

## Chapter 7: Triangles

NCERT · Class 9 · Maths

### Exercise: 7.1 (

#### Question 1

In quadrilateral ACBD,  $AC = AD$  and  $AB$  bisect  $\angle A$  (see Fig. 7.16). Show that  $\triangle ABC \cong \triangle ABD$ . What can you say about  $BC$  and  $BD$ ?

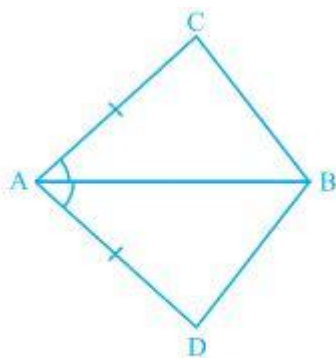


Fig. 7.16

#### Solution

It is given that  $AC$  and  $AD$  are equal i.e.  $AC = AD$  and the line segment  $AB$  bisects  $\angle A$ .

We will have to now prove that the two triangles  $ABC$  and  $ABD$  are similar i.e.  $\triangle ABC \cong \triangle ABD$

#### Proof:

Consider the triangles  $\triangle ABC$  and  $\triangle ABD$ ,

- (i)  $AC = AD$  (It is given in the question)
- (ii)  $AB = AB$  (Common)
- (iii)  $\angle CAB = \angle DAB$  (Since  $AB$  is the bisector of angle  $A$ )

So, by **SAS congruency criterion**,  $\triangle ABC \cong \triangle ABD$ .

For the 2<sup>nd</sup> part of the question,  $BC$  and  $BD$  are of equal lengths by the rule of C.P.C.T.

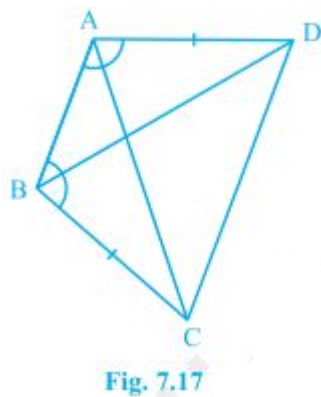
**Question 2**

ABCD is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$  (see Fig. 7.17). Prove that

(i)  $\triangle ABD \cong \triangle BAC$

(ii)  $BD = AC$

(iii)  $\angle ABD = \angle BAC$ .

**Solution**

The given parameters from the questions are  $\angle DAB = \angle CBA$  and  $AD = BC$ .

(i)  $\triangle ABD$  and  $\triangle BAC$  are similar by SAS congruency as

$AB = BA$  (It is the common arm)

$\angle DAB = \angle CBA$  and  $AD = BC$  (These are given in the question)

So, triangles ABD and BAC are similar i.e.  $\triangle ABD \cong \triangle BAC$ . (Hence proved).

(ii) It is now known that  $\triangle ABD \cong \triangle BAC$  so,

$BD = AC$  (by the rule of CPCT).

(iii) Since  $\triangle ABD \cong \triangle BAC$  so,

Angles  $\angle ABD = \angle BAC$  (by the rule of CPCT).

**Question 3**

AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

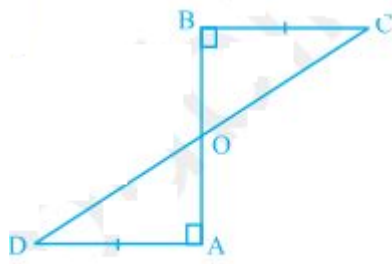


Fig. 7.18

### Solution

It is given that AD and BC are two equal perpendiculars to AB.

We will have to prove that **CD is the bisector of AB**

Now,

Triangles  $\triangle AOD$  and  $\triangle BOC$  are similar by AAS congruency since:

- (i)  $\angle A = \angle B$  (They are perpendiculars)
- (ii)  $AD = BC$  (As given in the question)
- (iii)  $\angle AOD = \angle BOC$  (They are vertically opposite angles)

$\therefore \triangle AOD \cong \triangle BOC$ .

So,  $AO = OB$  (by the rule of CPCT).

Thus, CD bisects AB (Hence proved).

### Question 4

$l$  and  $m$  are two parallel lines intersected by another pair of parallel lines  $p$  and  $q$  (see Fig. 7.19). Show that  $\triangle ABC \cong \triangle CDA$ .

