

Chapter 14: Factorisation

NCERT · Class 8 · Maths

Exercise 14.1 · Textbook page 208

Question 1

Find the common factors of the given terms.

(i) $12x$, 36

(ii) $2y$, $22xy$

(iii) $14pq$, $28p^2q^2$

(iv) $2x$, $3x^2$, 4

(v) $6abc$, $24ab^2$, $12a^2b$

(vi) $16x^3$, $-4x^2$, $32x$

(vii) $10pq$, $20qr$, $30rp$

(viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

Solution

(i) Factors of $12x$ and 36

$$12x = 2 \times 2 \times 3 \times x$$

$$36 = 2 \times 2 \times 3 \times 3$$

Common factors of $12x$ and 36 are $2, 2, 3$

$$\text{and } 2 \times 2 \times 3 = 12$$

(ii) Factors of $2y$ and $22xy$

$$2y = 2 \times y$$

$$22xy = 2 \times 11 \times x \times y$$

Common factors of $2y$ and $22xy$ are $2, y$

and, $2 \times y = 2y$

(iii) Factors of $14pq$ and $28p^2q^2$

$$14pq = 2 \times 7 \times p \times q$$

$$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$$

Common factors of $14pq$ and $28p^2q^2$ are $2, 7, p, q$

$$\text{and, } 2 \times 7 \times p \times q = 14pq$$

(iv) Factors of $2x, 3x^2$ and 4

$$2x = 2 \times x$$

$$3x^2 = 3 \times x \times x$$

$$4 = 2 \times 2$$

Common factors of $2x, 3x^2$ and 4 is 1 .

(v) Factors of $6abc, 24ab^2$ and $12a^2b$

$$6abc = 2 \times 3 \times a \times b \times c$$

$$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$$

$$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$$

Common factors of $6abc, 24ab^2$ and $12a^2b$ are $2, 3, a, b$

$$\text{and, } 2 \times 3 \times a \times b = 6ab$$

(vi) Factors of $16x^3, -4x^2$ and $32x$

$$16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$$

$$-4x^2 = -1 \times 2 \times 2 \times x \times x$$

$$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$$

Common factors of $16x^3, -4x^2$ and $32x$ are $2, 2, x$

$$\text{and, } 2 \times 2 \times x = 4x$$

(vii) Factors of $10pq, 20qr$ and $30rp$

$$10\ pq = 2 \times 5 \times p \times q$$

$$20qr = 2 \times 2 \times 5 \times q \times r$$

$$30rp = 2 \times 3 \times 5 \times r \times p$$

Common factors of 10 pq, 20qr and 30rp are 2, 5

and, $2 \times 5 = 10$

(viii) Factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$

$$3x^2y^3 = 3 \times x \times x \times y \times y \times y$$

$$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$$

$$6x^2y^2z = 3 \times 2 \times x \times x \times y \times y \times z$$

Common factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$ are x^2 , y^2

and, $x^2 \times y^2 = x^2y^2$

Question 2

Factorise the following expressions.

(i) $7x-42$

(ii) $6p-12q$

(iii) $7a^2+14a$

(iv) $-16z+20z^3$

(v) $20l^2m+30alm$

(vi) $5x^2y-15xy^2$

(vii) $10a^2-15b^2+20c^2$

(viii) $-4a^2+4ab-4ca$

(ix) $x^2yz+xy^2z+xyz^2$

(x) ax^2y+bxy^2+cxyz

Solution

$$(i) \quad 7x = 7 \times x$$

$$42 = 2 \times 3 \times 7$$

The common factor is 7

$$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$$

$$(ii) \quad 6p = 2 \times 3 \times p$$

$$12q = 2 \times 2 \times 3 \times q$$

The common factors are 2 and 3

$$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$$

$$= 2 \times 3 [p - (2 \times q)]$$

$$= 6(p - 2q)$$

$$(iii) \quad 7a^2 = 7 \times a \times a$$

$$14a = 2 \times 7 \times a$$

The common factors are 7 and a

$$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$$

$$= 7 \times a [a + 2] = 7a(a + 2)$$

$$(iv) \quad 16z = 2 \times 2 \times 2 \times 2 \times z$$

$$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$$

The common factors are 2, 2, and z.

