

## Chapter 9: Differential Equations

NCERT · Class 12 · Maths

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### Exercise 9.1

#### Question 1

Determine order and degree (if defined) of differential equation  $\frac{d^4y}{dx^4} + \sin(y') = 0$ .

#### Solution

Ans: Rewrite the equation  $\frac{d^4y}{dx^4} + \sin(y') = 0$ . as:

$$\Rightarrow y' + \sin(y''') = 0$$

The highest order between the two terms is of  $y'$  which is four.

The differential equation contains a trigonometric derivative term and is not completely polynomial in its derivative, thus degree is not defined.

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#### Question 2

Determine order and degree (if defined) of differential equation  $y' + 5y = 0$ .

#### Solution

Ans: The given differential equation is  $y' + 5y = 0$ .

The highest order term is  $y'$ , thus the order is one.

As the derivative is of completely polynomial nature is and highest power of derivative is of  $y'$  which is one. Thus degree is one.

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### Question 3

Determine order and degree (if defined) of differential equation  $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ .

#### Solution

Ans: The given differential equation is  $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ .

The highest order term is  $\frac{d^2s}{dt^2}$ , thus the order is two.

As the derivative is of completely polynomial nature is and highest power of derivative term  $\frac{d^2s}{dt^2}$  which is one. Thus the degree is one.

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### Question 4

Determine order and degree (if defined) of differential equation  $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ .

#### Solution

Ans: The given differential equation is  $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ .

The highest order term is  $\frac{d^2y}{dx^2}$ , thus the order is two.

The differential equation contains a trigonometric derivative term and is not completely polynomial in its derivative, thus degree is not defined.

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### Question 5

Determine order and degree (if defined) of differential equation  $\left(\frac{d^2y}{dx^2}\right) - \cos 3x + \sin 3x = 0$ .

#### Solution

Ans: The given differential equation is  $\left(\frac{d^2y}{dx^2}\right) - \cos 3x + \sin 3x = 0$ .

The highest order term is  $\frac{d^2y}{dx^2}$ , thus the order is two.

As the derivative is of completely polynomial nature is and highest power of derivative term  $\frac{d^2y}{dx^2}$  which is one. Thus degree is one.

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### Question 6

**Determine order and degree (if defined) of differential equation  $(y')^2 + (y')^3 + (y')^4 + y^5 = 0$ .**

#### Solution

Ans: The given differential equation is  $(y')^2 + (y')^3 + (y')^4 + y^5 = 0$ .

The highest order term is  $(y''')^2$ , thus the order is three.

The differential equation is of the polynomial form and the power of highest order term  $y'$  is two, thus the degree is two.

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### Question 7

**Determine order and degree (if defined) of differential equation  $y'' + 2y' + y' = 0$ .**

#### Solution

Ans: The given differential equation is  $y'' + 2y' + y' = 0$ .

The highest order derivative in the differential equation is  $y'''$ . Thus its order is three.

The differential equation is polynomial with the highest order term  $y'''$  having a degree one. Thus the degree is one.

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### Question 8

**Determine order and degree (if defined) of differential equation  $y'' + y = e'$ .**

#### Solution

Ans: The given differential equation is  $y'' + y = e'$ . Therefore:

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$$\Rightarrow y' + y - e' = 0$$

The highest order derivative in the differential equation is  $y'$ . Thus its order is one.

The given equation is of polynomial form with the highest order term  $y'$  with degree one. Thus the degree is one.

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### Question 9

**Determine order and degree (if defined) of differential equation  $y' + (y')^2 + 2y = 0$ .**

### Solution

Ans: The given differential equation is  $y' + (y')^2 + 2y = 0$ .

The highest order derivative in the differential equation is  $y'$ . Thus its order is two.

The given equation is of polynomial form with the highest order term  $y'$  with highest degree one. Thus the degree is one.

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### Question 10

**Determine order and degree (if defined) of differential equation  $y' + 2y' + \sin y = 0$ .**

### Solution

Ans: The given differential equation is  $y' + 2y' + \sin y = 0$ .

The highest order derivative in the differential equation is  $y'$ . Thus its order is two.

The given equation is of polynomial form with the highest order term  $y'$  with the highest degree one. Thus the degree is one.

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### Question 11

**Find the degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ .**

A.

3

B.

2

C.

1

D.

not defined

### Solution

Ans: The given differential equation is  $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ .

The differential equation is not polynomial in its derivative because of the term  $\sin\left(\frac{dy}{dx}\right)$  thus its order is not defined.

The correct answer is (D).

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### Question 12

Find the order of the differential equation  $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$ .

A.

2

B.

1

C.

0

**D.**

not defined

### Solution

Ans: The given differential equation is  $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$ .

The highest order term of the equation is  $\frac{d^2y}{dx^2}$ , thus the order is two.

The correct answer is (A).

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## Exercise 9.2

### Question 1

Verify the function  $y=e^x+1$  is solution of differential equation  $y'-y=0$ .

### Solution

Ans: The given function is  $y=e^x + 1$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx}(e^x + 1)$$

$$\Rightarrow y' = e^x \quad \dots\dots(1)$$

Take the derivative of the above equation:

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$

$$\Rightarrow y'' = e^x$$

Using result from equation (1):

$$y'' - y' = 0$$

Thus the given function is solution of differential equation  $y'-y=0$ .

### Question 2

Verify the function  $y=x^2+2x+C$  is solution of differential equation  $y'-2x-2=0$ .

#### Solution

Ans: The given function is  $y=x^2+2x+C$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 2x + C)$$

$$\Rightarrow y' = 2x + 2$$

$$\Rightarrow y' - 2x - 2 = 0$$

Thus the given function is solution of differential equation  $y'-2x-2=0$ .

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### Question 3

Verify the function  $y=\cos x + C$  is solution of differential equation  $y'+\sin x =0$ .

#### Solution

Ans: The given function is  $y=\cos x + C$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx}(\cos x + C)$$

$$\Rightarrow y' = -\sin x$$

$$\Rightarrow y' + \sin x = 0$$

Thus the given function is solution of differential equation  $y' + \sin x = 0$ .

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### Question 4

Verify the function  $y=\sqrt{1+x^2}$  is solution of differential equation  $y'=\frac{xy}{1+x^2}$ .

**Solution**

Ans: The given function is  $y = \sqrt{1 + x^2}$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{1 + x^2} \right)$$

$$y' = \frac{1}{2\sqrt{1+x^2}} \times \frac{d}{dx} (1 + x^2)$$

$$y' = \frac{2}{2\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{1}{\sqrt{1+x^2}}$$

Multiply numerator and denominator by  $\sqrt{1+x^2}$ :

$$y' = \frac{1}{\sqrt{1+x^2}} \times \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}$$

Substitute  $y = \sqrt{1+x^2}$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

Thus the given function is solution of differential equation  $y' = \frac{xy}{1+x^2}$ .

**Question 5**

Verify the function  $y = Ax$  is solution of differential equation  $xy' = y$  ( $x \neq 0$ ).

**Solution**

Ans: The given function is  $y = Ax$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx} (Ax)$$

$$\Rightarrow y' = A$$

Multiply by  $x$  on both side:

$$xy' = Ax$$

Substitute  $y = Ax$ :

$$\Rightarrow xy' = y$$

Thus the given function is solution of differential equation  $xy' = y$  ( $x \neq 0$ ).

### Question 6

Verify the function  $y = x \sin x$  is solution of differential equation

$$xy' = y + x\sqrt{x^2 - y^2} \quad (x \neq 0 \text{ and } x > y \text{ or } x < -y).$$

### Solution

Ans: The given function is  $y = x \sin x$ .

Take its derivative:

$$\frac{dy}{dx} = \frac{d}{dx}(x \sin x)$$

$$\Rightarrow y' = \sin x \frac{d}{dx}(x) + x \frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x \cos x$$

Multiply by  $x$  on both side:

$$xy' = x(\sin x + x \cos x)$$

$$xy' = x \sin x + x^2 \cos x$$

Substitute  $y = x \sin x$ :

$$\Rightarrow xy' = y + x^2 \cos x$$

Use  $\sin x = \frac{y}{x}$  and substitute  $\cos x$ :

$$xy' = y + x^2 \sqrt{1 - \sin^2 x}$$

$$xy' = y + x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2}$$

$$\Rightarrow xy' = y + x\sqrt{y^2 - x^2}$$

Thus the given function is solution of differential equation  $xy' = y + x\sqrt{y^2 - x^2}$ .

**Question 7**

Verify the function  $xy = \log y + C$  is solution of differential equation  $y' = \frac{y^2}{1-xy}$  ( $xy \neq 1$ ).

**Solution**

Ans: The given function is  $xy = \log y + C$ .

Take derivative on both side:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y \frac{d}{dx}(x) + x \frac{dy}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y}y'$$

$$\Rightarrow y^2 + xy y' = y'$$

Shift the  $y'$  terms on one side and take it common.

$$\Rightarrow (xy - 1) y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

Thus the given function is solution of differential equation  $y' = \frac{y^2}{1-xy}$ .

**Question 8**

Verify the function  $y - \cos y = x$  is solution of differential equation

$$(y \sin y + \cos y + x) y' = 1$$

.

**Solution**

Ans: The given function is  $y - \cos y = x$ .

Take derivative on both side:

$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$

$$\Rightarrow y' + y' \sin y = 1$$

$$\Rightarrow y'(1 + \sin y) = 1$$

$$\Rightarrow y' = \frac{1}{1 + \sin y}$$

Multiply by

$$(y \sin y + \cos y + x)$$

on both side:

$$(y \sin y + \cos y + x) y' = \frac{(y \sin y + \cos y + x)}{1 + \sin y}$$

Substitute  $y = \cos y + x$  in the numerator:

$$(y \sin y + \cos y + x) y' = \frac{(y \sin y + y)}{1 + \sin y}$$

$$(y \sin y + \cos y + x) y' = \frac{y(\sin y + 1)}{1 + \sin y}$$

$$\Rightarrow (y \sin y + \cos y + x) y' = y$$

Thus the given function is solution of differential equation  $(y \sin y + \cos y + x) y' = y$ .

### Question 9

Verify the function  $x + y = \tan^{-1} y$  is solution of differential equation

$$y^2 y' + y^2 + 1 = 0$$

.

### Solution

Ans: The given function is  $x + y = \tan^{-1} y$ .

Take derivative on both side:

$$\frac{d}{dx}(x + y) = \frac{d}{dx}(\tan^{-1}y)$$

$$1 + y' = \left(\frac{1}{1+y^2}\right) y'$$

$$\Rightarrow y' \left[ \frac{1}{1+y^2} - 1 \right] = 1$$

$$\Rightarrow y' \left[ \frac{1-(1+y^2)}{1+y^2} \right] = 1$$

$$\Rightarrow y' \left[ \frac{-y^2}{1+y^2} \right] = 1$$

$$\Rightarrow -y^2 y' = 1 + y^2$$

$$\Rightarrow y^2 y' + y^2 + 1 = 0$$

Thus the given function is solution of differential equation  $y^2 y' + y^2 + 1 = 0$ .

### Question 10

Verify the function  $y = \sqrt{a^2 - x^2}$   $x \in (-a, a)$  is solution of differential equation

$$x + y \frac{dy}{dx} = 0 \quad (y \neq 0)$$

.

### Solution

Ans: The given function is:

$$y = \sqrt{a^2 - x^2} \quad x \in (-a, a)$$

.

Take derivative on both side:

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{a^2 - x^2} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx} (a^2 - x^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2\sqrt{a^2 - x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{a^2 - x^2}}$$

Substitute  $y = \sqrt{a^2 - x^2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

$$\Rightarrow x + y \frac{dy}{dx} = 0$$

Thus the given function is solution of differential equation:

$$x + y \frac{dy}{dx} = 0 \ (y \neq 0).$$

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### Question 11

Find the numbers of arbitrary constants in the general solution of a differential equation of fourth order.

A.

0

B.

2

C.

3

D.

4

### Solution

Ans: The number of arbitrary constants in the general solution of a differential equation is equal to its order. As the given differential equation is of fourth order, thus it has four arbitrary constants in its solution.

The correct answer is (D).

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**Question 12**

Find the numbers of arbitrary constants in the particular solution of a differential equation of third order.

A.

3

B.

2

C.

1

D.

0

**Solution**

Ans: The particular solution of any differential equation does not have any arbitrary constants. Thus it has zero constants in its solution.

The correct answer is (D).

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**Exercise 9.3****Question 1**

Find the general solution for  $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ .

**Solution**

Ans: The given differential equation is  $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ .

Use trigonometric half-angle identities to simplify:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \tan^2 \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 \frac{x}{2} - 1$$

Separate the differentials and integrate:

$$\int dy = \int \left( \sec^2 \frac{x}{2} - 1 \right) dx$$

$$\Rightarrow y = \int \sec^2 \frac{x}{2} dx - \int dx$$

$$\Rightarrow y = 2 \tan \frac{x}{2} - x + C$$

Thus the general solution of given differential equation is  $y = 2 \tan \frac{x}{2} - x + C$ .

## Question 2

Find the general solution for  $\frac{dy}{dx} = \sqrt{4 - y^2} (-2 - y + 2)$ .

### Solution

Ans: The given differential equation is  $\frac{dy}{dx} = \sqrt{4 - y^2} (-2 - y + 2)$ .

Simplify the expression:

$$\frac{dy}{dx} = \sqrt{4 - y^2}$$

$$\Rightarrow \frac{dy}{\sqrt{4 - y^2}} = dx$$

Use standard integration:

$$\int \frac{dy}{\sqrt{4 - y^2}} = \int dx$$

$$\Rightarrow \sin^{-1} \frac{y}{2} = x + C$$

$$\Rightarrow \frac{y}{2} = \sin(x + C)$$

$$\Rightarrow y = 2 \sin(x + C)$$

Thus the general solution of given differential equation is  $y = 2 \sin(x + C)$ .

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### Question 3

Find the general solution for  $\frac{dy}{dx} + y = 1$  ( $y \neq 1$ ).

### Solution

Ans: The given differential equation is  $\frac{dy}{dx} + y = 1$  ( $y \neq 1$ ).

Simplify the expression:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{1-y} = dx$$

Use standard integration:

$$\int \frac{dy}{1-y} = \int dx$$

$$\Rightarrow -\log(1-y) = x + C$$

$$\Rightarrow \log(1-y) = -(x + C)$$

$$\Rightarrow 1-y = e^{-(x+C)}$$

$$y = 1 - Ae^{-x} \quad (A = e^{-C})$$

Thus the general solution of given differential equation is  $y = 1 - Ae^{-x}$ .

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### Question 4

Find the general solution for  $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$ .