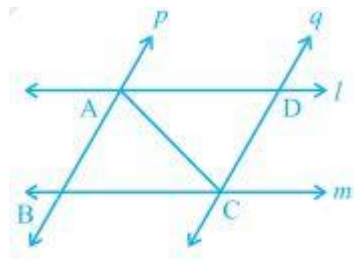


Fig. 7.19

**Solution**

It is given that  $p \parallel q$  and  $l \parallel m$

**To prove:**

Triangles ABC and CDA are similar i.e.  $\triangle ABC \cong \triangle CDA$

**Proof:**

Consider the  $\triangle ABC$  and  $\triangle CDA$ ,

- (i)  $\angle BCA = \angle DAC$  and  $\angle BAC = \angle DCA$  Since they are alternate interior angles
- (ii)  $AC = CA$  as it is the common arm

So, by **ASA congruency criterion**,  $\triangle ABC \cong \triangle CDA$ .

**Question 5**

Line  $l$  is the bisector of an angle  $\angle A$  and  $B$  is any point on  $l$ .  $BP$  and  $BQ$  are perpendiculars from  $B$  to the arms of  $\angle A$  (see Fig. 7.20). Show that:

- (i)  $\triangle APB \cong \triangle AQB$
- (ii)  $BP = BQ$  or  $B$  is equidistant from the arms of  $\angle A$ .

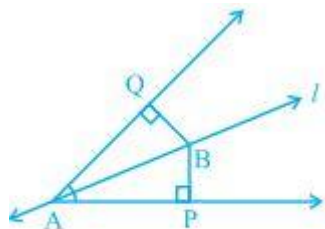


Fig. 7.20

**Solution**

It is given that the line " $l$ " is the bisector of angle  $\angle A$  and the line segments  $BP$  and  $BQ$  are perpendiculars drawn from  $l$ .

- (i)  $\triangle APB$  and  $\triangle AQB$  are similar by AAS congruency because:

$\angle P = \angle Q$  (They are the two right angles)

$AB = AB$  (It is the common arm)

$\angle BAP = \angle BAQ$  (As line  $l$  is the bisector of angle  $A$ )

So,  $\triangle APB \cong \triangle AQB$ .

(ii) By the rule of CPCT,  $BP = BQ$ . So, it can be said the point  $B$  is equidistant from the arms of  $\angle A$ .

### Question 6

In Fig. 7.21,  $AC = AE$ ,  $AB = AD$  and  $\angle BAD = \angle EAC$ . Show that  $BC = DE$ .

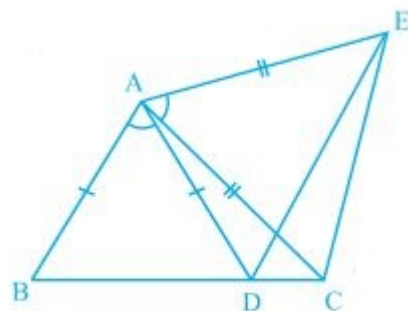


Fig. 7.21

### Solution

It is given in the question that  $AB = AD$ ,  $AC = AE$ , and  $\angle BAD = \angle EAC$

**To prove:**

The line segment  $BC$  and  $DE$  are similar i.e.  $BC = DE$

**Proof:**

We know that  $\angle BAD = \angle EAC$

Now, by adding  $\angle DAC$  on both sides we get,

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC$$

This implies,  $\angle BAC = \angle EAD$

Now,  $\triangle ABC$  and  $\triangle ADE$  are similar by SAS congruency since:

(i)  $AC = AE$  (As given in the question)

(ii)  $\angle BAC = \angle EAD$

(iii)  $AB = AD$  (It is also given in the question)

$\therefore$  Triangles  $ABC$  and  $ADE$  are similar i.e.  $\triangle ABC \cong \triangle ADE$ .

So, by the rule of CPCT, it can be said that  $BC = DE$ .

### Question 7

$AB$  is a line segment and  $P$  is its mid-point.  $D$  and  $E$  are points on the same side of  $AB$  such that  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$  (see Fig. 7.22). Show that

(i)  $\triangle DAP \cong \triangle EBP$

(ii)  $AD = BE$

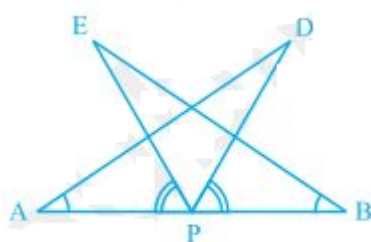


Fig. 7.22

### Solution

In the question, it is given that  $P$  is the mid-point of line segment  $AB$ . Also,  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$

(i) It is given that  $\angle EPA = \angle DPB$

Now, add  $\angle DPE$  on both sides,

$$\angle EPA + \angle DPE = \angle DPB + \angle DPE$$

This implies that angles  $DPA$  and  $EPB$  are equal i.e.  $\angle DPA = \angle EPB$

Now, consider the triangles  $DAP$  and  $EBP$ .

$$\angle DPA = \angle EPB$$

$AP = BP$  (Since  $P$  is the mid-point of the line segment  $AB$ )

$\angle BAD = \angle ABE$  (As given in the question)

So, by **ASA congruency**,  $\triangle DAP \cong \triangle EBP$ .