$$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$$

$$= (2 \times 2 \times z) [-(2 \times 2) + (5 \times z \times z)]$$

$$= 4z(-4 + 5z^2)$$

$$(v) \quad 20l^2m = 2 \times 2 \times 5 \times l \times l \times m$$

$$30alm = 2 \times 3 \times 5 \times a \times l \times m$$

The common factors are 2, 5, l and m

$$\therefore 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$$

$$= (2 \times 5 \times l \times m) [(2 \times l) + (3 \times a)]$$

$$= 10lm(2l + 3a)$$

$$(vi) \quad 5x^2y = 5 \times x \times x \times y$$

$$15xy^2 = 3 \times 5 \times x \times y \times y$$

The common factors are 5, x, and y

$$\therefore 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$$

$$= 5 \times x \times y [x - (3 \times y)]$$

$$= 5xy(x - 3y)$$

$$(vii) \quad 10a^2 - 15b^2 + 20c^2$$

$$10a^2 = 2 \times 5 \times a \times a$$

$$-15b^2 = -1 \times 3 \times 5 \times b \times b$$

$$20c^2 = 2 \times 2 \times 5 \times c \times c$$

Common factor of $10a^2$, $15b^2$ and $20c^2$ is 5

$$10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$$

$$(viii) -4a^2 + 4ab - 4ca$$

$$-4a^2 = -1 \times 2 \times 2 \times a \times a$$

$$4ab = 2 \times 2 \times a \times b$$

$$-4ca = -1 \times 2 \times 2 \times c \times a$$

Common factor of $-4a^2$, $4ab$, $-4ca$ are 2, 2, a i.e. $4a$

So,

$$-4a^2 + 4ab - 4ca = 4a(-a + b - c)$$

$$(ix) x^2yz + xy^2z + xyz^2$$

$$x^2yz = x \times x \times y \times z$$

$$xy^2z = x \times y \times y \times z$$

$$xyz^2 = x \times y \times z \times z$$

Common factor of x^2yz , xy^2z and xyz^2 are x, y, z i.e. xyz

$$\text{Now, } x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$$

$$(x) ax^2y + bxy^2 + cxyz$$

$$ax^2y = a \times x \times x \times y$$

$$bxy^2 = b \times x \times y \times y$$

$$cxyz = c \times x \times y \times z$$

Common factors of ax^2y , bxy^2 and $cxyz$ are xy

$$\text{Now, } ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$$

Question 3

Factorise.

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

Solution

$$\begin{aligned}
 (i) \quad x^2 + xy + 8x + 8y &= x \times x + x \times y + 8 \times x + 8 \times y \\
 &= x(x + y) + 8(x + y) \\
 &= (x + y)(x + 8)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad 15xy - 6x + 5y - 2 &= 3 \times 5 \times x \times y - 3 \times 2 \times x + 5xy - 2 \\
 &= 3x(5y - 2) + 1(5y - 2) \\
 &= (5y - 2)(3x + 1)
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad ax + bx - ay - by &= a \times x + b \times x - a \times y - b \times y \\
 &= x(a + b) - y(a + b) \\
 &= (a + b)(x - y)
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad 15pq + 15 + 9q + 25p &= 15pq + 9q + 25p + 15 \\
 &= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5 \\
 &= 3q(5p + 3) + 5(5p + 3) \\
 &= (5p + 3)(3q + 5)
 \end{aligned}$$

$$\begin{aligned}
 (v) \quad z - 7 + 7xy - xyz &= z - x \times y \times z - 7 + 7 \times x \times y \\
 &= z(1 - xy) - 7(1 - xy) \\
 &= (1 - xy)(z - 7)
 \end{aligned}$$

Exercise 14.2 · Textbook page 223**Question 1**

Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l+m)^2 - 4lm$ (Hint: Expand $(l+m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Solution