### Solution

Ans: The given differential equation is:

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0.$$

Divide both side by  $\tan x \tan y$ :

$$\begin{aligned} \frac{\sec^2 x \tan y dx + \sec^2 y \tan x dy}{\tan x \tan y} &= 0 \\ \Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy &= 0 \end{aligned}$$

Integrate both side:

$$\begin{aligned} \int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy &= 0 \\ \Rightarrow \int \frac{\sec^2 y}{\tan y} dy &= - \int \frac{\sec^2 x}{\tan x} dx \quad \dots\dots(1) \end{aligned}$$

Use a substitution method for integration. Substitute  $\tan x = u$ :

For integral on RHS:

$$\begin{aligned} \Rightarrow \tan x &= u \\ \Rightarrow \sec^2 x dx &= du \\ \Rightarrow \int \frac{\sec^2 x}{\tan x} dx &= \int \frac{du}{u} \\ \Rightarrow \int \frac{\sec^2 x}{\tan x} dx &= \log u \\ \Rightarrow \int \frac{\sec^2 x}{\tan x} dx &= \log(\tan x) \end{aligned}$$

Thus evaluating result form (1):

$$\begin{aligned} \Rightarrow \log(\tan y) &= -\log(\tan x) + \log(C) \\ \Rightarrow \log(\tan y) &= \log\left(\frac{C}{\tan x}\right) \\ \Rightarrow \tan x \tan y &= C \end{aligned}$$

Thus the general solution of given differential equation is  $\tan x \tan y = C$ .

### Question 5

Find the general solution for  $(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$ .

## Solution

Ans: The given differential equation is:

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0.$$

Simplify the expression:

$$dy = \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx$$

Integrate both side:

$$\int dy = \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx \dots\dots(1)$$

Use a substitution method for integration. Substitute

$$e^x + e^{-x} = t$$

:

For integral on RHS:

$$\Rightarrow e^x + e^{-x} = t$$

$$\Rightarrow (e^x - e^{-x}) dx = dt$$

$$\Rightarrow \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx = \int \frac{dt}{t}$$

$$\Rightarrow \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx = \ln t + C$$

$$\Rightarrow \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx = \log(e^x + e^{-x}) + C$$

Thus evaluating result form (1):

$$y = \log(e^x + e^{-x}) + C$$

Thus the general solution of given differential equation is  $y = \log(e^x + e^{-x}) + C$ .

## Question 6

Find the general solution for  $\frac{dy}{dx} = (1+x^2)(1+y^2)$ .

**Solution**

Ans: The given differential equation is:

$$\frac{dy}{dx} = (1 + x^2) (1 + y^2).$$

Simplify the expression:

$$\frac{dy}{1+y^2} = (1 + x^2) dx$$

Integrate both side:

$$\int \frac{dy}{1+y^2} = \int (1 + x^2) dx$$

Use standard integration:

$$\tan^{-1}y = \int dx + \int x^2 dx$$

$$\Rightarrow \tan^{-1}y = x + \frac{x^3}{3} + C$$

Thus the general solution of given differential equation is  $\tan^{-1}y = x + \frac{x^3}{3} + C$ .

**Question 7**

Find the general solution for  $y \log y \, dx - x \, dy = 0$ .

**Solution**

Ans: The given differential equation is:

$$y \log y \, dx - x \, dy = 0.$$

Simplify the expression:

$$y \log y \, dx = x \, dy$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y \log y}$$

Integrate both side:

$$\int \frac{dx}{x} = \int \frac{dy}{y \log y} \dots\dots(1)$$

Use substitution method for integration on LHS. Substitute

$$\log y = t$$

:

$$\log y = t$$

$$\Rightarrow \frac{1}{y} dy = dt$$

$$\Rightarrow \int \frac{dy}{y \log y} = \int \frac{dt}{t}$$

$$\Rightarrow \int \frac{dy}{y \log y} = \log t$$

$$\Rightarrow \int \frac{dy}{y \log y} = \log(\log y)$$

Evaluating expression (1):

$$\log(\log y) = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

Thus the general solution of given differential equation is  $y = e^{Cx}$ .

### Question 8

Find the general solution for  $x^5 \frac{dy}{dx} = -y^5$ .

### Solution

Ans: The given differential equation is:

$$x^5 \frac{dy}{dx} = -y^5.$$

Simplify the expression:

$$\frac{dy}{y^5} = -\frac{dx}{x^5}$$

Integrate both side:

$$\int \frac{dy}{y^5} = - \int \frac{dx}{x^{-5}}$$

$$\Rightarrow \int y^{-5} dy = - \int x^{-5} dx$$

$$\Rightarrow \frac{y^{-5+1}}{-5+1} = - \frac{x^{-5+1}}{-5+1} + C$$

$$\Rightarrow \frac{y^{-4}}{-4} = - \frac{x^{-4}}{-4} + C$$

$$\Rightarrow x^{-4} + y^{-4} = -4C$$

$$\Rightarrow x^{-4} + y^{-4} = A \quad (A = -4C)$$

Thus the general solution of given differential equation is  $x^{-4} + y^{-4} = A$ .

### Question 9

Find the general solution for  $\frac{dy}{dx} = \sin^{-1} x$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x.$$

Simplify the expression:

$$dy = \sin^{-1} x dx$$

Integrate both side:

$$\int dy = \int \sin^{-1} x dx$$

$$\Rightarrow y = \int 1 \times \sin^{-1} x dx$$

Use product rule of integration:

$$\int \sin^{-1} x dx = \sin^{-1} x \int dx - \int \left( \frac{1}{\sqrt{1-x^2}} \int dx \right) dx$$

$$\Rightarrow \int \sin^{-1} x dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

Substitute  $1-x^2=t^2$

$$1 - x^2 = t^2$$

$$\Rightarrow -2x dx = 2t dt$$

$$\Rightarrow -x dx = t dt$$

Evaluating the integral:

$$\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \int \sin^{-1} x dx = x \sin^{-1} x + \int \frac{t dt}{\sqrt{t^2}}$$

$$\Rightarrow \int \sin^{-1} x dx = x \sin^{-1} x + t + C$$

$$\Rightarrow \int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1-x^2} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$

Thus the general solution of given differential equation is  $y = x \sin^{-1} x + \sqrt{1-x^2} + C$ ,

### Question 10

Find the general solution for  $e^x \tan y dx + (1-e^x) \sec^2 y dy = 0$ .

### Solution

Ans: The given differential equation is:

$$e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0.$$

Simplify the expression:

$$(1 - e^x) \sec^2 y dy = -e^x \tan y dx$$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{(1-e^x)} dx$$

Integrate both side:

$$\int \frac{\sec^2 y}{\tan y} dy = - \int \frac{e^x}{(1-e^x)} dx \dots\dots(1)$$

Substitute  $\tan y = u$

$$\tan y = u$$

$$\Rightarrow \sec^2 y = du$$

Evaluating the LHS integral of (1):

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \int \frac{du}{u}$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \log u$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \log(\tan y)$$

Substitute  $1 - e^x = v$

$$1 - e^x = v$$

$$\Rightarrow -e^x dx = dv$$

Evaluating the RHS integral of (1):

$$\Rightarrow - \int \frac{e^x}{(1 - e^x)} dx = \int \frac{dv}{v}$$

$$\Rightarrow - \int \frac{e^x}{(1 - e^x)} dx = \log v$$

$$\Rightarrow - \int \frac{e^x}{(1 - e^x)} dx = \log(1 - e^x)$$

Therefore the integral (1) will be:

$$\log(\tan y) = \log(1 - e^x) + \log C$$

$$\Rightarrow \log(\tan y) = \log C (1 - e^x)$$

$$\Rightarrow \tan y = C (1 - e^x)$$

Thus the general solution of given differential equation is  $\tan y = C (1 - e^x)$ ,

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### Question 11

Find the particular solution of  $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$ ;  $y=1, x=0$  to satisfy the given condition.

## Solution

Ans: The given differential equation is:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x; \quad y = 1, \quad x = 0.$$

Simplify the expression:

$$\frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x + x^2 + 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{(x+1)(x^2+1)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)(x^2+1)} dx$$

Integrate both side:

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \dots\dots(1)$$

Use partial fraction method to simplify the RHS:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow 2x^2 + x = A(x^2 + 1) + (Bx + C)(x + 1)$$

$$\Rightarrow 2x^2 + x = (A + B)x^2 + (B + C)x + (A + C)$$

By comparing coefficients:

$$A + B = 2$$

$$B + C = 1$$

$$A + C = 0$$

Solving this we get:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{\left(\frac{1}{2}\right)}{x+1} + \frac{\left(\frac{3}{2}\right)x + \left(-\frac{1}{2}\right)}{x^2+1}$$

$$\Rightarrow \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{1}{2} \left( \frac{1}{x+1} + \frac{3x-1}{x^2+1} \right)$$

Rewriting the integral(1):

$$y = \frac{1}{2} \int \left( \frac{1}{x+1} + \frac{3x-1}{x^2+1} \right) dx$$

$$\Rightarrow y = \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{3x-1}{x^2+1} dx$$

$$\Rightarrow y = \frac{1}{2} \log(x+1) + \frac{3}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx$$

$$\Rightarrow y = \frac{1}{2} \log(x+1) + \frac{3}{4} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow y = \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C$$

For  $y = 1$  when  $x=0$ .

$$1 = \frac{1}{2} \log(0+1) + \frac{3}{4} \log(0+1) - \frac{1}{2} \tan^{-1} 0 + C$$

$$\Rightarrow C = 1$$

Thus the required particular solution is :

$$\Rightarrow y = \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + 1.$$

## Question 12

Find the particular solution of  $x(x^2-1) \frac{dy}{dx} = 1$ ;  $y=0$  when  $x=2$  to satisfy the given condition.

**Solution**

Ans: The given differential equation is:

$$x(x^2 - 1) \frac{dy}{dx} = 1; y = 0 \text{ when } x = 2$$

Simplify the expression:

$$x(x^2 - 1) \frac{dy}{dx} = 1$$

$$\Rightarrow dy = \frac{dx}{x(x^2-1)}$$

$$\Rightarrow dy = \frac{dx}{x(x-1)(x+1)}$$

Integrate both side:

$$\int dy = \int \frac{dx}{x(x-1)(x+1)} \dots\dots(1)$$

Use partial fraction method to simplify the RHS:

$$\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\Rightarrow 1 = A(x^2 - 1) + Bx(x + 1) + Cx(x - 1)$$

$$\Rightarrow 1 = (A + B + C)x^2 + (B - C)x - A$$

By comparing coefficients:

$$A + B + C = 0$$

$$B - C = 0$$

$$-A = 1$$

Solving this we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{(-1)}{x} + \frac{\left(\frac{1}{2}\right)}{x-1} + \frac{\left(\frac{1}{2}\right)}{x+1}$$

$$\Rightarrow \frac{1}{x(x-1)(x+1)} = -\frac{1}{x} + \frac{1}{2} \left( \frac{1}{x-1} + \frac{1}{x+1} \right)$$

Rewriting the integral(1):

$$y = \int \left( -\frac{1}{x} + \frac{1}{2} \left( \frac{1}{x-1} + \frac{1}{x+1} \right) \right) dx$$

$$\Rightarrow y = - \int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x+1} dx$$

$$\Rightarrow y = -\log x + \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1) + \log C$$

$$\Rightarrow y = -\frac{2}{2} \log x + \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1) + \frac{2}{2} \log C$$

$$\Rightarrow y = \frac{1}{2} \left( -\log x^2 + \log(x-1) + \log(x+1) + \log C^2 \right)$$

$$\Rightarrow y = \frac{1}{2} \log \left[ \frac{C^2 (x^2 - 1)}{x^2} \right]$$

For  $y = 0$  when  $x=2$ .

$$0 = \frac{1}{2} \log \left[ \frac{C^2 (2^2 - 1)}{2^2} \right]$$

$$\Rightarrow 0 = \log \left[ \frac{3C^2}{4} \right]$$

$$\Rightarrow \frac{3C^2}{4} = 1$$

$$\Rightarrow C^2 = \frac{4}{3}$$

Thus the required particular solution is :

$$y = \frac{1}{2} \log \left[ \frac{4(x^2-1)}{3x^2} \right].$$

---

### Question 13

Find the particular solution of  $\cos \left( \frac{dy}{dx} \right) = a$  ( $a \in \mathbb{R}$ );  $y=1$  when  $x=0$  to satisfy the given condition.

### Solution

Ans: The given differential equation is:

$$\cos \left( \frac{dy}{dx} \right) = a \quad (a \in \mathbb{R}); y = 1 \text{ when } x = 0$$

Simplify the expression:

$$\cos \left( \frac{dy}{dx} \right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a dx$$

Integrate both side:

$$\int dy = \int \cos^{-1} a dx$$

$$\Rightarrow y = \cos^{-1} a \int dx$$

$$\Rightarrow y = x \cos^{-1} a + C$$

For  $y = 1$  when  $x=0$ .

$$1 = 0 \cos^{-1} a + C$$

$$\Rightarrow C = 1$$

Thus the required particular solution is:

$$y = x \cos^{-1} a + 1$$

.

$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

---

### Question 14

Find the particular solution of  $\frac{dy}{dx} = y \tan x$ ;  $y=1$  when  $x=0$  to satisfy the given condition.

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} = y \tan x; y = 1 \text{ when } x = 0$$

Simplify the expression:

$$\frac{dy}{dx} = y \tan x$$

$$\Rightarrow \frac{dy}{y} = \tan x dx$$

Integrate both side:

$$\int \frac{dy}{y} = \int \tan x dx$$

$$\Rightarrow \log y = \log(\sec x) + \log C$$

$$\Rightarrow \log y = \log(C \sec x)$$

$$\Rightarrow y = C \sec x$$

For  $y=1$  when  $x=0$ .

$$1 = C \sec 0$$

$$\Rightarrow C = 1$$

Thus the required particular solution is :

$$y = \sec x$$

.

### Question 15

Find the equation of a curve passing through the point (0, 0) and whose differential equation is  $y' = e^x \sin x$ .

### Solution

Ans: The given differential equation is:

$$y' = e^x \sin x$$

The curve passes through (0, 0).

Simplify the expression:

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x dx$$

Integrate both side:

$$\int dy = \int e^x \sin x dx$$

Use product rules for integration of RHS. Let:

$$I = \int e^x \sin x dx$$

$$\Rightarrow I = \sin x \int e^x dx - \int (\cos x \int e^x dx) dx$$

$$\Rightarrow I = e^x \sin x - \int e^x \cos x dx$$

$$\Rightarrow I = e^x \sin x - (\cos x \int e^x dx + \int (\sin x \int e^x dx) dx)$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - \int (e^x \sin x) dx$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow I = \frac{e^x}{2}(\sin x - \cos x)$$

Thus integral will be:

$$y = \frac{e^x}{2}(\sin x - \cos x) + C$$

Thus as the curve passes through (0,0)

$$0 = \frac{e^0}{2}(\sin 0 - \cos 0) + C$$

$$\Rightarrow 0 = \frac{1}{2}(0 - 1) + C$$

$$\Rightarrow C = \frac{1}{2}$$

Thus the equation of the curve will be:

$$y = \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2}$$

$$\Rightarrow y = \frac{e^x}{2}(\sin x - \cos x + 1)$$

---

### Question 16

For the differential equation  $xy \frac{dy}{dx} = (x+2)(y+2)$  find the solution curve passing through the point (1,-1).

### Solution

Ans: The given differential equation is:

$$xy \frac{dy}{dx} = (x+2)(y+2)$$

The curve passes through (1, -1).

