

Chapter 6: Lines and Angles

NCERT · Class 9 · Maths

Exercise: 6.1 (

Question 1

In Fig. 6.13, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.

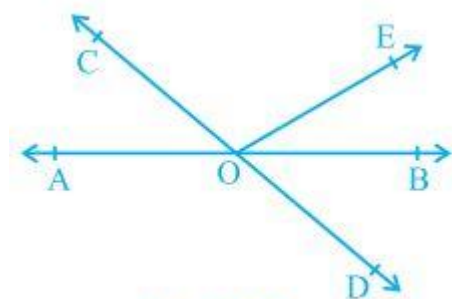


Fig. 6.13

Solution

From the diagram, we have

$(\angle AOC + \angle BOE + \angle COE)$ and $(\angle COE + \angle BOD + \angle BOE)$ forms a straight line.

So, $\angle AOC + \angle BOE + \angle COE = \angle COE + \angle BOD + \angle BOE = 180^\circ$

Now, by putting the values of $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$ we get

$\angle COE = 110^\circ$ and $\angle BOE = 30^\circ$

So, reflex $\angle COE = 360^\circ - 110^\circ = 250^\circ$

Question 2

In Fig. 6.14, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.

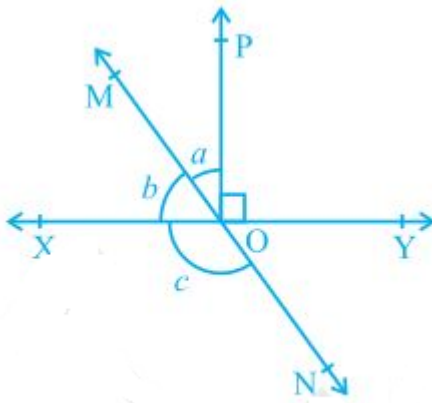


Fig. 6.14

Solution

We know that the sum of linear pair is always equal to 180°

So,

$$\angle POY + a + b = 180^\circ$$

Putting the value of $\angle POY = 90^\circ$ (as given in the question), we get,

$$a + b = 90^\circ$$

Now, it is given that $a:b = 2:3$, so

Let a be $2x$ and b be $3x$

$$\therefore 2x + 3x = 90^\circ$$

Solving this, we get

$$5x = 90^\circ$$

$$\text{So, } x = 18^\circ$$

$$\therefore a = 2 \times 18^\circ = 36^\circ$$

Similarly, b can be calculated, and the value will be

$$b = 3 \times 18^\circ = 54^\circ$$

From the diagram, $b+c$ also forms a straight angle, so

$$b + c = 180^\circ$$

$$c + 54^\circ = 180^\circ$$

$$\therefore c = 126^\circ$$

Question 3

In Fig. 6.15, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

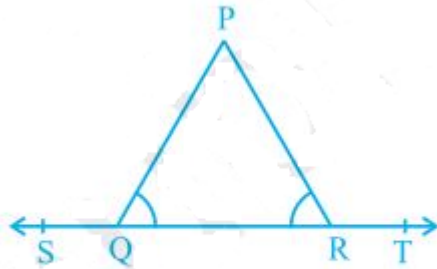


Fig. 6.15

Solution

Since ST is a straight line, so

$$\angle PQS + \angle PQR = 180^\circ \text{ (linear pair) and}$$

$$\angle PRT + \angle PRQ = 180^\circ \text{ (linear pair)}$$

$$\text{Now, } \angle PQS + \angle PQR = \angle PRT + \angle PRQ = 180^\circ$$

Since $\angle PQR = \angle PRQ$ (as given in the question)

$$\angle PQS = \angle PRT. \text{ (Hence proved).}$$

Question 4

In Fig. 6.16, if $x + y = w + z$, then prove that AOB is a line.