(ii) By the rule of CPCT,  $AD = BE$ .

**Question 8**

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that  $DM = CM$ . Point D is joined to point B (see Fig. 7.23). Show that:

(i)  $\triangle AMC \cong \triangle BMD$

(ii)  $\angle DBC$  is a right angle.

(iii)  $\triangle DBC \cong \triangle ACB$

(iv)  $CM = \frac{1}{2} AB$

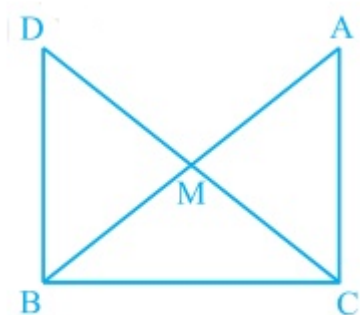


Fig. 7.23

**Solution**

It is given that M is the mid-point of the line segment AB,  $\angle C = 90^\circ$ , and  $DM = CM$

(i) Consider the triangles  $\triangle AMC$  and  $\triangle BMD$ :

$AM = BM$  (Since M is the mid-point)

$CM = DM$  (Given in the question)

$\angle CMA = \angle DMB$  (They are vertically opposite angles)

So, by **SAS congruency criterion**,  $\triangle AMC \cong \triangle BMD$ .

(ii)  $\angle ACM = \angle BDM$  (by CPCT)

$\therefore AC \parallel BD$  as alternate interior angles are equal.

Now,  $\angle ACB + \angle DBC = 180^\circ$  (Since they are co-interiors angles)

$\Rightarrow 90^\circ + \angle B = 180^\circ$

$\therefore \angle DBC = 90^\circ$

(iii) In  $\triangle DBC$  and  $\triangle ACB$ ,

$BC = CB$  (Common side)

$\angle ACB = \angle DBC$  (They are right angles)

$DB = AC$  (by CPCT)

So,  $\triangle DBC \cong \triangle ACB$  by **SAS congruency**.

(iv)  $DC = AB$  (Since  $\triangle DBC \cong \triangle ACB$ )

$\Rightarrow DM = CM = AM = BM$  (Since M is the mid-point)

So,  $DM + CM = BM + AM$

Hence,  $CM + CM = AB$

$\Rightarrow CM = \left(\frac{1}{2}\right) AB$

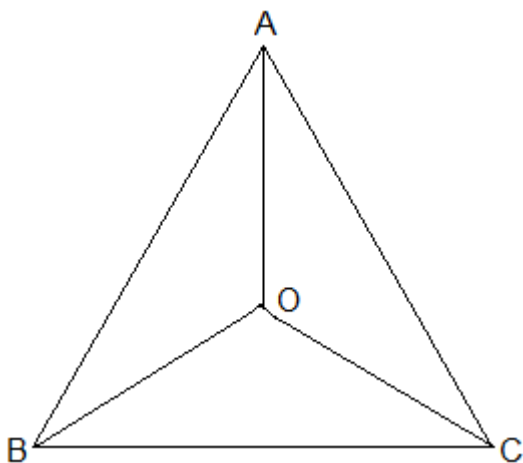
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## Exercise: 7.2 (

### Question 1

In an isosceles triangle  $ABC$ , with  $AB = AC$ , the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$ . Join  $A$  to  $O$ . Show that:

(i)  $OB = OC$  (ii)  $AO$  bisects  $\angle A$



### Solution

Given:

$AB = AC$  and

the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$

(i) Since  $ABC$  is an isosceles with  $AB = AC$ ,

$$\angle B = \angle C$$

$$\frac{1}{2} \angle B = \frac{1}{2} \angle C$$

$$\Rightarrow \angle OBC = \angle OCB \text{ (Angle bisectors)}$$

$\therefore OB = OC$  (Side opposite to the equal angles are equal.)

(ii) In  $\triangle AOB$  and  $\triangle AOC$ ,

$AB = AC$  (Given in the question)

$AO = AO$  (Common arm)

$OB = OC$  (As Proved Already)

So,  $\triangle AOB \cong \triangle AOC$  by SSS congruence condition.

$\angle BAO = \angle CAO$  (by CPCT)

Thus,  $AO$  bisects  $\angle A$ .

## Question 2

In  $\triangle ABC$ ,  $AD$  is the perpendicular bisector of  $BC$  (see Fig. 7.30). Show that  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ .

