

Chapter 9: Algebraic Expressions and Identities

NCERT · Class 8 · Maths

Exercise 9.1 · Textbook page 140

Question 1

Identify the terms, their coefficients for each of the following expressions. (i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$ (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$ (iv) $3 - pq + qr - p$ (v) $(x/2) + (y/2) - xy$ (vi) $0.3a - 0.6ab + 0.5b$

Solution

Sl. No.	Expression	Term	Coefficient
i)	$5xyz^2 - 3zy$	Term: $5xyz^2$ Term: $-3zy$	5 -3
ii)	$1 + x + x^2$	Term: 1 Term: x Term: x^2	1 1 1
iii)	$4x^2y^2 - 4x^2y^2z^2 + z^2$	Term: $4x^2y^2$ Term: $-4x^2y^2z^2$ Term: z^2	4 -4 1
iv)	$3 - pq + qr - p$	Term: 3 -pq qr -p	3 -1 1 -1
v)	$(x/2) + (y/2) - xy$	Term: $x/2$ $y/2$ -xy	$\frac{1}{2}$ $\frac{1}{2}$ -1
vi)	$0.3a - 0.6ab + 0.5b$	Term: 0.3a -0.6ab 0.5b	0.3 -0.6 0.5

Question 2

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories? $x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Solution

Let us first define the classifications of these 3 polynomials:

Monomials contain only one term.

Binomials contain only two terms.

Trinomials contain only three terms.

$x + y$	two terms	Binomial
1000	one term	Monomial
$x + x^2 + x^3 + x^4$	four terms	Polynomial, and it does not fit in the listed three categories
$2y - 3y^2$	two terms	Binomial
$2y - 3y^2 + 4y^3$	three terms	Trinomial
$5x - 4y + 3xy$	three terms	Trinomial
$4z - 15z^2$	two terms	Binomial
$ab + bc + cd + da$	four terms	Polynomial, and it does not fit in the listed three categories
pqr	one term	Monomial
$p^2q + pq^2$	two terms	Binomial
$2p + 2q$	two terms	Binomial
$7 + y + 5x$	three terms	Trinomial

Question 3

Add the following.

(i) $ab - bc, bc - ca, ca - ab$

(ii) $a - b + ab, b - c + bc, c - a + ac$

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

Solution

$$\text{i) } (ab - bc) + (bc - ca) + (ca - ab)$$

$$= ab - bc + bc - ca + ca - ab$$

$$= ab - ab - bc + bc - ca + ca$$

$$= 0$$

$$\text{ii) } (a - b + ab) + (b - c + bc) + (c - a + ac)$$

$$= a - b + ab + b - c + bc + c - a + ac$$

$$= a - a + b - b + c - c + ab + bc + ca$$

$$= 0 + 0 + 0 + ab + bc + ca$$

$$= ab + bc + ca$$

$$\text{iii) } 2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$$

$$= (2p^2q^2 - 3pq + 4) + (5 + 7pq - 3p^2q^2)$$

$$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$$

$$= -p^2q^2 + 4pq + 9$$

$$\text{iv) } (l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl)$$

$$= l^2 + l^2 + m^2 + m^2 + n^2 + n^2 + 2lm + 2mn + 2nl$$

$$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$$

Question 4

$$\text{(a) Subtract } 4a - 7ab + 3b + 12 \text{ from } 12a - 9ab + 5b - 3$$

$$\text{(b) Subtract } 3xy + 5yz - 7zx \text{ from } 5xy - 2yz - 2zx + 10xyz$$

$$\text{(c) Subtract } 4p^2q - 3pq + 5pq^2 - 8p + 7q - 10 \text{ from } 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$$

Solution

$$\text{(a) } (12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$$

$$= 12a - 9ab + 5b - 3 - 4a + 7ab - 3b - 12$$

$$= 12a - 4a - 9ab + 7ab + 5b - 3b - 3 - 12$$

$$= 8a - 2ab + 2b - 15$$

$$\text{b) } (5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)$$

$$= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx$$

$$= 5xy - 3xy - 2yz - 5yz - 2zx + 7zx + 10xyz$$

$$= 2xy - 7yz + 5zx + 10xyz$$

$$\text{c) } (18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$$

$$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$$

$$= 18 + 10 - 3p + 8p - 11q - 7q + 5pq + 3pq - 2pq^2 - 5pq^2 + 5p^2q - 4p^2q$$

$$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$$

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Question 1

Find the product of the following pairs of monomials.

