

Chapter 11: Dual Nature of Radiation and Matter

NCERT · Class 12 · Physics

Questions

Question 11.6

In an experiment on the photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Solution

Given that the slope of cut-off voltage (V) versus frequency (ν) is:

$$\frac{V}{\nu}$$

-15

V and frequency are related by the equation:

$$h\nu = eV$$

Where,

$$e = \text{Charge on an electron} = 1.6 \times 10^{-19} \text{ C}$$

$$h = \text{Planck's constant}$$

Therefore, $h = e \times$

$$\frac{V}{\nu}$$
$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34} \text{ Js}$$

Therefore, 6.592×10^{-34} Js is the Planck's constant that is determined from the above equation.

Question 11.7

A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of sodium light is 589 nm. (a) What is the energy per photon associated with sodium light? (b) At what rate are the photons delivered to the sphere?

Solution

Power of the sodium lamp **P = 100W**

Wavelength of the emitted sodium light,

$$\lambda$$

$$= 589 \times 10^{-9} \text{ m}$$

Planck's constant, **h = 6.626 x 10⁻³⁴ Js**

Speed of light, c = 3 x 10⁸

(a)

The energy per photon associated with the sodium light is given as:

$$E =$$

$$\frac{hc}{\lambda}$$

$$E =$$

$$\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}}$$

$$= 3.37 \times 10^{-19} \text{ J} =$$

$$\frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= \mathbf{2.11 \text{ eV}}$$

(b)

Number of photons delivered to the sphere = n

The equation for power can be written as:

$$\mathbf{P = nE}$$

Therefore, n=

$$\frac{P}{E}$$

$$=$$

$$\frac{100}{3.37 \times 10^{-19}}$$

$$20$$

Therefore, 2.96×10^{20} photons are delivered every second to the sphere.

Question 11.8

The threshold frequency for a certain metal is 3.3×10^{14} Hz. If the light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Solution

Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}$ Hz

Frequency of light incident on the metal, $\nu = 8.2 \times 10^{14}$ Hz

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Cut-off voltage for the photoelectric emission from the metal = V_0

The equation for the cut-off energy is given as:

$$eV_0 = h(\nu - \nu_0)$$

$$V_0 =$$

$$\frac{h(\nu - \nu_0)}{e}$$

=

$$\frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}}$$

$$= 2.0291 \text{ V}$$

Therefore, the cut-off voltage for the photoelectric emission is 2.0291 V.

Question 11.9

The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution

Work function of the metal,

$$\Phi_0$$

$$= 4.2 \text{ eV}$$

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.626 \times 10^{-34} \text{ Js}$

Wavelength of the incident radiation,

$$\lambda$$

$$= 330 \times 10^{-9} \text{ m}$$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

The energy of the incident photon is given as:

$$E =$$

$$\frac{hc}{\lambda}$$

$$=$$

$$\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}}$$

$$= 6.0 \times 10^{-19} \text{ J} =$$

$$\frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 3.76 \text{ eV}$$

The energy of the incident radiation is less than the work function of the metal. Hence, there is no photoelectric emission takes place.

Question 11.10

Light of frequency $7.21 \times 10^{14} \text{ Hz}$ is incident in a metal surface. Electrons with a maximum speed of $6.0 \times 10^5 \text{ m/s}$ are ejected from the surface. What is the threshold frequency for the photoemission of electrons?

Solution

Frequency of the incident photon, $\nu = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$

The maximum speed of the electrons, $v = 6.0 \times 10^5 \text{ m/s}$

Planck's constant, $h = 6.626 \times 10^{-34} \text{ Js}$

Mass of an electron, $m = 9.1 \times 10^{-31} \text{ Kg}$

For threshold frequency ν_0 , the relation for kinetic energy is written as:

$$\frac{1}{2}mv^2$$

0

$$V_0 = V -$$

$$\frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14} -$$

$$\frac{(9.1 \times 10^{-31}) \times (6 \times 10^5)^2}{2 \times (6.626 \times 10^{-34})}$$

$$= 7.21 \times 10^{14} - 2.472 \times 10^{14} = \mathbf{4.738 \times 10^{14} \text{ Hz}}$$

Therefore, $4.738 \times 10^{14} \text{ Hz}$ is the threshold frequency for the photoemission of the electrons.

Question 11.11

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Solution

The wavelength of light produced by the argon laser,

$$\lambda$$

$$= 488 \times 10^{-9} \text{ m}$$

Stopping potential of the photoelectrons, $V_0 = 0.38 \text{ V}$

$$1\text{eV} = 1.6 \times 10^{-19} \text{ J}$$

Therefore, $V_0 =$

$$\frac{0.38}{1.6 \times 10^{-19}}$$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Using Einstein's photoelectric effect, the following is the relation for the work function:

$$\Phi_0$$

$$eV_0 =$$

$$\frac{hc}{\lambda} - \phi_0$$

$$\begin{aligned}
 & \Phi_o \\
 & \frac{hc}{\lambda} \\
 & 0 \\
 & = \\
 & \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 488 \times 10^{-9}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}} \\
 & = 2.54 - 0.38 = \mathbf{2.16 \text{ eV}}
 \end{aligned}$$

Therefore, 2.16 eV is the work function of the material with which the emitter is made.

Question 11.12

Calculate the following:

(a) Momentum

(b) The de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Solution

The potential difference, $V = 56V$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$

Mass of an electron, $m = 9.1 \times 10^{-31} \text{ Kg}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

(a) At equilibrium, the kinetic energy of each electron is equal to the accelerating potential, i.e. we can write the relation of velocity (v) of each electron as:

$$\begin{aligned}
 & \frac{1}{2}mv^2 \\
 & v^2 = \frac{2eV}{m}
 \end{aligned}$$

Therefore, $v =$

$$\begin{aligned}
 & \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 56}{9.1 \times 10^{-31}}} \\
 & =
 \end{aligned}$$

$$\begin{aligned}
 & \sqrt{19.69 \times 10^{12}} \\
 & = 4.44 \times 10^6 \text{ m/s}
 \end{aligned}$$

The momentum of each accelerated electron is given as:

$$p = mv$$

$$= 9.1 \times 10^{-31} \times 4.44 \times 10^6 = 4.04 \times 10^{-24} \text{ Kg m/s}$$

Therefore, 4.04×10^{-24} Kg m/s is the momentum of each electron.

