

Chapter 3: Matrices

NCERT · Class 12 · Maths

Exercise 3.1

Question 1

In the matrix

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

, write

- The order of the matrix.
- The number of elements.
- Write the elements

$$a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$$

Solution

Ans: The order of a matrix is

$$m \times n$$

where

$$m$$

is the number of rows and

$$n$$

is the number of columns. Therefore, here the order is

$$3 \times 4$$

.

Ans: Since the order of the given matrix is

$$3 \times 4$$

therefore, the number of elements in it is

$$3 \times 4 = 12$$

.

Ans: The elements are given as

$$a_{mn}$$

. Therefore, here

$$a_{13} = 19$$

,

$$a_{21} = 35$$

,

$$a_{33} = -5$$

,

$$a_{24} = 12$$

,

$$a_{23} = \frac{5}{2}$$

.

Question 2

If a matrix has

$$24$$

elements, what are the possible order it can have? What if it has

$$13$$

elements?

Solution

Ans: The order of a matrix is

$$m \times n$$

where

$$m$$

is the number of rows and

$$n$$

is the number of columns. To find the possible orders of a matrix, we have to find all the ordered pairs of natural numbers whose product is

$$24$$

.

$$\therefore (1 \times 24), (24 \times 1), (2 \times 12), (12 \times 2), (3 \times 8), (8 \times 3), (4 \times 6), (6 \times 4)$$

are all the possible ordered pairs here.

If the matrix had

$$13$$

elements, then the ordered pairs would be

$$(1 \times 13)$$

and

$$(13 \times 1)$$

.

Question 3

If a matrix has

$$18$$

elements, what are the possible orders it can have? What if it has

$$5$$

elements?

Solution

Ans: The order of a matrix is

$$m \times n$$

where

$$m$$

is the number of rows and

$$n$$

is the number of columns. To find the possible orders of a matrix, we have to find all the ordered pairs of natural numbers whose product is

$$18$$

.

$$\therefore (1 \times 18), (18 \times 1), (2 \times 9), (9 \times 2), (3 \times 6), (6 \times 3)$$

are all the possible ordered pairs here.

If the matrix had

$$5$$

elements, then the ordered pairs would be

$$(1 \times 5)$$

and

$$(5 \times 1)$$

.

Question 4

Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

$$(i) \ a_{ij} = \frac{(i+j)^2}{2}$$

$$(ii) \ a_{ij} = \frac{i}{j}$$

$$(iii) \ a_{ij} = \frac{(i+2j)^2}{2}$$

Solution

Ans:

$$(i) \ a_{ij} = \frac{(i+j)^2}{2}$$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = 8$$

So, the required matrix is: $\begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$.

(ii) $a_{ij} = \frac{i}{j}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$

So, the required matrix is: $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$.

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Elements for 2×2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{11} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = 8$$

$$a_{22} = \frac{(2+4)^2}{2} = \frac{(6)^2}{2} = 18$$

So, the required matrix is: $\begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$.

Question 5**Construct a**

$$3 \times 4$$

matrix, whose elements are given by**i.**

$$a_{ij} = \frac{1}{2}|-3i + j|$$

ii.

$$a_{ij} = 2i - j$$

Solution

Ans: Given that

$$a_{ij} = \frac{1}{2}|-3i + j|$$

,

$$\therefore a_{11} = \frac{1}{2}|-3 \times 1 + 1| = 1$$

$$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = 4$$

$$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = 2$$

$$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = 0$$

$$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = 3$$

$$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2} |-3 \times 2 + 4| = 1$$

$$a_{34} = \frac{1}{2} |-3 \times 3 + 4| = \frac{5}{2}$$

Thus, the required matrix is

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

.