(i) $4, 7p$

(ii) $-4p, 7p$

(iii) $-4p, 7pq$

(iv) $4p^3, -3p$

(v) $4p, 0$

Solution

(i) $4, 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7) \times (p \times pq) = -28p^2q$

$$(iv) 4p^3 \times -3p = (4 \times -3) (p^3 \times p) = -12p^4$$

$$(v) 4p \times 0 = 0$$

Question 2

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths, respectively.

$$(p, q) ; (10m, 5n) ; (20x^2, 5y^2) ; (4x, 3x^2) ; (3mn, 4np)$$

Solution

Area of rectangle = Length $\times$ breadth. So, it is multiplication of two monomials.

The results can be written in square units.

$$(i) p \times q = pq$$

$$(ii) 10m \times 5n = 50mn$$

$$(iii) 20x^2 \times 5y^2 = 100x^2y^2$$

$$(iv) 4x \times 3x^2 = 12x^3$$

$$(v) 3mn \times 4np = 12mn^2p$$

Question 3

Complete the following table of products:

First monomial $\rightarrow$ Second monomial $\downarrow$	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Solution

First monomial	2x	-5y	3x ²	-4xy	7x ² y	-9x ² y ²
Second monomial						
2x	4x ²	-10xy	6x ³	-8x ² y	14x ³ y	-18x ³ y ²
-5y	-10xy	25y ²	-15x ² y	20xy ²	-35x ² y ²	45x ² y ³
3x ²	6x ³	-15x ² y	9x ⁴	-12x ³ y	21x ⁴ y	-27x ⁴ y ²
-4xy	-8x ² y	20xy ²	-12x ³ y	16x ² y ²	-28x ³ y ²	36x ³ y ³
7x ² y	14x ³ y	-35x ² y ²	21x ⁴ y	-28x ³ y ²	49x ⁴ y ²	-63x ⁴ y ³
-9x ² y ²	-18x ³ y ²	45x ² y ³	-27x ⁴ y ²	36x ³ y ³	-63x ⁴ y ³	81x ⁴ y ⁴

Question 4

Obtain the volume of rectangular boxes with the following length, breadth and height, respectively.

(i) $5a, 3a^2, 7a^4$

(ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$

Solution

Volume of rectangle = length $\times$ breadth $\times$ height. To evaluate volume of rectangular boxes, multiply all the monomials.

(i) $5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7) (a \times a^2 \times a^4) = 105a^7$

(ii) $2p \times 4q \times 8r = (2 \times 4 \times 8) (p \times q \times r) = 64pqr$

(iii) $y \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2) (x \times x^2 \times x \times y \times y \times y^2) = 4x^4y^4$

(iv) $a \times 2b \times 3c = (1 \times 2 \times 3) (a \times b \times c) = 6abc$

Question 5

Obtain the product of

(i) xy, yz, zx

(ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$

Solution

(i) $xy \times yz \times zx = x^2 y^2 z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024 y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2 b^2 c^2$

(v) $m \times -mn \times mnp = -m^3 n^2 p$

Exercise 9.3 · Textbook page 146

Question 1

Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q + r$

(ii) $ab, a - b$

(iii) $a + b, 7a^2b^2$

(iv) $a^2 - 9, 4a$

(v) $pq + qr + rp, 0$

Solution

(i) $4p(q + r) = 4pq + 4pr$

(ii) $ab(a - b) = a^2 b - a b^2$

(iii) $(a + b)(7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9)(4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = 0$ (Anything multiplied by zero is zero)