(b) de Broglie wavelength of an electron accelerating through a potential V is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}}$$

o

=

$$\frac{12.27}{\sqrt{56}}$$

$$^{-19} = 0.1639 \text{ nm}$$

Therefore, 0.1639 nm is the de Broglie wavelength of each electron.

Question 11.13

Find the following:

(a) Momentum,

(b) Speed, and

(c) De Broglie wavelength of an electron with a kinetic energy of 120 eV.

Solution

Kinetic energy of the electron, $E_K = 120 \text{ eV}$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$

Mass of an electron, $m = 9.1 \times 10^{-31} \text{ Kg}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

(a) For an electron, we can write the relation for kinetic energy as:

$$E_k =$$

$$\frac{1}{2}mv^2$$

Where, v = speed of the electron

Therefore,

$$\begin{aligned}
 v^2 &= \sqrt{\frac{2eE_k}{m}} \\
 &= \\
 &= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}}} \\
 &= \\
 &= \sqrt{42.198 \times 10^{12}} \\
 &= 6.496 \times 10^6 \text{ m/s}
 \end{aligned}$$

Momentum of the electron, $p = mv = 9.1 \times 10^{-31} \times 6.496 \times 10^6$

$$= 5.91 \times 10^{-24} \text{ kg m/s}$$

Therefore, $5.91 \times 10^{-24} \text{ Kg m/s}$ is the momentum of the electron.

(b) speed of the electron, $v = 6.496 \times 10^6 \text{ m/s}$

(c) de Broglie wavelength of an electron having a momentum p is given as:

$$\begin{aligned}
 \lambda &= \frac{h}{p} \\
 &= \\
 &= \frac{6.6 \times 10^{-34}}{5.91 \times 10^{-24}} \\
 &= 0.112 \text{ nm}
 \end{aligned}$$

Therefore, 0.112 nm is the de Broglie wavelength of the electron.

Question 11.14

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Solution

Wavelength of light of a sodium line,

$$\begin{aligned}
 \lambda &= \\
 &= 589 \text{ nm}
 \end{aligned}$$

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ Kg

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ Kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

(a) For the kinetic energy, K , of an electron accelerating with the velocity v , we have the relation:

$K =$

$$\frac{1}{2}mv^2$$

(1)

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

Therefore,

$$v^2 = \frac{h^2}{\lambda^2 m_e^2}$$

(2)

Substituting equation (2) in equation (1), we get the relation:

$K =$

$$\frac{1}{2} \frac{m_e h^2}{\lambda^2 m_e^2}$$

$=$

$$\frac{h^2}{2\lambda^2 m_e}$$

(3)

$=$

$$\frac{(6.6 \times 10^{-34})^2}{2 \times (589 \times 10^{-9})^2 \times 9.1 \times 10^{-31}}$$

$$= 6.9 \times 10^{-25} \text{ J} =$$

$$\frac{6.9 \times 10^{-25}}{1.6 \times 10^{-19}}$$

$$= 4.31 \times 10^{-6} \text{ eV}$$

Hence, the kinetic energy of the electron is 6.9×10^{-25} J.

(b) Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$=$

$$\frac{h^2}{2\lambda^2 m_n}$$

=

$$\frac{(6.6 \times 10^{-34})^2}{2 \times (589 \times 10^{-9})^2 \times 1.66 \times 10^{-27}}$$

$$= 3.78 \times 10^{-28} \text{ J}$$

=

$$\frac{3.78 \times 10^{-28}}{1.6 \times 10^{-19}}$$

$$= 2.36 \times 10^{-9} \text{ eV} = \mathbf{2.36 \text{ neV}}$$

Therefore, the neutron has a kinetic energy of $3.78 \times 10^{-28} \text{ J}$ or 2.36 neV .

Question 11.15

What is the de Broglie wavelength of:

- (a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s ,
- (b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s , and
- (c) a dust particle of mass $1.0 \times 10^{-9} \text{ kg}$ drifting with a speed of 2.2 m/s ?

Solution

(a) Mass of the bullet, $m = 0.040 \text{ Kg}$

The speed of the bullet, $v = 1.0 \text{ km/s} = 1000 \text{ m/s}$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$

de Broglie wavelength of the bullet is given by the relation:

$$\lambda = \frac{h}{mv}$$

=

$$\frac{6.6 \times 10^{-34}}{0.040 \times 1000}$$

$$= \mathbf{1.65 \times 10^{-35} \text{ m.}}$$

(b) Mass of the ball, $m = 0.060 \text{ Kg}$

The speed of the ball, $v = 1.0 \text{ m/s}$

de Broglie wavelength of the ball is given by the relation:

=

$$\lambda = \frac{h}{mv}$$

=

$$\frac{6.6 \times 10^{-34}}{0.060 \times 1}$$

$$= 1.1 \times 10^{-32} \text{ m.}$$

(c) Mass of the dust particle, $m = 1 \times 10^{-9} \text{ Kg}$

The speed of the dust particle, **$v = 2.2 \text{ m/s}$**

de Broglie wavelength of the dust particle is given by the relation:

=

$$\lambda = \frac{h}{mv}$$

=

$$\frac{6.6 \times 10^{-34}}{2.2 \times 1 \times 10^{-9}}$$

$$= 3.0 \times 10^{-25} \text{ m.}$$

Question 11.16

An electron and a photon each have a wavelength of 1.00 nm. Find the following:

(a) Momenta

(b) The energy of the photon

(c) The kinetic energy of the electron

Solution

Wavelength of an electron

$$\lambda_e$$

$$\lambda_p$$

$$\lambda_e$$

$$\lambda_p$$

$$\lambda$$

$$= 1 \times 10^{-9} \text{ m}$$

Planck's constant, $h = 6.63 \times 10^{-34} \text{ Js}$

(a) The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

Therefore, $p =$

$$\frac{6.63 \times 10^{-34}}{1 \times 10^{-9}}$$

$$= 6.63 \times 10^{-25} \text{ Kg m/s}$$

Therefore, both the electron and photon have an equal momentum of $6.63 \times 10^{-25} \text{ Kg m/s}$.

