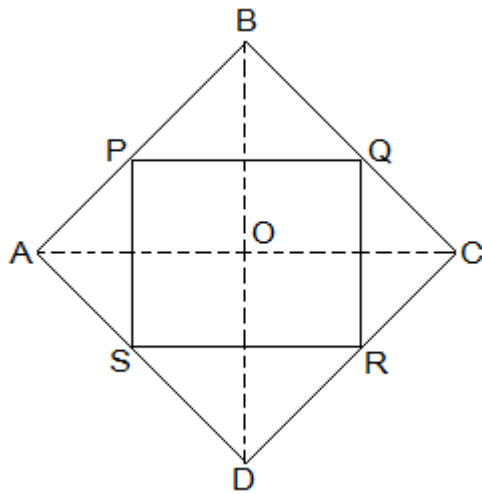
also, $PQ = SR$

, PQRS is a parallelogram.

Question 2

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rectangle.

Solution



Given in the question,

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively.

To Prove,

PQRS is a rectangle.

Construction,

Join AC and BD.

Proof:

In $\triangle DRS$ and $\triangle BPQ$,

$DS = BQ$ (Halves of the opposite sides of the rhombus)

$\angle SDR = \angle QBP$ (Opposite angles of the rhombus)

$DR = BP$ (Halves of the opposite sides of the rhombus)

, $\triangle DRS \cong \triangle BPQ$ [SAS congruency]

$RS = PQ$ [CPCT] ————— (i)

In $\triangle QCR$ and $\triangle SAP$,

$RC = PA$ (Halves of the opposite sides of the rhombus)

$\angle RCQ = \angle PAS$ (Opposite angles of the rhombus)

$CQ = AS$ (Halves of the opposite sides of the rhombus)

, $\triangle QCR \cong \triangle SAP$ [SAS congruency]

$RQ = SP$ [CPCT]————— (ii)

Now,

In $\triangle CDB$,

R and Q are the mid points of CD and BC, respectively.

$\Rightarrow QR \parallel BD$

also,

P and S are the mid points of AD and AB, respectively.

$\Rightarrow PS \parallel BD$

$\Rightarrow QR \parallel PS$

, PQRS is a parallelogram.

also, $\angle PQR = 90^\circ$

Now,

In PQRS,

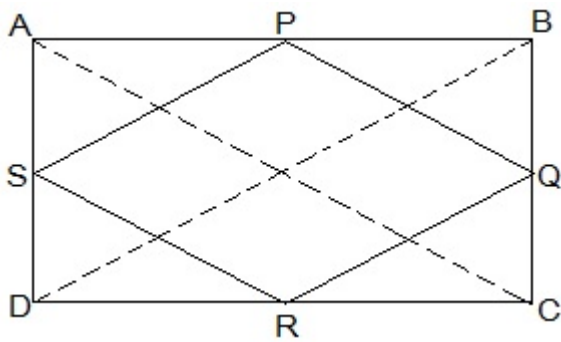
$RS = PQ$ and $RQ = SP$ from (i) and (ii)

$\angle Q = 90^\circ$

, PQRS is a rectangle.

Question 3

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rhombus.

Solution

Given in the question,

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA, respectively.

Construction,

Join AC and BD.

To Prove,

PQRS is a rhombus.

Proof:

In $\triangle ABC$

P and Q are the mid-points of AB and BC, respectively

, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (Midpoint theorem) — (i)

In $\triangle ADC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (Midpoint theorem) — (ii)

So, $PQ \parallel SR$ and $PQ = SR$

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

, $PS \parallel QR$ and $PS = QR$ (Opposite sides of parallelogram) — (iii)

Now,

In $\triangle BCD$,

Q and R are mid points of side BC and CD, respectively.

, $QR \parallel BD$ and $QR = \frac{1}{2} BD$ (Midpoint theorem) — (iv)

$AC = BD$ (Diagonals of a rectangle are equal) — (v)

From equations (i), (ii), (iii), (iv) and (v),

$$PQ = QR = SR = PS$$

So, PQRS is a rhombus.

Hence Proved

Question 4

ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

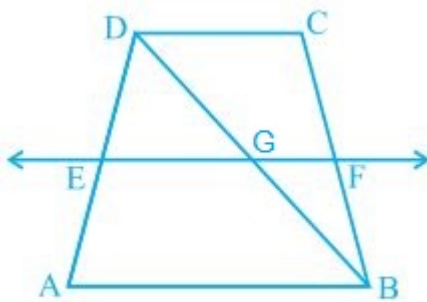


Fig. 8.30

Solution

Given that,

ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD.

To prove,

F is the mid-point of BC.

Proof,

BD intersected EF at G.

In $\triangle BAD$,

E is the mid point of AD and also $EG \parallel AB$.