Ans: A

$$3 \times 4$$

matrix is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Given that

$$a_{ij} = 2i - j$$

,

$$\therefore a_{11} = 2 \times 1 - 1 = 1$$

$$a_{21} = 2 \times 2 - 1 = 3$$

$$a_{31} = 2 \times 3 - 1 = 5$$

$$a_{12} = 2 \times 1 - 2 = 0$$

$$a_{22} = 2 \times 2 - 2 = 4$$

$$a_{32} = 2 \times 3 - 2 = 4$$

$$a_{13} = 2 \times 1 - 3 = -1$$

$$a_{23} = 2 \times 2 - 3 = 1$$

$$a_{33} = 2 \times 3 - 3 = 3$$

$$a_{14} = 2 \times 1 - 4 = -2$$

$$a_{24} = 2 \times 2 - 4 = 0$$

$$a_{34} = 2 \times 3 - 4 = 2$$

Thus, the required matrix is

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Question 6

Find the value of

$$x, y, z$$

from the following equation:

i.

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

ii.

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

iii.

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Solution

Ans: Given

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

Comparing the corresponding elements we get,

$$x = 1, y = 4, z = 3$$

Ans: Given

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

Comparing the corresponding elements we get,

$$x+y=6, xy=8, 5+z=5$$

Now,

$$\begin{aligned} \therefore 5+z &= 5 \\ \Rightarrow z &= 0 \end{aligned}$$

We know that,

$$\begin{aligned} (x-y)^2 &= (x+y)^2 - 4xy \\ \Rightarrow (x-y)^2 &= 36 - 32 \\ \Rightarrow (x-y) &= \pm 2 \end{aligned}$$

When

$$(x-y) = 2$$

and

$$(x+y) = 6$$

,

We get

$$x=4, y=2$$

When

$$(x-y) = -2$$

and

$$(x+y) = 6$$

,

We get

$$x=2, y=4$$

$$\therefore x = 4, y = 2, z = 0$$

or

$$\therefore x = 2, y = 4, z = 0$$

Ans: Given

$$\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Comparing the corresponding elements we get,

$$x + y + z = 9$$

...(1)

$$x + z = 5$$

...(2)

$$y + z = 7$$

...(3)

From equation (1) and (2),

$$y + 5 = 9$$

$$\Rightarrow y = 4$$

From equation (3) we have,

$$4 + z = 7$$

$$\Rightarrow z = 3$$

$$x + z = 5$$

$$\Rightarrow x = 2$$

$$\therefore x = 2, y = 4, z = 3$$

Question 7

Find the value of

$$a, b, c, d$$

from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution

Ans: Given

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Comparing the corresponding elements we get,

$$a - b = -1$$

...(1)

$$2a - b = 0$$

...(2)

$$2a + c = 5$$

...(3)

$$3c + d = 13$$

...(4)

From equation (2),

$$b = 2a$$

From equation (1),

$$a - 2a = -1$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

From equation (3),

$$2 \times 1 + c = 5$$

$$\Rightarrow c = 3$$

From equation (4),

$$3 \times 3 + d = 13$$

$$\Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3, d = 4$$

Question 8

$$A = [a_y]_{m \times n}$$

is a square matrix, if

1. $m < n$
2. $m > n$
3. $m = n$
4. None of these

Solution

Ans: A given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

$$\therefore A = [a_y]_{m \times n}$$

is a square matrix if,

$$m = n$$

.

Thus, option (C) is correct.

Question 9

Which of the given values of

$$x$$

and

$$y$$

make the following pair of matrices equal

$$\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix} = \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$$

1. $x = \frac{-1}{3}, y = 7$
2. Not possible to find

3. $y = 7, x = \frac{-2}{3}$

4. $x = \frac{-1}{3}, y = \frac{-2}{3}$

Solution

Ans: Given

$$\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix} = \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$$

Comparing the corresponding elements we get,

$$3x + 7 = 0$$

$$\Rightarrow x = -\frac{7}{3}$$

$$y - 2 = 5$$

$$\Rightarrow y = 7$$

$$y + 1 = 8$$

$$\Rightarrow y = 7$$

$$2 - 3x = 4$$

$$\Rightarrow x = -\frac{2}{3}$$

Since we get two different values of

$$x$$

,which is not possible. It is not possible to find the values of

$$x$$

and

$$y$$

for which the given matrices are equal.

Thus, the correct option is (B).

Question 10

The number of all possible matrices of order

$$3 \times 3$$

with each entry

0

or

1

is:

- | | |
|----|-----|
| 1. | 27 |
| 2. | 18 |
| 3. | 81 |
| 4. | 512 |

Solution

Ans: Given a matrix of the order

$$3 \times 3$$

has nine elements and each of these elements can be either

0

or

1

.

