

Chapter 8: Applications of Integrals

NCERT · Class 12 · Maths

Exercise 8.1

Question 1

Find the area of the region bounded by the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

.

Solution

Ans: Rewrite the given equation of ellipse

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

The intersection point of the ellipse and the x-axis are at

$$x = \pm 4$$

.

The intersection point of the ellipse and the y-axis are at

$$y = \pm 3$$

.

The given equation of the ellipse can be represented as

If we change

$$x$$

to

$$-x$$

or

$$y$$

to

$$y$$

the equation remains the same.

So, the ellipse is symmetrical about the x-axis and the y-axis.

Area bounded by ellipse

$$= 4 \times$$

Area of OABO

Area of OABO

$$= \int_0^4 y dx$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$\Rightarrow y^2 = 9 \left(1 - \frac{x^2}{16} \right)$$

$$\Rightarrow y = 3 \sqrt{1 - \frac{x^2}{16}}$$

Area of OABO

$$= \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} dx$$

Area of OABO

$$= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} dx$$

Area of OABO

$$= \frac{3}{4} \int_0^4 \sqrt{4^2 - x^2} dx$$

Apply the formula

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

in above equation

Area of OABO

$$= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4$$

Area of OABO

$$= \frac{3}{4} [0 + 8 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0)]_0^4$$

Area of OABO

$$= \frac{3}{4} \left(\frac{8\pi}{2} \right)$$

Area of OABO

$$= 3\pi$$

Therefore, the area bounded by ellipse

$$= 4 \times 3\pi$$

The area bounded by ellipse

$$= 12\pi \text{ sq. Units}$$

Question 2

Find the area of the region bounded by the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

.

Solution

Ans: Rewrite the given equation of the ellipse

$$\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

The intersection point of the ellipse and the x-axis are at

$$x = \pm 2$$

.

The intersection point of the ellipse and the y-axis are at

$$y = \pm 3$$

.

The given equation of the ellipse can be represented as a vertical ellipse.

It can be observed that the ellipse is symmetrical about the x-axis and y-axis.

Since, if we change

$$x$$

to

$$-x$$

or

$$y$$

to

$$-y$$

the equation remains the same.

Area bounded by ellipse

$$= 4 \times$$

Area of OABO

Area of OABO

$$= \int_0^2 y dx$$

.....(A)

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$\Rightarrow y^2 = 9 \left(1 - \frac{x^2}{4} \right)$$

$$\Rightarrow y = 3 \sqrt{1 - \frac{x^2}{4}}$$

$$\Rightarrow y = \frac{3}{2} \sqrt{4 - x^2}$$

.....(1)

Area of OABO

$$= \int_0^2 \frac{3}{2} \sqrt{4 - x^2} dx$$

using equation (1) in (A)

Area of OABO

$$= \frac{3}{2} \int_0^2 \sqrt{2^2 - x^2} dx$$

Apply the formula

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

in above equation

Area of OABO

$$= \frac{3}{2} \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2$$

Area of OABO

$$= \frac{3}{2} \left[\frac{2\pi}{2} \right]$$

Area of OABO

$$= \frac{3\pi}{2}$$

Area bound by the ellipse

$$= 4 \times \frac{3\pi}{2}$$

Area bound by the ellipse

$$= 6\pi \text{ sq. Units}$$

Choose the correct answer in the following Exercises 3 and 4.

Question 3

Area lying in the first quadrant and bounded by the circle

$$x^2 + y^2 = 4$$

and the lines

$$x = 0$$

and

$$x = 2$$

is

- | | |
|----|-----------------|
| 1. | π |
| 2. | $\frac{\pi}{2}$ |
| 3. | $\frac{\pi}{3}$ |
| 4. | $\frac{\pi}{4}$ |

Solution

Ans:

Area of

$$OAB = \int_0^2 y dx$$

Since,

$$x^2 + y^2 = 4$$

[Equation of circle]

$$y^2 = 4 - x^2$$

$$y = \sqrt{4 - x^2}$$

So,

Area of

$$OAB = \int_0^2 \sqrt{4 - x^2} dx$$

Area of

$$OAB = \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2$$

Area of

$$OAB = \left[\frac{2}{2} \sqrt{4 - 2^2} + \frac{4}{2} \sin^{-1} \frac{2}{2} - \left(\frac{0}{2} \sqrt{4 - 0^2} + \frac{4}{2} \sin^{-1} \frac{0}{2} \right) \right]$$

Area of

$$OAB = [\sqrt{4 - 4} + 2 \sin^{-1} 1 - 0]$$

Area of

$$OAB = \left[2 \times \frac{\pi}{2} \right]$$

Area of

$$OAB = \pi$$

square units

Hence, the area lying in the first quadrant and bounded by the circle

$$x^2 + y^2 = 4$$

and the lines

$$x = 0$$

and

$$x = 2$$

is

$$\pi$$

square units.

Hence, option (A) is the correct answer.

