

Chapter 4: Determinants

NCERT · Class 12 · Maths

Exercise 4.1

Question 1

Evaluate the determinant: $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Solution

Ans: Solving the determinant $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ we have:

$$\Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) \Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = -2 + 20 \Rightarrow \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 18$$

Question 2

Evaluate the determinants.

i.

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

ii.

$$\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$$

Solution

Ans: Solving the determinant

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

$$\text{We have: } \Rightarrow \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = (\cos \theta)(\cos \theta) - (-\sin \theta)(\sin \theta)$$

$$\Rightarrow \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos^2 \theta + \sin^2 \theta$$

We know,

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = 1$$

Ans: Solving the determinant

$$\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix},$$

$$\text{We have: } \Rightarrow \begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = (x^2-x+1)(x+1) - (x-1)(x+1)$$

$$\Rightarrow \begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = x^3-x^2+x+x^2-x+1 - (x^2-1)$$

$$\text{So, } \begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = x^3+1-x^2+1$$

$$\therefore \begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = x^3-x^2+2.$$

Question 3

If $A =$

$$\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$$

, then show that

$$|2A| = 4|A|$$

.

Solution

Ans: Given that,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

Multiplying A by 2, we have:

$$\Rightarrow 2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$\therefore$

L.H.S

$$= |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$$

$$\Rightarrow |2A| = 2 \times 4 - 4 \times 8$$

$$\Rightarrow |2A| = 8 - 32$$

$$\therefore |2A| = -24$$

The value of determinant A is

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$$

$$\Rightarrow |A| = 2 - 8$$

$$\therefore |A| = -6$$

R.H.S is given as $4|A|$.

$$\therefore 4|A| = 4 \times (-6) = -24$$

Hence, we have $L.H.S = R.H.S$

$$\therefore |2A| = 4|A|$$

.

Question 4

If

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

, then show that

$$|3A| = 27 |A|$$

.

Solution

Ans: Given,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

Determining the value of determinant A, by expanding along the first column, i.e.,

C1

, we get:

$$\Rightarrow |A| = 1 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$

$$\Rightarrow |A| = 1(4 - 0) - 0 + 0$$

$$\Rightarrow |A| = 4$$

$$\therefore 27|A| = 27 \times 4 = 108$$

.....(1) The value of $|3A|$ is obtained as:

$$\Rightarrow 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow 3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = 3 \begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$\Rightarrow 3(36 - 0) + 0 + 0$$

$$\Rightarrow |3A| = 3 \times 36$$

Thus,

$$|3A| = 108$$

.....(2) From equations (1) and (2), we have:

$$|3A| = 27|A|$$

Hence proved.

Question 5

Evaluate the determinants

i.

$$\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

ii.

$$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

iii.

$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

$$\text{iv. } \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

Solution

Ans: Let

$$A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

Determining the value of A by expanding along the second row, we have:

$$\begin{aligned} \Rightarrow |A| &= -0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0 \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} \\ \Rightarrow |A| &= (-15+3) \\ \therefore |A| &= -12 \end{aligned}$$

Ans: Let

$$A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

Determining the value of A by expanding along the first row, we have:

$$\Rightarrow |A| = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$\begin{aligned}\Rightarrow |A| &= 3(1+6) + 4(1+4) + 5(3-2) \\ \Rightarrow |A| &= 21 + 20 + 5 \\ \therefore |A| &= 46\end{aligned}$$

Ans: Let

$$A = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

Determining the value of A by expanding along the first row, we have:

$$\begin{aligned}\Rightarrow |A| &= 0 \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix} \\ \Rightarrow |A| &= 0(9) - (-6) + 2(-3) \\ \therefore |A| &= 0\end{aligned}$$

Ans: Let $A = \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Determining the value of A by expanding along the first column, we have:

$$\begin{aligned}\Rightarrow |A| &= 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} \\ \Rightarrow |A| &= 2(-5) - 0 + 3(5) \\ \Rightarrow |A| &= -10 + 15 \\ \therefore |A| &= 5\end{aligned}$$

Question 6

If

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

, find

$$|A|$$

.

Solution

Ans: Given,

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

Determining the value of A by expanding along the first row, we have:

$$\begin{aligned} \Rightarrow |A| &= 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} \\ \Rightarrow |A| &= 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5) \\ \Rightarrow |A| &= 3 + 3 - 6 \\ \therefore |A| &= 0 \end{aligned}$$

Question 7

Find values of

$$x$$

, if

i.

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

ii.

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Solution

Ans: Given,

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

Solving it, we have:

$$\Rightarrow (2 \times 1) - (5 \times 4) = (2x \times x) - (6 \times 4)$$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow -18 + 24 = 2x^2$$

$$\Rightarrow 3 = x^2$$

Applying square root on both the sides, we obtain:

$$\Rightarrow x = \pm \sqrt{3}$$

Ans: Given,

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Solving it, we have:

$$\Rightarrow (2 \times 5) - (3 \times 4) = (x \times 5) - (3 \times 2x)$$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

Multiplying by (-1) on both the sides, we obtain:

$$\Rightarrow x = 2$$

Question 8

If

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

, then

$$x$$

is equal to

1. 6
2. ± 6
3. -6
4. 0

Solution

Ans: Given,

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

Solving it, we have:

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow x^2 - 36 = 0$$

$$\Rightarrow x^2 = 36$$

Applying square root on both the sides, we obtain:

$$\Rightarrow x = \pm 6$$

Hence, B.

$$\pm 6$$

is the correct answer.

Exercise 4.2

Question 1

Find the area of the triangle with vertices at the point given in each of the following:

i.

$$(1,0), (6,0), (4,3)$$

ii.

$$(2,7), (1,1), (10,8)$$

iii.

$$(-2,-3), (3,2), (-1,-8)$$

Solution

Ans: Given vertices,

$$(1,0), (6,0), (4,3)$$

Thus, the area of the triangle is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [1(0-3) - 0(6-4) + 1(18-0)]$$

$$\Rightarrow \Delta = \frac{1}{2} [-3 + 18]$$

$$\Rightarrow \Delta = \frac{15}{2}$$

square units

∴ Area of the triangle with vertices

$$(1,0), (6,0), (4,3)$$

is

$$\frac{15}{2}$$

square units.

Ans: Given vertices,

$$(2,7), (1,1), (10,8)$$

The area of the triangle is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [2(1-8) - 7(1-10) + 1(8-10)]$$

$$\Rightarrow \Delta = \frac{1}{2} [2(-7) - 7(-9) + 1(-2)]$$

$$\Rightarrow \Delta = \frac{1}{2} [-16 + 63]$$

$$\Rightarrow \Delta = \frac{47}{2}$$

square units

∴ Area of the triangle with vertices

$$(2,7), (1,1), (10,8)$$

is

$$\frac{47}{2}$$

square units.

Ans: Given vertices,

$$(-2,-3), (3,2), (-1,-8)$$

The area of the triangle with vertices

$$(-2,-3), (3,2), (-1,-8)$$

is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [-2(2+8) + 3(3+1) + 1(-24+2)]$$

$$\Rightarrow \Delta = \frac{1}{2} [-20 + 12 - 22]$$

$$\Rightarrow \Delta = -\frac{30}{2}$$

$$\Rightarrow \Delta = -15$$

∴ The area of the triangle with vertices

$$(2,7), (1,1), (10,8)$$

is

$$|-15| = 15$$

square units.

Question 2

Show that points

$$A(a, b+c), B(b, c+a), C(c, a+b)$$

are collinear.

Solution

Ans: To show that the points

$$A(a, b+c), B(b, c+a), C(c, a+b)$$

are collinear, the area of the triangle formed by these points as vertices should be zero.

∴ Area of

$$\Delta ABC$$

is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

Applying the row operations,

$$R_2 \rightarrow R_2 - R_1$$

and

$$R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix}$$

Taking out $(a-b)$ and $(c-a)$ common from

$$R_2$$

and

$$R_3$$

respectively,

$$\Rightarrow \Delta = \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

Applying the row operation

$$R_3 \rightarrow R_3 + R_2$$

$$\Rightarrow \Delta = \frac{1}{2}(a-b)(c-a) \begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

We know that when all the elements of a row or a column in a determinant are zero then the value of the determinant is always zero.

$$\therefore \Delta = 0$$

Thus, the area of the triangle formed by points

A

,

B

and

C

is zero.

Hence, the points

$$A(a, b+c), B(b, c+a), C(c, a+b)$$

are collinear.