Now, each of the nine elements can be filled in two possible ways.

Therefore, the required number of possible matrices is

$$2^9 = 512$$

.

Exercise 3.2

Question 1

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following

i. $A + B$

ii. $A - B$ iii. $3A - C$ iv. AB **Solution**

Ans: Adding the matrices

$$A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

Ans: Subtracting the matrices

$$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

Ans: Subtracting the matrices

$$3A - C = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Ans: Matrix A has 2 columns. This number is equal to the number of rows in matrix B. Therefore, AB is defined as:

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2(1) + 4(-2) & 2(3) + 4(5) \\ 3(1) + 2(-2) & 3(3) + 2(5) \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ 1 & 19 \end{bmatrix}$$

Question 2**Compute the following:**

$$\text{i. } \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ -\mathbf{b} & \mathbf{a} \end{bmatrix} + \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{b} & \mathbf{a} \end{bmatrix}$$

$$\text{iii. } \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$\text{iv. } \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

Solution

Ans: Computing,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$\text{ii. } \begin{bmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

Ans: Computing,

$$\begin{bmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2 + b^2 + 2ab & b^2 + c^2 + 2bc \\ a^2 + c^2 - 2ac & a^2 + b^2 - 2ab \end{bmatrix}$$

$$\begin{bmatrix} a^2 + b^2 + 2ab & b^2 + c^2 + 2bc \\ a^2 + c^2 - 2ac & a^2 + b^2 - 2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

Ans: Computing,

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

Ans: Computing,

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$\begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, (\because \sin^2 x + \cos^2 x = 1)$$

Question 3

Compute the indicated products

$$\text{i. } \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ -\mathbf{b} & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{a} & -\mathbf{b} \\ \mathbf{b} & \mathbf{a} \end{bmatrix}$$

$$\text{ii. } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$\text{iii. } \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\text{iv. } \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\text{v. } \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\text{vi. } \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Solution

$$\text{Ans: Given: } \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} &= \begin{bmatrix} a(a) + b(b) & a(-b) + a(b) \\ -b(a) + b(a) & -b(-b) + a(a) \end{bmatrix} \\ &= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix} \end{aligned}$$

$$\text{Ans: Given: } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4]$$

$$\begin{aligned} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4] &= \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} \\ &= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix} \end{aligned}$$

$$\text{Ans: Given: } \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} &= \begin{bmatrix} 1(1) - 2(2) & 1(2) - 2(3) & 1(3) - 2(1) \\ 2(1) + 3(2) & 2(2) + 3(3) & 2(3) + 3(1) \end{bmatrix} \\ &= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix} \end{aligned}$$

Ans: Given: $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(0) + 4(3) & 2(-3) + 3(2) + 4(0) & 2(5) + 3(4) + 4(5) \\ 3(1) + 4(0) + 5(3) & 3(-3) + 4(2) + 5(0) & 3(5) + 4(4) + 5(5) \\ 4(1) + 5(0) + 6(3) & 4(-3) + 5(2) + 6(0) & 4(5) + 5(4) + 6(5) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Ans: Given: $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1) + 1(-1) & 2(0) + 1(2) & 2(1) + 1(1) \\ 3(1) + 2(-1) & 3(0) + 2(2) & 3(1) + 2(1) \\ -1(1) + 1(-1) & -1(0) + 1(2) & -1(1) + 1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

Ans: Given: $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3(2) - 1(1) + 3(3) & 3(-3) - 1(0) + 3(1) \\ -1(2) + 0(1) + 2(3) & -1(-3) + 0(0) + 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Question 4

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then Compute $(A + B)$ and $(B - C)$.