(b) The energy of a photon is given by the relation:

$E =$

$$\frac{hc}{\lambda}$$

Where,

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Therefore, $E =$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$$

$$= 1243.1 \text{ eV} = 1.243 \text{ keV}$$

Therefore, the energy of the photon is 1.243 keV.

(c) The kinetic energy (K) of an electron having momentum p , is given by the relation:

$K =$

$$\frac{1}{2} \frac{p^2}{m}$$

Where, $m =$ Mass of the electron $= 9.1 \times 10^{-31} \text{ Kg}$

$p = 6.63 \times 10^{-25} \text{ Kg m/s}$

Therefore, $K =$

$$\frac{1}{2} \times \frac{(6.63 \times 10^{-25})^2}{9.1 \times 10^{-31}}$$

$$= 2.415 \times 10^{-19} \text{ J}$$

=

$$\frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 1.51 \text{ eV}$$

Therefore, 1.51eV is the kinetic energy of the electron.

Question 11.17

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be $1.40 \times 10^{-10} \text{ m}$?

(b) Also find the de Broglie wavelength of a neutron in thermal equilibrium with matter, having an average kinetic energy of $(3/2) kT$ at 300 K.

Solution

(a) de Broglie wavelength of the neutron,

$$\lambda = 1.40 \times 10^{-10} \text{ m}$$

$$\text{Mass of a neutron, } m_n = 1.66 \times 10^{-27} \text{ Kg}$$

$$\text{Planck's constant, } h = 6.63 \times 10^{-34} \text{ Js}$$

Kinetic energy (K) and velocity (v) are related as:

$$K =$$

$$\frac{1}{2} m_n v^2$$

..... (1)

de Broglie wavelength (

$$\lambda = \frac{h}{m_n v}$$

..... (2)

Using equation (2) and equation (1), we get

$$K =$$

$$\frac{1}{2} \frac{m_n h^2}{\lambda^2 m_n^2}$$

=

$$\frac{h^2}{2\lambda^2 m_n}$$

$$=$$

$$\frac{(6.63 \times 10^{-34})^2}{2 \times (1.40 \times 10^{-10})^2 \times 1.66 \times 10^{-27}}$$

$$= 6.75 \times 10^{-21} \text{ J}$$

$$=$$

$$\frac{6.75 \times 10^{-21}}{1.6 \times 10^{-19}}$$

$$= 4.219 \times 10^{-2} \text{ eV}$$

Hence, the kinetic energy of the neutron is $6.75 \times 10^{-21} \text{ J}$ or $4.219 \times 10^{-2} \text{ eV}$.

(b) Temperature of the neutron, $T = 300\text{K}$

Boltzmann constant, $k = 1.38 \times 10^{-23} \text{ Kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$

The average kinetic energy of the neutron:

$$K' =$$

$$\frac{3}{2}$$

$$=$$

$$\frac{3}{2}$$

$$\times 10^{-23}$$

$$= 6.21 \times 10^{-21} \text{ J}$$

The relation for the de Broglie wavelength is given as:

$$\lambda' = \frac{h}{\sqrt{2K'm_n}}$$

Where,

$$m_n = 1.66 \times 10^{-27} \text{ Kg}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$K' = 6.21 \times 10^{-21} \text{ J}$$

Therefore,

$$\lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}}$$

$$= 1.46 \times 10^{-10} \text{ m} = \mathbf{0.146 \text{ nm}}$$

Therefore, 0.146nm is the de Broglie wavelength of the neutron.

Question 11.18

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution

The momentum of a photon having energy ($h\nu$) is given as:

$$p =$$

$$\frac{h\nu}{c}$$

$$\frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

..... (i)

Where,

$$\lambda$$

c = speed of light

h = Planck's constant

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

But, **p = mv**

Therefore,

$$\lambda = \frac{h}{p}$$

.....(ii)

Where, m = mass of the photon

v = velocity of the photon

From equations (i) and (ii), it can be concluded that the wavelength of the electromagnetic radiation and the de Broglie wavelength of the photon is equal.

Question 11.19

What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Solution

The temperature of the nitrogen molecule, $T = 300 \text{ K}$

The atomic mass of nitrogen = **14.0076 u**

Hence, the mass of the nitrogen molecule, $m = 2 \times 14.0076 = \mathbf{28.0152 \text{ u}}$

But, $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

Therefore, $m = 28.0152 \times 1.66 \times 10^{-27} \text{ kg}$

Planck's constant, $h = 6.63 \times 10^{-34} \text{ Js}$

Boltzmann constant, $k = 1.38 \times 10^{-23} \text{ J/K}$

We have the expression that relates mean kinetic energy

$$\left(\frac{3}{2}kT\right)$$

rms

$$\frac{1}{2}mv_{rms}^2$$

$$\left(\frac{3}{2}kT\right)$$

$$V_{rms} =$$

$$\sqrt{\frac{3kT}{m}}$$

For nitrogen molecule, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{mv_{rms}}$$

$$\frac{h}{\sqrt{3mkT}}$$

=

$$\frac{6.63 \times 10^{-34}}{\sqrt{3 \times 28.0152 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.028 \times 10^{-9} \text{ m} = \mathbf{0.028 \text{ nm}}$$

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.

Question 11.20

(a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the low initial speeds of the electrons. The specific charge of the electron, i.e. its e/m , is given to be $1.76 \times 10^{11} \text{ C kg}^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for a collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Solution

(a) Potential difference of the evacuated tube = 500 V

Specific charge of the electron, $e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$

Kinetic energy = $(1/2) mv^2 = eV$

Speed of the emitted electron, $v = (2Ve/m)^{1/2}$

$$= (2 \times 500 \times 1.76 \times 10^{11})^{1/2}$$

$$= 1.32 \times 10^7 \text{ m/s.}$$

(b) Collector potential, $V = 10 \text{ MV} = 10 \times 10^6 \text{ V}$.