Question 2

Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

Solution

	First expression	Second expression	Product
(i)	a	$b + c + d$	$a(b+c+d)$ $= a \times b + a \times c + a \times d$ $= ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5xy(x + y - 5)$ $= 5xy \times x + 5xy \times y - 5xy \times 5$ $= 5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$p(6p^2 - 7p + 5)$ $= p \times 6p^2 - p \times 7p + p \times 5$ $= 6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^2q^2 \times (p^2 - q^2)$ $= 4p^4q^2 - 4p^2q^4$

(v)	$a + b + c$	abc	$abc(a + b + c)$ $= abc \times a + abc \times b + abc \times c$ $= a^2bc + ab^2c + abc^2$
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Question 3

Find the product.

i) $a^2 \times (2a^{22}) \times (4a^{26})$

ii) $(\frac{2}{3}xy) \times (-\frac{9}{10}x^2y^2)$

(iii) $(-\frac{10}{3}pq^3) \times (\frac{6}{5}p^3q)$

(iv) $(x) \times (x^2) \times (x^3) \times (x^4)$

Solution

i) $a^2 \times (2a^{22}) \times (4a^{26})$

$$= (2 \times 4) (a^2 \times a^{22} \times a^{26})$$

$$= 8 \times a^{2+22+26}$$

$$= 8a^{50}$$

ii) $(\frac{2xy}{3}) \times (-\frac{9x^2y^2}{10})$

$$= (\frac{2}{3} \times -\frac{9}{10}) (x \times x^2 \times y \times y^2)$$

$$= (-\frac{3}{5}x^3y^3)$$

iii) $(-\frac{10pq^3}{3}) \times (\frac{6p^3q}{5})$

$$= (-\frac{10}{3} \times \frac{6}{5}) (p \times p^3 \times q^3 \times q)$$

$$= (-4p^4q^4)$$

iv) $(x) \times (x^2) \times (x^3) \times (x^4)$

$$= x^{1+2+3+4}$$

$$= x^{10}$$

Question 4

(a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$ (ii) $x = 1/2$

(b) Simplify $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$, (ii) $a = 1$ (iii) $a = -1$.

Solution

a) $3x(4x - 5) + 3$

$$= 3x(4x) - 3x(5) + 3$$

$$= 12x^2 - 15x + 3$$

(i) Putting $x=3$ in the equation we get $12x^2 - 15x + 3 = 12(3^2) - 15(3) + 3$

$$= 108 - 45 + 3$$

$$= 66$$

(ii) Putting $x=1/2$ in the equation we get

$$12x^2 - 15x + 3 = 12(1/2)^2 - 15(1/2) + 3$$

$$= 12(1/4) - 15/2 + 3$$

$$= 3 - 15/2 + 3$$

$$= 6 - 15/2$$

$$= (12 - 15) / 2$$

$$= -3/2$$

b) $a(a^2 + a + 1) + 5$

$$= a \times a^2 + a \times a + a \times 1 + 5 = a^3 + a^2 + a + 5$$

(i) putting $a=0$ in the equation we get $0^3 + 0^2 + 0 + 5 = 5$

(ii) putting $a=1$ in the equation we get $1^3 + 1^2 + 1 + 5 = 1 + 1 + 1 + 5 = 8$

(iii) Putting $a = -1$ in the equation we get $(-1)^3 + (-1)^2 + (-1) + 5 = -1 + 1 - 1 + 5 = 4$

Question 5

(a) Add: $p (p - q)$, $q (q - r)$ and $r (r - p)$

(b) Add: $2x (z - x - y)$ and $2y (z - y - x)$

(c) Subtract: $3l (l - 4 m + 5 n)$ from $4l (10 n - 3 m + 2 l)$

(d) Subtract: $3a (a + b + c) - 2 b (a - b + c)$ from $4c (- a + b + c)$

Solution

$$a) p (p - q) + q (q - r) + r (r - p)$$

$$= (p^2 - pq) + (q^2 - qr) + (r^2 - pr)$$

$$= p^2 + q^2 + r^2 - pq - qr - pr$$

$$b) 2x (z - x - y) + 2y (z - y - x)$$

$$= (2xz - 2x^2 - 2xy) + (2yz - 2y^2 - 2xy)$$

$$= 2xz - 4xy + 2yz - 2x^2 - 2y^2$$

$$c) 4l (10 n - 3 m + 2 l) - 3l (l - 4 m + 5 n)$$

$$= (40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$$

$$= 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$$

$$= 25ln + 5l^2$$

$$d) 4c (- a + b + c) - (3a (a + b + c) - 2 b (a - b + c))$$

$$= (-4ac + 4bc + 4c^2) - (3a^2 + 3ab + 3ac - (2ab - 2b^2 + 2bc))$$

$$= -4ac + 4bc + 4c^2 - (3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc)$$