Also verify that $A + (B - C) = (A + B) - C$

Solution

Ans:

$$\text{Computing } (A + B) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$\text{Computing } (B - C) = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

To verify $A + (B - C) = (A + B) - C$,We calculate $A + (B - C)$ and $(A + B) - C$ first, and then check that LHS = RHSComputing $A + (B - C)$

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}, \text{ and Computing } (A + B) - C$$

$$\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence, we have verified that $A + (B - C) = (A + B) - C$.**Question 5**

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ then calculate $3A - 5B$

Solution

$$\text{Ans: Evaluating } 3A - 5B, \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 6

Simplify $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Solution

Ans: Simplifying $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

Question 7

Find X and Y, if

i. $\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{X} - \mathbf{Y} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

ii. $2\mathbf{X} + 3\mathbf{Y} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3\mathbf{X} + 2\mathbf{Y} = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$:

Solution

Ans: Given:

$$\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \dots (1)$$

$$\mathbf{X} - \mathbf{Y} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \dots (2)$$

Adding these two equations, we get $2\mathbf{X} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$2\mathbf{X} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \quad \text{Putting } \mathbf{X} \text{ in (1)} \quad \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + \mathbf{Y} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Ans: Given:

$$\begin{aligned}
2X + 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots (1) \\
3X + 2Y &= \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \dots
\end{aligned}$$

Multiplying (1) with 2, we have $4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \dots (3)$ Multiplying (2)

with 3, we have $9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \dots (4)$ From (3) and (4), we have

$$-5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} \therefore X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} \text{ Putting X in (1)}$$

$$2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \therefore 3Y = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} \therefore Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Question 8

Find X , if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution

Ans: Given:

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}, \text{ where } Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \therefore 2X = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question 9

Find x and y , if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Solution

Ans: Given: $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

Question 10

Solve the equation for x, y, z and t if: $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Solution

Ans: Given:

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \quad \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$\begin{aligned} 2x+3 &= 9 & 2y &= 12 & 2z-3 &= 15 \Rightarrow z = 9 & 2t+6 &= 18 \Rightarrow t = 6 \\ \Rightarrow x &= 3 & \Rightarrow y &= 6 & & & & \therefore x = 3, y = 6, z = 9 \text{ and } t = 6. \end{aligned}$$

Question 11

If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of x and y .

Solution

Ans: Given:

$$x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \quad \text{Comparing the corresponding elements of these two matrices, we get}$$

$$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$2x - y = 10 \text{ and } 3x + y = 5$$

Adding these two equations, we have:

$$\begin{aligned} 5x &= 15 & y &= 5 - 3x \\ \Rightarrow x &= 3 & \text{Now, putting } x = 3 \text{ in any equation, we have: } & \Rightarrow y = -4 \\ & & & \therefore x = 3 \text{ and } y = -4 \end{aligned}$$

Question 12

Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w .

Solution

Ans: Simplifying

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+y+x \\ -1+z+w & 2w+3 \end{bmatrix} \quad \text{Comparing the corresponding elements of these two matrices,}$$

$$\text{we get: } 3x = x+4 \Rightarrow x=2 \quad 3y = 6+x+y \Rightarrow y=4 \quad 3w = 2w+3 \Rightarrow w=3 \quad 3z = -1+z+w$$

$$\Rightarrow z=1$$

$$\therefore x=2, y=4, z=1 \text{ and } w=3$$

Question 13

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$

Solution

Ans: To show $F(x)F(y) = F(x+y)$,

We first calculate $F(x)F(y)$ and $F(x+y)$, and check that both are equal LHS

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RHS

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LHS = RHS, proved.

Question 14**Show that:**

$$\text{i. } \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\text{ii. } \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution**Ans:** To show, we first calculate LHS and RHS

LHS

$$\begin{aligned} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} &= \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix} \quad \text{RHS} \\ \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} &= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \\ \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} &= \begin{bmatrix} 2(5) + 1(6) & 2(-1) + 1(7) \\ 3(5) + 4(6) & 3(-1) + 4(7) \end{bmatrix} \\ \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} &= \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \\ \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} &\therefore \text{LHS} \neq \text{RHS} \end{aligned}$$

Ans: To show, we first calculate LHS and RHS

LHS

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(-1) + 2(0) + 3(2) & 1(1) + 2(-1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(-1) + 1(0) + 0(2) & 0(1) + 1(-1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(-1) + 1(0) + 0(2) & 1(1) + 1(-1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