(i) $a^2 + 8a + 16$

$$= a^2 + 2 \times 4 \times a + 4^2$$

$$= (a+4)^2$$

Using the identity $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $p^2 - 10p + 25$

$$= p^2 - 2 \times 5 \times p + 5^2$$

$$= (p-5)^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $25m^2 + 30m + 9$

$$= (5m)^2 + 2 \times 5m \times 3 + 3^2$$

$$= (5m+3)^2$$

Using the identity $(x+y)^2 = x^2 + 2xy + y^2$

(iv) $49y^2 + 84yz + 36z^2$

$$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$$

$$= (7y+6z)^2$$

Using the identity $(x+y)^2 = x^2 + 2xy + y^2$

$$(v) 4x^2 - 8x + 4$$

$$= (2x)^2 - 2 \times 4x + 2^2$$

$$= (2x - 2)^2$$

Using the identity $(x - y)^2 = x^2 - 2xy + y^2$

$$(vi) 121b^2 - 88bc + 16c^2$$

$$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$$

$$= (11b - 4c)^2$$

Using the identity $(x - y)^2 = x^2 - 2xy + y^2$

$$(vii) (l + m)^2 - 4lm \text{ (Hint: Expand } (l + m)^2 \text{ first)}$$

Expand $(l + m)^2$ using the identity $(x + y)^2 = x^2 + 2xy + y^2$

$$(l + m)^2 - 4lm = l^2 + m^2 + 2lm - 4lm$$

$$= l^2 + m^2 - 2lm$$

$$= (l - m)^2$$

Using the identity $(x - y)^2 = x^2 - 2xy + y^2$

$$(viii) a^4 + 2a^2b^2 + b^4$$

$$= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$$

$$= (a^2 + b^2)^2$$

Using the identity $(x + y)^2 = x^2 + 2xy + y^2$

Question 2

Factorise.

$$(i) 4p^2 - 9q^2$$

$$(ii) 63a^2 - 112b^2$$

$$(iii) 49x^2 - 36$$

(iv) $16x^5 - 144x^3$ differ

(v) $(l+m)^2 - (l-m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution

(i) $4p^2 - 9q^2$

$$= (2p)^2 - (3q)^2$$

$$= (2p-3q)(2p+3q)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

(ii) $63a^2 - 112b^2$

$$= 7(9a^2 - 16b^2)$$

$$= 7((3a)^2 - (4b)^2)$$

$$= 7(3a+4b)(3a-4b)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

(iii) $49x^2 - 36$

$$= (7x)^2 - 6^2$$

$$= (7x+6)(7x-6)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

(iv) $16x^5 - 144x^3$

$$= 16x^3(x^2 - 9)$$

$$= 16x^3(x^2 - 9)$$

$$= 16x^3(x-3)(x+3)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(v) (l+m)^2 - (l-m)^2$$

$$= \{(l+m) - (l-m)\} \{(l+m) + (l-m)\}$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$= (l+m - l + m)(l+m + l - m)$$

$$= (2m)(2l)$$

$$= 4ml$$

$$(vi) 9x^2y^2 - 16$$

$$= (3xy)^2 - 4^2$$

$$= (3xy - 4)(3xy + 4)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(vii) (x^2 - 2xy + y^2) - z^2$$

$$= (x-y)^2 - z^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

$$= \{(x-y) - z\} \{(x-y) + z\}$$

$$= (x-y-z)(x-y+z)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(viii) 25a^2 - 4b^2 + 28bc - 49c^2$$

$$= 25a^2 - (4b^2 - 28bc + 49c^2)$$

$$= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$$

$$= (5a)^2 - (2b - 7c)^2$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$, we have

$$= (5a + 2b - 7c)(5a - 2b + 7c)$$

Question 3

Factorise the expressions.

(i) ax^2+bx

(ii) $7p^2+21q^2$

(iii) $2x^3+2xy^2+2xz^2$

(iv) $am^2+bm^2+bn^2+an^2$

(v) $(lm+l)+m+1$

(vi) $y(y+z)+9(y+z)$

(vii) $5y^2-20y-8z+2yz$

(viii) $10ab+4a+5b+2$

(ix) $6xy-4y+6-9x$

Solution

(i) $ax^2+bx = x(ax+b)$

(ii) $7p^2+21q^2 = 7(p^2+3q^2)$

(iii) $2x^3+2xy^2+2xz^2 = 2x(x^2+y^2+z^2)$

(iv) $am^2+bm^2+bn^2+an^2 = m^2(a+b)+n^2(a+b) = (a+b)(m^2+n^2)$

(v) $(lm+l)+m+1 = lm+m+l+1 = m(l+1)+(l+1) = (m+1)(l+1)$

(vi) $y(y+z)+9(y+z) = (y+9)(y+z)$

(vii) $5y^2-20y-8z+2yz = 5y(y-4)+2z(y-4) = (y-4)(5y+2z)$

(viii) $10ab+4a+5b+2 = 5b(2a+1)+2(2a+1) = (2a+1)(5b+2)$

(ix) $6xy-4y+6-9x = 6xy-9x-4y+6 = 3x(2y-3)-2(2y-3) = (2y-3)(3x-2)$

Question 4

Factorise.