Speed of electron = $v = (2Ve/m)^{1/2}$

$$= (2 \times 10^7 \times 1.76 \times 10^{11})^{1/2}$$

$$= 1.88 \times 10^9 \text{ m/s.}$$

This answer is not correct. Since the value is greater than the speed of light (c), the expression $(1/2)mv^2$ for energy should be used in the non-relativistic limit, i.e., $v \ll c$.

In the relativistic limits, the total energy is given as

$$E = mc^2$$

Here,

m is the relativistic mass

$$m = m_0 (1 - v^2/c^2)^{1/2}$$

m_0 = mass of the particle at rest

Kinetic energy is given as

$$K = mc^2 - m_0c^2.$$

Question 11.21

(a) A monoenergetic electron beam with an electron speed of $5.20 \times 10^6 \text{ m s}^{-1}$ is subject to a magnetic field of $1.30 \times 10^{-4} \text{ T}$ normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals $1.76 \times 10^{11} \text{ C kg}^{-1}$?

(b) Is the formula you employ in (a) valid for calculating the radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

[Note: Exercises 11.20 (b) and 11.21 (b) take you to relativistic mechanics, which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

Solution

Speed of the electron, $v = 5.20 \times 10^6 \text{ m s}^{-1}$

Magnetic field experienced by the electron, $B = 1.30 \times 10^{-4} \text{ T}$

Specific charge, $e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$

Here,

e = charge on the electron = $1.6 \times 10^{-19} \text{ C}$

m = mass of the electron = $9.1 \times 10^{-31} \text{ kg}$

Force exerted on the electron is given as

$$F = e \left| \vec{v} \times \vec{B} \right|$$

$$= evB \sin \theta$$

θ is the angle between the magnetic field and the velocity of the beam. The magnetic field is normal in the direction of the beam.

$$\theta = 90^\circ$$

$$F = evB$$

The normal magnetic field provides the centripetal force.

Therefore, $evB = mv^2/r$

$$r = mv/eB = v/(e/m)B$$

$$= (5.20 \times 10^6) / (1.76 \times 10^{11}) \times (1.30 \times 10^{-4}) = 0.227 \text{ m} = 22.7 \text{ cm}$$

Therefore, the radius of the circular path is 22.7 cm.

(b) Energy of the electron beam, $E = 20 \text{ Mev} = 20 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$

The energy of the electron beam, $E = (1/2) mv^2$

$\Rightarrow$

$$v = \sqrt{\frac{2E}{m}}$$

$$v = \sqrt{\frac{2 \times 20 \times 10^6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.652 \times 10^9 \text{ m/s}$$

The result is greater than the speed of light. Therefore, it is wrong. The expression $(1/2) mv^2$ for energy should be used in the non-relativistic limit, i.e., $v \ll c$.

In the relativistic limits, the total mass is given as

$$m = m_0 (1 - v^2/c^2)^{-1/2}$$

m_0 = mass of the particle at rest

Therefore, the radius of the circular path is

$$r = mv/eB$$

$$r = \frac{m_0 v}{eB \sqrt{\frac{c^2 - v^2}{c^2}}}$$

Question 11.22

An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure ($\sim 10^{-2}$ mm of Hg). A magnetic field of $2.83 \times 10^{-4} \text{ T}$ curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.

Solution

Potential of the collector, $V = 100 \text{ V}$

Magnetic field experienced by the electron, $B = 2.83 \times 10^{-4} \text{ T}$

Radius of the circular orbit, $r = 12 \text{ cm} = 12.0 \times 10^{-2} \text{ m}$

Kinetic energy, $(1/2)mv^2 = eV$

$$v^2 = 2eV/m \text{ ———(1)}$$

The magnetic field that curves the path of the electron provides the centripetal force

$$evB = mv^2/r$$

$$eB = mv/r$$

$$v = eBr/m \text{ ———(2)}$$

Substituting (2) in (1)

$$\begin{aligned} \frac{2eV}{m} &= \frac{e^2 B^2 r^2}{m^2} \\ \frac{e}{m} &= \frac{2V}{B^2 r^2} \\ \frac{e}{m} &= \frac{2 \times 100}{(2.83 \times 10^{-4})^2 (12 \times 10^{-2})^2} = 1.73 \times 10^{11} \text{ Ckg}^{-1} \end{aligned}$$

Therefore, the specific charge ration e/m is $1.73 \times 10^{11} \text{ Ckg}^{-1}$.

Question 11.23

Question 11. 23: (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength ending at $0.45 \text{ \AA}$. What is the maximum energy of a photon in radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Solution

(a) Wavelength produced by the X-ray tube, $\lambda = 0.45 \text{ \AA} = 0.45 \times 10^{-10} \text{ m}$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Planck's constant, $h = 6.626 \times 10^{-34} \text{ Js}$

The maximum energy of a photon is given as

$$E_{\max} = hc/\lambda_{\min}$$

$$= (6.626 \times 10^{-34})(3 \times 10^8 \text{ m/s})/(0.45 \times 10^{-10} \text{ m} \times 1.6 \times 10^{-19})$$

$$= 19.878 \times 10^{-26} / 0.72 \times 10^{-29}$$

$$= 27.60 \times 10^3 \text{ eV} = 27.6 \text{ keV.}$$

(b) To incident electron should have an energy of 27.6 keV to get an X-ray of 27.6 keV. Therefore, the accelerating voltage of the order of 30 keV is required for producing X-rays.

Question 11.24

In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as the annihilation of an electron-positron pair of total energy 10.2 BeV into two γ -rays of equal energy. What is the wavelength associated with each γ -ray? (1BeV = 10^9 eV)

Solution

Total energy of the electron-positron pair, $E = 10.2 \text{ BeV} = 10.2 \times 10^9 \text{ eV} = 10.2 \times 10^9 \times 1.6 \times 10^{-19} \text{ J}$

Hence, the energy of each γ -ray, $E' = E/2 = (10.2 \times 10^9 \times 1.6 \times 10^{-19})/2 = 8.16 \times 10^{-10} \text{ J}$

Energy and wavelength relation is given as,

$$E' = hc/\lambda$$

Therefore, $\lambda = hc/E'$

Here, $h = 6.626 \times 10^{-34} \text{ Js}$

$c = 3 \times 10^8 \text{ m/s}$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-10}} = 2.436 \times 10^{-16} \text{ m}$$

Therefore, the wavelength associated with each γ -ray is $2.436 \times 10^{-16} \text{ m}$.