RHS

$$\begin{aligned} & \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ = & \begin{bmatrix} -1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) - 1(0) + 1(1) & 0(2) - 1(1) + 1(1) & 0(3) - 1(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix} \\ & = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \end{aligned}$$

Question 15

Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Solution

Ans: We know that, $A^2 = A \times A$

$$\begin{aligned} & \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2(2) + 0(2) + 1(1) & 2(0) + 0(1) + 1(-1) & 2(1) + 0(3) + 1(0) \\ 2(2) + 1(2) + 3(1) & 2(0) + 1(1) + 3(-1) & 2(1) + 1(3) + 3(0) \\ 1(2) - 1(2) + 0(1) & 1(0) - 1(1) + 0(-1) & 1(1) - 1(3) + 0(0) \end{bmatrix} \\ & \begin{bmatrix} 4 + 0 + 1 & 0 + 0 - 1 & 2 + 0 + 0 \\ 4 + 2 + 3 & 0 + 1 - 3 & 2 + 3 + 0 \\ 2 - 2 + 0 & 0 - 1 + 0 & 1 - 3 + 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} \end{aligned}$$

Substituting value of A^2 , A , I in $A^2 - 5A + 6I$

$$\Rightarrow \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5-10+6 & -1 & 2-5 \\ 9-10 & -2-5+6 & 5-15 \\ -5+0 & -1+5 & -2-0+6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Question 16

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, Prove that $A^3 - 6A^2 + 7A + 2I = 0$

Solution

Ans: We know that, $A^2 = A \times A$ and $A^3 = A^2 \times A$ For A^2 ,

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1(1)+0(0)+2(2) & 1(0)+0(2)+2(0) & 1(2)+0(1)+2(3) \\ 0(1)+2(0)+1(2) & 0(0)+2(2)+1(0) & 0(2)+2(1)+1(3) \\ 2(1)+0(0)+3(2) & 2(0)+0(2)+3(0) & 2(2)+0(1)+3(3) \end{bmatrix}$$

$$\begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

For A^3 ,

Substituting value of A^3 , A^2 , A , I in $A^3 - 6A^2 + 7A + 2I = 0$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 2I = 0$

Question 17

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solution

Ans: We know that, $A^2 = A \times A$

$$\begin{aligned} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} &= \begin{bmatrix} 3(3) - 2(4) & 3(-2) - 2(-2) \\ 4(3) - 2(4) & 4(-2) - 2(-2) \end{bmatrix} \quad \text{Now } A^2 = kA - 2I \\ &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k - 2 & -2k \\ 4k & -2k - 2 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements, we have:

Comparing the corresponding elements, we have:

$$\Rightarrow 3k - 2 = 1$$

$$\Rightarrow k + 1$$

Thus, the value of k is 1 .

Question 18

If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2 , show that

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Solution

Ans: LHS

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

RHS

$$\begin{aligned} (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ \Rightarrow \begin{bmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \cos \alpha \tan \frac{\alpha}{2} \\ -\cos \alpha \tan \frac{\alpha}{2} + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
&\Rightarrow \begin{bmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} \\ -\left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin^2 \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - \tan \frac{\alpha}{2} \\ -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin^2 \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}
\end{aligned}$$

We get LHS = RHS.

Question 19

A trust fund has Rs30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of :

(a) Rs 1,800

Solution

Ans: Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond pays Rs $(30,000 - x)$

It is given that the first bond pays 5% interest per year and the second bond pay 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1,800 , we have:

$$\Rightarrow [x \quad 30,000 - x] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1,800$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30,000 - x)}{100} = 1,800$$

$$\Rightarrow 5x + 2,10,000 - 7x = 1,80,000$$

$$\Rightarrow 2x = 30,000$$

$$\Rightarrow x = 15,000$$

Thus, in order to obtain an annual total interest of Rs 1,800 , the trust fund should invest Rs 15,000 in the first bond and the remaining Rs 15,000 in the second bond.