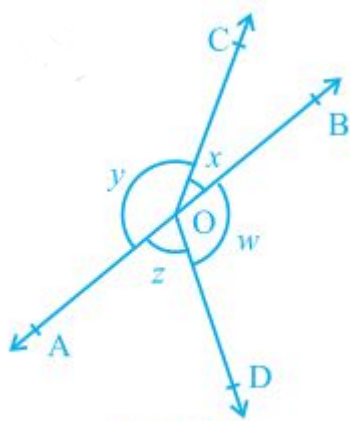


Fig. 6.16

Solution

To prove AOB is a straight line, we will have to prove $x+y$ is a linear pair

$$\text{i.e. } x+y = 180^\circ$$

We know that the angles around a point are 360° , so

$$x+y+w+z = 360^\circ$$

In the question, it is given that,

$$x+y = w+z$$

$$\text{So, } (x+y)+(x+y) = 360^\circ$$

$$2(x+y) = 360^\circ$$

$$\therefore (x+y) = 180^\circ \text{ (Hence proved).}$$

Question 5

In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$.

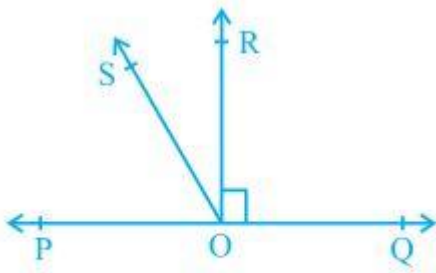


Fig. 6.17

Solution

In the question, it is given that $(OR \perp PQ)$ and $\angle POQ = 180^\circ$

We can write it as $\angle ROP = \angle ROQ = 90^\circ$

We know that

$$\angle ROP = \angle ROQ$$

It can be written as

$$\angle POS + \angle ROS = \angle ROQ$$

$$\angle POS + \angle ROS = \angle QOS - \angle ROS$$

$$\angle SOR + \angle ROS = \angle QOS - \angle POS$$

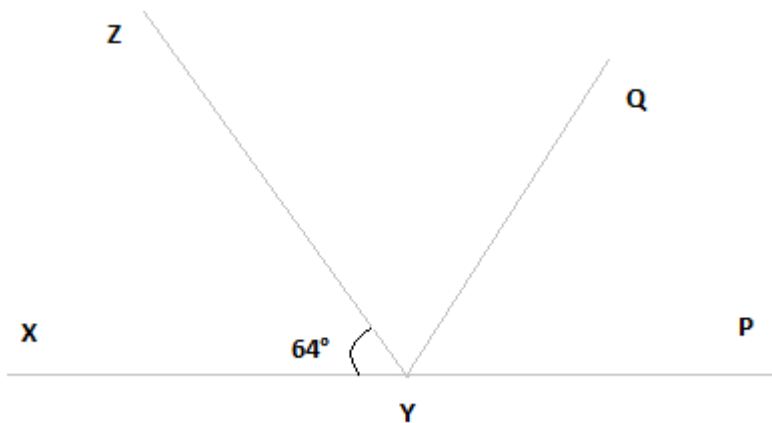
So we get

$$2\angle ROS = \angle QOS - \angle POS$$

Or, $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$ (Hence proved).

Question 6

It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution

Here, XP is a straight line

So, $\angle XYZ + \angle ZYP = 180^\circ$

Putting the value of $\angle XYZ = 64^\circ$, we get

$$64^\circ + \angle ZYP = 180^\circ$$

$$\therefore \angle ZYP = 116^\circ$$

From the diagram, we also know that $\angle ZYP = \angle ZYQ + \angle QYP$

Now, as YQ bisects $\angle ZYP$,

$$\angle ZYQ = \angle QYP$$

$$\text{Or, } \angle ZYP = 2\angle ZYQ$$

$$\therefore \angle ZYQ = \angle QYP = 58^\circ$$

$$\text{Again, } \angle XYQ = \angle XYZ + \angle ZYQ$$

By putting the value of $\angle XYZ = 64^\circ$ and $\angle ZYQ = 58^\circ$, we get.

$$\angle XYQ = 64^\circ + 58^\circ$$

$$\text{Or, } \angle XYQ = 122^\circ$$

$$\text{Now, reflex } \angle QYP = 180^\circ - \angle XYQ$$

We computed that the value of $\angle XYQ = 122^\circ$.

So,

$$\angle QYP = 180^\circ + 122^\circ$$

$$\therefore \angle QYP = 302^\circ$$

Exercise: 6.2 (

Question 1

In Fig. 6.28, find the values of x and y and then show that $AB \parallel CD$.