Question 11.25

Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons. The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a medium wave transmitter of 10 kW power, emitting radio waves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($\sim 10^{-10} \text{ W m}^{-2}$). Take the area of the pupil to be about 0.4 cm^2 , and the average frequency of white light to be about $6 \times 10^{14} \text{ Hz}$.

Solution

(a) Power of the medium wave transmitter, $P = 10 \text{ kW} = 10^4 \text{ W}$

The energy emitted by the transmitter per second, $E = 10^4$

The wavelength of the radio waves, $\lambda = 500 \text{ m}$

The energy of the wave is given as, $E' = hc/\lambda$

$$E' = (6.6 \times 10^{-34} \times 3 \times 10^8)/500$$

$$= 3.96 \times 10^{-28} \text{ J}$$

Let n be the number of photons emitted by the transmitter

$$nE' = E$$

$$n = E/E'$$

$$= 10^4/(3.96 \times 10^{-28})$$

$$= 0.2525 \times 10^{32}$$

The energy E' of the radio photon is very less, but the number of photons emitted is large. The total energy of the radio waves can be considered as continuous, and the existence of the minimum quantum energy can be ignored.

(b) Intensity of the light perceived by the human eye, $I = 10^{-10} \text{ W m}^{-2}$

Area of the pupil, $A = 0.4 \text{ cm}^2 = 0.4 \times 10^{-4} \text{ m}^2$

Frequency of the white light, $\nu = 6 \times 10^{14} \text{ Hz}$

h = Planck's constant = $6.6 \times 10^{-34} \text{ Js}$

The energy of the emitted photon, $E = h\nu$

$$= 6.6 \times 10^{-34} \times 6 \times 10^{14}$$

$$= 3.96 \times 10^{-19} \text{ J}$$

Let n be the total number of photons falling per unit area per unit time. The total energy per unit for n photons is

$$E = n \times 3.96 \times 10^{-19} \text{ J/s/m}^2$$

The total energy per unit for n photons is equal to the intensity of the light.

$$E = I$$

$$I = n \times 3.96 \times 10^{-19} \text{ J/s/m}^2$$

$$n = I / 3.96 \times 10^{-19}$$

$$= 10^{-10} / 3.96 \times 10^{-19}$$

$$= 2.52 \times 10^8 \text{ m}^2/\text{s}$$

The total number of photons entering the pupil is given as,

$$nA = 2.52 \times 10^8 \times 0.4 \times 10^{-4}$$

$$= 1.008 \times 10^4 \text{ s}^{-1}$$

The number is large, so the human eye can never count the number of individual photons.

Question 11.26

Ultraviolet light of wavelength $2271 \text{ \AA}$ from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is -1.3 V , estimate the work function of the metal. How would the photo-cell respond to high-intensity ($\sim 10^5 \text{ W m}^{-2}$) red light of wavelength $6328 \text{ \AA}$ produced by a He-Ne laser?

Solution

Wavelength of ultraviolet light, $\lambda = 2271 \text{ \AA} = 2271 \times 10^{-10} \text{ m}$

Stopping potential of the metal, $V_0 = 1.3 \text{ V}$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ J}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

From the photoelectric effect, we have the photon-electron relation as,

The work function of the metal, $\Phi_0 = h\nu - eV_0$

$$= (hc/\lambda) - eV_0$$

=

$$\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10}} - 1.6 \times 10^{-19} \times 1.3$$

$$= 8.72 \times 10^{-19} - 2.08 \times 10^{-19}$$

$$= 6.64 \times 10^{-19} \text{ J}$$

Therefore, the threshold frequency of the metal

$$\Phi_0 = h\nu_0$$

$$\nu_0 = \Phi_0/h = 6.64 \times 10^{-19} / 6.6 \times 10^{-34}$$

$$= 1.006 \times 10^{15} \text{ Hz}$$

Wavelength of the red light, $\lambda_r = 6328 \times 10^{-10} \text{ m}$

$$\text{Frequency of the red light, } \nu_r = c/\lambda_r = 3 \times 10^8 / 6328 \times 10^{-10}$$

$$= 4.74 \times 10^{14} \text{ Hz}$$

$$\nu_0 > \nu_r$$

The photocell will not respond to the red light produced by the He-Ne laser.

Question 11.27

Monochromatic radiation of wavelength 640.2 nm ($1\text{nm} = 10^{-9} \text{ m}$) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source, and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Solution

Wavelength of the monochromatic radiation, $\lambda = 640.2 \text{ nm} = 640.2 \times 10^{-9} \text{ m}$

The stopping potential of the neon lamp, $V_0 = 0.54 \text{ V}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$

From the photoelectric effect, we have the photon-electron relation as $eV_0 = h\nu - \Phi_0$

The work function of the metal, $\Phi_0 = h\nu - eV_0$

$$= (hc/\lambda) - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{640.2 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.54$$

$$= 3.093 \times 10^{-19} - 0.864 \times 10^{-19}$$

$$= 2.229 \times 10^{-19} \text{ J}$$

The wavelength of the radiation emitted by the iron source, $\lambda' = 427.2 \text{ nm} = 427.2 \times 10^{-9} \text{ m}$

Let V_0' be the new stopping potential

$$\text{Therefore, } eV_0' = (hc/\lambda') - \Phi_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{427.2 \times 10^{-9}} - 2.229 \times 10^{-19}$$

$$= 4.63 \times 10^{-19} - 2.229 \times 10^{-19}$$

$$= 2.401 \times 10^{-19} \text{ J}$$

$$V_0' = 2.401 \times 10^{-19} \text{ J} / 1.6 \times 10^{-19} \text{ J}$$

$$= 1.5 \text{ eV}$$

Therefore, the new stopping potential = 1.50 eV.