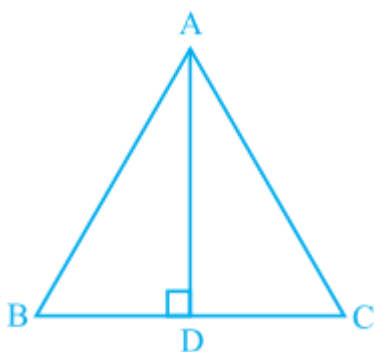


**Fig. 7.30**

### Solution

It is given that AD is the perpendicular bisector of BC

**To prove:**

$$AB = AC$$

**Proof:**

In  $\triangle ADB$  and  $\triangle ADC$ ,

$$AD = AD \text{ (It is the Common arm)}$$

$$\angle ADB = \angle ADC$$

$$BD = CD \text{ (Since AD is the perpendicular bisector)}$$

So,  $\triangle ADB \cong \triangle ADC$  by **SAS congruency criterion**.

Thus,

$$AB = AC \text{ (by CPCT)}$$

### Question 3

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

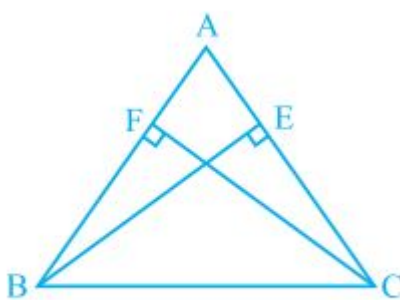


Fig. 7.31

### Solution

Given:

(i) BE and CF are altitudes.

(ii)  $AC = AB$

**To prove:**

$BE = CF$

**Proof:**

Triangles  $\triangle AEB$  and  $\triangle AFC$  are similar by AAS congruency since

$\angle A = \angle A$  (It is the common arm)

$\angle AEB = \angle AFC$  (They are right angles)

$AB = AC$  (Given in the question)

$\therefore \triangle AEB \cong \triangle AFC$  and so,  $BE = CF$  (by CPCT).

#### Question 4

$ABC$  is a triangle in which altitudes  $BE$  and  $CF$  to sides  $AC$  and  $AB$  are equal (see Fig. 7.32). Show that

(i)  $\triangle ABE \cong \triangle ACF$

(ii)  $AB = AC$ , i.e.,  $ABC$  is an isosceles triangle.

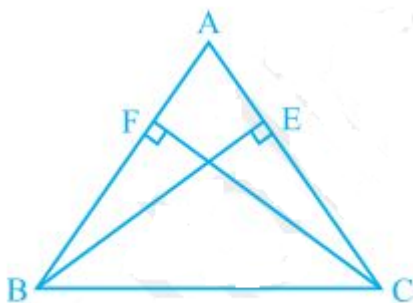


Fig. 7.32

#### Solution

It is given that  $BE = CF$

(i) In  $\triangle ABE$  and  $\triangle ACF$ ,

$\angle A = \angle A$  (It is the common angle)

$\angle AEB = \angle AFC$  (They are right angles)

$BE = CF$  (Given in the question)

$\therefore \triangle ABE \cong \triangle ACF$  by **AAS congruency condition**.

(ii)  $AB = AC$  by CPCT and so, ABC is an isosceles triangle.

### Question 5

ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that  $\angle ABD = \angle ACD$ .

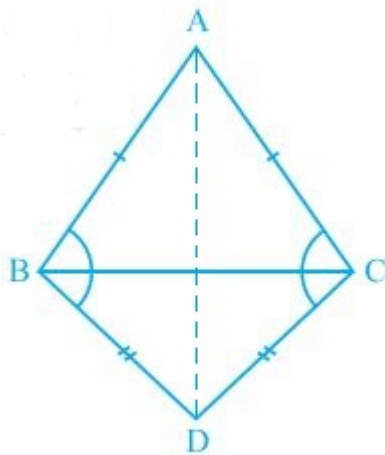


Fig. 7.33

### Solution

In the question, it is given that ABC and DBC are two isosceles triangles.

We will have to show that  $\angle ABD = \angle ACD$

#### Proof:

Triangles  $\triangle ABD$  and  $\triangle ACD$  are similar by SSS congruency since

$AD = AD$  (It is the common arm)

$AB = AC$  (Since ABC is an isosceles triangle)

$BD = CD$  (Since BCD is an isosceles triangle)

So,  $\triangle ABD \cong \triangle ACD$ .

$\therefore \angle ABD = \angle ACD$  by CPCT.

**Question 6**

$\triangle ABC$  is an isosceles triangle in which  $AB = AC$ . Side  $BA$  is produced to  $D$  such that  $AD = AB$  (see Fig. 7.34). Show that  $\angle BCD$  is a right angle.

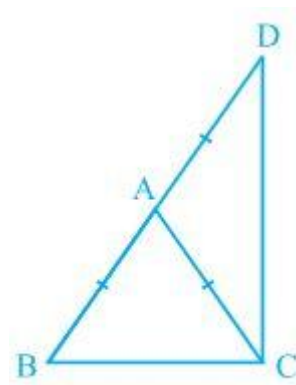


Fig. 7.34

**Solution**

It is given that  $AB = AC$  and  $AD = AB$

We will have to now prove  $\angle BCD$  is a right angle.