Simplify the expression:

$$\Rightarrow \left( \frac{y}{y+2} \right) dy = \frac{(x+2)}{x} dx$$

$$\Rightarrow \left( 1 - \frac{2}{y+2} \right) dy = \frac{(x+2)}{x} dx$$

Integrate both side:

$$\int \left( 1 - \frac{2}{y+2} \right) dy = \int \frac{(x+2)}{x} dx$$

$$\Rightarrow \int dy - 2 \int \frac{1}{y+2} dy = \int \frac{x}{x} dx + \int \frac{2}{x} dx$$

$$\Rightarrow y - 2 \log(y+2) = x + 2 \log x + C$$

$$\Rightarrow y - x = 2 \log(y+2) + 2 \log x + C$$

$$\Rightarrow y - x = 2 \log[x(y+2)] + C$$

$$\Rightarrow y - x = \log[x^2(y+2)^2] + C$$

Thus as the curve passes through (1,-1)

$$\Rightarrow -1 - 1 = \log[(1)^2(-1+2)^2] + C$$

$$\Rightarrow -2 = \log 1 + C$$

$$\Rightarrow C = -2$$

Thus the equation of the curve will be:

$$y - x = \log[x^2(y+2)^2] - 2$$

$$\Rightarrow y - x + 2 = \log(x^2(y+2)^2)$$

---

### Question 17

Find the equation of a curve passing through the point (0,-2) given that at any point (x,y) on the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point.

### Solution

Ans: According to , the equation is given by:

$$y \frac{dy}{dx} = x$$

The curve passes through (0, -2).

Simplify the expression:

$$\Rightarrow y dy = x dx$$

Integrate both side:

$$\int y dy = \int x dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow y^2 - x^2 = 2C$$

Thus as the curve passes through (0,-2)

$$\Rightarrow (-2)^2 - 0^2 = 2C$$

$$\Rightarrow 4 = 2C$$

$$\Rightarrow C = 2$$

Thus the equation of the curve will be:

$$y^2 - x^2 = 2(2)$$

$$\Rightarrow y^2 - x^2 = 4.$$

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### Question 18

At any point (x,y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point(-4,-3) . Find the equation of the curve given that it passes through (-2,1).

### Solution

Ans: Let the point of contact of the tangent be (x,y). Then the slope of the segment joining point of contact and (-4,-3):

$$m = \frac{y+3}{x+4}$$

According to the for the slope of tangent  $\frac{dy}{dx}$  it follows:

$$\frac{dy}{dx} = 2m$$

$$\Rightarrow \frac{dy}{dx} = 2 \left( \frac{y+3}{x+4} \right)$$

Simplify the expression:

$$\frac{dy}{dx} = 2 \left( \frac{y+3}{x+4} \right)$$

$$\frac{dy}{y+3} = \frac{2}{x+4} dx$$

Integrate both side:

$$\int \frac{dy}{y+3} = \int \frac{2}{x+4} dx$$

$$\Rightarrow \log(y+3) = 2\log(x+4) + \log C$$

$$\Rightarrow \log(y+3) = \log(x+4)^2 + \log C$$

$$\Rightarrow \log(y+3) = \log C(x+4)^2$$

$$\Rightarrow y+3 = C(x+4)^2$$

Thus as the curve passes through (-2,1)

$$1+3 = C(-2+4)^2$$

$$\Rightarrow 4 = 4C$$

$$\Rightarrow C = 1$$

Thus the equation of the curve will be:

$$y+3 = (x+4)^2$$

.

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### Question 19

The volume of spherical balloons being inflated changes at a constant rate. If initially its radius is 3 units and after

3

seconds it is 6 units. Find the radius of balloon after  $t$  seconds.

**Solution**

Ans: Let the volume of spherical balloon be  $V$  and its radius  $r$ . Let the rate of change of volume be  $k$ .

$$\frac{dV}{dt} = k$$

$$\Rightarrow \frac{d}{dt} \left( \frac{4}{3} \pi r^3 \right) = k$$

$$\Rightarrow \frac{4}{3} \pi \frac{d}{dt} (r^3) = k$$

$$\Rightarrow \frac{4}{3} \pi (3r^2) \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k dt$$

Integrate both side:

$$\int 4\pi r^2 dr = \int k dt$$

$$\Rightarrow 4\pi \int r^2 dr = kt + C$$

$$\Rightarrow \frac{4}{3} \pi r^3 = kt + C$$

At initial time,  $t=0$  and  $r=3$ :

$$\frac{4}{3} \pi 3^3 = k(0) + C$$

$$\Rightarrow C = 36\pi$$

At  $t=3$  the radius  $r=6$ :

$$\frac{4}{3} \pi (6^3) = k(3) + 36\pi$$

$$\Rightarrow 3k = 288\pi - 36\pi$$

$$\Rightarrow k = 84\pi$$

Thus the radius-time relation can be given by:

$$\frac{4}{3} \pi r^3 = 84\pi t + 36\pi$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

The radius of balloon after  $t$  seconds is given by:  $r = (63t + 27)^{\frac{1}{3}}$ .

### Question 20

In a bank, principal increases continuously at the rate of  $r\%$  per year. Find the value of  $r$  if Rs. 100 doubles itself in 10 years ( $\log_e 2 = 0.6931$ ).

### Solution

Ans: Let the principal be  $p$ , according to :

$$\frac{dp}{dt} = \left( \frac{r}{100} \right) p$$

Simplify the expression:

$$\frac{dp}{p} = \left( \frac{r}{100} \right) dt$$

Integrate both side:

$$\int \frac{dp}{p} = \int \left( \frac{r}{100} \right) dt$$

$$\Rightarrow \log p = \frac{rt}{100} + c$$

$$\Rightarrow p = e^{\frac{rt}{100} + c}$$

$$\Rightarrow p = Ae^{\frac{rt}{100}} \quad (A = e^c)$$

At  $t=0$ ,  $p=100$ :

$$100 = Ae^{\frac{r(0)}{100}}$$

$$\Rightarrow A = 100$$

Thus the principle and rate of interest relation:

$$p = 100e^{\frac{rt}{100}}$$

At  $t=10$ ,  $p=2 \times 100=200$ :

$$200 = 100e^{\frac{r(10)}{100}}$$

$$\Rightarrow 2 = e^{\frac{r}{10}}$$

Take logarithm on both side:

$$\log\left(e^{\frac{r}{10}}\right) = \log(2)$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$

Thus the rate of interest  $r=6.931$ .

### Question 21

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ( $e^{0.5}=1.648$ ).

### Solution

Ans: Let the principal be  $p$ , according to principle increases at the rate of 5% per year.

$$\frac{dp}{dt} = \left(\frac{5}{100}\right)p$$

Simplify the expression:

$$\frac{dp}{dt} = \frac{p}{20}$$

$$\Rightarrow \frac{dp}{p} = \frac{1}{20}dt$$

Integrate both side:

$$\int \frac{dp}{p} = \int \frac{1}{20}dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20} + C}$$

$$\Rightarrow p = Ae^{\frac{t}{20}} \quad (A = e^C)$$

At  $t=0$ ,  $p=1000$ :

$$1000 = Ae^{\frac{0}{20}}$$

$$\Rightarrow A = 1000$$

Thus the relation of principal and time relation:

$$\Rightarrow p = 1000e^{\frac{t}{20}}$$

At  $t=10$ :

$$p = 1000e^{\frac{10}{20}}$$

$$\Rightarrow p = 1000e^{0.5}$$

$$\Rightarrow p = 1000 \times 1.648$$

$$\Rightarrow p = 1648$$

Thus after 10 this year the amount will become Rs. 1648.

---

### Question 22

In a culture, the bacteria count is 100000 . The number is increased by 10 in 2 hours. In how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present?

### Solution

Ans: Let the number of bacteria be  $y$  at time  $t$ . According to :

$$\frac{dy}{dt} \propto y$$

$$\frac{dy}{dt} = cy$$

Here  $c$  is constant.

Simplify the expression:

$$\frac{dy}{y} = cdt$$

Integrate both side:

$$\int \frac{dy}{y} = \int cdt$$

$$\Rightarrow \log y = ct + D$$

$$\Rightarrow y = e^{ct+D}$$

$$\Rightarrow y = Ae^{ct} \quad (A = e^D)$$

At  $t=0$ ,  $y=100000$ :

$$100000 = Ae^{c(0)}$$

$$\Rightarrow A = 100000$$

At  $t=2$ ,  $y=\frac{11}{10}(100000) = 110000$ :

$$y = 100000e^{ct}$$

$$\Rightarrow 110000 = 100000e^{c(2)}$$

$$\Rightarrow e^{2c} = \frac{11}{10}$$

$$\Rightarrow 2c = \log\left(\frac{11}{10}\right)$$

$$\Rightarrow c = \frac{1}{2}\log\left(\frac{11}{10}\right) \dots\dots(1)$$

For  $y=200000$ :

$$200000 = 100000e^{ct}$$

$$\Rightarrow e^{ct} = 2$$

$$\Rightarrow ct = \log 2$$

$$\Rightarrow t = \frac{\log 2}{c}$$

Back substituting using expression (1):

$$t = \frac{\log 2}{\frac{1}{2}\log\left(\frac{11}{10}\right)}$$

$$\Rightarrow t = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Thus time required for bacteria to reach 200000 is

$$t = \frac{\log 2}{\frac{1}{2}\log\left(\frac{11}{10}\right)}$$

hrs.

### Question 23

Find the general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$ .

A.

$$e^x + e^{-y} = C$$

B.

$$e^x + e^y = C$$

C.

$$e^{-x} + e^y = C$$

D.

$$e^{-x} + e^{-y} = C$$

### Solution

Ans: The given differential equation is  $\frac{dy}{dx} = e^{x+y}$ . Simplify the expression:

$$\frac{dy}{dx} = e^x e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrate both side:

$$\int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + D$$

$$\Rightarrow e^x + e^{-y} = -D$$

$$\Rightarrow e^x + e^{-y} = C \quad (C = -D)$$

Thus the general solution of given differential equation is  $e^x + e^{-y} = C$

Thus the correct option is (A).

## Exercise 9.4

### Question 1

Show that, differential equation  $(x^2 + xy) dy = (x^2 + y^2) dx$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Checking for homogeneity:

$$F(x, y) = \frac{x^2 + y^2}{x^2 + xy}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda^2(x^2 + y^2)}{\lambda^2(x^2 + xy)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{x^2 + y^2}{x^2 + xy}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{x^2 + (vx)^2}{x^2 + x(vx)}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x^2(1+v^2)}{x^2(1+v)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2 - v - v^2}{1+v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1-v}{1+v}$$

Separate the differentials:

$$\frac{1+v}{1-v} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{1+v}{1-v} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{2-(1-v)}{1-v} dv = \log x - \log k$$

$$\Rightarrow \int \frac{2}{1-v} dv - \int \frac{1-v}{1-v} dv = \log \frac{x}{k}$$

$$\Rightarrow -2 \log(1-v) - \int dv = \log \frac{x}{k}$$

$$\Rightarrow -2 \log(1-v) - v = \log \frac{x}{k}$$

$$\Rightarrow v = -\log \frac{x}{k} - 2 \log(1-v)$$

$$\Rightarrow v = \log \left( \frac{k}{x(1-v)^2} \right)$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{y}{x} = \log \left( \frac{k}{x \left(1 - \frac{y}{x}\right)^2} \right)$$

$$\Rightarrow \frac{y}{x} = \log \left( \frac{kx}{(x-y)^2} \right)$$

$$\Rightarrow e^{\frac{y}{x}} = \frac{kx}{(x-y)^2}$$

$$\Rightarrow (x-y)^2 = kxe^{-\frac{y}{x}}$$

The solution of the given differential equation  $(x-y)^2 = kxe^{-\frac{y}{x}}$ .

## Question 2

Show that, differential equation  $y' = \frac{x+y}{x}$  is homogenous and solves it.

## Solution

Ans: Rewrite the equation in standard form:

$$\frac{dy}{dx} = \frac{x+y}{x}$$

Checking for homogeneity:

$$F(x, y) = \frac{x+y}{x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda(x+y)}{\lambda(x)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{x+y}{x}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\frac{d(vx)}{dx} = \frac{x+(vx)}{x}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x(1+v)}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1$$

Separate the differentials:

$$dv = \frac{dx}{x}$$

Integrate both side:

$$\int dv = \int \frac{dx}{x}$$

$$\Rightarrow \int dv = \log x + \log k$$

$$\Rightarrow v = \log kx$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{y}{x} = \log kx$$

$$\Rightarrow y = x \log kx$$

The solution of the given differential equation  $y = x \log kx$

### Question 3

Show that, differential equation  $(x-y) dy - (x+y) dx = 0$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Checking for homogeneity:

$$F(x, y) = \frac{x+y}{x-y}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda(x+y)}{\lambda(x-y)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{x+y}{x-y}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{x+(vx)}{x-vx}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x(1+v)}{x(1-v)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1+v}{1-v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v}{1-v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v-v+v^2}{1-v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2}{1-v}$$

Separate the differentials:

$$\frac{1-v}{1+v^2} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{1}{1+v^2} dv - \int \frac{v}{1+v^2} dv = \log x + C$$

$$\Rightarrow \tan^{-1} v - \frac{1}{2} \int \frac{2v}{1+v^2} dv = \log x + C$$

$$\Rightarrow \tan^{-1} v - \frac{1}{2} \log(1+v^2) = \log x + C$$

$$\Rightarrow \tan^{-1} v = \log x + \frac{1}{2} \log(1+v^2) + C$$

$$\Rightarrow \tan^{-1} v = \frac{1}{2} \log x^2 + \frac{1}{2} \log(1+v^2) + C$$

$$\Rightarrow \tan^{-1} v = \frac{1}{2} \log[x^2(1+v^2)] + C$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \tan^{-1} \frac{y}{x} = \frac{1}{2} \log \left[ x^2 \left( 1 + \frac{y^2}{x^2} \right) \right] + C$$

$$\Rightarrow \tan^{-1} \frac{y}{x} = \frac{1}{2} \log(x^2 + y^2) + C$$

The solution of the given differential equation  $\tan^{-1} \frac{y}{x} = \frac{1}{2} \log(x^2 + y^2) + C$ .

#### Question 4

Show that, differential equation  $(x^2 - y^2) dx + 2xy dy = 0$  is homogenous and solves it.

#### Solution

Ans: Rewrite the equation in standard form:

$$\frac{dy}{dx} = -\frac{x^2 - y^2}{2xy}$$

Checking for homogeneity:

$$F(x, y) = -\frac{x^2 - y^2}{2xy}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{(\lambda x)^2 - (\lambda y)^2}{2(\lambda x)(\lambda y)}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{\lambda^2 x^2 - \lambda^2 y^2}{2\lambda^2 xy}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{\lambda^2(x^2 - y^2)}{\lambda^2(2xy)}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{x^2 - y^2}{2xy}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = -\frac{x^2 - (vx)^2}{2x(vx)}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x^2(v^2 - 1)}{x^2(2v)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = -\frac{1 + v^2}{2v}$$

Separate the differentials:

$$\frac{2v}{1 + v^2} dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \frac{2v}{1 + v^2} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log(1 + v^2) = -\log x + C$$

$$\Rightarrow \log(1 + v^2) + \log x = C$$

$$\Rightarrow \log[x(1 + v^2)] = C$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \log\left[x\left(1 + \frac{y^2}{x^2}\right)\right] = C$$

$$\Rightarrow \left(\frac{x^2 + y^2}{x}\right) = k \quad k = e^C$$

$$\Rightarrow x^2 + y^2 = kx$$

The solution of the given differential equation  $x^2 + y^2 = kx$ .