$$= -4ac + 4bc + 4c^2 - 3a^2 - 3ab - 3ac + 2ab - 2b^2 + 2bc$$

$$= -7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$$

Exercise 9.4 · Textbook page 148

Question 1

Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$

(ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

(iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(vi) $(\frac{3}{4}a^2 + 3b^2)$ and $4(a^2 - \frac{2}{3}b^2)$

Solution

(i) $(2x + 5)(4x - 3)$

$$= 2x \times 4x - 2x \times 3 + 5 \times 4x - 5 \times 3$$

$$= 8x^2 - 6x + 20x - 15$$

$$= 8x^2 + 14x - 15$$

ii) $(y - 8)(3y - 4)$

$$= y \times 3y - 4y - 8 \times 3y + 32$$

$$= 3y^2 - 4y - 24y + 32$$

$$= 3y^2 - 28y + 32$$

(iii) $(2.5l - 0.5m)(2.5l + 0.5m)$

$$= 2.5l \times 2.5l + 2.5l \times 0.5m - 0.5m \times 2.5l - 0.5m \times 0.5m$$

$$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$$

$$= 6.25l^2 - 0.25m^2$$

iv) $(a + 3b)(x + 5)$

$$= ax + 5a + 3bx + 15b$$

v) $(2pq + 3q^2)(3pq - 2q^2)$

$$= 2pq \times 3pq - 2pq \times 2q^2 + 3q^2 \times 3pq - 3q^2 \times 2q^2$$

$$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$$

$$= 6p^2q^2 + 5pq^3 - 6q^4$$

$$(vi) \left(\frac{3}{4}a^2 + 3b^2\right) \text{ and } 4\left(a^2 - \frac{2}{3}b^2\right)$$

$$= \left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right)$$

$$= \left(\frac{3}{4}a^2 + 3b^2\right) \times (4a^2 - \frac{8}{3}b^2)$$

$$= \frac{3}{4}a^2 \times (4a^2 - \frac{8}{3}b^2) + 3b^2 \times (4a^2 - \frac{8}{3}b^2)$$

$$= \frac{3}{4}a^2 \times 4a^2 - \frac{3}{4}a^2 \times \frac{8}{3}b^2 + 3b^2 \times 4a^2 - 3b^2 \times \frac{8}{3}b^2$$

$$= 3a^4 - 2a^2b^2 + 12a^2b^2 - 8b^4$$

$$= 3a^4 + 10a^2b^2 - 8b^4$$

Question 2

Find the product.

$$(i) (5 - 2x)(3 + x)$$

$$(ii) (x + 7y)(7x - y)$$

$$(iii) (a^2 + b)(a + b^2)$$

$$(iv) (p^2 - q^2)(2p + q)$$

Solution

$$(i) (5 - 2x)(3 + x)$$

$$= 5(3 + x) - 2x(3 + x)$$

$$= 15 + 5x - 6x - 2x^2$$

$$= 15 - x - 2x^2$$

$$(ii) (x + 7y)(7x - y)$$

$$= x(7x - y) + 7y(7x - y)$$

$$= 7x^2 - xy + 49xy - 7y^2$$

$$= 7x^2 - 7y^2 + 48xy$$

$$\text{iii) } (a^2 + b)(a + b^2)$$

$$= a^2(a + b^2) + b(a + b^2)$$

$$= a^3 + a^2b^2 + ab + b^3$$

$$= a^3 + b^3 + a^2b^2 + ab$$

$$\text{iv) } (p^2 - q^2)(2p + q)$$

$$= p^2(2p + q) - q^2(2p + q)$$

$$= 2p^3 + p^2q - 2pq^2 - q^3$$

$$= 2p^3 - q^3 + p^2q - 2pq^2$$

Question 3

Simplify.