**Proof:**

Consider  $\triangle ABC$ ,

$AB = AC$  (It is given in the question)

Also,  $\angle ACB = \angle ABC$  (They are angles opposite to the equal sides and so, they are equal)

Now, consider  $\triangle ACD$ ,

$AD = AB$

Also,  $\angle ADC = \angle ACD$  (They are angles opposite to the equal sides and so, they are equal)

Now,

In  $\triangle ABC$ ,

$$\angle CAB + \angle ACB + \angle ABC = 180^\circ$$

$$\text{So, } \angle CAB + 2\angle ACB = 180^\circ$$

$$\Rightarrow \angle CAB = 180^\circ - 2\angle ACB \text{ — (i)}$$

Similarly, in  $\triangle ADC$ ,

$$\angle CAD = 180^\circ - 2\angle ACD \text{ — (ii)}$$

also,

$$\angle CAB + \angle CAD = 180^\circ \text{ (BD is a straight line.)}$$

Adding (i) and (ii) we get,

$$\angle CAB + \angle CAD = 180^\circ - 2\angle ACB + 180^\circ - 2\angle ACD$$

$$\Rightarrow 180^\circ = 360^\circ - 2\angle ACB - 2\angle ACD$$

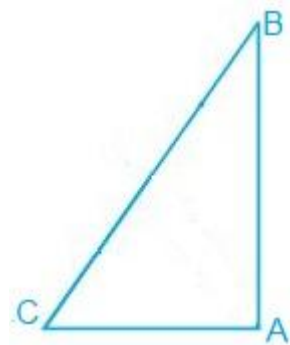
$$\Rightarrow 2(\angle ACB + \angle ACD) = 180^\circ$$

$$\Rightarrow \angle BCD = 90^\circ$$

### Question 7

ABC is a right-angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$ .

### Solution



In the question, it is given that

$$\angle A = 90^\circ \text{ and } AB = AC$$

$$AB = AC$$

$$\Rightarrow \angle B = \angle C \text{ (They are angles opposite to the equal sides and so, they are equal)}$$

Now,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Since the sum of the interior angles of the triangle)}$$

$$\therefore 90^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 90^\circ$$

$$\Rightarrow \angle B = 45^\circ$$

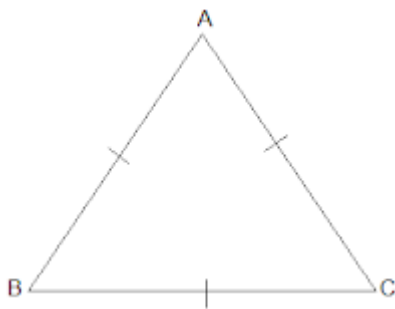
$$\text{So, } \angle B = \angle C = 45^\circ$$

### Question 8

Show that the angles of an equilateral triangle are  $60^\circ$  each.

#### Solution

Let ABC be an equilateral triangle as shown below:



Here,  $BC = AC = AB$  (Since the length of all sides is same)

$$\Rightarrow \angle A = \angle B = \angle C \text{ (Sides opposite to the equal angles are equal.)}$$

Also, we know that

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 3\angle A = 180^\circ$$

$$\Rightarrow \angle A = 60^\circ$$

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

So, the angles of an equilateral triangle are always  $60^\circ$  each.

### Exercise: 7.3 (

#### Question 1

$\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i)  $\triangle ABD \cong \triangle ACD$

(ii)  $\triangle ABP \cong \triangle ACP$

(iii) AP bisects  $\angle A$  as well as  $\angle D$ .

(iv) AP is the perpendicular bisector of BC.

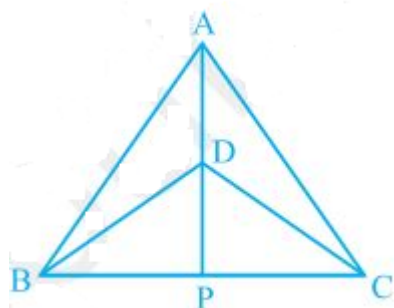


Fig. 7.39

### Solution

In the above question, it is given that  $\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles.

(i)  $\triangle ABD$  and  $\triangle ACD$  are similar by SSS congruency because:

$AD = AD$  (It is the common arm)

$AB = AC$  (Since  $\triangle ABC$  is isosceles)

$BD = CD$  (Since  $\triangle DBC$  is isosceles)

$\therefore \triangle ABD \cong \triangle ACD$ .