Question 3

Find values of

$$k$$

if area of triangle is

$$4$$

square units and vertices are:

i.

$$(k,0), (4,0), (0,2)$$

ii.

$$(-2,0), (0,4), (0,k)$$

Solution

Ans: Given vertices are

$$(k,0), (4,0), (0,2)$$

.

We know, the area of the triangle is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [k(0-2) - 0(4-0) + 1(8-0)]$$

$$\Rightarrow \Delta = \frac{1}{2} [-2k + 8]$$

$$\therefore \Delta = -k + 4$$

Since the area is given to be

$$4$$

square units, thus

$$-k + 4 = \pm 4$$

When

$$-k + 4 = -4$$

$$\therefore k = 8$$

.

When

$$-k + 4 = 4$$

$$\therefore k = 0$$

.

Hence,

$$k=0,8$$

.

Ans: Given vertices are

$$(-2,0), (0,4), (0,k)$$

.

The area of the triangle is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [-2(4 - k)]$$

$$\Rightarrow \Delta = k - 4$$

Since the area is given to be

$$4$$

square units, thus

$$k - 4 = \pm 4$$

When

$$k - 4 = -4$$

$$\therefore k = 0$$

.

When

$$k - 4 = 4$$

$$\therefore k = 8$$

.

Hence,

$$k = 0, 8$$

.

Question 4

Determine the following:

i. Find the equation of line joining

$$(1,2)$$

and

$$(3,6)$$

using determinants.

ii. Find the equation of line joining

$$(3,1)$$

and

$$(9,3)$$

using determinants.

Solution

Ans: Let us assume a point,

$$P(x, y)$$

on the line joining points

$$A(1,2)$$

and

$$B(3,6)$$

.

Then, the points

$$A$$

, B and P are collinear.

Thus, the area of the triangle

ABP

will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [1(6-y) - 2(3-x) + 1(3y-6x)] = 0$$

$$\Rightarrow 6-y-6+2x+3y-6x=0$$

$$\Rightarrow 2y-4x=0$$

$$\Rightarrow y=2x$$

$\therefore$ The equation of the line joining the given points is

$$y = 2x$$

.

Ans: Let us assume a point,

P (x, y)

on the line joining points

A (3,1)

and

B (9,3)

.

Then, the points

A

, B and P are collinear.

Thus, the area of the triangle

ABP

will be zero.

$$\therefore \frac{1}{2} \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [3(3-y) - 1(9-x) + 1(9y-3x)] = 0$$

$$\Rightarrow 9-3y-9+x+9y-3x=0$$

$$\Rightarrow 6y-2x=0$$

$$\Rightarrow x-3y=0$$

$\therefore$ The equation of the line joining the given points is

$$x-3y=0$$

.

Question 5

If the area of triangle is

$$35$$

square units with vertices

$$(2,-6)$$

,

$$(5,4)$$

and

$$(k,4)$$

. Then

$$k$$

is

1. 12
2. -2
3. -12,-2
4. 12,-2

Solution

Ans: Given vertices,

$$(2,-6)$$

,

$$(5,4)$$

and

$$(k,4)$$

The area of the triangle is given by,

$$\Rightarrow \Delta = \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{2} [2(4-4) + 6(5-k) + 1(20-4k)]$$

$$\Rightarrow \Delta = \frac{1}{2} [50-10k]$$

$$\Rightarrow \Delta = 25-5k$$

Given, the area of the triangle is

$$35$$

square units.

Thus, we have:

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5 - k) = \pm 35$$

$$\Rightarrow 5 - k = \pm 7$$

.

When

$$5 - k = 7$$

$$\therefore k = -2$$

.

When

$$5 - k = -7$$

$$\therefore k = 12$$

.

Hence,

$$k = 12, -2$$

.

Thus, D.

$$12, -2$$

is the correct option.

Exercise 4.3

Question 1

Write Minors and Cofactors of the elements of following determinants:

1.

$$\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$$

Solution

Ans: Given,

$$\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$$

Minor of an element is termed as the determinant obtained by removing the row and the column in which that element is present.

Minor of element

$$a_{ij}$$

is denoted by

$$M_{ij}$$

,where

i and j denotes the row and the column of the determinant respectively.

$$\therefore M_{11}=3$$

$$M_{12}=0$$

$$M_{21}=-4$$

$$M_{22}=2$$

Cofactor of an element is termed as the determinant obtained by removing the row and the column in which that element is present preceded by a negative or a positive sign based on the position of the element.

Thus,

Cofactor of

$$a_{ij}$$

is

$$A_{ij}=(-1)^{i+j}M_{ij}$$

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11}$$

$$\Rightarrow A_{11} = (-1)^2 (3)$$

$$\therefore A_{11}=3$$

Similarly,

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12}$$

$$\Rightarrow A_{12}=(-1)^3 (0)$$

$$\therefore A_{12}=0$$

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21}$$

$$\Rightarrow A_{21}=(-1)^3 (-4)$$

$$\therefore A_{21}=4$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22}$$

$$\Rightarrow A_{22}=(-1)^4 (2)$$

$$\therefore A_{22}=2$$

1.

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

Ans: Given,

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

Minor of element

$$a_{ij}$$

is denoted by

$$M_{ij}$$

.

$$\therefore M_{11}=d$$

$$M_{12}=b$$

$$M_{21}=c$$

$$M_{22}=a$$

Cofactor of

$$a_{ij}$$

is

$$A_{ij}=(-1)^{i+j}M_{ij}$$

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11}$$

$$\Rightarrow A_{11} = (-1)^2 (d)$$

$$\therefore A_{11}=d$$

Similarly,

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12}$$

$$\Rightarrow A_{12}=(-1)^3 (b)$$

$$\therefore A_{12}= - b$$

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21}$$

$$\Rightarrow A_{21}=(-1)^3 (c)$$

$$\therefore A_{21}= - c$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22}$$

$$\Rightarrow A_{22} = (-1)^4 (a)$$

$$\therefore A_{22} = a$$

Question 2

Write Minors and Cofactors of the elements of following determinants:

1.

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Solution

Ans: Given determinant,

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

.

Minor of element

$$a_{ij}$$

is denoted by

$$M_{ij}$$

.

$$\therefore M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\Rightarrow M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = 1$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = 0$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = 0$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = 0$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = 1$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = 0$$

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = 0$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = 0$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = 1$$

1.

$$\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

Ans: Given determinant,

$$\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

Minor of element

$$a_{ij}$$

is denoted by

$$M_{ij}$$

.

$$\Rightarrow M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$$

$$\Rightarrow M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$$

$$\Rightarrow M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$\Rightarrow M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$$

$$\Rightarrow M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\Rightarrow M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$$

$$\Rightarrow M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$$

$$\Rightarrow M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$$

Cofactor of

$$a_{ij}$$

is

$$A_{ij}=(-1)^{i+j}M_{ij}$$

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11} = 11$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12} = 6$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13} = 3$$

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21} = -4$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22} = 2$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23} = 1$$

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31} = -20$$

$$\Rightarrow A_{32}=(-1)^{3+2}M_{32} = -13$$

$$\Rightarrow A_{33}=(-1)^{3+3}M_{33} = 5$$

Question 3

Using Cofactors of elements of second row, evaluate

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Solution

Ans: Given determinant,

$$\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Determining the minors and cofactors, we get:

$$\Rightarrow M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$$

$$\therefore A_{21}=(-1)^{2+1}M_{21}=7$$

$$\Rightarrow M_{22} = \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$$

$$\therefore A_{22} = (-1)^{2+2} M_{22} = 7$$

$$\Rightarrow M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$$

$$\therefore A_{23} = (-1)^{2+1} M_{23} = -7$$

Since,

$$\Delta$$

is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

$$\Rightarrow \Delta = 2(7) + 0(7) + 1(-7)$$

Hence,

$$\Delta = -7$$

.

Question 4

Using Cofactors of elements of third column, evaluate

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

.