(b) Rs 2,000

Ans: Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond pays Rs $(30,000 - x)$

It is given that the first bond pays 5% interest per year and the second bond pay 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 2,000 , we have:

$$\Rightarrow [x \quad 30,000 - x] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2,000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30,000 - x)}{100} = 2,000$$

$$\Rightarrow 5x + 2,10,000 - 7x = 2,00,000$$

$$\Rightarrow 2x = 10,000$$

$$\Rightarrow x = 5,000$$

Thus, in order to obtain an annual total interest of Rs 2,000 , the trust fund should invest Rs 5,000 in the first bond and the remaining Rs 25,000 in the second bond.

Question 20

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution

Ans: The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books. The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60 and Rs 40 . The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$\begin{aligned} &\Rightarrow 12 [10 \quad 8 \quad 10] \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} \\ &\Rightarrow 12[10 \times 80 + 8 \times 60 + 10 \times 40] \\ &\Rightarrow 12(800 + 480 + 400) \\ &\Rightarrow 12(1680) \\ &\Rightarrow 20160 \end{aligned}$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question 21

Assume X, Y, Z, W and P are the matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively. The restriction on n, k and p so that $PY + WY$ will be defined.

A.

$$k = 3, p = n$$

B.

$$k \text{ is arbitrary, } p = 2$$

C.

$$p \text{ is arbitrary, } k = 2$$

D.

$$k = 2, p = 3$$

Solution

Ans: The correct option is Option(A)

Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

Therefore, matrix PY will be defined if $k = 3$.

Consequently, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y , matrix WY is well-defined and is of the order $n \times k$.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$. Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$, are the restrictions on n, k and p so that $PY + WY$ will be defined.

Question 22

Assume X, Y, Z, W and P are the matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively. If $n = p$, then the order of the matrix $7X - 5Z$ is

A.

$$p \times 2$$

B.

$$2 \times n$$

C.

$$n \times 3$$

D.

$$p \times n$$

Answer

Correct option: B

Solution

Ans: The correct answer is B.

Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$ or $2 \times n$ (Because $n = p$)

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

Exercise 3.3

Question 1

Find the transpose of each of the following matrices:

$$(i) \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$$

Solution

Ans:

Find transpose

$$(i) A = \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 6 \\ 6 & 6 & -1 \end{bmatrix}$$

Question 2

If

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

, then verify that

$$(i) (A + B)' = A' + B'$$

$$(ii) (A - B)' = A' - B'$$

Solution

Ans:

We have

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} \quad B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$(i) A + B = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix} \Rightarrow (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that

$$(A + B)' = A' + B'$$

$$(ii) A - B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix} \Rightarrow (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that

$$(A - B)' = A' - B'$$

Question 3

If

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

, then verify that

$$(i) (A + B)' = A' + B'$$

$$(ii) (A - B)' = A' - B'$$

Solution

Ans:

We have

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} \quad B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$(i) A + B = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix} \Rightarrow (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Hence, we have verified that

$$(A + B)' = A' + B'$$

$$(ii) A - B = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} \Rightarrow (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Hence, we have verified that

$$(A - B)' = A' - B'$$

Question 4

If

$$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

, then find

$$(A + 2B)'$$

Solution

Solution coming soon.

Question 5

For the matrix

A & B

, verify that

$$(AB)' = B'A'$$

where

$$(i) A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1 \quad 2 \quad 1]$$

$$(ii) A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = [1 \quad 5 \quad 7]$$

Solution

Ans:

$$(i) AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} [-1 \quad 2 \quad 1] = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \quad -4 \quad 3] = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified

$$(AB)' = B'A'$$

$$\begin{aligned}
 (ii) \quad AB &= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix} \\
 \therefore (AB)' &= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \\
 B'A' &= \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}
 \end{aligned}$$

Hence, we have verified

$$(AB)' = B'A'$$

Question 6

If

$$\begin{aligned}
 (i) \quad A &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}, \text{ then verify that } A^{-1} = A \\
 (ii) \quad A &= \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}, \text{ then verify that } A^{-1} = A
 \end{aligned}$$

Solution

Ans:

Solve for the condition

1.

$$\begin{aligned}
 A &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\
 \therefore A' &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

$$\begin{aligned}
 A &= \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\
 \therefore A' &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\
 A'A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

Question 7

(i) Show that the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix (ii) Show that the matrix