(i) a^4-b^4

$$(ii) p^4 - 81$$

$$(iii) x^4 - (y+z)^4$$

$$(iv) x^4 - (x-z)^4$$

$$(v) a^4 - 2a^2b^2 + b^4$$

Solution

$$(i) a^4 - b^4$$

$$= (a^2)^2 - (b^2)^2$$

$$= (a^2 - b^2)(a^2 + b^2)$$

$$= (a - b)(a + b)(a^2 + b^2)$$

$$(ii) p^4 - 81$$

$$= (p^2)^2 - (9)^2$$

$$= (p^2 - 9)(p^2 + 9)$$

$$= (p^2 - 3^2)(p^2 + 9)$$

$$= (p - 3)(p + 3)(p^2 + 9)$$

$$(iii) x^4 - (y+z)^4 = (x^2)^2 - [(y+z)^2]^2$$

$$= \{x^2 - (y+z)^2\} \{x^2 + (y+z)^2\}$$

$$= \{(x - (y+z))(x + (y+z))\} \{x^2 + (y+z)^2\}$$

$$= (x - y - z)(x + y + z) \{x^2 + (y+z)^2\}$$

$$(iv) x^4 - (x-z)^4 = (x^2)^2 - \{(x-z)^2\}^2$$

$$= \{x^2 - (x-z)^2\} \{x^2 + (x-z)^2\}$$

$$= \{x - (x-z)\} \{x + (x-z)\} \{x^2 + (x-z)^2\}$$

$$= z(2x - z) \{x^2 + x^2 - 2xz + z^2\}$$

$$= z(2x - z) (2x^2 - 2xz + z^2)$$

$$\begin{aligned} \text{(v)} \quad a^4 - 2a^2b^2 + b^4 &= (a^2)^2 - 2a^2b^2 + (b^2)^2 \\ &= (a^2 - b^2)^2 \\ &= ((a-b)(a+b))^2 \\ &= (a-b)^2 (a+b)^2 \end{aligned}$$

Question 5

Factorise the following expressions.

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Solution

(i) $p^2 + 6p + 8$

We observed that $8 = 4 \times 2$ and $4 + 2 = 6$

$p^2 + 6p + 8$ can be written as $p^2 + 2p + 4p + 8$

Taking Common terms, we get

$$p^2 + 6p + 8 = p^2 + 2p + 4p + 8 = p(p+2) + 4(p+2)$$

Again, $p+2$ is common in both the terms.

$$= (p+2)(p+4)$$

This implies that $p^2 + 6p + 8 = (p+2)(p+4)$

(ii) $q^2 - 10q + 21$

We observed that $21 = -7 \times -3$ and $-7 + (-3) = -10$

$$q^2 - 10q + 21 = q^2 - 3q - 7q + 21$$

$$= q(q-3) - 7(q-3)$$

$$= (q-7)(q-3)$$

This implies that $q^2 - 10q + 21 = (q-7)(q-3)$

(iii) $p^2 + 6p - 16$

We observed that $-16 = -2 \times 8$ and $8 + (-2) = 6$

$$p^2 + 6p - 16 = p^2 - 2p + 8p - 16$$

$$= p(p-2) + 8(p-2)$$

$$= (p+8)(p-2)$$

$$\text{So, } p^2 + 6p - 16 = (p+8)(p-2)$$

Exercise 14.3 · Textbook page 227

Question 1

Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Solution

$$(i) 28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$$

$$56x = 2 \times 2 \times 2 \times 7 \times x$$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

$$(ii) -36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

$$(iii) 66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

$$(iv) 34x^3y^3z^3 \div 51xy^2z^3 = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2}{3} x^2y$$

$$(v) 12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2 a^2 b^4$$

Question 2

Divide the given polynomial by the given monomial.

$$(i) (5x^2 - 6x) \div 3x$$

$$(ii) (3y^8 - 4y^6 + 5y^4) \div y^4$$

$$(iii) 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$$

$$(iv) (x^3 + 2x^2 + 3x) \div 2x$$

$$(v) (p^3q^6 - p^6q^3) \div p^3q^3$$

Solution

$$(i) \ 5x^2 - 6x = x(5x - 6)$$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

$$(ii) \ 3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

$$(iii) \ 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$$

$$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$$

$$(iv) \ x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^2 + 2x + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

$$(v) \ p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$$

Question 3

Work out the following divisions.