Solution

Ans: Given determinant,

$$\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Determining the minors and cofactors, we get:

$$\Rightarrow M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$\therefore A_{13} = (-1)^{1+3} M_{13} = (z - y)$$

$$\Rightarrow M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$$

$$\therefore A_{23} = (-1)^{2+3} M_{23} = (x - z)$$

$$\Rightarrow M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

$$\therefore A_{33} = (-1)^{3+3} M_{33} = (y - x)$$

Since,

$$\Delta$$

is equal to the sum of the product of the elements of the first row with their corresponding cofactors.

$$\therefore \Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

$$\Rightarrow \Delta = yz(z - y) + zx(x - z) + xy(y - x)$$

$$\Rightarrow \Delta = yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y$$

$$\Rightarrow \Delta = (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$$

$$\Rightarrow \Delta = (x - y) [zx + zy - z^2 - xy]$$

$$\Rightarrow \Delta = (x - y) [z(x - z) + y(z - x)]$$

Thus,

$$\Delta = (x - y)(y - z)(z - x)$$

Question 5

For the matrices

$$\mathbf{A}$$

and

$$\mathbf{B}$$

, verify that

$$(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$$

where

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij} , then value of Δ is given by

1. $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
2. $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
3. $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
4. $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Solution

Ans: It is given that $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

The value of Δ by expanding along first column is obtained as,

$$a_{11}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{21}(a_{12} \cdot a_{33} - a_{13} \cdot a_{32}) + a_{31}(a_{12} \cdot a_{23} - a_{13} \cdot a_{22})$$

..... 1

Now, the cofactor A_{ij} of element a_{ij} is given by $(-1)^{i+j}M_{ij}$, where M_{ij} is the minor. Minor is the determinant obtained by cancelling the i th row and j th column of the original matrix.

Now for element a_{11} , the minor is $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22} \cdot a_{33} - a_{23} \cdot a_{32}$ and the cofactor is $A_{11} = (-1)^{1+1}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) \Rightarrow A_{11} = (a_{22} \cdot a_{33} - a_{23} \cdot a_{32})$.

Next for element a_{21} , the minor is $M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12} \cdot a_{33} - a_{13} \cdot a_{32}$ and the cofactor is $A_{21} = (-1)^{2+1}(a_{12} \cdot a_{33} - a_{13} \cdot a_{32}) \Rightarrow A_{21} = -(a_{12} \cdot a_{33} - a_{13} \cdot a_{32})$.

Next for element a_{31} , the minor is $M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12} \cdot a_{23} - a_{13} \cdot a_{22}$ and the cofactor is $A_{31} = (-1)^{3+1}(a_{12} \cdot a_{23} - a_{13} \cdot a_{22}) \Rightarrow A_{31} = (a_{12} \cdot a_{23} - a_{13} \cdot a_{22})$.

Now substituting the terms as obtained from above computation in equation 1,

$$a_{11}A_{11} - a_{21}(-A_{21}) + a_{31}A_{31}$$

$$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$$

This matches with option d.

Exercise 4.4

Question 1

Find the adjoint of each of the matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 4$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -3$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -2$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = 1$$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

Thus,

$$\text{adj}A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Question 2

Find adjoint of each of the matrices

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$$

$$\Rightarrow A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

Similarly,

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1 - 0) = 1$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$$

$$\Rightarrow A_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = (0 - 2) = -2$$

and

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$$

$$\Rightarrow A_{31} = \begin{vmatrix} -1 & 2 \\ 2 & 5 \end{vmatrix} = -5 - 4 = -9$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow A_{32} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5 - 4) = -1$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$$

.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

Thus,

$$\text{adj}A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -9 & -1 & 5 \end{bmatrix}$$

Question 3

Verify

$$\mathbf{A} (\text{adj} \mathbf{A}) = (\text{adj} \mathbf{A}) \mathbf{A} = |\mathbf{A}| \mathbf{I}$$

.

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Solution

Ans: Given,

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

$$\therefore |\mathbf{A}| = -12 - (-12)$$

$$\Rightarrow |\mathbf{A}| = 0$$

Hence,

$$|\mathbf{A}| \mathbf{I} = 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow |\mathbf{A}| \mathbf{I} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Then,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = -6$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = 4$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -3$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = 2$$

Cofactor matrix is $\begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}$.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

Thus,

$$\text{adj } A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

Now, multiplying A with its adjoint, we have:

$$\Rightarrow A (\text{adj } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow A (\text{adj } A) = \begin{bmatrix} -12 + 12 & -6 + 6 \\ 24 - 24 & 12 - 12 \end{bmatrix}$$

$$\therefore A (\text{adj } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Similarly, multiplying

$$(\text{adj } A)$$

with

$$A$$

, we get:

$$\Rightarrow (\text{adj } A) A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

$$\Rightarrow (\text{adj } A) A = \begin{bmatrix} -12 + 12 & -18 + 18 \\ 8 - 8 & 12 - 12 \end{bmatrix}$$

$$\therefore (\text{adj } A) A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Thus,

$$A (\text{adj } A) = (\text{adj } A) A = |A| I$$

Hence verified.

Question 4

Verify

$$\mathbf{A} (\text{adj} \mathbf{A}) = (\text{adj} \mathbf{A}) \mathbf{A} = |\mathbf{A}| \mathbf{I}$$

.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

.

Solution

Ans: Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow |\mathbf{A}| = 1(0-0) + 1(9+2) + 2(0-0)$$

$$\therefore |\mathbf{A}| = 11$$

$$|\mathbf{A}| \mathbf{I} = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 0$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -(9+2) = -11$$

$$\begin{aligned}\Rightarrow A_{13} &= (-1)^{1+3} M_{13} = (-1)^4 M_{13} \\ &\Rightarrow A_{13} = 0\end{aligned}$$

Similarly,

$$\begin{aligned}\Rightarrow A_{21} &= (-1)^{2+1} M_{21} = (-1)^3 M_{21} \\ &\Rightarrow A_{21} = -(-3 + 0) = 3 \\ \Rightarrow A_{22} &= (-1)^{2+2} M_{22} = (-1)^4 M_{22} \\ &\Rightarrow A_{22} = 3 - 2 = 1 \\ \Rightarrow A_{23} &= (-1)^{2+3} M_{23} = (-1)^5 M_{23} \\ &\Rightarrow A_{23} = -(0 + 1) = -1\end{aligned}$$

and

$$\begin{aligned}\Rightarrow A_{31} &= (-1)^{3+1} M_{31} = (-1)^4 M_{31} \\ &\Rightarrow A_{31} = 2 - 0 = 2 \\ \Rightarrow A_{32} &= (-1)^{3+2} M_{32} = (-1)^5 M_{32} \\ &\Rightarrow A_{32} = -(-2 - 6) = 8 \\ \Rightarrow A_{33} &= (-1)^{3+3} M_{33} = (-1)^6 M_{33} \\ &\Rightarrow A_{33} = 0 + 3 = 3\end{aligned}$$

.

Cofactor matrix is

$$\begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$$

.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\begin{aligned}\Rightarrow \text{adj} A &= \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}^T \\ \therefore \text{adj} A &= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\end{aligned}$$

Now, multiplying A with its adjoint, we have:

$$\begin{aligned}\Rightarrow A (\text{adj } A) &= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \\ \Rightarrow A (\text{adj } A) &= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix} \\ \therefore A (\text{adj } A) &= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}\end{aligned}$$

Similarly, multiplying

$$(\text{adj } A)$$

with

$$A$$

, we get:

$$\begin{aligned}\Rightarrow (\text{adj } A) A &= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \\ \Rightarrow (\text{adj } A) A &= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix} \\ \therefore (\text{adj } A) A &= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}\end{aligned}$$

Thus,

$$A (\text{adj } A) = (\text{adj } A) A = |A| I$$

Hence verified.

Question 5

Find the inverse of each of the matrices (if it exists). $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Solution

Ans: Let

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\Rightarrow |A| = 6 + 8$$

$$\therefore |A| = 14$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Then,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 3$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -4$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = 2$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = 2$$

Cofactor matrix is $\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj}A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Hence, the inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Question 6

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow |A| = -2 + 15$$

$$\therefore |A| = 13$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Then,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 2$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = 3$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -5$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = -1$$

Cofactor matrix is $\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj}A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Hence, the inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Question 7

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Then,

$$\Rightarrow |A| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0)$$

$$\therefore |A| = 10$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij}=(-1)^{i+j}M_{ij}$$

.