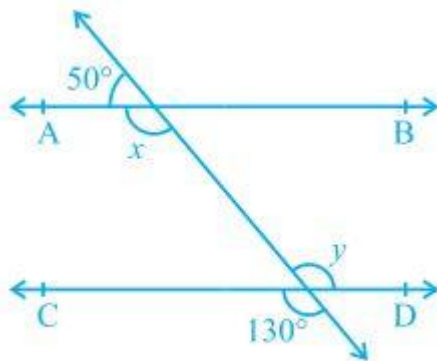


Fig. 6.28

Solution

We know that a linear pair is equal to 180° .

$$\text{So, } x + 50^\circ = 180^\circ$$

$$\therefore x = 130^\circ$$

We also know that vertically opposite angles are equal.

$$\text{So, } y = 130^\circ$$

In two parallel lines, the alternate interior angles are equal. In this,

$$x = y = 130^\circ$$

This proves that alternate interior angles are equal, so $AB \parallel CD$.

Question 2

In Fig. 6.29, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x .

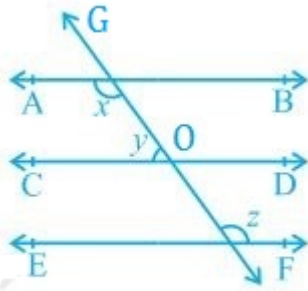


Fig. 6.29

Solution

It is known that $AB \parallel CD$ and $CD \parallel EF$

As the angles on the same side of a transversal line sum up to 180° ,

$$x + y = 180^\circ \text{ —(i)}$$

Also,

$$\angle O = z \text{ (Since they are corresponding angles)}$$

$$\text{and, } y + \angle O = 180^\circ \text{ (Since they are a linear pair)}$$

$$\text{So, } y + z = 180^\circ$$

$$\text{Now, let } y = 3w \text{ and hence, } z = 7w \text{ (As } y : z = 3 : 7)$$

$$\therefore 3w + 7w = 180^\circ$$

$$\text{Or, } 10w = 180^\circ$$

$$\text{So, } w = 18^\circ$$

$$\text{Now, } y = 3 \times 18^\circ = 54^\circ$$

$$\text{and, } z = 7 \times 18^\circ = 126^\circ$$

Now, angle x can be calculated from equation (i)

$$x + y = 180^\circ$$

$$\text{Or, } x + 54^\circ = 180^\circ$$

$$\therefore x = 126^\circ$$

Question 3

In Fig. 6.30, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.

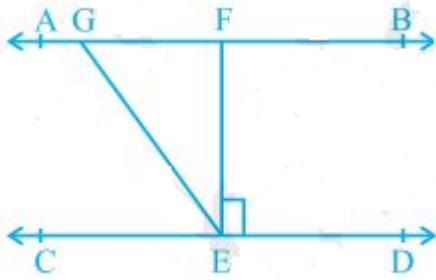


Fig. 6.30

Solution

Since $AB \parallel CD$, GE is a transversal.

It is given that $\angle GED = 126^\circ$

So, $\angle GED = \angle AGE = 126^\circ$ (As they are alternate interior angles)

Also,

$$\angle GED = \angle GEF + \angle FED$$

As $EF \perp CD$, $\angle FED = 90^\circ$

$$\therefore \angle GED = \angle GEF + 90^\circ$$

$$\text{Or, } \angle GEF = 126^\circ - 90^\circ = 36^\circ$$

Again, $\angle FGE + \angle GED = 180^\circ$ (Transversal)

Putting the value of $\angle GED = 126^\circ$, we get

$$\angle FGE = 54^\circ$$

So,

$$\angle AGE = 126^\circ$$

$$\angle GEF = 36^\circ \text{ and}$$

$$\angle FGE = 54^\circ$$

Question 4

In Fig. 6.31, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.

[Hint : Draw a line parallel to ST through point R.]

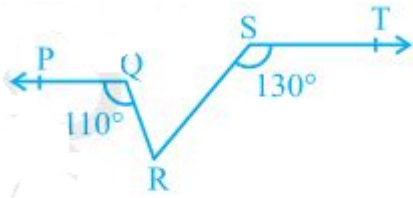
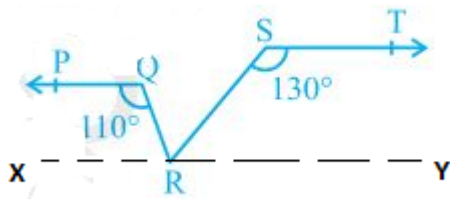


Fig. 6.31

Solution

First, construct a line XY parallel to PQ.