Question 11.28

A mercury lamp is a convenient source for studying the frequency dependence of photoelectric emission since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

$$\lambda_1 = 3650 \text{ \AA},$$

$$\lambda_2 = 4047 \text{ \AA},$$

$$\lambda_3 = 4358 \text{ \AA},$$

$$\lambda_4 = 5461 \text{ \AA},$$

$$\lambda_5 = 6907 \text{ \AA},$$

The stopping voltages, respectively, were measured to be: $V_{01} = 1.28 \text{ V}$, $V_{02} = 0.95 \text{ V}$, $V_{03} = 0.74 \text{ V}$, $V_{04} = 0.16 \text{ V}$, $V_{05} = 0 \text{ V}$. Determine the value of Planck's constant h , the threshold frequency and work function for the material.

[Note: You will notice that to get h from the data, you will need to know e (which you can take to be 1.6

$\times 10^{-19}$ C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of e (from the oil-drop experiment), confirmed Einstein's photoelectric equation and, at the same time, gave an independent estimate of the value of h .]

Solution

From the photoelectric effect, we have the photon-electron relation as $eV_0 = h\nu - \Phi_0$

The work function of the metal, $\Phi_0 = h\nu - eV_0$

$$\Phi_0 = (hc/\lambda) - eV_0$$

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Here,

V_0 = Stopping potential

h = Planck's constant

e = Charge on an electron

ν = Frequency of radiation

Φ_0 = Work function of a material

Stopping potential is directly proportional to the frequency.

Frequency, $\nu = \text{Speed of light (c)}/\text{Wavelength } (\lambda)$

Using this equation, we can find the frequency of various lines

$$\nu_1 = c/\lambda_1 = 3 \times 10^8 / 3650 \times 10^{-10}$$

$$= 8.219 \times 10^{14} \text{ Hz}$$

$$\nu_2 = c/\lambda_2 = 3 \times 10^8 / 4047 \times 10^{-10}$$

$$= 7.412 \times 10^{14} \text{ Hz}$$

$$\nu_3 = c/\lambda_3 = 3 \times 10^8 / 4358 \times 10^{-10}$$

$$= 6.88 \times 10^{14} \text{ Hz}$$

$$\nu_4 = c/\lambda_4 = 3 \times 10^8 / 5461 \times 10^{-10}$$

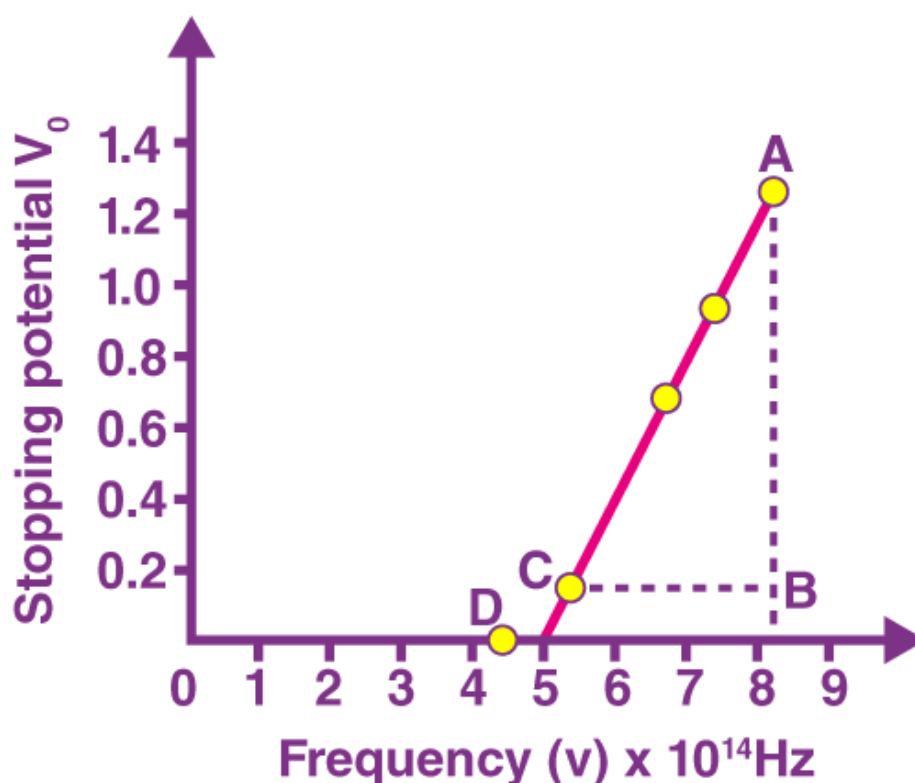
$$= 5.493 \times 10^{14} \text{ Hz}$$

$$\nu_5 = c/\lambda_5 = 3 \times 10^8 / 6907 \times 10^{-10}$$

$$= 4.343 \times 10^{14} \text{ Hz}$$

Frequency	8.219	7.412	6.884	5.493	4.343
Stopping potential	1.28	0.95	0.74	0.16	0

The above values can be plotted in a graph



The graph is a straight line, and it intersects the y-axis at $5 \times 10^{14} \text{ Hz}$. This is the threshold frequency. Point D is the frequency less than the threshold frequency.

$$\text{Slope of the straight line} = AB/CB = (1.28 - 0.16)/(8.214 - 5.493) \times 10^{14}$$

From equation (1), the slope is written as

$$h/e = (1.28 - 0.16)/(8.214 - 5.493) \times 10^{14}$$

$$h = (1.12 \times 1.6 \times 10^{-19})/(2.726 \times 10^{14})$$

$$= 6.573 \times 10^{-34} \text{ Js}$$

The work function of the metal is,

$$\Phi_0 = h\nu_0$$

$$= (6.573 \times 10^{-34} \times 5 \times 10^{14})$$

$$= 3.286 \times 10^{-19} \text{ J}$$

$$= 3.286 \times 10^{-19} / 1.6 \times 10^{-19}$$

$$\Phi_0 = 2.054 \text{ eV.}$$

Question 11.29

The work function for the following metals is given: Na: 2.75 eV, K: 2.30 eV, Mo: 4.17 eV, Ni: 5.15 eV. Which of these metals will not give photoelectric emission for radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Solution

$$\text{Wavelength } \lambda = 3300 \text{ Å}$$

$$\text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$\text{Planck's constant} = 6.63 \times 10^{-34} \text{ Js}$$