Thus,

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11} = (-1)^2M_{11}$$

$$\Rightarrow A_{11}=10 - 0 = 10$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12} = (-1)^3M_{12}$$

$$\Rightarrow A_{12} = -(0 + 0) = 0$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13} = (-1)^4M_{13}$$

$$\Rightarrow A_{13}=0$$

Similarly,

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21} = (-1)^3M_{21}$$

$$\Rightarrow A_{21} = -(10 - 0) = -10$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22} = (-1)^4M_{22}$$

$$\Rightarrow A_{22} = 5 - 0 = 5$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23} = (-1)^5M_{23}$$

$$\Rightarrow A_{23} = -(0 - 0) = 0$$

And

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31} = (-1)^4M_{31}$$

$$\Rightarrow A_{31} = 8 - 6 = 2$$

$$\Rightarrow A_{32}=(-1)^{3+2}M_{32} = (-1)^5M_{32}$$

$$\Rightarrow A_{32} = -(4 - 0) = -4$$

$$\Rightarrow A_{33}=(-1)^{3+3}M_{33} = (-1)^6M_{33}$$

$$\Rightarrow A_{33} = 2 - 0 = 2$$

.

Cofactor matrix is $\begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}.$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj}A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Hence, the inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Question 8

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Then,

$$\Rightarrow |A| = 1(-3 - 0) - 0 + 0$$

$$\therefore |A| = -3$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11} = (-1)^2M_{11}$$

$$\Rightarrow A_{11} = -3 - 0 = -3$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12} = (-1)^3M_{12}$$

$$\Rightarrow A_{12} = -(-3 - 0) = 3$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13} = (-1)^4M_{13}$$

$$\Rightarrow A_{13} = 6 - 15 = -9$$

Similarly,

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21} = (-1)^3M_{21}$$

$$\Rightarrow A_{21} = -(0 + 0) = 0$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22} = (-1)^4M_{22}$$

$$\Rightarrow A_{22} = -1 - 0 = -1$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23} = (-1)^5M_{23}$$

$$\Rightarrow A_{23} = -(2 - 0) = -2$$

and

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31} = (-1)^4M_{31}$$

$$\Rightarrow A_{31} = 0 - 0 = 0$$

$$\Rightarrow A_{32}=(-1)^{3+2}M_{32} = (-1)^5M_{32}$$

$$\Rightarrow A_{32} = -(0 - 0) = 0$$

$$\Rightarrow A_{33}=(-1)^{3+3}M_{33} = (-1)^6M_{33}$$

$$\Rightarrow A_{33} = 3 - 0 = 3$$

Cofactor matrix is $\begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow adjA = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Hence, the inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Question 9

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Thus,

$$\Rightarrow |A| = 2(-1-0) - 1(4-0) + 3(8-7)$$

$$\Rightarrow |A| = 2(-1) - 1(4) + 3(1)$$

$$\therefore |A| = -3$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11} = (-1)^2M_{11}$$

$$\Rightarrow A_{11}= -1 - 0 = -1$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12} = (-1)^3M_{12}$$

$$\Rightarrow A_{12}= -(4 - 0) = -4$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13} = (-1)^4M_{13}$$

$$\Rightarrow A_{13}= 8 - 7 = 1$$

Similarly,

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21} = (-1)^3M_{21}$$

$$\Rightarrow A_{21}= -(1 - 6) = 5$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22} = (-1)^4M_{22}$$

$$\Rightarrow A_{22}= 2 + 21 = 23$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23} = (-1)^5M_{23}$$

$$\Rightarrow A_{23}= -(4 + 7) = -11$$

and

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31} = (-1)^4M_{31}$$

$$\Rightarrow A_{31}= 0 + 3 = 3$$

$$\Rightarrow A_{32}=(-1)^{3+2}M_{32} = (-1)^5M_{32}$$

$$\Rightarrow A_{32}= -(0 - 12) = 12$$

$$\Rightarrow A_{33}=(-1)^{3+3}M_{33} = (-1)^6M_{33}$$

$$\Rightarrow A_{33}= -2 - 4 = -6$$

Cofactor matrix is $\begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}$.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow adj A = \begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Hence, the inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Question 10

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Expanding along column

$$C_1$$

,

$$\Rightarrow |A| = 1(8 - 6) - 0 + 3(3 - 4)$$

$$\therefore |A| = -1$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11} = (-1)^2M_{11}$$

$$\Rightarrow A_{11}=8-6=2$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12} = (-1)^3M_{12}$$

$$\Rightarrow A_{12} = -(0 + 9) = -9$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13} = (-1)^4M_{13}$$

$$\Rightarrow A_{13} = 0 - 6 = -6$$

Similarly,

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21} = (-1)^3M_{21}$$

$$\Rightarrow A_{21} = -(-4 + 4) = 0$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22} = (-1)^4M_{22}$$

$$\Rightarrow A_{22} = 4 - 6 = -2$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23} = (-1)^5M_{23}$$

$$\Rightarrow A_{23} = -(-2 + 3) = -1$$

and

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31} = (-1)^4M_{31}$$

$$\Rightarrow A_{31} = 3 - 4 = -1$$

$$\Rightarrow A_{32}=(-1)^{3+2}M_{32} = (-1)^5M_{32}$$

$$\Rightarrow A_{32} = -(-3 - 0) = 3$$

$$\Rightarrow A_{33}=(-1)^{3+3}M_{33} = (-1)^6M_{33}$$

$$\Rightarrow A_{33} = 2 - 0 = 2$$

.

Cofactor matrix is

$$\begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj}A = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}^T$$

$$\Rightarrow \text{adj}A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

The inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = -1 \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Question 11

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

Solution

Ans: Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

Expanding along column,

$$C_1$$

$$\Rightarrow |A| = 1 (-\cos^2 a - \sin^2 a)$$

$$\Rightarrow |A| = -(\cos^2 a + \sin^2 a)$$

$$\therefore |A| = -1$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = -\cos^2 a - \sin^2 a = -1$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = 0$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$$

$$\Rightarrow A_{13} = 0$$

Similarly,

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = 0$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = -\cos a$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$$

$$\Rightarrow A_{23} = -\sin a$$

and

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$$

$$\Rightarrow A_{31} = 0$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow A_{32} = -\sin a$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow A_{33} = \cos a$$

.

Cofactor matrix is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj}A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

The inverse of the matrix A is given by,

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\therefore A^{-1} = -1 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos a & -\sin a \\ 0 & -\sin a & \cos a \end{bmatrix}$$

Hence,

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

.

Question 12

Let

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

. Verify that

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1}$$

.

Solution

Ans: Let

$$\mathbf{A} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

Thus, determining the value of

$$|\mathbf{A}|$$

,

$$\Rightarrow |\mathbf{A}| = 15 - 14$$

$$\therefore |\mathbf{A}| = 1$$

Since, Cofactor of

$$a_{ij}$$

is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

.

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 5$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -2$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -7$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = 3$$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj}A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}^T$$

$$\Rightarrow \text{adj}A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

The inverse of a matrix is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

Hence,

$$A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

For

$$B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$\Rightarrow |B| = 54 - 56$$

$$\therefore |B| = -2$$

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = 9$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -7$$

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -8$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = 6$$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\therefore \text{adj}A = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}^T$$

$$\Rightarrow \text{adj}A = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Hence,

$$\text{adj}B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj}B = -\frac{1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Thus,

$$B^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$$

Now, multiplying B^{-1} and A^{-1} , we get:

$$\Rightarrow B^{-1}A^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow B^{-1}A^{-1} = \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

.....(1)

Similarly, multiplying the matrices A and B , we get:

$$\Rightarrow AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

The value of

$$|AB|$$

is

$$\Rightarrow |AB| = 67 \times 61 - 87 \times 47$$

$$\Rightarrow |AB| = 4087 - 4089$$

$$\therefore |AB| = -2$$

The adjoint of (AB) is given by,

$$\Rightarrow \text{adj}(AB) = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

Thus, the inverse is,

$$\Rightarrow (AB)^{-1} = \frac{1}{|AB|} \text{adj}(AB)$$

$$\Rightarrow (AB)^{-1} = \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$\therefore (AB)^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

..... (2)

From (1) and (2), we have:

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence proved.

Question 13

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1} .

Solution

Ans: Given,

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

We can write,

$$\begin{aligned} A^2 &= A \cdot A \\ \Rightarrow A^2 &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \\ &\therefore \end{aligned}$$

The value of

$$A^2 - 5A + 7I$$

is:

$$\begin{aligned} \Rightarrow A^2 - 5A + 7I &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow A^2 - 5A + 7I &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ \therefore A^2 - 5A + 7I &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{aligned} A^2 - 5A + 7I &= 0 \\ \Rightarrow A^2 - 5A &= -7I \end{aligned}$$

Multiplying by

$$A^{-1}$$

on both the sides, we have:

$$\begin{aligned} \Rightarrow AA(A^{-1}) - 5AA^{-1} &= -7IA^{-1} \\ \Rightarrow A(AA^{-1}) - 5I &= -7IA^{-1} \\ \Rightarrow AI - 5I &= -7IA^{-1} \end{aligned}$$

$$\begin{aligned}\Rightarrow A^{-1} &= -\frac{1}{7}(A - 5I) \\ \Rightarrow A^{-1} &= \frac{1}{7}(5I - A) \\ \Rightarrow A^{-1} &= \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) \\ \therefore A^{-1} &= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\end{aligned}$$

Question 14

For the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

find the number

a

and

b

such that.

$$A^2 + aA + bI = 0$$

.