(ii)  $\triangle ABP$  and  $\triangle ACP$  are similar as:

$AP = AP$  (It is the common side)

$\angle PAB = \angle PAC$  (by CPCT since  $\triangle ABD \cong \triangle ACD$ )

$AB = AC$  (Since  $\triangle ABC$  is isosceles)

So,  $\triangle ABP \cong \triangle ACP$  by SAS congruency condition.

(iii)  $\angle PAB = \angle PAC$  by CPCT as  $\triangle ABD \cong \triangle ACD$ .

AP bisects  $\angle A$ . — (i)

Also,  $\triangle BPD$  and  $\triangle CPD$  are similar by SSS congruency as

$PD = PD$  (It is the common side)

$BD = CD$  (Since  $\triangle DBC$  is isosceles.)

$BP = CP$  (by CPCT as  $\triangle ABP \cong \triangle ACP$ )

So,  $\triangle BPD \cong \triangle CPD$ .

Thus,  $\angle BDP = \angle CDP$  by CPCT. — (ii)

Now by comparing (i) and (ii) it can be said that AP bisects  $\angle A$  as well as  $\angle D$ .

(iv)  $\angle BPD = \angle CPD$  (by CPCT as  $\triangle BPD \cong \triangle CPD$ )

and  $BP = CP$  — (i)

also,

$\angle BPD + \angle CPD = 180^\circ$  (Since BC is a straight line.)

$$\Rightarrow 2\angle BPD = 180^\circ$$

$$\Rightarrow \angle BPD = 90^\circ \text{ —(ii)}$$

Now, from equations (i) and (ii), it can be said that

AP is the perpendicular bisector of BC.

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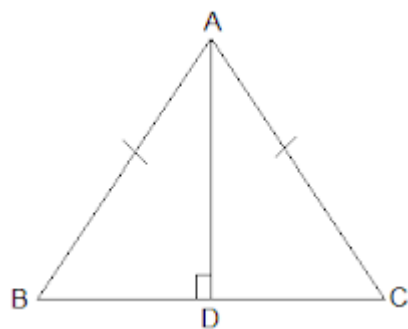
## Question 2

AD is an altitude of an isosceles triangle ABC in which  $AB = AC$ . Show that

(i) AD bisects BC (ii) AD bisects  $\angle A$ .

## Solution

It is given that AD is an altitude and  $AB = AC$ . The diagram is as follows:



(i) In  $\triangle ABD$  and  $\triangle ACD$ ,

$$\angle ADB = \angle ADC = 90^\circ$$

$AB = AC$  (It is given in the question)

$AD = AD$  (Common arm)

$\therefore \triangle ABD \cong \triangle ACD$  by RHS congruence condition.

Now, by the rule of CPCT,

$$BD = CD.$$

So, AD bisects BC

(ii) Again, by the rule of CPCT,  $\angle BAD = \angle CAD$

Hence, AD bisects  $\angle A$ .

### Question 3

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of  $\triangle PQR$  (see Fig. 7.40). Show that:

(i)  $\triangle ABM \cong \triangle PQN$

(ii)  $\triangle ABC \cong \triangle PQR$

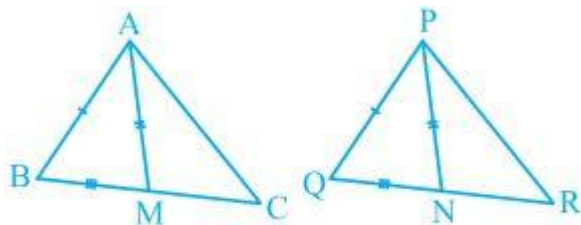


Fig. 7.40

### Solution

Given parameters are:

$$AB = PQ,$$

$$BC = QR \text{ and}$$

$$AM = PN$$

(i)  $\frac{1}{2} BC = BM$  and  $\frac{1}{2} QR = QN$  (Since AM and PN are medians)

Also,  $BC = QR$

So,  $\frac{1}{2} BC = \frac{1}{2} QR$

$\Rightarrow BM = QN$

In  $\triangle ABM$  and  $\triangle PQN$ ,

$AM = PN$  and  $AB = PQ$  (As given in the question)

$BM = QN$  (Already proved)

$\therefore \triangle ABM \cong \triangle PQN$  by SSS congruency.

(ii) In  $\triangle ABC$  and  $\triangle PQR$ ,

$AB = PQ$  and  $BC = QR$  (As given in the question)

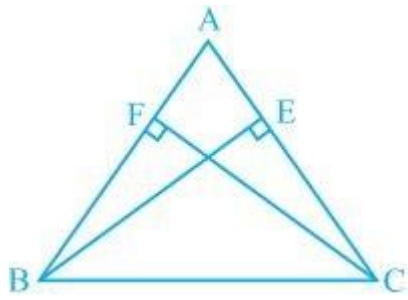
$\angle ABC = \angle PQR$  (by CPCT)

So,  $\triangle ABC \cong \triangle PQR$  by SAS congruency.