Thus, G is the mid point of BD (Converse of mid point theorem)

Now,

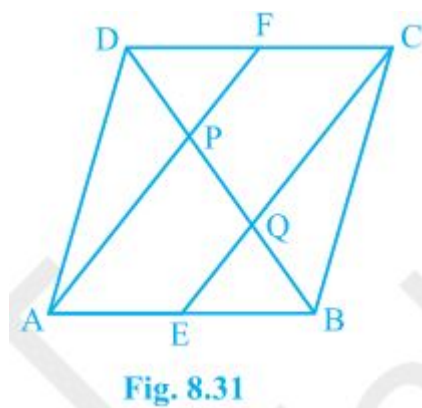
In $\triangle BDC$,

G is the mid point of BD and also $GF \parallel AB \parallel DC$.

Thus, F is the mid point of BC (Converse of mid point theorem)

Question 5

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD, respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.



Solution

Given that,

ABCD is a parallelogram. E and F are the mid-points of sides AB and CD, respectively.

To show,

AF and EC trisect the diagonal BD.

Proof,

ABCD is a parallelogram

, $AB \parallel CD$

also, $AE \parallel FC$

Now,

$AB = CD$ (Opposite sides of parallelogram ABCD)

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$$

$\Rightarrow AE = FC$ (E and F are midpoints of side AB and CD)

AECF is a parallelogram (AE and CF are parallel and equal to each other)

$AF \parallel EC$ (Opposite sides of a parallelogram)

Now,

In $\triangle DQC$,

F is mid point of side DC and $FP \parallel CQ$ (as $AF \parallel EC$).

P is the mid-point of DQ (Converse of mid-point theorem)

$\Rightarrow DP = PQ$ — (i)

Similarly,

In $\triangle APB$,

E is midpoint of side AB and $EQ \parallel AP$ (as $AF \parallel EC$).

Q is the mid-point of PB (Converse of mid-point theorem)

$\Rightarrow PQ = QB$ — (ii)

From equations (i) and (ii),

$DP = PQ = BQ$

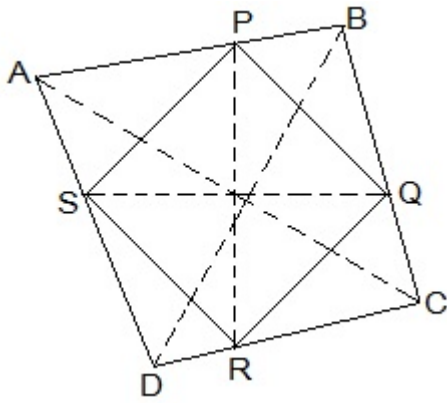
Hence, the line segments AF and EC trisect the diagonal BD.

Hence Proved.

Question 6

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:



Let ABCD be a quadrilateral and P, Q, R and S the mid points of AB, BC, CD and DA, respectively.

Now,

In $\triangle ACD$,

R and S are the mid points of CD and DA, respectively.

, $SR \parallel AC$.

Similarly we can show that,

$PQ \parallel AC$,

$PS \parallel BD$ and

$QR \parallel BD$

, PQRS is parallelogram.

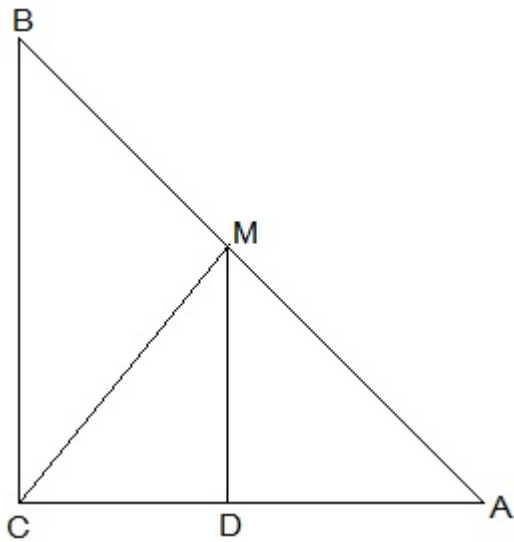
PR and QS are the diagonals of the parallelogram PQRS. So, they will bisect each other.

7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2} AB$

Solution

(i) In $\triangle ACB$,

M is the midpoint of AB and $MD \parallel BC$

, D is the midpoint of AC (Converse of mid point theorem)

(ii) $\angle ACB = \angle ADM$ (Corresponding angles)

also, $\angle ACB = 90^\circ$

, $\angle ADM = 90^\circ$ and $MD \perp AC$

(iii) In $\triangle AMD$ and $\triangle CMD$,

$AD = CD$ (D is the midpoint of side AC)

$\angle ADM = \angle CDM$ (Each 90°)

$DM = DM$ (common)

, $\triangle AMD \cong \triangle CMD$ [SAS congruency]

$AM = CM$ [CPCT]

also, $AM = \frac{1}{2} AB$ (M is midpoint of AB)

Hence, $CM = MA = \frac{1}{2} AB$