(i) $(10x-25) \div 5$

(ii) $(10x-25) \div (2x-5)$

(iii) $10y(6y+21) \div 5(2y+7)$

(iv) $9x^2y^2(3z-24) \div 27xy(z-8)$

(v) $96abc(3a-12)(5b-30) \div 144(a-4)(b-6)$

Solution

(i) $(10x-25) \div 5 = 5(2x-5)/5 = 2x-5$

(ii) $(10x-25) \div (2x-5) = 5(2x-5)/(2x-5) = 5$

(iii) $10y(6y+21) \div 5(2y+7) = 10y \times 3(2y+7)/5(2y+7) = 6y$

(iv) $9x^2y^2(3z-24) \div 27xy(z-8) = 9x^2y^2 \times 3(z-8)/27xy(z-8) = xy$

$$(v) \underline{96abc(3a-12)(5b-30)} \div 144(a-4)(b-6) = \frac{96abc \times 3(a-4) \times 5(b-6)}{144(a-4)(b-6)} = 10abc$$

Question 4

Divide as directed.

(i) $5(2x+1)(3x+5) \div (2x+1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4)$

(iii) $52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$

(iv) $20(y+4)(y^2+5y+3) \div 5(y+4)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

Solution

$$\begin{aligned} (i) \quad 5(2x+1)(3x+5) \div (2x+1) &= \frac{5(2x+1)(3x+5)}{(2x+1)} \\ &= 5(3x+5) \end{aligned}$$

$$\begin{aligned} (ii) \quad 26xy(x+5)(y-4) \div 13x(y-4) &= \frac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)} \\ &= 2y(x+5) \end{aligned}$$

$$\begin{aligned} (iii) \quad 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p) \\ &= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)} \\ &= \frac{1}{2}r(p+q) \end{aligned}$$

$$\begin{aligned} (iv) \quad 20(y+4)(y^2+5y+3) &= 2 \times 2 \times 5 \times (y+4)(y^2+5y+3) \\ 20(y+4)(y^2+5y+3) \div 5(y+4) &= \frac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)} \\ &= 4(y^2+5y+3) \end{aligned}$$

$$\begin{aligned} (v) \quad x(x+1)(x+2)(x+3) \div x(x+1) &= \frac{x(x+1)(x+2)(x+3)}{x(x+1)} \\ &= (x+2)(x+3) \end{aligned}$$

Question 5

Factorise the expressions and divide them as directed.

(i) $(y^2+7y+10) \div (y+5)$

(ii) $(m^2-14m-32) \div (m+2)$

(iii) $(5p^2-25p+20) \div (p-1)$

(iv) $4yz(z^2+6z-16) \div 2y(z+8)$

(v) $5pq(p^2-q^2) \div 2p(p+q)$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