The energy of the photon of the incident light

$$E = hc/\lambda = (6.63 \times 10^{-34} \times 3 \times 10^8) / 3300 \times 10^{-10}$$

$$\Rightarrow 6.018 \times 10^{-19} \text{ J}$$

$$\Rightarrow (6.018 \times 10^{-19} \text{ J}) / 1.6 \times 10^{-19}$$

$$= 3.7 \text{ eV}$$

The energy of the incident radiation is greater than the work function of Na and K. It is lesser for Mo and Ni. Therefore, Mo and Ni will not show a photoelectric effect.

If the laser is brought nearer and placed 50 cm away, then the intensity of the radiation will increase. The energy of the radiation will not be affected. Therefore, the result will be the same. However, the photoelectrons from Na and K will increase in proportion to intensity.

Question 11.30

Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of a surface area of 2 cm^2 . Assuming that the top 5 layers of sodium absorb the incident energy, estimate the time required for photoelectric emission in the wave picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Solution

The intensity of the light = 10^{-5} W m^{-2}

The surface area of the sodium photocell, $A = 2 \text{ cm}^2$

Incident power of the light, $P = I \times A$

$$= 10^{-5} \times 2 \times 10^{-4}$$

$$= 2 \times 10^{-9} \text{ W}$$

Work function of the metal, $\Phi_0 = 2 \text{ eV}$

$$= 2 \times 1.6 \times 10^{-19}$$

$$= 3.2 \times 10^{-19} \text{ J}$$

The number of layers of sodium that absorbs the incident energy, $n = 5$

The atomic area of the sodium atom, A_e , is 10^{-20} m^2

Hence, the number of conduction electrons in n layers is given as:

$$n' = n \times (A/A_e)$$

$$= 5 \times [(2 \times 10^{-4})/10^{-20}] = 10^{17}$$

The incident power is absorbed by all the electrons continuously. The amount of energy absorbed per electron per second is

$$E = P/n'$$

$$= (2 \times 10^{-9})/10^{17}$$

$$= 2 \times 10^{-26} \text{ J/s}$$

The time for photoelectric emission

$$t = \Phi_0/E$$

$$= (3.2 \times 10^{-19})/(2 \times 10^{-26}) = 1.6 \times 10^7 \text{ s} \approx 0.507 \text{ years}$$

The time required for the photoelectric emission is almost half a year. This is not practical. Therefore, the wave function is in disagreement with the given experiment.

Question 11.31

Crystal diffraction experiments can be performed using X-rays or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of interatomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31} \text{ kg}$).

Solution

For electrons, kinetic energy, $K.E = (1/2) m_e v^2$

$$= (m_e v)^2 / 2m$$

$$K.E = p^2 / 2m_e$$

$\Rightarrow$

$$p = \sqrt{2m_e K.E}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K.E}}$$

$$\lambda^2 = \frac{h^2}{2m_e K.E}$$

$$K.E = \frac{h^2}{2m_e \lambda^2}$$

$$K.E = \frac{(6.64 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$K.E = 2.4 \times 10^{-17} \text{ J}$$

$$K.E =$$

$$\frac{2.4 \times 10^{-17}}{1.6 \times 10^{-19}}$$

$$= 149.375 \text{ eV}$$

For photon of X-rays, energy, $E = hc/\lambda$

$$= (6.63 \times 10^{-34} \times 3 \times 10^8) / (10^{-10} \times 1.6 \times 10^{-19})$$

$$= 12.375 \times 10^3 \text{ eV}$$

$$= 12.375 \text{ keV}$$

Hence, the energy of the photons of X-rays is more than the energy of the electron.

Question 11.32

(a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in question 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27} \text{ kg}$).

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Also, explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Solution

(a) Kinetic energy of the neutron = 150 eV

$$= 150 \times 1.6 \times 10^{-19}$$

$$= 2.4 \times 10^{-17} \text{ J}$$

Mass of the neutron, $m_n = 1.675 \times 10^{-27} \text{ kg}$

The kinetic energy of the neutron is given by the relation

$$\text{K.E} = (1/2) m_e v^2$$

$$\text{K.E} = p^2/2m_e$$

$\Rightarrow$

$$p = \sqrt{2m_e \text{K.E}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e \text{K.E}}}$$

Wavelength and mass are inversely proportional. Wavelength decreases with an increase in mass and vice versa.

$$\lambda = \frac{(6.63 \times 10^{-34})}{\sqrt{2.24 \times 10^{-17} \times 1.675 \times 10^{-27}}}$$

$$= 2.327 \times 10^{-12} \text{ m}$$

In question 11.31, it is given inter-atomic spacing of the crystal is about 1 Å, i.e., 10^{-10} m . The interatomic spacing is about 100 times greater. Therefore, a neutron of kinetic energy is 150 eV is not

good for a diffraction experiment.

$$(b) \text{ Room temperature} = 27^{\circ} \text{C} = 27 + 273 = 300 \text{ K}$$

Average kinetic energy of the neutron, $E = (3/2) kT$

here, k = Boltzmann constant = $1.38 \times 10^{-23} \text{ J/mol/K}$

The wavelength of the neutron is

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m_n E}} \\ \lambda &= \frac{h}{\sqrt{2m_n \frac{3}{2} kT}} \\ \lambda &= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \\ &= 1.447 \times 10^{-10} \text{ m} \end{aligned}$$

This wavelength is comparable to the inter-atomic spacing of the crystal. Hence, a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Question 11.33

An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution

Electrons are accelerated by a voltage = 50 kV

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Mass of the electron, $m_e = 9.11 \times 10^{-31} \text{ kg}$

Wavelength of the yellow light = $5.9 \times 10^{-7} \text{ m}$

The kinetic energy of the electron, $E = eV$

$$= (1.6 \times 10^{-19}) \times (50 \times 10^3)$$

$$= 8 \times 10^{-15} \text{ J}$$

De Broglie wavelength of an electron is given as

$$\lambda = \frac{h}{\sqrt{2m_e E}}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}$$

$$= 5.467 \times 10^{-12} \text{ m}$$

The wavelength is 10^5 times lesser than the wavelength of yellow light. The resolving power of the microscope and the wavelength of the light used is inversely proportional. Therefore, the resolving power of the electron microscope is 10^5 times greater than the optical microscope.