$$\text{(i) } (x^2 - 5)(x + 5) + 25$$

$$\text{(ii) } (a^2 + 5)(b^3 + 3) + 5$$

$$\text{(iii) } (t + s^2)(t^2 - s)$$

$$\text{(iv) } (a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$$

$$\text{(v) } (x + y)(2x + y) + (x + 2y)(x - y)$$

$$\text{(vi) } (x + y)(x^2 - xy + y^2)$$

$$\text{(vii) } (1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$$

$$\text{(viii) } (a + b + c)(a + b - c)$$

Solution

$$\text{i) } (x^2 - 5)(x + 5) + 25$$

$$= x^3 + 5x^2 - 5x - 25 + 25$$

$$= x^3 + 5x^2 - 5x$$

$$\text{ii) } (a^2 + 5)(b^3 + 3) + 5$$

$$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$$

$$= a^2b^3 + 5b^3 + 3a^2 + 20$$

$$\text{iii) } (t + s^2)(t^2 - s)$$

$$= t(t^2 - s) + s^2(t^2 - s)$$

$$= t^3 - st + s^2t^2 - s^3$$

$$= t^3 - s^3 - st + s^2t^2$$

$$\text{iv) } (a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$$

$$= (a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$$

$$= (ac - ad + bc - bd) + (ac + ad - bc - bd) + (2ac + 2bd)$$

$$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$$

$$= 4ac$$

$$\text{v) } (x + y)(2x + y) + (x + 2y)(x - y)$$

$$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$$

$$= 3x^2 + 4xy - y^2$$

$$\text{vi) } (x + y)(x^2 - xy + y^2)$$

$$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$$

$$= x^3 + y^3$$

$$\text{vii) } (1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$$

$$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y = 2.25x^2 - 16y^2$$

$$\text{viii) } (a + b + c)(a + b - c)$$

$$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$$

$$= a^2 + b^2 - c^2 + 2ab$$

Exercise 9.5 · Textbook page 151

Question 1

Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$

(ii) $(2y + 5)(2y + 5)$

(iii) $(2a - 7)(2a - 7)$

(iv) $(3a - \frac{1}{2})(3a - \frac{1}{2})$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

(vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$

(viii) $(-a + c)(-a + c)$

(ix) $(\frac{1}{2}x + \frac{3}{4}y)(\frac{1}{2}x + \frac{3}{4}y)$

(x) $(7a - 9b)(7a - 9b)$

Solution

(i) $(x + 3)(x + 3) = (x + 3)^2$

$$= x^2 + 6x + 9$$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$$= 4y^2 + 20y + 25$$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

iii) $(2a - 7)(2a - 7) = (2a - 7)^2$

$$= 4a^2 - 28a + 49$$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

iv) $(3a - \frac{1}{2})(3a - \frac{1}{2}) = (3a - \frac{1}{2})^2$

$$= 9a^2 - 3a + \frac{1}{4}$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab$$

$$\text{v) } (1.1m - 0.4)(1.1m + 0.4)$$

$$= 1.21m^2 - 0.16$$

$$\text{Using } (a - b)(a + b) = a^2 - b^2$$

$$\text{vi) } (a^2 + b^2)(-a^2 + b^2)$$

$$= (b^2 + a^2)(b^2 - a^2)$$

$$= -a^4 + b^4$$

$$\text{Using } (a - b)(a + b) = a^2 - b^2$$

$$\text{vii) } (6x - 7)(6x + 7)$$

$$= 36x^2 - 49$$

$$\text{Using } (a - b)(a + b) = a^2 - b^2$$

$$\text{viii) } (-a + c)(-a + c) = (-a + c)^2$$

$$= c^2 + a^2 - 2ac$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab$$

$$\text{ix) } \left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$$

$$= (x^2/4) + (9y^2/16) + (3xy/4)$$

$$\text{Using } (a+b)^2 = a^2 + b^2 + 2ab$$

$$\text{x) } (7a - 9b)(7a - 9b) = (7a - 9b)^2$$

$$= 49a^2 - 126ab + 81b^2$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab$$