(vii) $39y^3(50y^2-98) \div 26y^2(5y+7)$

Solution

(i) $(y^2+7y+10) \div (y+5)$

First, solve the equation $(y^2+7y+10)$

$$(y^2+7y+10) = y^2+2y+5y+10 = y(y+2)+5(y+2) = (y+2)(y+5)$$

$$\text{Now, } (y^2+7y+10) \div (y+5) = (y+2)(y+5)/(y+5) = y+2$$

(ii) $(m^2-14m-32) \div (m+2)$

Solve for $m^2-14m-32$, we have

$$m^2-14m-32 = m^2+2m-16m-32 = m(m+2)-16(m+2) = (m-16)(m+2)$$

$$\text{Now, } (m^2-14m-32) \div (m+2) = (m-16)(m+2)/(m+2) = m-16$$

(iii) $(5p^2-25p+20) \div (p-1)$

Step 1: Take 5 common from the equation, $5p^2-25p+20$, we get

$$5p^2-25p+20 = 5(p^2-5p+4)$$

Step 2: Factorise p^2-5p+4

$$p^2-5p+4 = p^2-p-4p+4 = (p-1)(p-4)$$

Step 3: Solve original equation

$$(5p^2-25p+20) \div (p-1) = 5(p-1)(p-4)/(p-1) = 5(p-4)$$

(iv) $4yz(z^2 + 6z-16) \div 2y(z+8)$

Factorising $z^2+6z-16$,

$$z^2+6z-16 = z^2-2z+8z-16 = (z-2)(z+8)$$

$$\text{Now, } 4yz(z^2+6z-16) \div 2y(z+8) = 4yz(z-2)(z+8)/2y(z+8) = 2z(z-2)$$

$$\text{(v) } 5pq(p^2-q^2) \div 2p(p+q)$$

p^2-q^2 can be written as $(p-q)(p+q)$ using the identity.

$$5pq(p^2-q^2) \div 2p(p+q) = 5pq(p-q)(p+q)/2p(p+q) = 5q(p-q)/2$$

$$\text{(vi) } 12xy(9x^2-16y^2) \div 4xy(3x+4y)$$

Factorising $9x^2-16y^2$, we have

$$9x^2-16y^2 = (3x)^2-(4y)^2 = (3x+4y)(3x-4y) \text{ using the identity } p^2-q^2 = (p-q)(p+q)$$

$$\text{Now, } 12xy(9x^2-16y^2) \div 4xy(3x+4y) = 12xy(3x+4y)(3x-4y) / 4xy(3x+4y) = 3(3x-4y)$$

$$\text{(vii) } 39y^3(50y^2-98) \div 26y^2(5y+7)$$

st solve for $50y^2-98$, we have

$$50y^2-98 = 2(25y^2-49) = 2((5y)^2-7^2) = 2(5y-7)(5y+7)$$

$$\text{Now, } 39y^3(50y^2-98) \div 26y^2(5y+7) =$$

$$\frac{3 \times 13 \times y^3 \times 2(5y-7)(5y+7)}{2 \times 13 \times y^2(5y+7)} = 3y(5y-7)$$

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Question 1

$$4(x-5) = 4x-5$$

Solution

$$4(x-5) = 4x - 20 \neq 4x - 5 = \text{RHS}$$

The correct statement is $4(x-5) = 4x-20$

Question 2

$$x(3x+2) = 3x^2+2$$

Solution

$$\text{LHS} = x(3x+2) = 3x^2+2x \neq 3x^2+2 = \text{RHS}$$

The correct solution is $x(3x+2) = 3x^2+2x$

Question 3

$$2x+3y = 5xy$$

Solution

$$\text{LHS} = 2x+3y \neq \text{R. H. S}$$

The correct statement is $2x+3y = 2x+3y$

Question 4

$$x+2x+3x = 5x$$

Solution

$$\text{LHS} = x+2x+3x = 6x \neq \text{RHS}$$

The correct statement is $x+2x+3x = 6x$

Question 5

$$5y+2y+y-7y = 0$$

Solution

$$\text{LHS} = 5y+2y+y-7y = y \neq \text{RHS}$$

The correct statement is $5y+2y+y-7y = y$

Question 6

$$3x+2x = 5x^2$$

Solution

$$\text{LHS} = 3x+2x = 5x \neq \text{RHS}$$

The correct statement is $3x+2x = 5x$

Question 7

$$(2x)^2+4(2x)+7 = 2x^2+8x+7$$

Solution

$$\text{LHS} = (2x)^2+4(2x)+7 = 4x^2+8x+7 \neq \text{RHS}$$

The correct statement is $(2x)^2+4(2x)+7 = 4x^2+8x+7$

Question 8

$$(2x)^2+5x = 4x+5x = 9x$$

Solution

$$\text{LHS} = (2x)^2+5x = 4x^2+5x \neq 9x = \text{RHS}$$

The correct statement is $(2x)^2+5x = 4x^2+5x$

Question 9

$$(3x + 2)^2 = 3x^2+6x+4$$

Solution

$$\text{LHS} = (3x+2)^2 = (3x)^2+2^2+2 \times 2 \times 3x = 9x^2+4+12x \neq \text{RHS}$$

The correct statement is $(3x + 2)^2 = 9x^2+4+12x$

Question 10

Substituting $x = -3$ in

$$(a) \ x^2 + 5x + 4 \text{ gives } (-3)^2+5(-3)+4 = 9+2+4 = 15$$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = 9 - 15 = -6$