### Question 5

Show that, differential equation  $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$$

Checking for homogeneity:

$$F(x, y) = \frac{x^2 - 2y^2 + xy}{x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda^2 x^2 - 2\lambda^2 y^2 + (\lambda x)(\lambda y)}{\lambda^2 x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda^2 (x^2 - 2y^2 + xy)}{\lambda^2 (x^2)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{x^2 - 2y^2 + xy}{x^2}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{x^2 - 2(vx)^2 + x(vx)}{x^2}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x^2(1 - 2v^2 + v)}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 - 2v^2 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1 - 2v^2$$

Separate the differentials:

$$\frac{1}{1-2v^2} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{1}{1-2v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{\frac{1}{2}-v^2} dv = \log x + C$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} dv = \log x + C$$

$$\Rightarrow \frac{1}{2} \left( \frac{1}{2 \times \frac{1}{\sqrt{2}}} \right) \log \frac{\left| \frac{1}{\sqrt{2}} + v \right|}{\left| \frac{1}{\sqrt{2}} - v \right|} = \log x + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \frac{|1+\sqrt{2}v|}{|1-\sqrt{2}v|} = \log x + C$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \frac{\left| 1 + \sqrt{2} \left( \frac{y}{x} \right) \right|}{\left| 1 - \sqrt{2} \left( \frac{y}{x} \right) \right|} = \log x + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \frac{|x+\sqrt{2}y|}{|x-\sqrt{2}y|} = \log x + C$$

The solution of the given differential equation  $\frac{1}{2\sqrt{2}} \log \frac{|x+\sqrt{2}y|}{|x-\sqrt{2}y|} = \log x + C$ .

### Question 6

Show that, differential equation  $xdy - ydx = \sqrt{x^2 + y^2} dx$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$xdy = \sqrt{x^2 + y^2} dx + ydx$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$$

Checking for homogeneity:

$$F(x, y) = \frac{\sqrt{x^2 + y^2} + y}{x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\sqrt{\lambda^2 x^2 + \lambda^2 y^2 + \lambda y}}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\sqrt{\lambda^2(x^2 + y^2) + \lambda y}}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda(\sqrt{x^2 + y^2} + y)}{\lambda(x)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\sqrt{x^2 + y^2} + y}{x}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{\sqrt{x^2 + (vx)^2} + vx}{x}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x(\sqrt{1+v^2} + v)}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = \sqrt{1+v^2} + v$$

$$\Rightarrow x \frac{dv}{dx} = \sqrt{1+v^2}$$

Separate the differentials:

$$\frac{1}{\sqrt{1+v^2}} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{1}{\sqrt{1+v^2}} dv = \int \frac{dx}{x}$$

$$\Rightarrow \log|v + \sqrt{1+v^2}| = \log x + \log C$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \log\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right| = \log Cx$$

$$\Rightarrow \log\left|\frac{y + \sqrt{y^2 + x^2}}{x}\right| = \log Cx$$

$$y + \sqrt{y^2 + x^2} = Cx^2$$

### Question 7

Show that, differential equation  $\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x dy$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x dy$$

$$\frac{dy}{dx} = \frac{\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y}{\left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x}$$

Checking for homogeneity:

$$F(x, y) = \frac{\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y}{\left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\left\{\lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right)\right\} \lambda y}{\left\{\lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right)\right\} \lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda^2 \left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y}{\lambda^2 \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y}{\left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\frac{d(vx)}{dx} = \frac{\left\{x \cos\left(\frac{xv}{x}\right) + vx \sin\left(\frac{vx}{x}\right)\right\} vx}{\left\{vx \sin\left(\frac{vx}{x}\right) - x \cos\left(\frac{vx}{x}\right)\right\} x}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x^2 \{\cos v + v \sin v\} v}{x^2 \{v \sin v - \cos v\}}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v - v^2 \sin v + v \cos v}{v \sin v - \cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$$

Separate the differentials:

$$\frac{v \sin v - \cos v}{v \cos v} dv = 2 \frac{dx}{x}$$

Integrate both side:

$$\int \frac{v \sin v - \cos v}{v \cos v} dv = 2 \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{v \sin v}{v \cos v} dv - \int \frac{\cos v}{v \cos v} dv = 2 \int \frac{dx}{x}$$

$$\Rightarrow \int \tan v dv - \int \frac{1}{v} dv = 2 \log|x| + \log C$$

$$\Rightarrow \log|\sec v| - \log|v| = \log C|x|^2$$

$$\Rightarrow \log \frac{|\sec v|}{|v|} = \log C|x|^2$$

$$\Rightarrow \sec v = Cvx^2$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \sec \frac{y}{x} = C \left( \frac{y}{x} \right) x^2$$

$$\Rightarrow \cos \frac{y}{x} = \frac{k}{xy} \quad k = \frac{1}{C}$$

The solution of the given differential equation  $\cos \frac{y}{x} = \frac{k}{xy}$ .

### Question 8

Show that, differential equation  $x \frac{dy}{dx} - y + x \sin \left( \frac{y}{x} \right) = 0$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$

Checking for homogeneity:

$$F(x, y) = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda y - \lambda x \sin\left(\frac{\lambda y}{\lambda x}\right)}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda \left(y - x \sin\left(\frac{y}{x}\right)\right)}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{vx - x \sin\left(\frac{vx}{x}\right)}{x}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x(v - \sin v)}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \sin v$$

$$\Rightarrow x \frac{dv}{dx} = -\sin v$$

Separate the differentials:

$$\frac{1}{\sin v} dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \operatorname{cosec} v dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log|\operatorname{cosec} v - \cot v| = -\log x + \log C$$

$$\Rightarrow \log |\operatorname{cosec} v - \cot v| = \log \frac{C}{x}$$

$$\Rightarrow \operatorname{cosec} v - \cot v = \frac{C}{x}$$

$$\Rightarrow \frac{1}{\sin v} - \frac{\cos v}{\sin v} = \frac{C}{x}$$

$$\Rightarrow 1 - \cos v = \frac{C}{x} \sin v$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow 1 - \cos \frac{y}{x} = \frac{C}{x} \sin \frac{y}{x}$$

$$\Rightarrow x \left( 1 - \cos \frac{y}{x} \right) = C \sin \left( \frac{y}{x} \right)$$

The solution of the given differential equation  $x \left( 1 - \cos \frac{y}{x} \right) = C \sin \left( \frac{y}{x} \right)$ .

### Question 9

Show that, differential equation  $ydx + x \log \left( \frac{y}{x} \right) dy - 2xdy = 0$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$ydx = 2xdy - x \log \left( \frac{y}{x} \right) dy$$

$$\frac{dy}{dx} = \frac{y}{2x - x \log \left( \frac{y}{x} \right)}$$

Checking for homogeneity:

$$F(x, y) = \frac{y}{2x - x \log \left( \frac{y}{x} \right)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda y}{2\lambda x - \lambda x \log \left( \frac{\lambda y}{\lambda x} \right)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda \left( 2x - x \log \left( \frac{y}{x} \right) \right)}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{y}{2x - x \log \left( \frac{y}{x} \right)}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\frac{d(vx)}{dx} = \frac{vx}{2x - x \log\left(\frac{vx}{x}\right)}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{xv}{x(2-\log(v))}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v}{2-\log(v)}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{2-\log v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v-2v+v \log v}{2-\log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \log v - v}{2-\log v}$$

Separate the differentials:

$$\frac{2-\log v}{v \log v - v} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{2-\log v}{v \log v - v} dv = \int \frac{dx}{x}$$

$$\int \frac{1+1-\log v}{v(\log v - 1)} dv = \log x + \log C$$

$$\Rightarrow \int \frac{1}{v(\log v - 1)} dv + \int \frac{1-\log v}{v(\log v - 1)} dv = \log x + \log C$$

$$\Rightarrow \int \frac{1}{v(\log v - 1)} dv - \int \frac{1}{v} dv = \log Cx \dots\dots(1)$$

Solving :

$$\int \frac{1}{v(\log v - 1)} dv$$

Substitute  $\log v - 1 = t$ :

$$\log v - 1 = t$$

$$\Rightarrow \frac{1}{v} dv = dt$$

Thus the integral will be:

$$\Rightarrow \int \frac{1}{v(\log v - 1)} dv = \int \frac{dt}{t}$$

$$\Rightarrow \int \frac{1}{v(\log v - 1)} dv = \log t$$

$$\Rightarrow \int \frac{1}{v(\log v - 1)} dv = \log(\log v - 1)$$

Using above result for solving (1):

$$\Rightarrow \log(\log v - 1) - \log v = \log Cx$$

$$\Rightarrow \log \frac{\log v - 1}{v} = \log Cx$$

$$\Rightarrow \frac{\log v - 1}{v} = Cx$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{\log \frac{y}{x} - 1}{\frac{y}{x}} = Cx$$

$$\Rightarrow \log \frac{y}{x} - 1 = Cx \left( \frac{y}{x} \right)$$

$$\Rightarrow \log \frac{y}{x} - 1 = Cy$$

The solution of the given differential equation  $\log \frac{y}{x} - 1 = Cy$ .

### Question 10

Show that, differential equation  $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$  is homogenous and solves it.

### Solution

Ans: Rewrite the equation in standard form:

$$\left(1 + e^{\frac{x}{y}}\right) dx = -e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy$$

$$\frac{dx}{dy} = -\frac{e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$$

Checking for homogeneity:

$$F(x, y) = -\frac{e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{e^{\frac{\lambda x}{\lambda y}} \left(1 - \frac{\lambda x}{\lambda y}\right)}{\left(1 + e^{\frac{\lambda x}{\lambda y}}\right)}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{\left(1 + e^{\frac{x}{y}}\right)}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $x=vy$ :

$$\frac{d(vy)}{dy} = -\frac{e^{\frac{vy}{y}} \left(1 - \frac{vy}{y}\right)}{1 + e^{\frac{vy}{y}}}$$

$$\Rightarrow v \frac{dy}{dy} + y \frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v}$$

$$\Rightarrow v + y \frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v}$$

$$\Rightarrow y \frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v} - v$$

$$\Rightarrow y \frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1+e^v}$$

$$\Rightarrow y \frac{dv}{dy} = \frac{-(e^v + v)}{1+e^v}$$

Separate the differentials:

$$\frac{1+e^v}{e^v+v} dv = -\frac{dy}{y}$$

Integrate both side:

$$\int \frac{1+e^v}{e^v+v} dv = -\int \frac{dy}{y}$$

$$\int \frac{e^v + 1}{e^v + v} dv = -\log y + \log C$$

.....(1)

Solving the LHS integral. Substitute  $e^v + v = t$ :

$$e^v + v = t$$

$$\Rightarrow (e^v + 1) dv = dt$$

Solving the expression (1):

$$\Rightarrow \int \frac{1}{t} dt = \log \frac{C}{y}$$

$$\Rightarrow \log(t) = \log \frac{C}{y}$$

$$\Rightarrow \log(e^v + v) = \log \frac{C}{y}$$

$$\Rightarrow e^v + v = \frac{C}{y}$$

Back substitute  $v = \frac{x}{y}$ :

$$\Rightarrow e^{\frac{x}{y}} + \frac{x}{y} = \frac{C}{y}$$

$$\Rightarrow ye^{\frac{x}{y}} + x = C$$

The solution of the given differential equation

$$ye^{\frac{x}{y}} + x = C$$

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### Question 11

For the differential equation  $(x+y) dy + (x-y) dx = 0$ . Find the particular solution for the condition  $y=1$  when  $x=1$ .

### Solution

Ans: Given differential equation is:

$$(x + y) dy + (x - y) dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x-y}{x+y}$$

Checking for homogeneity:

$$F(x, y) = -\frac{x-y}{x+y}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{\lambda x - \lambda y}{\lambda x + \lambda y}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{\lambda(x-y)}{\lambda(x+y)}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{x-y}{x+y}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\frac{d(vx)}{dx} = -\frac{x-(vx)}{x+(vx)}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{x(v-1)}{x(v+1)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v-1-v^2-v}{v+1}$$

$$\Rightarrow x \frac{dv}{dx} = -\frac{1+v^2}{v+1}$$

Separate the differentials:

$$\frac{v+1}{1+v^2} dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \frac{v+1}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{v}{1+v^2} dv + \int \frac{1}{1+v^2} dv = -\log x + k$$

$$\Rightarrow \frac{1}{2} \log(1+v^2) + \tan^{-1} v + \log x = k$$

$$\Rightarrow \frac{1}{2} \log[x(1+v^2)] + \tan^{-1} v = k$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{1}{2} \log \left[ x \left( 1 + \frac{y^2}{x^2} \right) \right] + \tan^{-1} \frac{y}{x} = k$$

$$\Rightarrow \frac{1}{2} \log \left[ \frac{x^2+y^2}{x} \right] + \tan^{-1} \frac{y}{x} = k$$

Now  $y=1$  and  $x=1$ :

$$\Rightarrow \frac{1}{2} \log \left[ \frac{1^2+1^2}{1} \right] + \tan^{-1} \frac{1}{1} = k$$

$$k = \frac{1}{2} \log 2 + \frac{\pi}{4}$$

The required particular solution:

$$\Rightarrow \frac{1}{2} \log \left[ \frac{x^2+y^2}{x} \right] + \tan^{-1} \frac{y}{x} = \frac{1}{2} \log 2 + \frac{\pi}{4}$$

### Question 12

For the differential equation  $x^2 dy + (xy + y^2) dx = 0$ . Find the particular solution for the condition  $y=1$  when  $x=1$ .

## Solution

Ans: Given differential equation is:

$$x^2 dy + (xy + y^2) dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{xy+y^2}{x^2}$$

Checking for homogeneity:

$$F(x, y) = -\frac{xy+y^2}{x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{(\lambda x)(\lambda y) + \lambda^2 y^2}{\lambda^2 x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{\lambda^2(xy+y^2)}{\lambda^2 x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = -\frac{xy+y^2}{x^2}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\frac{d(vx)}{dx} = -\frac{x(vx)+(vx)^2}{x^2}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = -\frac{vx^2 + v^2 x^2}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = -\frac{x^2(v+v^2)}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = -v - v^2$$

$$\Rightarrow x \frac{dv}{dx} = -v^2 - 2v$$

Separate the differentials:

$$\frac{1}{v^2+2v} dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \frac{1}{v^2+2v} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \int \frac{v+2-v}{v(v+2)} dv = -\log x + \log C$$

$$\Rightarrow \frac{1}{2} \int \frac{v+2}{v(v+2)} dv - \frac{1}{2} \int \frac{v}{v(v+2)} dv = -\log x + \log C$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{v} dv - \frac{1}{2} \int \frac{1}{v+2} dv = -\log x + \log C$$

$$\Rightarrow \frac{1}{2} \log v - \frac{1}{2} \log(v+2) = \log \frac{C}{x}$$

$$\Rightarrow \frac{1}{2} \log \frac{v}{v+2} = \log \frac{C}{x}$$

$$\Rightarrow \frac{v}{v+2} = \left( \frac{C}{x} \right)^2$$

$$\Rightarrow \frac{v}{v+2} = \frac{C^2}{x^2}$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \frac{\frac{y}{x}}{\frac{y}{x} + 2} = \frac{C^2}{x^2}$$

$$\Rightarrow \frac{x^2 y}{y + 2x} = C^2$$

Now  $y=1$  and  $x=1$ :

$$\Rightarrow \frac{1^2(1)}{1+2(1)} = C^2$$

$$\Rightarrow C^2 = \frac{1}{3}$$

The required particular solution:

$$\Rightarrow \frac{x^2 y}{y + 2x} = \frac{1}{3}$$

$$\Rightarrow y + 2x = 3x^2 y$$

### Question 13

For the differential equation  $\left[ x \sin^2 \left( \frac{x}{y} \right) - y \right] dx + x dy = 0$ . Find the particular solution for the condition  $y = \frac{\pi}{4}$  when  $x = 1$ .