Question 2

Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

(i) $(x + 3)(x + 7)$

(ii) $(4x + 5)(4x + 1)$

(iii) $(4x - 5)(4x - 1)$

(iv) $(4x + 5)(4x - 1)$

(v) $(2x + 5y)(2x + 3y)$

(vi) $(2a^2 + 9)(2a^2 + 5)$

(vii) $(xyz - 4)(xyz - 2)$

Solution

(i) $(x + 3)(x + 7)$

$$= x^2 + (3+7)x + 21$$

$$= x^2 + 10x + 21$$

ii) $(4x + 5)(4x + 1)$

$$= 16x^2 + 4x + 20x + 5$$

$$= 16x^2 + 24x + 5$$

iii) $(4x - 5)(4x - 1)$

$$= 16x^2 - 4x - 20x + 5$$

$$= 16x^2 - 24x + 5$$

iv) $(4x + 5)(4x - 1)$

$$= 16x^2 + (5-1)4x - 5$$

$$= 16x^2 + 16x - 5$$

v) $(2x + 5y)(2x + 3y)$

$$= 4x^2 + (5y + 3y)2x + 15y^2$$

$$= 4x^2 + 16xy + 15y^2$$

vi) $(2a^2 + 9)(2a^2 + 5)$

$$= 4a^4 + (9+5)2a^2 + 45$$

$$= 4a^4 + 28a^2 + 45$$

$$\text{vii) } (xyz - 4)(xyz - 2)$$

$$= x^2y^2z^2 + (-4 - 2)xyz + 8$$

$$= x^2y^2z^2 - 6xyz + 8$$

Question 3

Find the following squares by using the identities.

(i) $(b - 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 - 5y)^2$

(iv) $[(2m/3) + (3n/2)]^2$

(v) $(0.4p - 0.5q)^2$

(vi) $(2xy + 5y)^2$

Solution

Using identities:

$$(a - b)^2 = a^2 + b^2 - 2ab \quad (a + b)^2 = a^2 + b^2 + 2ab$$

(i) $(b - 7)^2 = b^2 - 14b + 49$

(ii) $(xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2 = 36x^4 - 60x^2y + 25y^2$

(iv) $[(2m/3) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$

(v) $(0.4p - 0.5q)^2 = 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = 4x^2y^2 + 20xy^2 + 25y^2$

Question 4**Simplify.**

(i) $(a^2 - b^2)^2$

(ii) $(2x + 5)^2 - (2x - 5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Solution

i) $(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$

ii) $(2x + 5)^2 - (2x - 5)^2$
 $= 4x^2 + 20x + 25 - (4x^2 - 20x + 25) = 4x^2 + 20x + 25 - 4x^2 + 20x - 25 = 40x$

iii) $(7m - 8n)^2 + (7m + 8n)^2$
 $= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
 $= 98m^2 + 128n^2$

iv) $(4m + 5n)^2 + (5m + 4n)^2$
 $= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
 $= 41m^2 + 80mn + 41n^2$

v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
 $= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$
 $= 4p^2 - 4q^2$

vi) $(ab + bc)^2 - 2ab^2c = a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c = a^2b^2 + b^2c^2$

vii) $(m^2 - n^2m)^2 + 2m^3n^2$
 $= m^4 - 2m^3n^2 + m^2n^4 + 2m^3n^2$
 $= m^4 + m^2n^4$

Question 5**Show that.**

$$(i) (3x + 7)^2 - 84x = (3x - 7)^2$$

$$(ii) (9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$(iii) (4/3m - 3/4n)^2 + 2mn = 16/9 m^2 + 9/16 n^2$$

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$(v) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Solution

$$i) LHS = (3x + 7)^2 - 84x$$

$$= 9x^2 + 42x + 49 - 84x$$

$$= 9x^2 - 42x + 49$$

$$= RHS$$

$$LHS = RHS$$

$$ii) LHS = (9p - 5q)^2 + 180pq$$

$$= 81p^2 - 90pq + 25q^2 + 180pq$$

$$= 81p^2 + 90pq + 25q^2$$

$$RHS = (9p + 5q)^2$$

$$= 81p^2 + 90pq + 25q^2$$

$$LHS = RHS$$

$$(iii) LHS = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$$

$$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$$

$$= \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$= RHS$$

$$LHS = RHS$$

$$iv) LHS = (4pq + 3q)^2 - (4pq - 3q)^2$$

$$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$$

$$= 48pq^2$$

$$= \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

$$\text{v) LHS} = (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$$

$$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$= 0$$

$$= \text{RHS}$$

Question 6

Using identities, evaluate.