We know that the angles on the same side of the transversal is equal to 180° .

$$\text{So, } \angle PQR + \angle QRX = 180^\circ$$

$$\text{Or, } \angle QRX = 180^\circ - 110^\circ$$

$$\therefore \angle QRX = 70^\circ$$

Similarly,

$$\angle RST + \angle SRY = 180^\circ$$

$$\text{Or, } \angle SRY = 180^\circ - 130^\circ$$

$$\therefore \angle SRY = 50^\circ$$

Now, for the linear pairs on the line XY-

$$\angle QRX + \angle QRS + \angle SRY = 180^\circ$$

Putting their respective values, we get

$$\angle QRS = 180^\circ - 70^\circ - 50^\circ$$

$$\text{Hence, } \angle QRS = 60^\circ$$

Question 5

In Fig. 6.32, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y .

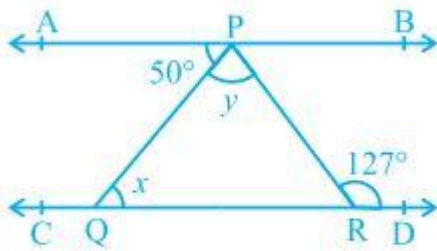


Fig. 6.32

Solution

From the diagram,

$$\angle APQ = \angle PQR \text{ (Alternate interior angles)}$$

Now, putting the value of $\angle APQ = 50^\circ$ and $\angle PQR = x$, we get

$$x = 50^\circ$$

Also,

$$\angle APR = \angle PRD \text{ (Alternate interior angles)}$$

$$\text{Or, } \angle APR = 127^\circ \text{ (As it is given that } \angle PRD = 127^\circ)$$

We know that

$$\angle APR = \angle APQ + \angle QPR$$

Now, putting values of $\angle QPR = y$ and $\angle APR = 127^\circ$, we get

$$127^\circ = 50^\circ + y$$

$$\text{Or, } y = 77^\circ$$

Thus, the values of x and y are calculated as:

$$x = 50^\circ \text{ and } y = 77^\circ$$

Question 6

In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.

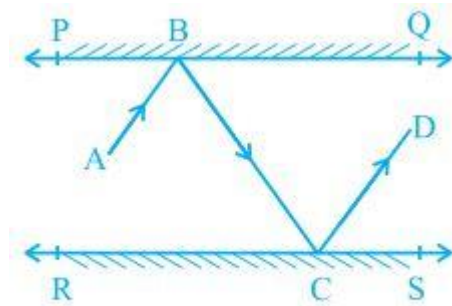


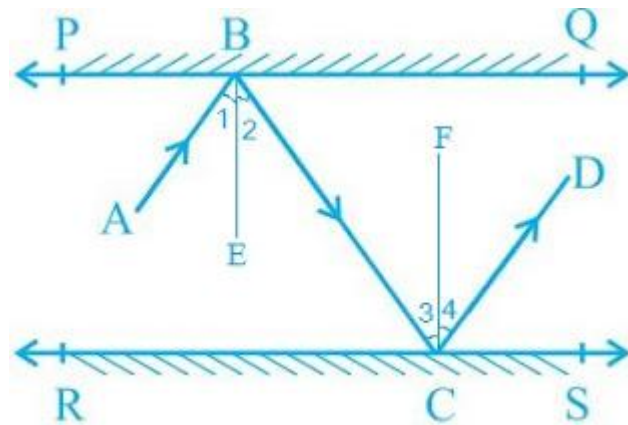
Fig. 6.33

Solution

First, draw two lines, BE and CF, such that $BE \perp PQ$ and $CF \perp RS$.

Now, since $PQ \parallel RS$,

So, $BE \parallel CF$



We know that,

Angle of incidence = Angle of reflection (By the law of reflection)

So,

$$\angle 1 = \angle 2 \text{ and}$$

$$\angle 3 = \angle 4$$

We also know that alternate interior angles are equal. Here, $BE \perp CF$ and the transversal line BC cuts them at B and C

So, $\angle 2 = \angle 3$ (As they are alternate interior angles)

$$\text{Now, } \angle 1 + \angle 2 = \angle 3 + \angle 4$$

$$\text{Or, } \angle ABC = \angle DCB$$

So, $AB \parallel CD$ (alternate interior angles are equal)

Exercise: 6.3 (

Question 1

In Fig. 6.39, sides QP and RQ of $\triangle PQR$ are produced to points S and T , respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.