### Solution

Ans: Given differential equation is:

$$\left[ x \sin^2 \left( \frac{x}{y} \right) - y \right] dx + x dy = 0$$

$$\Rightarrow \frac{dy}{dx} = - \frac{\left[ x \sin^2 \left( \frac{x}{y} \right) - y \right]}{x}$$

Checking for homogeneity:

$$F(x, y) = - \frac{x \sin^2 \left( \frac{x}{y} \right) - y}{x}$$

$$\Rightarrow F(\lambda x, \lambda y) = - \frac{\lambda x \sin^2 \left( \frac{\lambda x}{\lambda y} \right) - \lambda y}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = - \frac{\lambda \left( x \sin^2 \left( \frac{y}{x} \right) - y \right)}{\lambda x}$$

$$\Rightarrow F(\lambda x, \lambda y) = - \frac{x \sin^2 \left( \frac{y}{x} \right) - y}{x}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = - \frac{x \sin^2 \left( \frac{vx}{x} \right) - vx}{x}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{-x \sin^2(v) + vx}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = -\sin^2 v + v$$

$$\Rightarrow x \frac{dv}{dx} = -\sin^2 v$$

Separate the differentials:

$$\operatorname{cosec}^2 v dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \operatorname{cosec}^2 v dv = -\int \frac{dx}{x}$$

$$\Rightarrow -\cot v = -\log x - \log C$$

$$\Rightarrow \cot v = \log Cx$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \cot \frac{y}{x} = \log Cx$$

Now  $y = \frac{\pi}{4}$  and  $x = 1$ :

$$\cot \frac{\pi}{4} = \log C (1)$$

$$\Rightarrow \log C = \cot \frac{\pi}{4}$$

$$\Rightarrow \log C = 1$$

$$\Rightarrow C = e$$

The required particular solution:

$$\Rightarrow \cot \frac{y}{x} = \log |ex|$$

### Question 14

For the differential equation  $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$ . Find the particular solution for the condition  $y=0$  when  $x=1$ .

### Solution

Ans: Given differential equation is:

$$\begin{aligned}\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) &= 0 \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)\end{aligned}$$

Checking for homogeneity:

$$\begin{aligned}F(x, y) &= \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right) \\ \Rightarrow F(\lambda x, \lambda y) &= \frac{\lambda y}{\lambda x} - \operatorname{cosec}\left(\frac{\lambda y}{\lambda x}\right) \\ \Rightarrow F(\lambda x, \lambda y) &= \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right) \\ \Rightarrow F(x, y) &= F(\lambda x, \lambda y)\end{aligned}$$

Thus it is an homogenous equation.

Let  $y=vx$ :

$$\begin{aligned}\frac{d(vx)}{dx} &= v - \operatorname{cosec}(v) \\ \Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} &= v - \operatorname{cosec}(v)\end{aligned}$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \operatorname{cosec}(v)$$

$$\Rightarrow x \frac{dv}{dx} = -\operatorname{cosec}(v)$$

Separate the differentials:

$$\sin v dv = -\frac{dx}{x}$$

Integrate both side:

$$\int \sin v dv = - \int \frac{dx}{x}$$

$$\Rightarrow -\cos v = -\log|x| - \log C$$

$$\cos v = \log|Cx|$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow \cos \frac{y}{x} = \log|Cx|$$

Now  $y=0$  and  $x=1$ :

$$\Rightarrow \cos \frac{0}{1} = \log|C1|$$

$$\Rightarrow \log C = 1$$

$$\Rightarrow C = e$$

The required particular solution:

$$\Rightarrow \cos \frac{y}{x} = \log|ex|$$

.

### Question 15

For the differential equation  $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$ . Find the particular solution for the condition  $y=2$  when  $x=1$ .

### Solution

Ans: Given differential equation is:

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

Checking for homogeneity:

$$F(x, y) = \frac{2xy + y^2}{2x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{2(\lambda x)(\lambda y) + \lambda^2 y^2}{2\lambda^2 x^2}$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{2xy + y^2}{2x^2}$$

$$\Rightarrow F(x, y) = F(\lambda x, \lambda y)$$

Thus it is an homogenous equation.

Let  $y = vx$ :

$$\frac{d(vx)}{dx} = \frac{2x(vx) + (vx)^2}{2x^2}$$

$$\Rightarrow v \frac{dx}{dx} + x \frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \frac{v^2}{2}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^2}{2}$$

Separate the differentials:

$$\frac{dv}{v^2} = \frac{1}{2} \left( \frac{dx}{x} \right)$$

Integrate both side:

$$2 \int \frac{dv}{v^2} = \int \left( \frac{dx}{x} \right)$$

$$\Rightarrow \frac{v^{-2+1}}{-2+1} = \log|x| + C$$

$$-\frac{2}{v} = \log|x| + C$$

Back substitute  $v = \frac{y}{x}$ :

$$\Rightarrow -\frac{2x}{y} = \log|x| + C$$

Now  $y=2$  and  $x=1$ :

$$\Rightarrow -\frac{2(1)}{2} = \log|1| + C$$

$$\Rightarrow C = -1$$

The required particular solution:

$$\Rightarrow -\frac{2x}{y} = \log|x| - 1.$$

$$\Rightarrow y = \frac{2x}{1 - \log|x|} \quad (x \neq 0, e)$$

---

### Question 16

What substitution should be used for solving homogeneous differential equation  $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ .

**A.**

$$y = vx$$

**B.**

$$v = yx$$

**C.**

$$x = vy$$

**D.**

$$x = v$$

**Solution**

Ans: The required substitution will be:

$$\frac{x}{y} = v$$

$$\Rightarrow x = vy$$

The correct answer is (C).

**Question 17**

Which of the following equation is homogeneous:

1.  $(4x+6y+5) dy - (3y+2x+4) dx=0$

2.  $(xy) dx - (x^3+y^3) dy=0$

3.  $(x^3+2y^2) dx + 2xydy=0$

4.  $y^2dx + (x^2-xy-y^2) dy=0$

**Solution**

Ans: For option (A):

$$F(x, y) = \frac{3y+2x+4}{4x+6y+5}$$

$$F(\lambda x, \lambda y) = \frac{3\lambda y+2\lambda x+4}{4\lambda x+6\lambda y+5}$$

$$F(\lambda x, \lambda y) \neq F(x, y)$$

For option (B):

$$F(x, y) = \frac{xy}{x^3+y^3}$$

$$F(\lambda x, \lambda y) = \frac{(\lambda x)(\lambda y)}{(\lambda x)^3 + (\lambda y)^3}$$

$$F(\lambda x, \lambda y) = \frac{xy}{\lambda(x+y)}$$

$$F(\lambda x, \lambda y) \neq F(x, y)$$

For option (C):

$$F(x, y) = -\frac{x^3 + 2y^2}{2xy}$$

$$F(\lambda x, \lambda y) = -\frac{\lambda^3 x^3 + 2\lambda^2 y^2}{2(\lambda x)(\lambda y)}$$

$$F(\lambda x, \lambda y) = -\frac{\lambda x^3 + 2y^2}{2xy}$$

$$F(\lambda x, \lambda y) \neq F(x, y)$$

For option (D):

$$F(x, y) = -\frac{y^2}{x^2 - xy - y^2}$$

$$F(\lambda x, \lambda y) = -\frac{\lambda^2 y^2}{\lambda^2 x^2 - (\lambda x)(\lambda y) - \lambda^2 y^2}$$

$$F(\lambda x, \lambda y) = -\frac{y^2}{x^2 - xy - y^2}$$

$$F(\lambda x, \lambda y) = F(x, y)$$

Thus the correct answer is option (D).

## Exercise 9.5

### Question 1

Find the general solution for the differential equation  $\frac{dy}{dx} + 2y = \sin x$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + 2y = \sin x$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = 2$$

$$Q = \sin x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int 2 dx}$$

$$\Rightarrow I.F = e^{2x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$ye^{2x} = \int \sin x (e^{2x}) dx + C$$

$$\Rightarrow ye^{2x} = I + C \quad \left( I = \int \sin x (e^{2x}) dx \right)$$

.....(1)

$$I = \int \sin x (e^{2x}) dx$$

$$\Rightarrow I = (\sin x) \int e^{2x} dx - \int ((\sin x)' \int e^{2x} dx) dx$$

$$\Rightarrow I = \frac{e^{2x}}{2} \sin x - \int \left( \cos x \left( \frac{e^{2x}}{2} \right) \right) dx$$

$$\Rightarrow I = \frac{e^{2x}}{2} \sin x - \frac{1}{2} \left[ \cos x \int e^{2x} dx - \int ((\cos x)' (\int e^{2x} dx)) dx \right]$$

$$\Rightarrow I = \frac{e^{2x}}{2} \sin x - \frac{1}{2} \left[ \frac{e^{2x}}{2} \cos x + \frac{1}{2} \int e^{2x} (\sin x) dx \right]$$

$$\Rightarrow I = \frac{e^{2x}}{2} \sin x - \frac{1}{2} \left[ \frac{e^{2x}}{2} \cos x + \frac{1}{2} I \right]$$

$$\Rightarrow I = \frac{e^{2x}}{2} \sin x - \frac{e^{2x}}{4} \cos x - \frac{1}{4} I$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x}}{2} \sin x - \frac{e^{2x}}{4} \cos x$$

$$\Rightarrow I = \frac{2e^{2x}}{5} \sin x - \frac{e^{2x}}{5} \cos x$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x]$$

Back substituting I in expression (1):

$$\Rightarrow ye^{2x} = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

$$\Rightarrow y = \frac{1}{5} (2 \sin x - \cos x) + Ce^{-2x}$$

The general solution for given differential equation is

$$y = \frac{1}{5} (2 \sin x - \cos x) + Ce^{-2x}$$

## Question 2

Find the general solution for the differential equation  $\frac{dy}{dx} + 3y = e^{-2x}$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + 3y = e^{-2x}$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = 3$$

$$Q = e^{-2x}$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int 3 dx}$$

$$\Rightarrow I.F = e^{3x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow ye^{3x} = \int e^{-2x} (e^{3x}) dx + C$$

$$\Rightarrow ye^{3x} = \int e^{-2x+3x} dx + C$$

$$\Rightarrow ye^{3x} = \int e^x dx + C$$

$$\Rightarrow ye^{3x} = e^x + C$$

$$\Rightarrow y = e^{-2x} + Ce^{-3x}$$

The general solution for given differential equation is  $y = e^{-2x} + Ce^{-3x}$ .

### Question 3

Find the general solution for the differential equation  $\frac{dy}{dx} + \frac{y}{x} = x^2$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \frac{1}{x}$$

$$Q = x^2$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \frac{1}{x} dx}$$

$$\Rightarrow I.F = e^{\log x}$$

$$\Rightarrow I.F = x$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow yx = \int x^2(x) dx + C$$

$$\Rightarrow xy = \int x^3 dx + C$$

$$\Rightarrow xy = \frac{x^{3+1}}{3+1} + C$$

$$\Rightarrow xy = \frac{x^4}{4} + C$$

The general solution for given differential equation is  $xy = \frac{x^4}{4} + C$ .

#### Question 4

Find the general solution for the differential equation  $\frac{dy}{dx} + (\sec x)y = \tan x$   $\left(0 \leq x \leq \frac{\pi}{2}\right)$ .

#### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + (\sec x)y = \tan x$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \sec x$$

$$Q = \tan x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \sec x dx}$$

$$\Rightarrow I.F = e^{\log(\sec x + \tan x)}$$

$$\Rightarrow I.F = (\sec x + \tan x)$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \int \tan x (\sec x + \tan x) dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \int \tan x \sec x dx + \int \tan^2 x dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \int (\sec^2 x - 1) dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \int \sec^2 x dx - \int dx + C$$

$$\Rightarrow y(\sec x + \tan x) = \sec x + \tan x - x + C$$

The general solution for given differential equation is  $y(\sec x + \tan x) = \sec x + \tan x - x + C$ .

### Question 5

Find the general solution for the differential equation  $\cos^2 x \frac{dy}{dx} + y = \tan x$   $\left(0 \leq x \leq \frac{\pi}{2}\right)$ .

### Solution

Ans: The given differential equation is:

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

$$\frac{dy}{dx} + (\sec^2 x) y = \sec^2 x \tan x$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \sec^2 x$$

$$Q = \sec^2 x \tan x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \sec^2 x dx}$$

$$\Rightarrow I.F = e^{\tan x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow ye^{\tan x} = \int e^{\tan x} (\sec^2 x \tan x) dx + C$$

$$\Rightarrow ye^{\tan x} = I + C \quad (I = \int e^{\tan x} (\sec^2 x \tan x) dx) \dots\dots(1)$$

Solving the integral I:

$$I = \int e^{\tan x} (\sec^2 x \tan x) dx$$

Substitute  $\tan x = t$  :

$$\tan x = t$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow I = \int e^t t dt$$

$$\Rightarrow I = t \int e^t dt - \int ((t)') \int e^t dt dt$$

$$\Rightarrow I = te^t - \int (e^t) dt$$

$$\Rightarrow I = te^t - e^t$$

Back substitute t:

$$I = \tan x e^{\tan x} - e^{\tan x}$$

Back substitute I in expression (1):

$$\Rightarrow ye^{\tan x} = I + C$$

$$\Rightarrow ye^{\tan x} = \tan x e^{\tan x} - e^{\tan x} + C$$

The general solution for given differential equation is  $ye^{\tan x} = \tan x e^{\tan x} - e^{\tan x} + C$ .

### Question 6

Find the general solution for the differential equation  $x \frac{dy}{dx} + 2y = x^2 \log x$ .