Solution

(a) Substituting $x = -3$ in $x^2 + 5x + 4$, we have

$$x^2 + 5x + 4 = (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2. \text{ This is the correct answer.}$$

(b) Substituting $x = -3$ in $x^2 - 5x + 4$

$$x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28. \text{ This is the correct answer}$$

(c) Substituting $x = -3$ in $x^2 + 5x$

$$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6. \text{ This is the correct answer}$$

Question 11

$$(y-3)^2 = y^2 - 9$$

Solution

LHS = $(y-3)^2$, which is similar to $(a-b)^2$ identity, where $(a-b)^2 = a^2 + b^2 - 2ab$

$$(y-3)^2 = y^2 + (3)^2 - 2y \times 3 = y^2 + 9 - 6y \neq y^2 - 9 = \text{RHS}$$

The correct statement is $(y-3)^2 = y^2 + 9 - 6y$

Question 12

$$(z+5)^2 = z^2 + 25$$

Solution

LHS = $(z+5)^2$, which is similar to $(a+b)^2$ identity, where $(a+b)^2 = a^2 + b^2 + 2ab$

$$(z+5)^2 = z^2 + 5^2 + 2 \times 5 \times z = z^2 + 25 + 10z \neq z^2 + 25 = \text{RHS}$$

The correct statement is $(z+5)^2 = z^2 + 25 + 10z$

Question 13

$$(2a+3b)(a-b) = 2a^2-3b^2$$

Solution

$$\text{LHS} = (2a+3b)(a-b) = 2a(a-b)+3b(a-b)$$

$$= 2a^2-2ab+3ab-3b^2$$

$$= 2a^2+ab-3b^2$$

$$\neq 2a^2-3b^2 = \text{RHS}$$

The correct statement is $(2a+3b)(a-b) = 2a^2+ab-3b^2$

Question 14

$$(a+4)(a+2) = a^2+8$$

Solution

$$\text{LHS} = (a+4)(a+2) = a(a+2)+4(a+2)$$

$$= a^2+2a+4a+8$$

$$= a^2+6a+8$$

$$\neq a^2+8 = \text{RHS}$$

The correct statement is $(a+4)(a+2) = a^2+6a+8$

Question 15

$$(a-4)(a-2) = a^2-8$$

Solution

$$\text{LHS} = (a-4)(a-2) = a(a-2)-4(a-2)$$

$$= a^2-2a-4a+8$$

$$= a^2-6a+8$$

$$\neq a^2-8 = \text{RHS}$$

The correct statement is $(a-4)(a-2) = a^2 - 6a + 8$

Question 16

$$3x^2/3x^2 = 0$$

Solution

$$\text{LHS} = 3x^2/3x^2 = 1 \neq 0 = \text{RHS}$$

The correct statement is $3x^2/3x^2 = 1$

Question 17

$$(3x^2+1)/3x^2 = 1 + 1 = 2$$

Solution

$$\text{LHS} = (3x^2+1)/3x^2 = (3x^2/3x^2) + (1/3x^2) = 1 + (1/3x^2) \neq 2 = \text{RHS}$$

The correct statement is $(3x^2+1)/3x^2 = 1 + (1/3x^2)$

Question 18

$$3x/(3x+2) = \frac{1}{2}$$

Solution

$$\text{LHS} = 3x/(3x+2) \neq \frac{1}{2} = \text{RHS}$$

The correct statement is $3x/(3x+2) = 3x/(3x+2)$

Question 19

$$3/(4x+3) = 1/4x$$

Solution

$$\text{LHS} = 3/(4x+3) \neq 1/4x$$

The correct statement is $3/(4x+3) = 3/(4x+3)$

Question 20

$$(4x+5)/4x = 5$$

Solution

$$\text{LHS} = (4x+5)/4x = 4x/4x + 5/4x = 1 + 5/4x \neq 5 = \text{RHS}$$

The correct statement is $(4x+5)/4x = 1 + (5/4x)$

$$\text{LHS} = (7x+5)/5 = (7x/5) + 5/5 = (7x/5) + 1 \neq 7x = \text{RHS}$$

The correct statement is $(7x+5)/5 = (7x/5) + 1$