Question 11.34

The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length scale of 10^{-15} m or less. This structure was first probed in the early 1970s using high-energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Solution

The wavelength of the proton or neutron, $\lambda \approx 10^{-15}$ m

Rest mass-energy of an electron:

$$m_0 c^2 = 0.511 \text{ MeV}$$

$$= 0.511 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 0.8176 \times 10^{-13} \text{ J}$$

$$\text{Planck's constant, } h = 6.6 \times 10^{-34} \text{ Js}$$

$$\text{Speed of light, } c = 3 \times 10^8 \text{ m/s}$$

The momentum of the proton or a neutron is given as

$$p = h/\lambda$$

$$= 6.6 \times 10^{-34} / 10^{-15}$$

$$= 6.6 \times 10^{-19} \text{ kg m/s}$$

The relativistic relation for energy (E) is given as

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$= (6.6 \times 10^{-19} \times 3 \times 10^8)^2 + (0.8176 \times 10^{-13})^2$$

$$= 392.04 \times 10^{-22} + 0.6685 \times 10^{-26}$$

$$\approx 392.04 \times 10^{-22}$$

$$\Rightarrow E = 19.8 \times 10^{-11}$$

$$= 19.8 \times 10^{-11} / 1.6 \times 10^{-19}$$

$$= 12.375 \times 10^8 \text{ eV}$$

Thus, the order of energy of these electron beams is $12.375 \times 10^8 \text{ eV}$.

Question 11.35

Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure, and compare it with the mean separation between two atoms under these conditions.

Solution

Room temperature, $T = 27^\circ \text{C} = 27 + 273 = 300 \text{ K}$

Atmospheric pressure, $P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

The atomic weight of the He atom = 4

Avogadro's number, $N_A = 6.023 \times 10^{23}$

Boltzmann's constant, $k = 1.38 \times 10^{-23} \text{ J/mol/K}$

The average energy of a gas at temperature T ,

De Broglie wavelength is given as

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E = (3/2) kT$$

m = mass of the He atom

$$= \text{Atomic weight} / N_A$$

$$= 4/(6.023 \times 10^{23})$$

$$= 6.64 \times 10^{-24} \text{ g}$$

$$m = 6.64 \times 10^{-27} \text{ kg}$$

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 6.64 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.7268 \times 10^{-10} \text{ m}$$

We have the ideal gas formula

$$PV = RT$$

$$PV = kNT$$

$$V/N = kT/P$$

Here,

V is the volume of the gas

N is the number of moles of the gas

The mean separation between the two atoms of the gas is given as

$$r = \left[\frac{V}{N} \right]^{1/3} = \left[\frac{kT}{P} \right]^{1/3}$$

$$r = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} \right]^{1/3}$$

$$= 3.35 \times 10^{-9} \text{ m}$$

Therefore, the mean separation between the atom is greater than the de Broglie wavelength.

Question 11.36

Compute the typical de Broglie wavelength of an electron in metal at 27 °C and compare it with the mean separation between two electrons in a metal which is given to be about $2 \times 10^{-10} \text{ m}$. [Note: Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishability has many fundamental implications, which you will explore in more advanced Physics courses.]

Solution

Temperature, $T = 27^\circ\text{C} = 27 + 273 = 300\text{ K}$

Mean separation between two electrons, $r = 2 \times 10^{-10}\text{ m}$

De Broglie wavelength of an electron is

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

h = Planck's constant = $6.6 \times 10^{-34}\text{ Js}$

m = Mass of an electron = $9.11 \times 10^{-31}\text{ kg}$

k = Boltzmann constant = $1.38 \times 10^{-23}\text{ J/mol/K}$.

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}}$$

$$\approx 6.2 \times 10^{-9}\text{ m}$$

Hence, the de Broglie wavelength is much greater than the given inter-electron separation.

Question 11.37

Answer the following questions:

- (a) Quarks inside protons and neutrons are thought to carry fractional charges $[(+2/3)e; (-1/3)e]$. Why do they not show up in Millikan's oil-drop experiment?
- (b) What is so special about the combination e/m ? Why do we not simply talk of e and m separately?
- (c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?
- (d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?
- (e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:

$$E = h\nu, p = h/\lambda$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

Solution

(a) Quarks inside protons and neutrons are thought to carry fractional charges $[(+2/3)e; (-1/3)e]$. This is because nuclear forces grow stronger if they are pulled apart. Therefore, it seems that fractional charges may exist in nature. The observable charges are an integral multiple of electrical charges (e).

(b) The relation between the electric field and the magnetic field,

$$eV = (1/2) mv^2 \text{ and } eBv = mv^2/r$$

Here,

e = electric charge

v = velocity

V = potential

r = Radius

B = magnetic field

From these equations, it can be understood that the dynamics of an electron can be determined only by the ratio e/m and not by e and m separately.

(c) At the atmospheric pressure, the ions in the gas do not have a chance of reaching their respective electrons due to collision and recombination with other molecules in the gas. At low pressures, ions have a chance to reach their respective electrons, which results in the flow of current.

(d) The minimum energy required for an electron in the conduction band to get out of the metal is called the work function. These electrons occupy different energy levels, because of which, for the same incident radiation, electrons come out with different energies.

(e) The absolute value of the energy of a particle is arbitrary within the additive constant. Therefore, the wavelength(λ) is significant, but the frequency (ν) of the electron does not have direct physical significance. Therefore, product $\nu\lambda$ has no physical significance.