Solution

Ans: Given

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

We can write,

$$\begin{aligned}A^2 &= A.A \\ \therefore A^2 &= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}\end{aligned}$$

$$\Rightarrow A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Solving

$$A^2 + aA + bI = 0$$

by multiplying the whole equation by

$$A^{-1}$$

.

$$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = 0$$

$$\Rightarrow A(AA^{-1}) + aI + b(IA^{-1}) = 0$$

$$\Rightarrow AI + aI + bA^{-1} = 0$$

$$\Rightarrow A + aI = -bA^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{b}(A + aI)$$

Now, determining the value of

$$A^{-1}$$

.

We know that the adjoint of a square matrix is the transpose of its cofactor matrix.

Hence, the adjoint of matrix A is:

$$\therefore \text{adj}A = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

The inverse is given by,

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

.

$$\Rightarrow A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

Thus,

$$\begin{aligned}\Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} &= -\frac{1}{b} \left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \right) \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} &= -\frac{1}{b} \begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} &= \begin{bmatrix} \frac{-3-a}{b} & \frac{2}{b} \\ -\frac{1}{b} & \frac{-1-a}{b} \end{bmatrix}\end{aligned}$$

Equating the corresponding elements of the two matrices, we get:

$$\begin{aligned}\Rightarrow -\frac{1}{b} &= -1 \\ \therefore b &= 1 \\ \Rightarrow \frac{-3-a}{b} &= 1 \\ \therefore a &= -4\end{aligned}$$

Thus,

$$-4$$

and

$$1$$

are the required values of

$$a$$

and

$$b$$

respectively.

Question 15

For the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show that

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 5\mathbf{A} + 11\mathbf{I} = \mathbf{0}$$

. Hence,

$$\mathbf{A}^{-1}$$

.

Solution

Ans: Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^2 = \begin{bmatrix} 1+1+1 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$\therefore \mathbf{A}^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^3 = \mathbf{A}^2 \cdot \mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^3 = \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$\therefore \mathbf{A}^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

Substituting the values for

$$\mathbf{A}^3$$

,

$$\mathbf{A}^2$$

and

$$\mathbf{A}$$

in

$$A^3 - 6A^2 + 5A + 11I$$

, we have:

$$\begin{aligned} \Rightarrow A^3 - 6A^2 + 5A + 11I &= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow A^3 - 6A^2 + 5A + 11I &= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 2 & -5 & 15 \end{bmatrix} + 11 \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \\ \Rightarrow A^3 - 6A^2 + 5A + 11I &= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} \\ \therefore A^3 - 6A^2 + 5A + 11I &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \end{aligned}$$

Thus,

$$A^3 - 6A^2 + 5A + 11I = 0$$

Since,

$$A^3 - 6A^2 + 5A + 11I = 0$$

.

Multiplying the whole equation by

$$A^{-1}$$

, we have:

$$\Rightarrow (AAA) A^{-1} - 6(AA) A^{-1} + 5AA^{-1} + 11IA^{-1} = 0$$

$$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = 11(IA^{-1})$$

$$\Rightarrow A^2 - 6A + 5I = 11A^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$$

.... (1)

Now,

$$A^2 - 6A + 5I$$

is given by:

$$\begin{aligned}\Rightarrow A^2 - 6A + 5I &= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow A^2 - 6A + 5I &= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & 6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ \Rightarrow A^2 - 6A + 5I &= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} \\ \therefore A^2 - 6A + 5I &= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}\end{aligned}$$

Substituting for

$$A^2 - 6A + 5I$$

equation (1), we get

$$\begin{aligned}\Rightarrow A^{-1} &= -\frac{1}{11} \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} \\ \therefore A^{-1} &= \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}\end{aligned}$$

Question 16

If

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

verify that

$$A^3 - 6A^2 + 9A + 4I = 0$$

and hence find

$$\mathbf{A}^{-1}$$

.

Solution

Ans: Given,

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$\therefore \mathbf{A}^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Similarly,

$$\Rightarrow \mathbf{A}^3 = \mathbf{A}^2 \mathbf{A} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \mathbf{A}^3 = \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$\therefore \mathbf{A}^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now,

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 9\mathbf{A} - 4\mathbf{I}$$

is given by:

$$\Rightarrow \mathbf{A}^3 - 6\mathbf{A}^2 + 9\mathbf{A} - 4\mathbf{I} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 9A - 4I = 0$$

Since,

$$A^3 - 6A^2 + 9A - 4I = 0$$

.

Multiplying the whole equation by

$$A^{-1}$$

, we have:

$$\Rightarrow (AAA) A^{-1} - 6(AA) A^{-1} + 9AA^{-1} - 4IA^{-1} = 0$$

$$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 9(AA^{-1}) = 4(IA^{-1})$$

$$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$$

$$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$$

..... (1)

Now,

$$A^2 - 6A + 9I$$

is given by:

$$A^2 - 6A + 9I = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 - 6A + 9I = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore A^2 - 6A + 9I = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 3 & 3 \end{bmatrix}$$

Substituting for

$$A^2 - 6A + 9I$$

equation (1), we get

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 3 & 3 \end{bmatrix}$$

Question 17

Let

$$\mathbf{A}$$

be nonsingular square matrix of order

$$3 \times 3$$

. Then

$$|\text{adj} \mathbf{A}|$$

is equal to

1. $|\mathbf{A}|$
2. $|\mathbf{A}|^2$
3. $|\mathbf{A}|^3$
4. $3 |\mathbf{A}|$

Solution

Ans: Given $\mathbf{A}$ is a nonsingular square matrix, i.e., it is a square matrix whose determinant is not equal to zero.

The inverse of a matrix is given as

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \text{adj} \mathbf{A}$$

$$\Rightarrow A^{-1}A = \frac{1}{|A|} \text{adj}A$$

$$\Rightarrow |A| I = \text{adj}A$$

The adjoint of matrix A is given by,

$$\Rightarrow (\text{adj}A) = A = |A| I = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$\Rightarrow |(\text{adj}A) A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$$

$$\Rightarrow |\text{adj}A| |A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3 (I)$$

$$\therefore |\text{adj}A| = |A|^3$$

Hence, B.

$$|A|^2$$

is the correct answer.

Question 18

If

$$A$$

is an invertible matrix of order

$$2$$

, then

$$\det(A^{-1})$$

is equal to

1. $\det(A)$

2. $\frac{1}{\det(A)}$

3.

1

4.

0

Solution

Ans: Since

$$A$$

is an invertible matrix, thus

$$A^{-1}$$

exists and it is given by:

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

.

As matrix

$$A$$

is of order

$$2$$

,

∴ Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

.

Hence,

$$|A| = ad - bc$$

.

The adjoint of

$$A$$

would be,

$$\text{adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

.

Now, the inverse of the matrix is given by:

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} (ad-bc)$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A^2|} \cdot |A|$$

$$\therefore |A^{-1}| = \frac{1}{|A|}$$

Thus,

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

.

Hence, B.

$$\frac{1}{\det(A)}$$

is the correct answer.

Exercise 4.5

Question 1

Examine the consistency of the system of equations.

$$x + 2y = 2$$

$$2x + 3y = 3$$

Solution

Ans: The given system of equations is:

$$x + 2y = 2$$

$$2x + 3y = 3$$

The given system of equations is:

$$x + 2y = 2$$

$$2x + 3y = 3$$

The given system of equations can be written in the form of $AX = B$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and

$$B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\text{Now, } |A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$$

$\therefore A$ is non-singular. Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 2

Examine the consistency of the system of equations.

$$2x - y = 5$$

$$x + y = 4$$

Solution

Ans: The given system of equations is:

$$2x - y = 5$$

$$x + y = 4$$

The given system of equation can be written in the form of $AX = B$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$

$\therefore A$ is non-singular. Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 3

Examine the consistency of the system of equations.

$$x + 3y = 5$$

$$2x + 6y = 8$$

Solution

Ans: The given system of equations is:

$$x + 3y = 5$$

$$2x + 6y = 8$$

The given system of equation can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\text{Now, } |A| = 1(6) - 3(2) = 6 - 6 = 0$$

$$\therefore A \text{ is a singular matrix. } (\text{adj } A) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

Thus, the solution of the given system of equations does not exist. Hence, the given system of equations is inconsistent.