### Solution

Ans: The given differential equation is:

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

$$\frac{dy}{dx} + \frac{2}{x}y = x \log x$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \frac{2}{x}$$

$$Q = x \log x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \frac{2}{x} dx}$$

$$\Rightarrow I.F = e^{2 \log x}$$

$$\Rightarrow I.F = e^{\log x^2}$$

$$\Rightarrow I.F = x^2$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow yx^2 = \int x^2 (x \log x) dx + C$$

$$\Rightarrow yx^2 = \int x^3 \log x dx + C$$

$$\Rightarrow yx^2 = I + C \left( I = \int x^3 \log x dx \right)$$

.....(1)

Solving the integral I:

$$I = \int x^3 \log x dx$$

$$\Rightarrow I = \log x \int x^3 dx - \int \left( (\log x)' \int x^3 dx \right) dx$$

$$\Rightarrow I = \frac{x^4}{4} \log x - \int \left( \frac{1}{x} \left( \frac{x^4}{4} \right) \right) dx$$

$$\Rightarrow I = \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 dx$$

$$\Rightarrow I = \frac{x^4}{4} \log x - \frac{1}{4} \left( \frac{x^4}{4} \right)$$

$$\Rightarrow I = \frac{x^4}{4} \log x - \frac{x^4}{16}$$

Back substitute I in expression (1):

$$\Rightarrow yx^2 = I + C$$

$$\Rightarrow yx^2 = \frac{x^4}{4} \log x - \frac{x^4}{16} + C$$

$$\Rightarrow y = \frac{x^2}{16} (4 \log x - 1) + Cx^{-2}$$

The general solution for given differential equation is

$$y = \frac{x^2}{16} (4 \log x - 1) + Cx^{-2}$$

.

### Question 7

Find the general solution for the differential equation  $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$ .

### Solution

Ans: The given differential equation is:

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x^2}$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \frac{1}{x \log x}$$

$$Q = \frac{2}{x^2}$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \frac{1}{x \log x} dx}$$

Substitute  $\log x = t$ :

$$\log x = t$$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow I.F = e^{\int \frac{1}{t} dt}$$

$$\Rightarrow I.F = e^{\log t}$$

$$\Rightarrow I.F = t$$

$$\Rightarrow I.F = \log x$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y \log x = \int \frac{2}{x^2} (\log x) dx + C$$

$$\Rightarrow y \log x = I + C \quad \left( I = \int \frac{2}{x^2} (\log x) dx \right) \dots\dots(1)$$

Solving the integral I:

$$I = \int \frac{2}{x^2} (\log x) dx$$

$$I = 2 \left[ \log x \int \frac{1}{x^2} dx - \int \left( (\log x)' \int \frac{1}{x^2} dx \right) dx \right]$$

$$I = 2 \left[ \log x \left( \frac{-1}{x} \right) - \int \left( \frac{1}{x} \left( \frac{-1}{x} \right) \right) dx \right]$$

$$I = 2 \left[ -\frac{\log x}{x} + \int \left( \frac{1}{x^2} \right) dx \right]$$

$$I = 2 \left[ -\frac{\log x}{x} - \frac{1}{x} \right]$$

Back substitute I in expression (1):

$$y \log x = I + C$$

$$\Rightarrow y \log x = 2 \left[ -\frac{\log x}{x} - \frac{1}{x} \right] + C$$

$$\Rightarrow y \log x = -\frac{2}{x} (\log x + 1) + C$$

The general solution for given differential equation is

$$y \log x = -\frac{2}{x} (\log x + 1) + C$$

.

**Question 8**

Find the general solution for the differential equation  $(1+x^2) dy + 2xydx = \cot x dx$  ( $x \neq 0$ ).

**Solution**

Ans: The given differential equation is:

$$(1+x^2) dy + 2xydx = \cot x dx \quad (x \neq 0)$$

$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \frac{2x}{1+x^2}$$

$$Q = \frac{\cot x}{1+x^2}$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \frac{2x}{1+x^2} dx}$$

Substitute  $\log x = t$ :

$$1+x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow I.F = e^{\int \frac{1}{t} dt}$$

$$\Rightarrow I.F = e^{\log t}$$

$$\Rightarrow I.F = t$$

$$\Rightarrow I.F = 1+x^2$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y(1+x^2) = \int \frac{\cot x}{1+x^2} (1+x^2) dx + C$$

$$\Rightarrow y(1+x^2) = \int \cot x dx + C$$

$$\Rightarrow y(1+x^2) = \log |\sin x| + C$$

The general solution for given differential equation is  $y(1+x^2) = \log |\sin x| + C$ .

### Question 9

Find the general solution for the differential equation  $x \frac{dy}{dx} + y - x + xy \cot x = 0$  ( $x \neq 0$ ).

### Solution

Ans: The given differential equation is:

$$x \frac{dy}{dx} + y - x + xy \cot x = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} - 1 + y \cot x = 0$$

$$\Rightarrow \frac{dy}{dx} + \left( \frac{1}{x} + \cot x \right) y = 1$$

It is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \left( \frac{1}{x} + \cot x \right)$$

$$Q = 1$$

Calculate the integrating factor:

$$I. F = e^{\int p dx}$$

$$\Rightarrow I. F = e^{\int \left( \frac{1}{x} + \cot x \right) dx}$$

$$\Rightarrow I. F = e^{\int \frac{1}{x} dx + \int \cot x dx}$$

$$\Rightarrow I. F = e^{\log x + \log(\sin x)}$$

$$\Rightarrow I. F = e^{\log(x \sin x)}$$

$$\Rightarrow I. F = x \sin x$$

General solution is of the form:

$$y (I. F) = \int (Q \times I. F) dx + C$$

$$\Rightarrow y (x \sin x) = \int (1) (x \sin x) dx + C$$

$$\Rightarrow xy \sin x = I + C \quad (I = \int x \sin x dx) \dots\dots(1)$$

Solving the integral I:

$$I = \int x \sin x dx$$

$$\Rightarrow I = x \int \sin x dx - \int \left( (x)' \int \sin x dx \right) dx$$

$$\Rightarrow I = x (-\cos x) + \int (\cos x) dx$$

$$\Rightarrow I = -x \cos x + \sin x$$

Back substitute I in expression (1):

$$xy \sin x = I + C$$

$$\Rightarrow xy \sin x = -x \cos x + \sin x + C$$

$$\Rightarrow y = -\cot x + \frac{1}{x} + \frac{C}{x \sin x}$$

The general solution for given differential equation is

$$y = -\cot x + \frac{1}{x} + \frac{C}{x \sin x}$$

.

### Question 10

Find the general solution for the differential equation  $(x+y) \frac{dy}{dx} = 1$ .

### Solution

Ans: The given differential equation is:

$$(x+y) \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$$

$$\Rightarrow \frac{dx}{dy} = x + y$$

$$\Rightarrow \frac{dx}{dy} - x = y$$

It is differential equation of the form  $\frac{dx}{dy} + px = Q$ , with:

$$p = -1$$

$$Q = y$$

Calculate the integrating factor:

$$I.F = e^{\int p dy}$$

$$\Rightarrow I.F = e^{\int -1 dy}$$

$$\Rightarrow I.F = e^{-y}$$

General solution is of the form:

$$x(I.F) = \int (Q \times I.F) dy + C$$

$$\Rightarrow x(e^{-y}) = \int (y)(e^{-y}) dy + C$$

$$\Rightarrow xe^{-y} = I + C \quad (I = \int ye^{-y} dy) \dots\dots(1)$$

Solving the integral I:

$$I = \int ye^{-y} dy$$

$$\Rightarrow I = y \int e^{-y} dy - \int ((y)') \int e^{-y} dy dy$$

$$\Rightarrow I = -ye^{-y} + \int ((1) e^{-y}) dy$$

$$\Rightarrow I = -ye^{-y} + \int e^{-y} dy$$

$$\Rightarrow I = -ye^{-y} - e^{-y}$$

Back substitute I in expression (1):

$$xe^{-y} = I + C$$

$$\Rightarrow xe^{-y} = -ye^{-y} - e^{-y} + C$$

$$\Rightarrow x = -y - 1 + Ce^y$$

$$\Rightarrow x + y + 1 = Ce^y$$

The general solution for given differential equation is

$$x + y + 1 = Ce^y$$

.

### Question 11

Find the general solution for the differential equation  $ydx + (x - y^2) dy = 0$ .

### Solution

Ans: The given differential equation is:

$$ydx + (x - y^2) dy = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{y^2 - x}{y}$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{y}\right)x = y$$

It is differential equation of the form  $\frac{dx}{dy} + px = Q$ , with:

$$p = \frac{1}{y}$$

$$Q = y$$

Calculate the integrating factor:

$$I.F = e^{\int p dy}$$

$$\Rightarrow I.F = e^{\int \frac{1}{y} dy}$$

$$\Rightarrow I.F = e^{\log y}$$

$$\Rightarrow I.F = y$$

General solution is of the form:

$$x(I.F) = \int (Q \times I.F) dy + C$$

$$\Rightarrow x(y) = \int (y)(y) dy + C$$

$$\Rightarrow xy = \int y^2 dy + C$$

$$\Rightarrow xy = \frac{y^3}{3} + C$$

$$\Rightarrow x = \frac{y^2}{3} + \frac{C}{y}$$

The general solution for given differential equation is  $x = \frac{y^2}{3} + \frac{C}{y}$ .

### Question 12

Find the general solution for the differential equation  $(x+3y^2) \frac{dy}{dx} = y$  ( $y_0$ ).

### Solution

Ans: The given differential equation is:

$$(x + 3y^2) \frac{dy}{dx} = y$$

$$\Rightarrow \frac{dx}{dy} = \frac{x+3y^2}{y}$$

$$\Rightarrow \frac{dx}{dy} - \left(\frac{1}{y}\right)x = 3y$$

It is differential equation of the form  $\frac{dx}{dy} + px = Q$ , with:

$$p = -\frac{1}{y}$$

$$Q = 3y$$

Calculate the integrating factor:

$$I.F = e^{\int p dy}$$

$$\Rightarrow I.F = e^{-\int \frac{1}{y} dy}$$

$$\Rightarrow I.F = e^{-\log y}$$

$$\Rightarrow I.F = e^{\log y^{-1}}$$

$$\Rightarrow I.F = \frac{1}{y}$$

General solution is of the form:

$$x(I.F) = \int (Q \times I.F) dy + C$$

$$\Rightarrow x \left( \frac{1}{y} \right) = \int (3y) \left( \frac{1}{y} \right) dy + C$$

$$\Rightarrow \frac{x}{y} = 3 \int dy + C$$

$$\Rightarrow \frac{x}{y} = 3y + C$$

$$\Rightarrow x = 3y^2 + Cy$$

The general solution for given differential equation is  $x = 3y^2 + Cy$ .

### Question 13

Find particular solution for  $\frac{dy}{dx} + 2y \tan x = \sin x$  satisfying  $y=0$  when  $x = \frac{\pi}{3}$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + 2y \tan x = \sin x$$

$$\frac{dy}{dx} + (2 \tan x) y = \sin x$$

It is differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = 2 \tan x$$

$$Q = \sin x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{2 \int \tan x dx}$$

$$\Rightarrow I.F = e^{2 \log |\sec x|}$$

$$\Rightarrow I.F = e^{\log (\sec x)^2}$$

$$\Rightarrow I.F = \sec^2 x$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y \sec^2 x = \int (\sin x) (\sec^2 x) dx + C$$

$$\Rightarrow y \sec^2 x = \int \tan x \sec x dx + C$$

$$\Rightarrow y \sec^2 x = \sec x + C$$

$$\Rightarrow y = \cos x + C \cos^2 x$$

Given  $y=0$  when  $x=\frac{\pi}{3}$ :

$$0 = \cos\left(\frac{\pi}{3}\right) + C \cos^2\left(\frac{\pi}{3}\right)$$

$$\Rightarrow 0 = \frac{1}{2} + C\left(\frac{1}{2}\right)^2$$

$$\Rightarrow C = -2$$

Therefore the particular solution will be:

$$\Rightarrow y = \cos x - 2 \cos^2 x$$

The particular solution for given differential equation satisfying the given conditions is  $y = \cos x - 2 \cos^2 x$ .

**Question 14**

Find particular solution for  $(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$  satisfying  $y=0$  when  $x=1$ .

**Solution**

Ans: The given differential equation is:

$$(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

$$\frac{dy}{dx} + \left( \frac{2x}{1+x^2} \right) y = \frac{1}{(1+x^2)^2}$$

It is differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = \frac{2x}{1+x^2}$$

$$Q = \frac{1}{(1+x^2)^2}$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{\int \frac{2x}{1+x^2} dx}$$

$$\Rightarrow I.F = e^{2 \log |\sec x|}$$

$$\Rightarrow I.F = e^{\log(1+x^2)}$$

$$\Rightarrow I.F = 1 + x^2$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y(1+x^2) = \int \left( \frac{1}{(1+x^2)^2} \right) (1+x^2) dx + C$$

$$\Rightarrow y(1+x^2) = \int \frac{1}{1+x^2} dx + C$$

$$\Rightarrow y(1+x^2) = \tan^{-1} x + C$$

Given  $y=0$  when  $x=1$ :

$$0(1+1) = \tan^{-1}(1) + C$$

$$\Rightarrow C + \frac{\pi}{4} = 0$$

$$\Rightarrow C = -\frac{\pi}{4}$$

Therefore the particular solution will be:

$$\Rightarrow y(1+x^2) = \tan^{-1}x - \frac{\pi}{4}$$

The particular solution for given differential equation satisfying the given conditions is

$$y(1+x^2) = \tan^{-1}x - \frac{\pi}{4}.$$

### Question 15

Find particular solution for  $\frac{dy}{dx} - 3y \cot x = \sin 2x$  satisfying  $y=2$  when  $x=\frac{\pi}{2}$ .

### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} - 3y \cot x = \sin 2x$$

$$\frac{dy}{dx} + (-3 \cot x)y = \sin 2x$$

It is differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = -3 \cot x$$

$$Q = \sin 2x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{-3 \int \cot x dx}$$

$$\Rightarrow I.F = e^{-3 \log |\sin x|}$$

$$\Rightarrow I.F = e^{\log \left( \frac{1}{\sin^3 x} \right)}$$

$$\Rightarrow I.F = \frac{1}{\sin^3 x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y \left( \frac{1}{\sin^3 x} \right) = \int (\sin 2x) \left( \frac{1}{\sin^3 x} \right) dx + C$$

$$\Rightarrow y \left( \frac{1}{\sin^3 x} \right) = 2 \int (\sin x \cos x) \left( \frac{1}{\sin^3 x} \right) dx + C$$

$$\Rightarrow \frac{y}{\sin^3 x} = 2 \int \left( \frac{\cos x}{\sin^2 x} \right) dx + C$$

$$\Rightarrow \frac{y}{\sin^3 x} = 2 \int \cot x \operatorname{cosec} x dx + C$$

$$\Rightarrow \frac{y}{\sin^3 x} = -2 \operatorname{cosec} x + C$$

$$\Rightarrow y = -2 \sin^2 x + C \sin^3 x$$

Given  $y=2$  when  $x=\frac{\pi}{2}$ :

$$2 = -2 \sin^2 \left( \frac{\pi}{2} \right) + C \sin^3 \left( \frac{\pi}{2} \right)$$

$$\Rightarrow C - 2 = 2$$

$$\Rightarrow C = 4$$

Therefore the particular solution will be:

$$\Rightarrow y = -2 \sin^2 x + 4 \sin^3 x$$

The particular solution for given differential equation satisfying the given conditions is  $y = -2 \sin^2 x + 4 \sin^3 x$ .

**Question 16**

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point  $(x, y)$  is equal to the sum of the coordinates of the point.