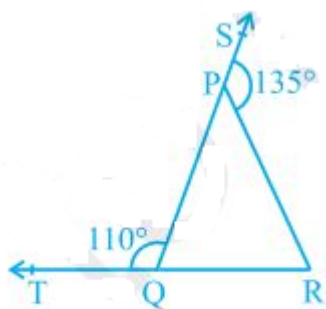


Fig. 6.39

Solution

It is given the TQR is a straight line, and so, the linear pairs (i.e. $\angle TQP$ and $\angle PQR$) will add up to 180°

$$\text{So, } \angle TQP + \angle PQR = 180^\circ$$

Now, putting the value of $\angle TQP = 110^\circ$, we get

$$\angle PQR = 70^\circ$$

Consider the $\triangle PQR$,

Here, the side QP is extended to S , and so $\angle SPR$ forms the exterior angle.

Thus, $\angle SPR$ ($\angle SPR = 135^\circ$) is equal to the sum of interior opposite angles. (Triangle property)

$$\text{Or, } \angle PQR + \angle PRQ = 135^\circ$$

Now, putting the value of $\angle PQR = 70^\circ$, we get

$$\angle PRQ = 135^\circ - 70^\circ$$

$$\text{Hence, } \angle PRQ = 65^\circ$$

Question 2

In Fig. 6.40, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$, respectively of ΔXYZ , find $\angle OZY$ and $\angle YOZ$.

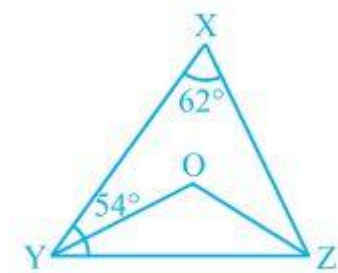


Fig. 6.40

Solution

We know that the sum of the interior angles of the triangle.

$$\text{So, } \angle X + \angle XYZ + \angle XZY = 180^\circ$$

Putting the values as given in the question, we get

$$62^\circ + 54^\circ + \angle XZY = 180^\circ$$

$$\text{Or, } \angle XZY = 64^\circ$$

Now, we know that ZO is the bisector, so

$$\angle OZY = \frac{1}{2} \angle XZY$$

$$\therefore \angle OZY = 32^\circ$$

Similarly, YO is a bisector, so

$$\angle OYZ = \frac{1}{2} \angle XYZ$$

Or, $\angle OYZ = 27^\circ$ (As $\angle XYZ = 54^\circ$)

Now, as the sum of the interior angles of the triangle,

$$\angle OZY + \angle OYZ + \angle O = 180^\circ$$

Putting their respective values, we get

$$\angle O = 180^\circ - 32^\circ - 27^\circ$$

Hence, $\angle O = 121^\circ$

Question 3

In Fig. 6.41, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.

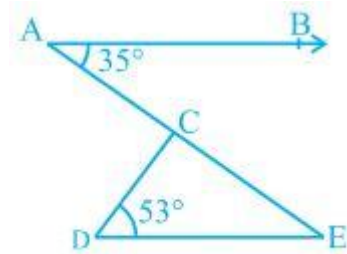


Fig. 6.41

Solution

We know that AE is a transversal since $AB \parallel DE$

Here $\angle BAC$ and $\angle AED$ are alternate interior angles.

Hence, $\angle BAC = \angle AED$

It is given that $\angle BAC = 35^\circ$

$$\angle AED = 35^\circ$$

Now consider the triangle CDE. We know that the sum of the interior angles of a triangle is 180° .

$$\therefore \angle DCE + \angle CED + \angle CDE = 180^\circ$$

Putting the values, we get

$$\angle DCE + 35^\circ + 53^\circ = 180^\circ$$

Hence, $\angle DCE = 92^\circ$

Question 4

In Fig. 6.42, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.