Question 4

Examine the consistency

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$ax + ay + 2az = 4$$

Solution

Ans: The given system of equations is:

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$ax + ay + 2az = 4$$

The system of equation can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\text{Now, } |A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$$

$$= 4a - 2a - a = 4a - 3a = a \neq 0$$

$\therefore A$ is a non-singular matrix. Therefore, A^{-1} exists. Hence, the given system of equation is consistent.

Question 5

Examine the consistency of the system of equations.

$$3x - y - 2z = 2$$

$$2y - z = -1$$

$$3x - 5y = 3$$

Solution

Ans: The given system of equation is:

$$3x - y - 2z = 2$$

$$2y - z = -1$$

$$3x - 5y = 3$$

This system of equations can be written in the form of $AX = B$,

where $A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Now, $|A| = 3(-5) - 0 + 3(1 + 4) = -15 + 15 = 0$

$\therefore A$ is a singular matrix.

Now $(\text{adj } A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$

$\therefore (\text{adj } A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Question 6

Examine the consistency of the system of equations.

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

Solution

Ans: The given system of equation is:

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

The system of equation can be written in the form of $AX = B$,

where $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

$$\begin{aligned}\text{Now, } |A| &= 5(18 + 10) + 1(12 - 25) + 4(-4 - 15) \\ &= 5(28) + 1(-13) + 4(-19) \\ &= 140 - 13 - 76 \\ &= 51 \neq 0\end{aligned}$$

$\therefore A$ is non-singular. Therefore, A^{-1} exists. Hence, the given system of equations is consistent.

Question 7

Solve the system of linear equations, using the matrix method.

$$5x + 2y = 4$$

$$7x + 3y = 5$$

Solution

Ans: The given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\text{Now } |A| = 15 - 14 - 1 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Hence, $x = 2$ and $y = -3$

Question 8

Solve the system of linear equations, using the matrix method.

$$2x - y = -2$$

$$3x + 4y = 3$$

Solution

Ans: The given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\text{Now, } |A| = 8 + 3 = 11 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$A^{-1} = |A|^{-1}(\text{adj } A) = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

$$\text{Hence, } x = -\frac{5}{11} \text{ and } y = \frac{12}{11}.$$

Question 9

Solve the system of linear equations, using the matrix method.

$$4x - 3y = 3$$

$$3x - 5y = 7$$

Solution

Ans: The given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\text{Now, } |A| = -20 + 9 = -11 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\text{Now, } A^{-1} = \frac{1}{|A|}(\text{adj } A) = -\frac{1}{11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{11} \begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\begin{aligned}\begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{11} \begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} \\ &= \frac{1}{11} \begin{bmatrix} 15 - 21 \\ 9 - 28 \end{bmatrix} \\ &= \frac{1}{11} \begin{bmatrix} -6 \\ -19 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix} \\ \text{Hence, } x &= \frac{-6}{11} \text{ and } y = \frac{-19}{11}\end{aligned}$$

Question 10

Solve the system of linear equations, using the matrix method.

$$5x + 2y = 3$$

$$3x + 2y = 5$$

Solution

Ans: The system of equation is

$$5x + 2y = 3$$

$$3x + 2y = 5$$

Writing the above equation as $AX = B$

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\text{Hence } A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \& B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Calculating $|A|$

$$\begin{aligned}|A| &= \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} \\ &= 5(2) - 3(2) = 10 - 6 = 4\end{aligned}$$

Since $|A| \neq 0$

The System of equation is consistent and has a unique solution

Now,

$$AX = B$$

$$X = A^{-1}B$$

Calculating A^{-1}

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

Interchange sign

$$\text{adj } A = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

Thus,

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2(3) + (-2)5 \\ -3(3) + 5(5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 6 - 10 \\ -9 + 25 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, $x = -1$ & $y = 4$

Question 11

Solve the system of linear equations, using the matrix method.

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

Solution

Ans: The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 2 & -4 & -2 \\ 0 & 3 & -5 \end{vmatrix}$$

$$= 2(20 + 6) - 1(-10 - 0) + 1(6 - 0)$$

$$= 52 + 10 + 6 = 68 \neq 0$$

Thus, A is non-singular, Therefore, its inverse exists.

Therefore, the given system is consistent and has a unique solution given by $X = A^{-1}B$

Cofactors of A are

$$A_{11} = 20 + 6 = 26$$

$$A_{12} = -(-10 + 0) = 10$$

$$A_{13} = 6 + 0 = 6$$

$$A_{21} = -(-5 - 3) = 8$$

$$A_{22} = -10 - 0 = -10$$

$$A_{23} = -(6 - 0) = -6$$

$$A_{31} = (-2 + 4) = 2$$

$$A_{32} = -(-4 - 2) = 6$$

$$A_{33} = -8 - 2 = -10$$

$$\text{adj}(A) = \begin{bmatrix} 26 & 10 & 6 \\ 8 & -10 & -6 \\ 2 & 6 & -10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{68} \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \frac{1}{68} \begin{bmatrix} 26 & 8 & 2 \\ 10 & -10 & 6 \\ 6 & -6 & -10 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68} \begin{bmatrix} 26 + 24 + 18 \\ 10 - 30 + 54 \\ 6 - 18 - 90 \end{bmatrix}$$

$$= \frac{1}{68} \begin{bmatrix} 68 \\ 34 \\ -102 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{-3}{2} \end{bmatrix}$$

Hence, $x = 1, y = \frac{1}{2}$ and $z = \frac{-3}{2}$

Question 12

Solve a system of linear equations, using matrix method.

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

Solution

Ans: The given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\text{Now, } |A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$$

Thus A is non-singular. Therefore, its inverse exists. Now, $A_{11} = 4, A_{12} = -5, A_{13} = 1$

$$A_{21} = 2, A_{22} = 0, A_{23} = -2$$

$$A_{31} = 2, A_{32} = 5, A_{33} = 3$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{aligned}
\therefore X = A^{-1}B &= \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix} \\
&= \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} \\
&= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}
\end{aligned}$$

Hence, $x = 2$, $y = -1$, & $z = 1$

Question 13

Solve the system of linear equations, using the matrix method. $2x + 3y + 3z = 5$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

Solution

Ans: The given system of equation can be written in the form of $AX = B$ where

$$\begin{aligned}
|A| &= 2(4 + 1) - 3(2 - 3) + 3(-1 + 6) \\
&= 2(5) - 3(-5) + 3(5) \\
&= 10 + 15 + 15 = 40 \neq 0
\end{aligned}$$

Thus, A is non-singular. Therefore, its inverse exists. Now.

$$A_{11} = 5, A_{21} = 3, A_{31} = 9$$

$$A_{12} = 3, A_{22} = -13, A_{32} = 1$$

$$A_{13} = 9, A_{23} = 1, A_{33} = -7$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\begin{aligned}\therefore X = A^{-1}B &= \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} y \\ z \end{bmatrix} &= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} \\ &= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\end{aligned}$$

Hence, $x = 1$, $y = 2$ and $z = -1$

Question 14

Solve the system of linear equations, using the matrix method.

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

Solution

Ans: The given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Now,

$$|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists. Now, $A_{11} = 7$, $A_{12} = -19$, $A_{13} = 11$

$$A_{21} = 1, A_{22} = -1, A_{23} = -1$$

$$A_{31} = -3, A_{32} = 11, A_{33} = 7$$

$$\therefore A^{-1} = |A|^{-1}(\text{adj } A) = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1$ and $z = 3$.

Question 15

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} Using A^{-1} solve the system of equations $2x - 3y + 5z = 11$

$$3x + 2y - 4z = -5 \quad x + y - 2z = -3$$

Solution

$$\text{Ans: } A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\therefore |A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$$

$$\text{Now, } A_{11} = 0, A_{22} = 2, A_{33} = 1$$

$$A_{31} = -1, A_{22} = -9, A_{23} = -5$$

$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = - \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, the given system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\text{The solution of the system of equations is given by } X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \text{ Using (1)}$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence $x = 1$, $y = 2$, and $z = 3$

Question 16

The cost of 4Kg onion, 3 kg wheat and 2 kg rice is Rs60. The cost of 2 kg onion, 4 kg wheat and 6Kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70 .