**Solution**

Ans: According to the slope of tangent  $\frac{dy}{dx}$  is equal to sum of the coordinates:

$$\frac{dy}{dx} = x + y$$

The given differential equation is:

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} + (-1)y = x$$

It is differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = -1$$

$$Q = x$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{-1 \int dx}$$

$$\Rightarrow I.F = e^{-x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y(e^{-x}) = \int x(e^{-x}) dx + C$$

$$\Rightarrow ye^{-x} = x \int e^{-x} dx - \int ((x)') \int e^{-x} dx dx + C$$

$$\Rightarrow ye^{-x} = -xe^{-x} + \int (e^{-x}) dx + C$$

$$\Rightarrow ye^{-x} = -xe^{-x} - e^{-x} + C$$

$$\Rightarrow y = -x - 1 + Ce^x$$

$$\Rightarrow y + x + 1 = Ce^x$$

Given  $y=0$  when  $x=0$  as it passes through origin:

$$0 + 0 + 1 = Ce^0$$

$$\Rightarrow C = 1$$

Therefore the equation of the required curve is  $y+x+1=e^x$ .

---

### Question 17

Find the equation of a curve passing through the point  $(0,2)$  given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5 .

### Solution

Ans: Let the slope of tangent be  $\frac{dy}{dx}$ .

According to :

$$x + y = \frac{dy}{dx} + 5$$

The given differential equation is:

$$x + y = \frac{dy}{dx} + 5$$

$$\frac{dy}{dx} + (-1)y = x - 5$$

It is differential equation of the form  $\frac{dy}{dx} + py = Q$ , with:

$$p = -1$$

$$Q = x - 5$$

Calculate the integrating factor:

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{-1 \int dx}$$

$$\Rightarrow I.F = e^{-x}$$

General solution is of the form:

$$y(I.F) = \int (Q \times I.F) dx + C$$

$$\Rightarrow y(e^{-x}) = \int (x - 5)(e^{-x}) dx + C$$

$$\Rightarrow y(e^{-x}) = \int x(e^{-x}) dx - 5 \int e^{-x} dx + C$$

$$\Rightarrow ye^{-x} = x \int e^{-x} dx - \int ((x)') \int e^{-x} dx dx - 5 \int e^{-x} dx C$$

$$\Rightarrow ye^{-x} = -xe^{-x} + \int (e^{-x}) dx + 5e^{-x} + C$$

$$\Rightarrow ye^{-x} = -xe^{-x} - e^{-x} + 5e^{-x} + C$$

$$\Rightarrow ye^{-x} = -xe^{-x} + 4e^{-x} + C$$

$$\Rightarrow y = -x + 4 + Ce^x$$

$$\Rightarrow y + x - 4 = Ce^x$$

Given as it passes through (0,2) :

$$2 + 0 - 4 = Ce^0$$

$$\Rightarrow C = -2$$

Therefore the equation of the required curve is:

$$y + x - 4 = -2e^x.$$

### Question 18

Find the integrating factor of the differential equation  $x \frac{dy}{dx} - y = 2x^2$ .

A.

$$e^{-x}$$

**B.**

$$e^{-y}$$

**C.**

$$\frac{1}{x}$$

**D.**

$$x$$

**Solution**

Ans: Given differential equation is:

$$x \frac{dy}{dx} - y = 2x^2$$

$$\Rightarrow \frac{dy}{dx} - \left(\frac{1}{x}\right)y = 2x^2$$

Thus it is a linear differential equation of the form  $\frac{dy}{dx} + py = Q$ :

$$p = -\frac{1}{x}$$

$$I.F = e^{\int p dx}$$

$$\Rightarrow I.F = e^{-\int \frac{1}{x} dx}$$

$$\Rightarrow I.F = e^{-\log|x|}$$

$$\Rightarrow I.F = e^{\log x^{-1}}$$

$$\Rightarrow I.F = \frac{1}{x}$$

Therefore integrating factor is  $\frac{1}{x}$ . Thus the correct option is (C).

**Question 19**

Find the integrating factor of the differential equation  $(1-y^2) \frac{dx}{dy} + yx = ay$  ( $-1 < y < 1$ ).

**A.**

$$\frac{1}{y^2-1}$$

**B.**

$$\frac{1}{\sqrt{y^2-1}}$$

**C.**

$$\frac{1}{1-y^2}$$

**D.**

$$\frac{1}{\sqrt{1-y^2}}$$

**Solution**

Ans: Given differential equation is:

$$(1 - y^2) \frac{dx}{dy} + yx = ay$$

$$\Rightarrow \frac{dx}{dy} + \left( \frac{y}{1-y^2} \right) x = \frac{ay}{(1-y^2)}$$

Thus it is a linear differential equation of the form  $\frac{dx}{dy} + px = Q$ :

$$p = \frac{y}{1-y^2}$$

$$I.F = e^{\int p dy}$$

$$\Rightarrow I.F = e^{\int \frac{y}{1-y^2} dy}$$

$$\Rightarrow I.F = e^{\frac{1}{-2} \int \frac{-2y}{1-y^2} dy}$$

$$\Rightarrow I.F = e^{\frac{1}{-2} \log(1-y^2)}$$

$$\Rightarrow I.F = e^{\log(1-y^2)^{-\frac{1}{2}}}$$

$$\Rightarrow I.F = \frac{1}{\sqrt{1-y^2}}$$

Therefore integrating factor is  $\frac{1}{\sqrt{1-y^2}}$ . Thus the correct option is (D).

## Miscellaneous Exercise

### Question 1

For each of the differential equations given below, indicate its order and degree (if defined).

(i)  $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$

(ii)  $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$

(iii)  $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$

### Solution

Ans: The given differential equation is:

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y - \log x = 0$$

The highest order derivative in the equation is of the term  $\frac{d^2y}{dx^2}$ , thus the order of the equation is 2 and its highest power is 1. Therefore its degree is 1.

Ans: The given differential equation is:

$$\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y - \sin x = 0$$

The highest order derivative in the equation is of the term  $\left(\frac{dy}{dx}\right)^3$ , thus the order of the equation is 1 and its highest power is 3. Therefore its degree is 3.

Ans: The given differential equation is:

$$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$$

The highest order derivative in the equation is of the term  $\frac{d^4y}{dx^4}$ , thus the order of the equation is 4.

As the differential equation is not polynomial in its derivative, therefore its degree is not defined.

## Question 2

For each of the exercises given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

$$1. xy = ae^x + be^{-x} + x^2 : x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy + x^2 - 2 = 0$$

## Solution

Ans: The given function is:

$$xy = ae^x + be^{-x} + x^2$$

Take derivative on both side:

$$\Rightarrow y + x \frac{dy}{dx} = ae^x - be^{-x} + 2x$$

Take derivative on both side:

$$\Rightarrow \frac{dy}{dx} + x \frac{d^2y}{dx^2} + \frac{dy}{dx} = ae^x + be^{-x} + 2$$

$$\Rightarrow x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = ae^x + be^{-x} + 2 \dots\dots(1)$$

The given differential equation is:

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy + x^2 - 2 = 0$$

Solving LHS:

Substitute  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx}$  from the result (1) and  $xy$ :

$$\Rightarrow \left( x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \right) - xy + x^2 - 2$$

$$\Rightarrow (ae^x + be^{-x} + 2) - (ae^x + be^{-x} + x^2) + x^2 - 2$$

$$\Rightarrow 2 - x^2 + x^2 - 2$$

$$\Rightarrow 0$$

Thus LHS=RHS, the given function is the solution of the given differential equation.

$$1. y = e^x (a \cos x + b \sin x) : \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

Ans: The given function is:

$$y = e^x (a \cos x + b \sin x)$$

Take derivative on both side:

$$\Rightarrow \frac{dy}{dx} = e^x (a \cos x + b \sin x) + e^x (-a \sin x + b \cos x)$$

$$\Rightarrow \frac{dy}{dx} = e^x ((a + b) \cos x + (b - a) \sin x)$$

Take derivative on both side:

$$\Rightarrow \frac{d^2 y}{dx^2} = e^x ((a + b) \cos x + (b - a) \sin x) + e^x (-(a + b) \sin x + (b - a) \cos x)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = e^x ((a + b + b - a) \cos x + (b - a - a - b) \sin x)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = e^x (2b \cos x - 2a \sin x)$$

The given differential equation is:

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$$

Solving LHS:

$$\Rightarrow e^x (2b \cos x - 2a \sin x) - 2e^x ((a + b) \cos x + (b - a) \sin x) + 2y$$

$$\Rightarrow e^x ((2b - 2a - 2b) \cos x + (-2a - 2b + 2a) \sin x) - 2y$$

$$\Rightarrow e^x (-2a \cos x - 2b \sin x) - 2y$$

$$\Rightarrow -2e^x (a \cos x + b \sin x) - 2y$$

$$\Rightarrow 0$$

Thus LHS=RHS, the given function is the solution of the given differential equation.

$$1. y = x \sin 3x : \frac{d^2 y}{dx^2} + 9y - 6 \cos 3x = 0$$

Ans: The given function is:

$$y = x \sin 3x$$

Take derivative on both side:

$$\Rightarrow \frac{dy}{dx} = \sin 3x + 3x \cos 3x$$

Take derivative on both side:

$$\begin{aligned}\Rightarrow \frac{d^2y}{dx^2} &= 3 \cos 3x + 3 (\cos 3x + x (-3 \sin 3x)) \\ \Rightarrow \frac{d^2y}{dx^2} &= 3 \cos 3x + 3 \cos 3x - 9x \sin 3x\end{aligned}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 6 \cos 3x - 9x \sin 3x$$

The given differential equation is:

$$\frac{d^2y}{dx^2} + 9y - 6 \cos 3x = 0$$

Solving LHS:

$$\begin{aligned}\Rightarrow \frac{d^2y}{dx^2} + 9y - 6 \cos 3x \\ \Rightarrow (6 \cos 3x - 9x \sin 3x) + 9(x \sin 3x) - 6 \cos 3x \\ \Rightarrow 6 \cos 3x - 9x \sin 3x + 9x \sin 3x - 6 \cos 3x \\ \Rightarrow 0\end{aligned}$$

Thus LHS=RHS, the given function is the solution of the given differential equation.

$$1. x^2 = 2y^2 \log y : (x^2 + y^2) \frac{dy}{dx} - xy = 0$$

Ans: The given function is:

$$x^2 = 2y^2 \log y$$

Take derivative on both side:

$$\Rightarrow 2x = 2 \left( 2y \log y + y^2 \left( \frac{1}{y} \right) \right) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{(2y \log y + y)}$$

Multiply numerator and denominator by y:

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{(2y^2 \log y + y^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{(x^2 + y^2)}$$

The given differential equation is:

$$(x^2 + y^2) \frac{dy}{dx} - xy = 0$$

Solving LHS:

$$\Rightarrow (x^2 + y^2) \frac{dy}{dx} - xy$$

$$\Rightarrow (x^2 + y^2) \left( \frac{xy}{x^2 + y^2} \right) - xy$$

$$\Rightarrow xy - xy$$

$$\Rightarrow 0$$

Thus LHS=RHS, the given function is the solution of the given differential equation.

### Question 3

**Prove that  $x^2 - y^2 = c(x^2 + y^2)^2$  is the general solution of differential equation  $(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$ , where c is a parameter.**

### Solution

Ans: Given differential equation:

$$(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

As it can be seen that this is an homogenous equation. Substitute  $y=vx$ :

$$\Rightarrow \frac{d(vx)}{dx} = \frac{x^3 - 3x(vx)^2}{(vx)^3 - 3x^2(vx)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{x^3(1-3v^2)}{x^3(v^3-3v)}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1-3v^2}{v^3-3v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1-3v^2-v^4+3v^2}{v^3-3v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1-v^4}{v^3-3v}$$

Separate the differentials:

$$\frac{v^3-3v}{1-v^4} dv = \frac{dx}{x}$$

Integrate both side:

$$\int \frac{v^3-3v}{1-v^4} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int \frac{v^3-3v}{1-v^4} dv = \log x + \log C$$

$$\Rightarrow I = \log x + \log C \quad \left( I = \int \frac{v^3-3v}{1-v^4} dv \right)$$

.....(1)

Solving integral I:

$$\Rightarrow I = \int \frac{v^3-3v}{1-v^4} dv$$

$$\Rightarrow \frac{v^3-3v}{1-v^4} = \frac{v^3-3v}{(1-v^2)(1+v^2)}$$

$$\Rightarrow \frac{v^3 - 3v}{1 - v^4} = \frac{v^3 - 3v}{(1 - v)(1 + v)(1 + v^2)}$$

Using partial fraction:

$$\Rightarrow \frac{v^3 - 3v}{1 - v^4} = \frac{A}{1 - v} + \frac{B}{1 + v} + \frac{Cv + D}{1 + v^2}$$

Solving for A,B,C and D:

$$A = -\frac{1}{2}$$

$$B = \frac{1}{2}$$

$$C = -2$$

$$D = 0$$

$$\Rightarrow \frac{v^3 - 3v}{1 - v^4} = \frac{-\frac{1}{2}}{1 - v} + \frac{\frac{1}{2}}{1 + v} + \frac{-2v + 0}{1 + v^2}$$

$$I = -\frac{1}{2} \int \frac{1}{1-v} dv + \frac{1}{2} \int \frac{1}{1+v} dv - \int \frac{2v}{1+v^2} dv$$

$$I = -\frac{1}{2}(-\log(1 - v)) + \frac{1}{2}(\log(1 + v)) - \log(1 + v^2)$$

$$I = \frac{1}{2}(\log(1 - v^2)) - \frac{2}{2}\log(1 + v^2)$$

$$I = \frac{1}{2} \left( \log \frac{(1 - v^2)}{(1 + v^2)^2} \right)$$

$$\Rightarrow I = \frac{1}{2} \left( \log \frac{\left(1 - \frac{y^2}{x^2}\right)}{\left(1 + \frac{y^2}{x^2}\right)^2} \right)$$

$$\Rightarrow I = \frac{1}{2} \left( \log \frac{x^2 (x^2 - y^2)}{(x^2 + y^2)^2} \right)$$

$$\Rightarrow I = \frac{1}{2} \left( \log \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \right) + \frac{1}{2} \log x^2$$

$$\Rightarrow I = \frac{1}{2} \left( \log \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \right) + \log x$$

Back substitute I in expression (1):

$$\Rightarrow I = \log x + \log C$$

$$\Rightarrow \frac{1}{2} \left( \log \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \right) + \log x = \log x + \log C$$

$$\Rightarrow \log \frac{(x^2 - y^2)}{(x^2 + y^2)^2} = 2 \log C$$

$$\Rightarrow \frac{x^2 - y^2}{(x^2 + y^2)^2} = C^2$$

$$\Rightarrow x^2 - y^2 = c(x^2 + y^2)^2 \quad (c = C^2)$$

Thus for given differential equation, its general solution is

$$x^2 - y^2 = c(x^2 + y^2)^2$$

#### Question 4

Find the general solution of the differential equation  $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ .

#### Solution

Ans: The given differential equation is:

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

Integrate both side:

$$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \sin^{-1}y = -\sin^{-1}x + C$$

$$\Rightarrow \sin^{-1}y + \sin^{-1}x = C$$

Thus the general solution of given differential equation is  $\sin^{-1}y + \sin^{-1}x = C$ .

#### Question 5

Show that the general solution of the differential equation  $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$  is given by  $(x+y+1) = A(1-x-y-2xy)$  where A is a parameter.