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#### Question 4

**BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.**



#### Solution

It is known that BE and CF are two equal altitudes.

Now, in  $\triangle BEC$  and  $\triangle CFB$ ,

$\angle BEC = \angle CFB = 90^\circ$  (Same Altitudes)

$BC = CB$  (Common side)

$BE = CF$  (Common side)

So,  $\triangle BEC \cong \triangle CFB$  by RHS congruence criterion.

Also,  $\angle C = \angle B$  (by CPCT)

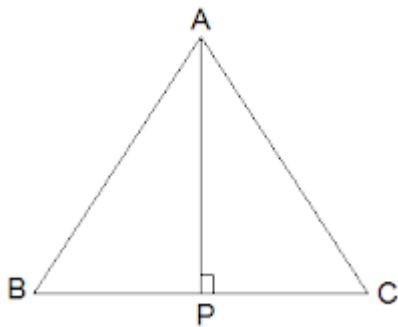
Therefore,  $AB = AC$  as sides opposite to the equal angles is always equal.

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### Question 5

ABC is an isosceles triangle with  $AB = AC$ . Draw  $AP \perp BC$  to show that  $\angle B = \angle C$ .

### Solution



In the question, it is given that  $AB = AC$

Now,  $\triangle ABP$  and  $\triangle ACP$  are similar by RHS congruency as

$\angle APB = \angle APC = 90^\circ$  (AP is altitude)

$AB = AC$  (Given in the question)

$AP = AP$  (Common side)

So,  $\triangle ABP \cong \triangle ACP$ .

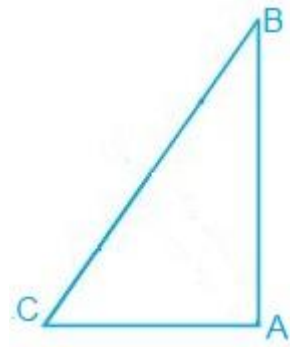
$\therefore \angle B = \angle C$  (by CPCT)

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## Exercise: 7.4 (

### Question 1

Show that in a right-angled triangle, the hypotenuse is the longest side.



### Solution

It is known that ABC is a triangle right angled at B.

We know that,

$$\angle A + \angle B + \angle C = 180^\circ$$

Now, if  $\angle B + \angle C = 90^\circ$  then  $\angle A$  has to be  $90^\circ$ .

Since A is the largest angle of the triangle, the side opposite to it must be the largest.

So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e.  $\triangle ABC$ .

### Question 2

In Fig. 7.48, sides AB and AC of  $\triangle ABC$  are extended to points P and Q respectively. Also,  $\angle PBC < \angle QCB$ . Show that  $AC > AB$ .

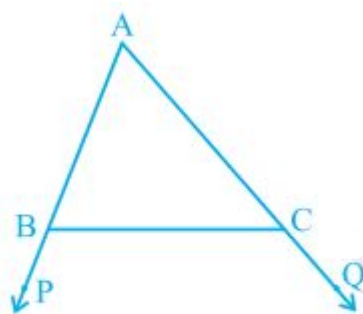


Fig. 7.48

### Solution

It is given that  $\angle PBC < \angle QCB$

We know that  $\angle ABC + \angle PBC = 180^\circ$

So,  $\angle ABC = 180^\circ - \angle PBC$

Also,

$$\angle ACB + \angle QCB = 180^\circ$$

Therefore  $\angle ACB = 180^\circ - \angle QCB$

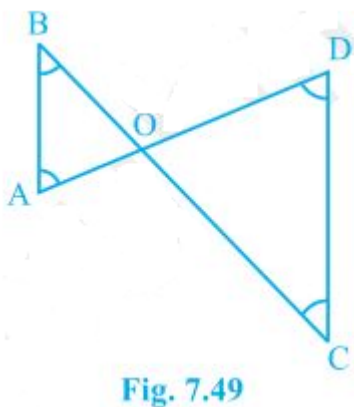
Now, since  $\angle PBC < \angle QCB$ ,

$$\therefore \angle ABC > \angle ACB$$

Hence,  $AC > AB$  as sides opposite to the larger angle is always larger.

### Question 3

In Fig. 7.49,  $\angle B < \angle A$  and  $\angle C < \angle D$ . Show that  $AD < BC$ .



### Solution

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e.  $\angle B < \angle A$  and  $\angle C < \angle D$ .