Find cost of each item per kg by matrix method

Solution

Ans: Let the cost of onions, wheat and rice per kg be Rs. X and Rs. Z respectively.

Then, the given situation can be represented by a system of equations as:

$$4x + 3y + 2z = 60$$

$$2x + 4y + 6z = 90$$

$$6x + 2y + 3z = 70$$

This system of equations can be written in the form of $AX = B$,

$$\text{where } A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$$

Now,

$$A_{11} = 0, A_{21} = -5, A_{31} = 10$$

$$A_{12} = 30, A_{22} = 0, A_{32} = -20$$

$$A_{13} = -20, A_{23} = 10, A_{33} = 10$$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore A^{-1} = |A|^{-1} \text{adj } A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now,

$$\begin{aligned}
 X &= A^{-1}B \\
 \Rightarrow X &= \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{50} \begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} \\
 &= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} \\
 &= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}
 \end{aligned}$$

$\therefore x = 5, y = 8, \text{ and } z = 8$

Hence, the cost of onions is 5Rs per kg, the cost of wheat is 8Rs per kg, and the cost of rice is 8Rs per kg.

Miscellaneous Exercise

Question 1

Prove that the determinant

$$\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

is independent of

$$\theta$$

.

Solution

Ans: Let

$$\Delta = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

To solve it, we have:

$$\begin{aligned}
\Rightarrow \Delta &= x(x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta) \\
\Rightarrow \Delta &= x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta \\
\Rightarrow \Delta &= x^3 - x + x(\sin^2 \theta + \cos^2 \theta) \\
\Rightarrow \Delta &= x^3 - x + x \\
\therefore \Delta &= x^3
\end{aligned}$$

Hence,

$$\Delta$$

is independent of

$$\theta$$

Question 2

Evaluate

$$\begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Solution

Ans: Let

$$\Delta = \begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Expanding along column

$$C_3$$

$$\begin{aligned}
\Rightarrow \Delta &= -\sin \alpha (-\sin \alpha \sin^2 \beta + \cos^2 \beta \sin \alpha) + \cos \alpha (\cos \alpha \cos^2 \beta + \cos \alpha \sin^2 \beta) \\
\Rightarrow \Delta &= \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta) + \cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) \\
\Rightarrow \Delta &= \sin^2 \alpha (1) + \cos^2 \alpha (1) \\
\therefore \Delta &= 1
\end{aligned}$$

Question 3

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Solution

Ans: The below result will be used for simplification,

$$(AB)^{-1} = B^{-1}A^{-1}$$

Finding the inverse of matrix B. So, the determinant is

$$|B| = 1(3 - 0) - 2(-1 - 0) - 2(2 - 0)$$

$$\Rightarrow |B| = 3 + 2 - 4$$

$$\therefore |B| = 1$$

Now finding the cofactors,

$$B_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} \Rightarrow B_{11} = 3$$

$$B_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow B_{12} = 1$$

$$B_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} \Rightarrow B_{13} = 2$$

$$B_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} \Rightarrow B_{21} = -(2 - 4) = 2$$

$$B_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} \Rightarrow B_{22} = 1$$

$$B_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} \Rightarrow B_{23} = 2$$

$$B_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} \Rightarrow B_{31} = 6$$

$$B_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} \Rightarrow B_{32} = 2$$

$$B_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \Rightarrow B_{33} = (3 + 2) = 5$$

The cofactor matrix is $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$. The adjoint will be the transpose of the cofactor matrix.

$$\text{adj}(B) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

The inverse is given by $B^{-1} = \frac{\text{adj}(B)}{|B|}$. So,

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now, it is already given that $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$.

So, we can compute $(AB)^{-1} = B^{-1}A^{-1}$ as below,

$$\begin{aligned} (AB)^{-1} &= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \\ \Rightarrow (AB)^{-1} &= \begin{bmatrix} 9 - 30 + 30 & -3 + 12 - 12 & 3 - 10 + 12 \\ 3 - 15 + 10 & -1 + 6 - 4 & 1 - 5 + 4 \\ 6 - 30 + 25 & -2 + 12 - 10 & 2 - 10 + 10 \end{bmatrix} \\ \therefore (AB)^{-1} &= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \end{aligned}$$

Question 4

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

verify that

(i)

$$[\text{adj} \mathbf{A}]^{-1} = \text{adj}(\mathbf{A}^{-1})$$

(ii)

$$\left(\mathbf{A}^{-1}\right)^{-1}=\mathbf{A}$$

Solution

Ans: Given,

$$\mathbf{A}=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow |\mathbf{A}|=1(15-1)-2(10-1)+1(2-3)$$

$$\Rightarrow |\mathbf{A}|=14-18-1$$

$$\therefore |\mathbf{A}|=-5$$

Thus,

$$\Rightarrow A_{11}=(-1)^{1+1}M_{11}=(-1)^2M_{11}$$

$$\Rightarrow A_{11}=14$$

$$\Rightarrow A_{12}=(-1)^{1+2}M_{12}=(-1)^3M_{12}$$

$$\Rightarrow A_{12}=-9$$

$$\Rightarrow A_{13}=(-1)^{1+3}M_{13}=(-1)^4M_{13}$$

$$\Rightarrow A_{13}=-1$$

Similarly,

$$\Rightarrow A_{21}=(-1)^{2+1}M_{21}=(-1)^3M_{21}$$

$$\Rightarrow A_{21}=-9$$

$$\Rightarrow A_{22}=(-1)^{2+2}M_{22}=(-1)^4M_{22}$$

$$\Rightarrow A_{22}=4$$

$$\Rightarrow A_{23}=(-1)^{2+3}M_{23}=(-1)^5M_{23}$$

$$\Rightarrow A_{23}=1$$

And

$$\Rightarrow A_{31}=(-1)^{3+1}M_{31}=(-1)^4M_{31}$$

$$\Rightarrow A_{31}=-1$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow A_{32} = 1$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow A_{33} = -1$$

Cofactor matrix is

$$\begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

We know that the adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj}A = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

Let us denote the adjoint of A as B. So,

$$B = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

The inverse of A is given by

$$A^{-1} = \frac{1}{|A|} (\text{adj}A)$$

$$\Rightarrow A^{-1} = -\frac{1}{5} \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} -14 & 9 & 1 \\ 9 & -4 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Now, we have to verify

$$[\text{adj}A]^{-1} = \text{adj}(A^{-1})$$

.

Let us compute the RHS first, i.e. the adjoint of

$$A^{-1} = \frac{1}{5} \begin{bmatrix} -14 & 9 & 1 \\ 9 & -4 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

or

$$A^{-1} = \begin{bmatrix} -\frac{14}{5} & \frac{9}{5} & \frac{1}{5} \\ \frac{9}{5} & -\frac{4}{5} & -\frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

.

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = -\frac{1}{5}$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = -\frac{2}{5}$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$$

$$\Rightarrow A_{13} = -\frac{1}{5}$$

Similarly,

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = -\frac{2}{5}$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = -\frac{3}{5}$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$$

$$\Rightarrow A_{23} = -\frac{1}{5}$$

And

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$$

$$\Rightarrow A_{31} = -\frac{1}{5}$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow A_{32} = -\frac{1}{5}$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow A_{33} = -1$$

.