(i) 71^2

(ii) 99^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(vi) 297×303

(vii) 78×82

(viii) 8.9^2

(ix) 10.5×9.5

Solution

i) 71^2

$$= (70+1)^2$$

$$= 70^2 + 140 + 1^2$$

$$= 4900 + 140 + 1$$

$$= 5041$$

ii) 99^2

$$= (100 - 1)^2$$

$$= 100^2 - 200 + 1^2$$

$$= 10000 - 200 + 1$$

$$= 9801$$

$$\text{iii) } 102^2$$

$$= (100 + 2)^2$$

$$= 100^2 + 400 + 2^2$$

$$= 10000 + 400 + 4 = 10404$$

$$\text{iv) } 998^2$$

$$= (1000 - 2)^2$$

$$= 1000^2 - 4000 + 2^2$$

$$= 1000000 - 4000 + 4$$

$$= 996004$$

$$\text{v) } 5.2^2$$

$$= (5 + 0.2)^2$$

$$= 5^2 + 2 + 0.2^2$$

$$= 25 + 2 + 0.04 = 27.04$$

$$\text{vi) } 297 \times 303$$

$$= (300 - 3)(300 + 3)$$

$$= 300^2 - 3^2$$

$$= 90000 - 9$$

$$= 89991$$

$$\text{vii) } 78 \times 82$$

$$= (80 - 2)(80 + 2)$$

$$= 80^2 - 2^2$$

$$= 6400 - 4$$

$$= 6396$$

viii) 8.9^2

$$= (9 - 0.1)^2$$

$$= 9^2 - 1.8 + 0.1^2$$

$$= 81 - 1.8 + 0.01$$

$$= 79.21$$

ix) 10.5×9.5

$$= (10 + 0.5)(10 - 0.5)$$

$$= 10^2 - 0.5^2$$

$$= 100 - 0.25$$

$$= 99.75$$

Question 7

Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Solution

i) $51^2 - 49^2$

$$= (51 + 49)(51 - 49) = 100 \times 2 = 200$$

ii) $(1.02)^2 - (0.98)^2$

$$= (1.02 + 0.98)(1.02 - 0.98) = 2 \times 0.04 = 0.08$$

$$\text{iii) } 153^2 - 147^2$$

$$= (153 + 147)(153 - 147) = 300 \times 6 = 1800$$

$$\text{iv) } 12.1^2 - 7.9^2$$

$$= (12.1 + 7.9)(12.1 - 7.9) = 20 \times 4.2 = 84$$

Question 8

Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103×104

(ii) 5.1×5.2

(iii) 103×98

(iv) 9.7×9.8

Solution

i) 103×104

$$= (100 + 3)(100 + 4)$$

$$= 100^2 + (3 + 4)100 + 12$$

$$= 10000 + 700 + 12$$

$$= 10712$$

ii) 5.1×5.2

$$= (5 + 0.1)(5 + 0.2)$$

$$= 5^2 + (0.1 + 0.2)5 + 0.1 \times 0.2$$

$$= 25 + 1.5 + 0.02$$

$$= 26.52$$

iii) 103×98

$$= (100 + 3)(100 - 2)$$

$$= 100^2 + (3-2)100 - 6$$

$$= 10000 + 100 - 6$$

$$= 10094$$

$$\text{iv) } 9.7 \times 9.8$$

$$= (9 + 0.7)(9 + 0.8)$$

$$= 9^2 + (0.7 + 0.8)9 + 0.56$$

$$= 81 + 13.5 + 0.56$$

$$= 95.06$$