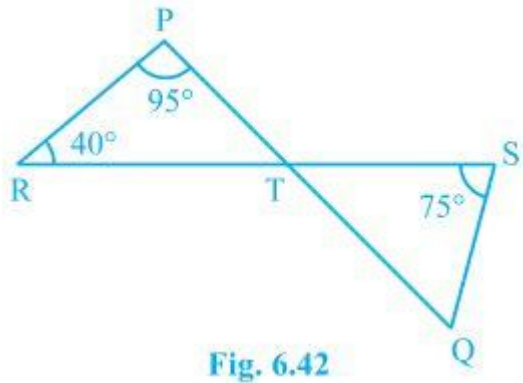


Fig. 6.42

Solution

Consider triangle PRT.

$$\angle PRT + \angle RPT + \angle PTR = 180^\circ$$

$$\text{So, } \angle PTR = 45^\circ$$

Now $\angle PTR$ will be equal to $\angle STQ$ as they are vertically opposite angles.

$$\text{So, } \angle PTR = \angle STQ = 45^\circ$$

Again, in triangle STQ,

$$\angle TSQ + \angle PTR + \angle SQT = 180^\circ$$

Solving this, we get

$$74^\circ + 45^\circ + \angle SQT = 180^\circ$$

$$\angle SQT = 60^\circ$$

Question 5

In Fig. 6.43, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.

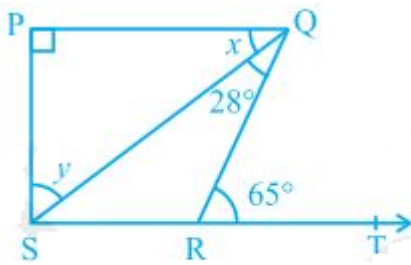


Fig. 6.43

Solution

$x + \angle SQR = \angle QRT$ (As they are alternate angles since QR is transversal)

$$\text{So, } x + 28^\circ = 65^\circ$$

$$\therefore x = 37^\circ$$

It is also known that alternate interior angles are the same, and so

$$\angle QSR = x = 37^\circ$$

Also, now

$$\angle QRS + \angle QRT = 180^\circ \text{ (As they are a Linear pair)}$$

$$\text{Or, } \angle QRS + 65^\circ = 180^\circ$$

$$\text{So, } \angle QRS = 115^\circ$$

Using the angle sum property in $\triangle SPQ$,

$$\angle SPQ + x + y = 180^\circ$$

Putting their respective values, we get

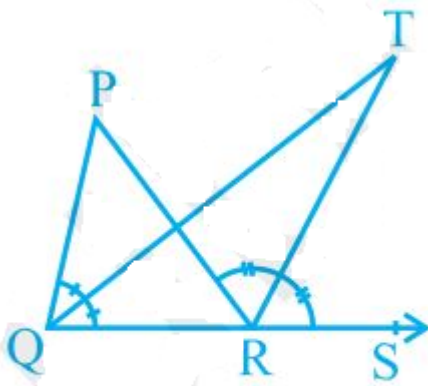
$$90^\circ + 37^\circ + y = 180^\circ$$

$$y = 180^\circ - 127^\circ = 53^\circ$$

$$\text{Hence, } y = 53^\circ$$

Question 6

In Fig. 6.44, the side QR of $\triangle PQR$ is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.

**Fig. 6.44****Solution**

Consider the ΔPQR . $\angle PRS$ is the exterior angle, and $\angle QPR$ and $\angle PQR$ are the interior angles.

So, $\angle PRS = \angle QPR + \angle PQR$ (According to triangle property)

$$\text{Or, } \angle PRS - \angle PQR = \angle QPR \text{ —————(i)}$$

Now, consider the ΔQRT ,

$$\angle TRS = \angle TQR + \angle QTR$$

$$\text{Or, } \angle QTR = \angle TRS - \angle TQR$$

We know that QT and RT bisect $\angle PQR$ and $\angle PRS$, respectively.

$$\text{So, } \angle PRS = 2 \angle TRS \text{ and } \angle PQR = 2 \angle TQR$$

$$\text{Now, } \angle QTR = \frac{1}{2} \angle PRS - \frac{1}{2} \angle PQR$$

$$\text{Or, } \angle QTR = \frac{1}{2} (\angle PRS - \angle PQR)$$

$$\text{From (i), we know that } \angle PRS - \angle PQR = \angle QPR$$

$$\text{So, } \angle QTR = \frac{1}{2} \angle QPR \text{ (hence proved).}$$