Now,

Since the side opposite to the smaller angle is always smaller

$$AO < BO \text{ — (i)}$$

$$\text{And } OD < OC \text{ —(ii)}$$

By adding equation (i) and equation (ii) we get

$$AO + OD < BO + OC$$

So,  $AD < BC$

#### Question 4

$AB$  and  $CD$  are respectively the smallest and longest sides of a quadrilateral  $ABCD$  (see Fig. 7.50).

Show that  $\angle A > \angle C$  and  $\angle B > \angle D$ .

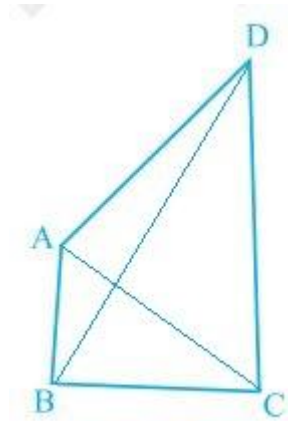


Fig. 7.50

#### Solution

In  $\triangle ABD$ , we see that

$$AB < AD < BD$$

So,  $\angle ADB < \angle ABD$  — (i) (Since angle opposite to longer side is always larger)

Now, in  $\triangle BCD$ ,

$$BC < DC < BD$$

Hence, it can be concluded that

$$\angle BDC < \angle CBD$$
 — (ii)

Now, by adding equation (i) and equation (ii) we get,

$$\angle ADB + \angle BDC < \angle ABD + \angle CBD$$

$$\angle ADC < \angle ABC$$

$$\angle B > \angle D$$

Similarly, In triangle ABC,

$\angle ACB < \angle BAC$  — (iii) (Since the angle opposite to the longer side is always larger)

Now, In  $\triangle ADC$ ,

$\angle DCA < \angle DAC$  — (iv)

By adding equation (iii) and equation (iv) we get,

$$\angle ACB + \angle DCA < \angle BAC + \angle DAC$$

$$\Rightarrow \angle BCD < \angle BAD$$

$$\therefore \angle A > \angle C$$

### Question 5

In Fig 7.51,  $PR > PQ$  and  $PS$  bisect  $\angle QPR$ . Prove that  $\angle PSR > \angle PSQ$ .

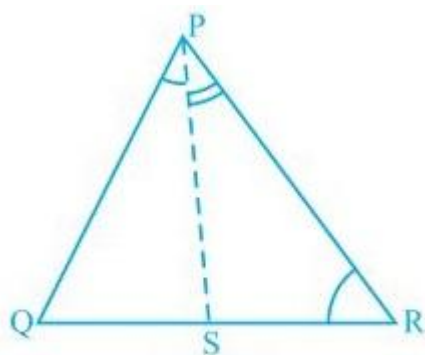


Fig. 7.51

### Solution

It is given that  $PR > PQ$  and  $PS$  bisects  $\angle QPR$

Now we will have to prove that angle PSR is smaller than PSQ i.e.  $\angle PSR > \angle PSQ$

**Proof:**

$$\angle QPS = \angle RPS \text{ — (ii) (As PS bisects } \angle QPR)$$

$$\angle PQR > \angle PRQ \text{ — (i) (Since } PR > PQ \text{ as angle opposite to the larger side is always larger)}$$

$$\angle PSR = \angle PQR + \angle QPS \text{ — (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)}$$

$$\angle PSQ = \angle PRQ + \angle RPS \text{ — (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles)}$$

By adding (i) and (ii)

$$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$$

Thus, from (i), (ii), (iii) and (iv), we get

$$\angle PSR > \angle PSQ$$

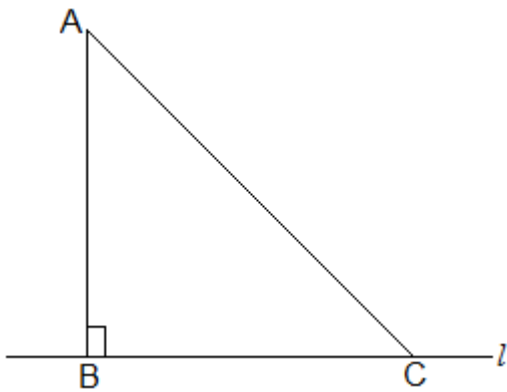
### Question 6

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

### Solution

First, let “ $l$ ” be a line segment and “ $B$ ” be a point lying on it. A line  $AB$  perpendicular to  $l$  is now drawn.

Also, let  $C$  be any other point on  $l$ . The diagram will be as follows:



**To prove:**

$$AB < AC$$

**Proof:**

$$\text{In } \triangle ABC, \angle B = 90^\circ$$

Now, we know that

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore \angle A + \angle C = 90^\circ$$

Hence,  $\angle C$  must be an acute angle which implies  $\angle C < \angle B$

So,  $AB < AC$  (As the side opposite to the larger angle is always larger)

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