Cofactor matrix is

$$\begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}$$

. We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj} A^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}^T$$

$$\therefore \text{adj} A^{-1} = \begin{bmatrix} -\frac{1}{5} & \frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}$$

Next, moving to the LHS, i.e. the inverse of the adjoint of A. We have an adjoint of A as matrix B,

$$B = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

The Determinant of B is

$$|B| = [14(-4 - 1) + 9(9 + 1) - 1(-9 + 4)]$$

$$\Rightarrow |B| = [-70 + 90 + 5]$$

$$\therefore |B| = 25$$

Now the cofactors,

Thus,

$$\Rightarrow B_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow B_{11} = -5$$

$$\Rightarrow B_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow B_{12} = -10$$

$$\Rightarrow B_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$$

$$\Rightarrow B_{13} = -5$$

Similarly,

$$\Rightarrow B_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow B_{21} = -10$$

$$\Rightarrow B_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow B_{22} = -15$$

$$\Rightarrow B_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$$

$$\Rightarrow B_{23} = -5$$

And

$$\Rightarrow B_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$$

$$\Rightarrow B_{31} = -5$$

$$\Rightarrow B_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow B_{32} = -5$$

$$\Rightarrow B_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow B_{33} = -25$$

Cofactor matrix is

$$\begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix}$$

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj}B = \begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix}^T$$

$$\therefore \text{adj}B = \begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix}$$

Now the inverse is found as

$$B^{-1} = \frac{1}{|B|} (\text{adj}B)$$

$$\Rightarrow B^{-1} = \frac{1}{25} \begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix}$$

$$\therefore B^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}$$

So, LHS is

$$[\text{adj}A]^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}$$

Since LHS=RHS, it is verified that

$$[\text{adj}A]^{-1} = \text{adj}(A^{-1})$$

Ans: Since

$$A^{-1} = \begin{bmatrix} -\frac{14}{5} & \frac{9}{5} & \frac{1}{5} \\ \frac{9}{5} & -\frac{4}{5} & -\frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

and

$$\text{adj}A^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix}$$

, thus,

$$\begin{aligned} \Rightarrow |A^{-1}| &= \left[-\frac{14}{5} \left(-\frac{4}{25} - \frac{1}{25} \right) - \frac{9}{5} \left(\frac{9}{25} + \frac{1}{25} \right) + \frac{1}{5} \left(-\frac{9}{25} + \frac{4}{25} \right) \right] \\ \Rightarrow |A^{-1}| &= \left[\frac{70}{125} - \frac{90}{125} - \frac{5}{125} \right] \\ \therefore |A^{-1}| &= -\frac{1}{5} \end{aligned}$$

Also, we know that an inverse of a matrix is given by:

$$\begin{aligned} \Rightarrow (A^{-1})^{-1} &= \frac{\text{adj}A^{-1}}{|A|} \\ \Rightarrow (A^{-1})^{-1} &= -5 \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & -1 \end{bmatrix} \end{aligned}$$

$$\Rightarrow (A^{-1})^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow (A^{-1})^{-1} = A$$

Hence verified that

$$(A^{-1})^{-1} = A$$

.

Question 5

Evaluate

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Solution

Ans: Given,

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\Delta = \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Taking

$$2(x+y)$$

common from

$$R_1$$

$$\Rightarrow \Delta = 2(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Applying the row operations

$$C_2 \rightarrow C_2 - C_1$$

and

$$C_3 \rightarrow C_3 - C_1$$

$$\Delta = 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix}$$

Expanding along

$$R_1$$

$$\Rightarrow \Delta = 2(x+y) [1((x \times (-x)) - (-y(x-y))) + 0 + 0]$$

$$\Rightarrow \Delta = 2(x+y) [1((-x^2) - (-yx + y^2))]$$

$$\Rightarrow \Delta = 2(x+y) [-x^2 + xy - y^2]$$

$$\Rightarrow \Delta = -2(x+y) [x^2 - xy + y^2]$$

Applying the identity,

$$\therefore \Delta = -2[x^3 + y^3]$$

Question 6

Evaluate

$$\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

Solution

Ans: Given,

$$\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$$

Applying the row operations

$$R_2 \rightarrow R_2 - R_1$$

and

$$R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$$

Expanding along

$$C_1$$

$$\Rightarrow \Delta = 1(xy - 0)$$

$$\therefore \Delta = xy$$

Question 7

Solve the system of the following equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} + \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 2$$

Solution

Ans: Given equations,

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} + \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 2$$

Let

$$\frac{1}{x} = p$$

,

$$\frac{1}{y} = q$$

and

$$\frac{1}{z} = r$$

Then the given system of equations becomes:

$$2p + 3q + 10r = 4$$

$$4p - 6q + 5r = 1$$

$$6p + 9q + 20r = 2$$

Let

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$$

,

$$X = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

and

$$B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

such that, this system can be written in the form of

$$AX = B$$

.

Now,

$$\Rightarrow |A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$

$$\Rightarrow |A| = 150 + 330 + 720$$

$$\Rightarrow |A| = 1200$$

Thus,

$$A$$

is a non-singular matrix.

∴ Its inverse exists.

So,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix}$$

$$\Rightarrow A_{11} = 75$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix}$$

$$\Rightarrow A_{12} = 110$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = (-1)^4 \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix}$$

$$\Rightarrow A_{13} = 72$$

Similarly,

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 \begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix}$$

$$\Rightarrow A_{21} = 150$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 \begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix}$$

$$\Rightarrow A_{22} = 100$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix}$$

$$\Rightarrow A_{23} = 0$$

and

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = (-1)^4 \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix}$$

$$\Rightarrow A_{31} = 75$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix}$$

$$\Rightarrow A_{32} = 30$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix}$$

$$\Rightarrow A_{33} = -24$$

The inverse of a matrix is given by:

$$\Rightarrow A^{-1} = \frac{1}{|A|} (\text{adj}A)$$

$$\therefore A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore P = \frac{1}{2}$$

,

$$q = \frac{1}{3}$$

and

$$r = \frac{1}{5}$$

Thus

$$x = 2$$

,

$$y = 3$$

and

$$z = 5$$

.

Question 8

Choose the correct answer.If

$$X, Y, Z$$

are nonzero real numbers, then the inverse of matrix

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

1.

$$\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

2.

$$xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

3.

$$\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

4.

$$\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

Ans: Given,

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$\Rightarrow |A| = x(yz - 0)$$

$$\therefore |A| = xyz \neq 0$$

Thus,

$$\Rightarrow A_{11} = (-1)^{1+1} M_{11} = (-1)^2 M_{11}$$

$$\Rightarrow A_{11} = yz$$

$$\Rightarrow A_{12} = (-1)^{1+2} M_{12} = (-1)^3 M_{12}$$

$$\Rightarrow A_{12} = 0$$

$$\Rightarrow A_{13} = (-1)^{1+3} M_{13} = (-1)^4 M_{13}$$

$$\Rightarrow A_{13} = 0$$

Similarly,

$$\Rightarrow A_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21}$$

$$\Rightarrow A_{21} = 0$$

$$\Rightarrow A_{22} = (-1)^{2+2} M_{22} = (-1)^4 M_{22}$$

$$\Rightarrow A_{22} = xz$$

$$\Rightarrow A_{23} = (-1)^{2+3} M_{23} = (-1)^5 M_{23}$$

$$\Rightarrow A_{23} = 0$$

and

$$\Rightarrow A_{31} = (-1)^{3+1} M_{31} = (-1)^4 M_{31}$$

$$\Rightarrow A_{31} = 0$$

$$\Rightarrow A_{32} = (-1)^{3+2} M_{32} = (-1)^5 M_{32}$$

$$\Rightarrow A_{32} = 0$$

$$\Rightarrow A_{33} = (-1)^{3+3} M_{33} = (-1)^6 M_{33}$$

$$\Rightarrow A_{33} = xy$$

.

Cofactor matrix is

$$\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

.

We know that adjoint of a matrix is the transpose of its cofactor matrix.

$$\Rightarrow \text{adj}A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}^T$$

$$\therefore \text{adj}A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

The inverse of a matrix is given by:

$$\Rightarrow A^{-1} = \frac{1}{|A|} (\text{adj}A)$$

$$\Rightarrow A^{-1} = \frac{1}{xyz} \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

Thus, A.

$$\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

is the correct answer.

Question 9

Choose the correct answer.

Let

$$A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$$

,

where

$$0 \leq \theta \leq 2\pi$$

, then

1. $\text{Det}(A) = 0$
2. $\text{Det}(A) \in (2, \infty)$
3. $\text{Det}(A) \in (2, 4)$
4. $\text{Det}(A) \in [2, 4]$

Solution

Ans: Given,

$$A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$$

$$\therefore |A| = 1(1 + \sin^2 \theta) - \sin \theta(-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1)$$

$$\Rightarrow |A| = 1 + \sin^2\theta + \sin^2\theta + 1$$

$$\Rightarrow |A| = 2 + 2\sin^2\theta$$

$$\Rightarrow |A| = 2(1 + \sin^2\theta)$$

We know,

$$0 \leq \sin\theta \leq 1$$

$$\Rightarrow 1 \leq 1 + \sin^2\theta \leq 2$$

$$\Rightarrow 2 \leq 2(1 + \sin^2\theta) \leq 4$$

Thus, D.

$$\text{Det}(A) [2,4]$$

is the correct answer.