**Solution**

Ans: The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} &= 0 \\ \Rightarrow \frac{dy}{dx} &= -\frac{y^2+y+1}{x^2+x+1} \\ \Rightarrow \frac{dy}{dx} &= -\frac{y^2+2\left(\frac{1}{2}\right)y+\frac{1}{4}-\frac{1}{4}+1}{x^2+2\left(\frac{1}{2}\right)x+\frac{1}{4}-\frac{1}{4}+1} \\ \Rightarrow \frac{dy}{dx} &= -\frac{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} \\ \Rightarrow \frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}} &= -\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\end{aligned}$$

Integrate both side:

$$\begin{aligned}\Rightarrow \int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} &= -\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} \\ \Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left[ \frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right] &= -\frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left[ \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right] + C \\ \Rightarrow \frac{2}{\sqrt{3}} \left( \tan^{-1} \left[ \frac{2y+1}{\sqrt{3}} \right] + \tan^{-1} \left[ \frac{2x+1}{\sqrt{3}} \right] \right) &= C \\ \Rightarrow \tan^{-1} \left[ \frac{2y+1}{\sqrt{3}} \right] + \tan^{-1} \left[ \frac{2x+1}{\sqrt{3}} \right] &= \frac{\sqrt{3}}{2} C\end{aligned}$$

Thus the general solution for given differential equation is  $\tan^{-1} \left[ \frac{2y+1}{\sqrt{3}} \right] + \tan^{-1} \left[ \frac{2x+1}{\sqrt{3}} \right] = \frac{\sqrt{3}}{2} C$ .

**Question 6**

Find the equation of the curve passing through the point  $\left(0, \frac{\pi}{4}\right)$  whose differential equation is  $\sin x \cos y dx + \cos x \sin y dy = 0$ .

**Solution**

Ans: Given differential equation is:

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

$$\Rightarrow \sin x \cos y dx + \cos x \sin y dy = 0$$

Divide both side by  $\cos x \cos y$ :

$$\Rightarrow \frac{\sin x \cos y dx + \cos x \sin y dy}{\cos x \cos y} = 0$$

$$\Rightarrow \tan x dx + \tan y dy = 0$$

$$\Rightarrow \tan y dy = -\tan x dx$$

Integrate both side:

$$\Rightarrow \int \tan y dy = -\int \tan x dx$$

$$\Rightarrow \log(\sec y) = -\log(\sec x) + C$$

$$\Rightarrow \log(\sec y) + \log(\sec x) = C$$

$$\Rightarrow \log(\sec x \sec y) = C$$

$$\Rightarrow \sec x \sec y = k \quad (k = e^C)$$

As curve passes through  $\left(0, \frac{\pi}{4}\right)$ :

$$\sec 0 \sec\left(\frac{\pi}{4}\right) = k$$

$$\Rightarrow k = \sqrt{2}$$

$$\Rightarrow \sec x \sec y = \sqrt{2}$$

Thus the equation of required curve is  $\sec x \sec y = \sqrt{2}$ .

### Question 7

Find the particular solution of the differential equation  $(1+e^{2x}) dy + (1+y^2) e^x dx = 0$  given that  $y=1$  when  $x=0$ .

### Solution

Ans: The given differential equation is:

$$(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0$$

Divide both side  $(1+e^{2x}) (1+y^2)$ :

$$\frac{dy}{(1+y^2)} + \frac{e^x}{(1+e^{2x})} dx = 0$$

$$\int \frac{dy}{(1+y^2)} = - \int \frac{e^x}{(1+e^{2x})} dx$$

$$\tan^{-1}y = - \int \frac{e^x}{(1+(e^x)^2)} dx$$

Substitute  $t = e^x$ :

$$dt = e^x dx$$

$$\Rightarrow \tan^{-1}y = - \int \frac{1}{(1+t^2)} dt$$

$$\Rightarrow \tan^{-1}y = -\tan^{-1}t + C$$

$$\Rightarrow \tan^{-1}y = -\tan^{-1}e^x + C$$

$$\Rightarrow \tan^{-1}y + \tan^{-1}e^x = C$$

As  $y=1$  when  $x=0$ :

$$\tan^{-1}(1) + \tan^{-1}(e^0) = C$$

$$\Rightarrow \frac{\pi}{4} + \frac{\pi}{4} = C$$

$$\Rightarrow C = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}y + \tan^{-1}e^x = \frac{\pi}{2}$$

Thus the required particular solution is  $\tan^{-1}y + \tan^{-1}e^x = \frac{\pi}{2}$ .

### Question 8

Solve the differential equation  $ye^{\frac{x}{y}} dx = (xe^{\frac{x}{y}} + y^2) dy$  ( $y \neq 0$ ).

### Solution

Ans: The given differential equation is:

$$ye^{\frac{x}{y}} dx = (xe^{\frac{x}{y}} + y^2) dy$$

$$ye^{\frac{x}{y}} \frac{dx}{dy} = xe^{\frac{x}{y}} + y^2$$

$$\Rightarrow ye^{\frac{x}{y}} \frac{dx}{dy} - xe^{\frac{x}{y}} = y^2$$

$$\Rightarrow e^{\frac{x}{y}} \frac{[y \frac{dx}{dy} - x]}{y^2} = 1$$

Substitute  $z = e^{\frac{x}{y}}$ :

$$z = e^{\frac{x}{y}}$$

$$\frac{d}{dy} z = \frac{d}{dy} e^{\frac{x}{y}}$$

$$\Rightarrow \frac{dz}{dy} = \frac{d}{dy} \left( e^{\frac{x}{y}} \right)$$

$$\Rightarrow \frac{dz}{dy} = e^{\frac{x}{y}} \frac{d}{dy} \left( \frac{x}{y} \right)$$

$$\Rightarrow \frac{dz}{dy} = e^{\frac{x}{y}} \left[ \left( \frac{1}{y} \right) \frac{dx}{dy} - \frac{x}{y^2} \right]$$

$$\Rightarrow \frac{dz}{dy} = e^{\frac{x}{y}} \left[ \frac{y \frac{dx}{dy} - x}{y^2} \right]$$

$$\Rightarrow \frac{dz}{dy} = 1$$

$$\Rightarrow dz = dy$$

$$\Rightarrow \int dz = \int dy$$

$$\Rightarrow z = y + C$$

$$\Rightarrow e^{\frac{x}{y}} = y + C$$

Thus the required general solution is

$$e^{\frac{x}{y}} = y + C$$

### Question 9

Find a particular solution of the differential equation  $(x-y)(dx+dy) = dx-dy$  given that  $y=-1$  when  $x=0$ . Hint (put  $x-y=t$ ).

### Solution

Ans: Given differential equation is:

$$(x-y)(dx+dy) = dx-dy$$

$$\Rightarrow (x-y)dx - dx = (y-x)dy - dy$$

$$\Rightarrow (x-y+1)dy = (1-x+y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$$

Put  $x-y=t$ :

$$x-y=t$$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 1 - \frac{dt}{dx} = \frac{dy}{dx}$$

$$\Rightarrow 1 - \frac{dt}{dx} = \frac{1-t}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = 1 - \frac{1-t}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = \frac{1+t-1+t}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$$

$$\Rightarrow \frac{1+t}{t} dt = 2dx$$

Integrate both side:

$$\Rightarrow \int \frac{1+t}{t} dt = 2 \int dx$$

$$\Rightarrow \int \frac{1}{t} dt + \int dt = 2x + C$$

$$\Rightarrow \log|t| + t = 2x + C$$

$$\Rightarrow \log|x - y| + x - y = 2x + C$$

$$\Rightarrow \log|x - y| - y = x + C$$

As  $y = -1$  when  $x = 0$ :

$$\Rightarrow \log|0 - (-1)| - (-1) = 0 + C$$

$$\Rightarrow \log 1 + 1 = C$$

$$\Rightarrow C = 1$$

Thus the required particular solution is:

$$\log|x - y| - y = x + 1.$$

### Question 10

Solve the differential equation  $\left[ \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right] \frac{dx}{dy} = 1$  ( $x \neq 0$ ).

### Solution

Ans: Given differential equation is:

$$\left[ \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right] \frac{dx}{dy} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

It is linear differential equation of the form  $\frac{dy}{dx} + py = Q$ :

$$p = \frac{1}{\sqrt{x}}$$

$$Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

Calculating integrating factor:

$$I.F = e^{\int p dx}$$

$$I.F = e^{\int \frac{1}{\sqrt{x}} dx}$$

$$I.F = e^{2\sqrt{x}}$$

The general solution is given by:

$$y \times I.F = \int (Q \times I.F) dx + C$$

$$y \times (e^{2\sqrt{x}}) = \int \left( \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}} \right) dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = \int \left( \frac{e^{-2\sqrt{x}+2\sqrt{x}}}{\sqrt{x}} \right) dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}} dx + C$$

$$\Rightarrow ye^{2\sqrt{x}} = 2\sqrt{x} + C$$

Thus the general solution for the given differential equation is

$$ye^{2\sqrt{x}} = 2\sqrt{x} + C.$$

### Question 11

Find a particular solution of the differential equation  $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$  ( $x \neq 0$ ) given that  $y=0$  when  $x=\frac{\pi}{2}$ .

### Solution

Ans:

The given differential equation is:

$$\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$$

It is linear differential equation of the form  $\frac{dy}{dx} + py = Q$ :

$$p = \cot x$$

$$Q = 4x \operatorname{cosec} x$$

Calculating integrating factor:

$$I.F = e^{\int p dx}$$

$$I.F = e^{\int \cot x dx}$$

$$I.F = e^{\log |\sin x|}$$

$$I.F = \sin x$$

The general solution is given by:

$$y \times I.F = \int (Q \times I.F) dx + C$$

$$\Rightarrow y \times \sin x = \int (4x \operatorname{cosec} x) \sin x dx + C$$

$$\Rightarrow y \sin x = 4 \int x dx + C$$

$$\Rightarrow y \sin x = 4 \left( \frac{x^2}{2} \right) + C$$

$$\Rightarrow y \sin x = 2x^2 + C$$

As  $y=0$  when  $x=\frac{\pi}{2}$ :

$$0 \times \sin\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right)^2 + C$$

$$C = -2\left(\frac{\pi^2}{4}\right)$$

$$C = -\frac{\pi^2}{2}$$

Thus the required particular solution is:

$$y \sin x = 2x^2 - \frac{\pi^2}{2}$$

### Question 12

Find a particular solution of the differential equation  $(x+1) \frac{dy}{dx} = 2e^{-y} - 1$  given that  $y=0$  when  $x=0$ .

### Solution

Ans:

The given differential equation is:

$$(x+1) \frac{dy}{dx} = 2e^{-y} - 1$$

$$\Rightarrow \frac{dy}{2e^{-y}-1} = \frac{dx}{x+1}$$

Integrate both side:

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \int \frac{dx}{x+1}$$

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \log(x+1) + \log C \dots\dots(1)$$

Evaluating LHS integral:

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \int \frac{e^y dy}{2-e^y}$$

Put  $t=2-e^y$ :

$$t = 2 - e^y$$

$$dt = -e^y dy$$

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = - \int \frac{dt}{t}$$

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = -\log(t)$$

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \log \frac{1}{t}$$

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \log \frac{1}{2-e^y}$$

Back substituting in expression (1):

$$\Rightarrow \int \frac{dy}{2e^{-y}-1} = \log(x+1) + \log C$$

$$\Rightarrow \log\left(\frac{1}{2-e^y}\right) = \log C (x+1)$$

$$\Rightarrow 2 - e^y = \frac{1}{C(x+1)}$$

As  $y=0$  when  $x=0$ :

$$\Rightarrow 2 - e^0 = \frac{1}{C(0+1)}$$

$$\Rightarrow 2 - 1 = \frac{1}{C}$$

$$\Rightarrow C = 1$$

Thus the required particular solution is:

$$\Rightarrow 2 - e^y = \frac{1}{(x+1)}$$

$$\Rightarrow e^y = 2 - \frac{1}{(x+1)}$$

$$\Rightarrow e^y = \frac{2x+2-1}{(x+1)}$$

$$\Rightarrow e^y = \frac{2x+1}{x+1}$$

$$\Rightarrow y = \log\left(\frac{2x+1}{x+1}\right)$$

Thus for given conditions the particular solution is  $y = \log\left(\frac{2x+1}{x+1}\right)$ .

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### Question 13

The general solution of the differential equation  $\frac{ydx - xdy}{y} = 0$ .

**A.**

$$xy = C$$

**B.**

$$x = Cy^2$$

**C.**

$$y = Cx$$

**D.**

$$y = Cx^2$$

**Solution**

Ans: Given differential equation:

$$\frac{ydx - xdy}{y} = 0$$

Divide both side by x :

$$\Rightarrow \frac{ydx - xdy}{xy} = 0$$

$$\Rightarrow \frac{dx}{x} - \frac{dy}{y} = 0$$

Integrate both side:

$$\Rightarrow \int \frac{dx}{x} - \int \frac{dy}{y} = 0$$

$$\Rightarrow \log|x| - \log|y| = \log k$$

$$\Rightarrow \log\left|\frac{x}{y}\right| = \log k$$

$$\Rightarrow \frac{x}{y} = k$$

$$\Rightarrow y = Cx \quad \left(C = \frac{1}{k}\right)$$

Thus the correct option is (C)

**Question 14**

Find the general solution of a differential equation of the type  $\frac{dx}{dy} + P_1x = Q_1$ .

**A.**

$$ye^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$$

**B.**

$$ye^{\int P_1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dy + C$$

**C.**

$$xe^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$$

**D.**

$$xe^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dx}) dy + C$$

**Solution**

Ans: The given differential equation is:

$$\frac{dx}{dy} + P_1 x = Q_1$$

It is a linear differential equation and its general solution is:

$$xe^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$$

With integrating factor  $I$ .  $F = e^{\int P_1 dy}$ .

Thus the correct option is (C).

**Question 15**

Find the general solution of the differential equation  $e^x dy + (ye^x + 2x) dx = 0$ .

**A.**

$$xe^y + x^2 = C$$

**B.**

$$xe^y + y^2 = C$$

**C.**

$$ye^x + x^2 = C$$

**D.**

$$ye^y + x^2 = C$$

**Solution**

Ans: The given differential equation is:

$$e^x dy + (ye^x + 2x) dx = 0$$

$$\Rightarrow e^x \frac{dy}{dx} + ye^x = -2x$$

$$\Rightarrow \frac{dy}{dx} + y = -2xe^{-x}$$

The given differential equation is of the form:

$$\frac{dy}{dx} + Py = Q$$

$$\Rightarrow P = 1$$

$$\Rightarrow Q = -2xe^{-x}$$

Calculating integrating factor:

$$I. F = e^{\int P dx}$$

$$\Rightarrow I. F = e^{\int dx}$$

$$\Rightarrow I. F = e^x$$

It is a linear differential equation and its general solution is:

$$y(I. F) = \int (Q \times I. F) dx + C$$

$$y(e^x) = \int (-2xe^{-x} \times e^x) dy + C$$

$$\Rightarrow ye^x = -2 \int x dx + C$$

$$\Rightarrow ye^x = -2 \left( \frac{x^2}{2} \right) + C$$

$$\Rightarrow ye^x + x^2 = C$$

Thus the correct answer is option (C).