Question 4

Area of the region bounded by the curve

$$y^2 = 4x$$

, y-axis and the line

$$y = 3$$

is

1. 2

2.

$$\frac{9}{4}$$

3.

$$\frac{9}{3}$$

4.

$$\frac{9}{2}$$

Solution

Ans:

The area bounded by the curve

$$y^2 = 4x$$

, y-axis and the line

$$y = 3$$

is the area of the region

$$OAB$$

.

Area of

$$OAB = \int_0^3 x dy$$

Area of

$$OAB = \int_0^3 \frac{y^2}{4} dy$$

Area of

$$OAB = \frac{1}{4} \int_0^3 y^2 dy$$

Area of

$$OAB = \frac{1}{4} \left[\frac{y^3}{3} \right]_0^3$$

Area of

$$OAB = \frac{1}{4} \left[\frac{3^3}{3} - 0 \right]$$

Area of

$$OAB = \frac{1}{4} \left[\frac{27}{3} \right]$$

Area of

$$OAB = \frac{1}{4} [9]$$

Area of

$$OAB = \frac{9}{4}$$

square units

Hence, option (B) is the correct answer.

Miscellaneous Exercise

Question 1

Find the area under the given curves and given lines:

(i)

$$y=x^2, x=1, x=2$$

and

x-axis

(ii)

$$y=x^4, x=1, x=5$$

and

x-axis

Solution

Ans:

$$\text{Area ADCBA} = \int_1^2 y dx$$

substitute $y = x^2$

$$= \int_1^2 x^2 dx$$

Substituting the limits,

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ units}$$

Ans:

$$\text{Area of ADCBA} = \int_1^5 x^4 dx$$

Integrating using the power rule,

$$= \left[\frac{x^5}{5} \right]_1^5$$

Substituting the limits,

$$= \frac{(5)^5}{5} - \frac{1}{5}$$

Simplifying,

$$= (5)^4 - \frac{1}{5}$$

$$= 625 - \frac{1}{5}$$

$$= 624.8 \text{ units}$$

Question 2

Sketch the graph of

$$y = |x+3|$$

and evaluate

$$\int_{-6}^0 |x+3| dx$$

.

Solution

Ans:

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

$$(x+3) \leq 0$$

for

$$-6 \leq x \leq -3$$

$$(x+3) \geq 0$$

for

$$-3 \leq x \leq 0$$

Therefore,

$$\int_{-6}^0 |(x+3)| dx = - \int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx$$

Integrating using the power rule

$$= - \left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x \right]_{-3}^0$$

Substituting the limits,

$$= - \left[\left(\frac{(-3)^2}{2} + 3(-3) \right) - \left(\frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3) \right) \right]$$

Simplifying,

$$= - \left[-\frac{9}{2} \right] - \left[-\frac{9}{2} \right]$$

$$= 9$$

Question 3

Find the area bounded by the curve

$$y = \sin x$$

between

$$x = 0$$

and

$$x = 2\pi$$

.

Solution

Ans:

Therefore,

$$\text{area} = \text{Area OABO} + \text{Area BCDB}$$

Area Bounded by Curve $y = \sin x$

$$= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$$

$$= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right|$$

Substituting the limits,

$$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$$

Simplifying,

$$=1+1+|(-1-1)|$$

$$=2+|-2|$$

$$=2+2$$

$$=4 \text{ units}$$

Question 4

Area bounded by the curve

$$y=x^3$$

, the

x-axis

and the ordinates

$$x = -2$$

and

$$x = 1$$

is

A.

$$-9$$

B.

$$-\frac{15}{4}$$

C.

$$\frac{15}{4}$$

D.

$$\frac{17}{4}$$

Solution

Ans:

As shown in the diagram, the required area is:

$$\begin{aligned}\text{Required area} &= \int_{-2}^1 y dx \\ &= \int_{-2}^1 x^3 dx\end{aligned}$$

Integrating using the power rule

$$= \left[\frac{x^4}{4} \right]_{-2}^1$$

Substituting the limits,

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4} \right]$$

Simplifying,

$$\begin{aligned}&= \left(\frac{1}{4} - 4 \right) \\ &= -\frac{15}{4} \text{ units}\end{aligned}$$

So, the correct answer is

$$-\frac{15}{4} \text{ units}$$

option B.

Question 5

The area bounded by the curve

$$y = x|x|, \text{ x-axis}$$

and the ordinate

$$x = 1$$

and

$$x = -1$$

is given by (Hint

$$y = x^2$$

if

$$x > 0$$

and

$$y = -x^2$$

if

$$x < 0$$

)

A.

$$0$$

B.

$$\frac{1}{3}$$

C.

$$\frac{2}{3}$$

D.

$$\frac{4}{3}$$

Solution

Ans:

$$\begin{aligned}\text{Required area} &= \int_{-1}^1 y dx \\ &= \int_{-1}^1 x|x| dx \\ &= - \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx\end{aligned}$$

Integrating using the power rule

$$= - \left[\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1$$

Substituting the limits,

$$\begin{aligned}&= - \left(-\frac{1}{3} \right) + \frac{1}{3} \\ &= \frac{2}{3} \text{ units}\end{aligned}$$

So, the correct answer is

$$\frac{2}{3} \text{ units}$$

option C.