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

is a skew symmetric matrix

Solution

Ans:

(i) We have

$$\begin{aligned}
 A' &= \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A \\
 \therefore A' &= A
 \end{aligned}$$

Hence, A is a symmetric matrix

(ii) We have

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$$\therefore A' = -A$$

Hence, A is a skew-symmetric matrix

Question 8

For the matrix

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

, verify that (i)

$$(A + A')$$

is a symmetric matrix (ii)

$$(A + A')$$

is a skew symmetric matrix

Solution

Ans:

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}, A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$(i) A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

Hence,

$$(A + A')$$

is a symmetric matrix

$$(ii) A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = A - A'$$

Hence,

$$(A - A')$$

is a skew-symmetric matrix

Question 9

Find

$$\frac{1}{2}(A + A') \quad \& \quad \frac{1}{2}(A - A')$$

, when

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Solution

Ans:

The given matrix is

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

, then

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$\begin{aligned}\frac{1}{2}(A + A') &= \frac{1}{2} \left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \right) \\ \frac{1}{2}(A + A') &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \frac{1}{2}(A - A') &= \frac{1}{2} \left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \right) \\ \frac{1}{2}(A - A') &= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\end{aligned}$$

Question 10

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Solution

Ans:

$$\begin{aligned}(i) \frac{1}{2}(A + A') &= \frac{1}{2} \left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right) \\ \frac{1}{2}(A + A') &= \frac{1}{2} \left(\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} \right) \\ \frac{1}{2}(A + A') &= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}\end{aligned}$$

Thus,

$$\frac{1}{2}(A + A')$$

is a symmetric matrix.

$$\begin{aligned}\frac{1}{2}(A - A') &= \frac{1}{2} \left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right) \\ \frac{1}{2}(A - A') &= \frac{1}{2} \left(\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} \right) \\ \frac{1}{2}(A - A') &= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\end{aligned}$$

Thus,

$$\frac{1}{2}(A - A')$$

is a skew-symmetric matrix.

Question 11

If A,B are symmetric matrix of same order, then

$$AB - BA$$

is a

1. Skew symmetric matrix
2. Symmetric matrix
3. Zero matrix
4. Identity matrix

Solution

Ans:

The correct answer is A

$$A \& B$$

are symmetric , therefore , we have

$$A' = A \& B' = B$$

Consider

$$\begin{aligned}
 (AB - BA)' &= (AB)' - (BA)' \\
 &= B'A' - A'B' \\
 &= BA - AB \\
 &= -(AB - BA) \\
 \therefore (AB - BA)' &= -(AB - BA)
 \end{aligned}$$

Thus,

$$(AB - BA)'$$

is a skew-symmetric matrix

Question 12

If

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

, then

$$A + A' = I$$

, if the value of

$$\alpha$$

is

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{3}$
- C. π
- D. $\frac{3\pi}{2}$

Solution

Ans:

The correct answer is B

$$\begin{aligned}
 A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\
 A' &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\
 A + A' &= I \\
 \therefore \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \begin{bmatrix} 2 \cos \alpha & 0 \\ 0 & 2 \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \alpha &= \frac{\pi}{3}
 \end{aligned}$$

Exercise 3.4

Question 1

Matrices

A

and

B

will be inverse of each other only if

1. $AB = BA$
2. $AB = BA = 0$
3. $AB = 0, BA = I$
4. $AB = BA = I$

Solution

Ans: Since, if

A

is a square matrix of order

m

, and if there exists another square matrix

$$B$$

of the same order

$$m$$

, such that

$$AB = BA = I$$

, then

$$B$$

is said to be the inverse of

$$A$$

. In such a case, it is clear that

$$A$$

is the inverse of

$$B$$

.

Thus, matrices

$$A$$

and

$$B$$

will be inverse of each other only if

$$AB = BA = I$$

.

Thus, option (D) is correct.

Miscellaneous Exercise

Question 1

If

$$A$$

and

$$B$$

are symmetric matrices, prove that

$$AB - BA$$

is a skew symmetric matrix.

Solution

Ans:

$$A$$

and

$$B$$

are symmetric matrix, therefore, we have:

$$A' = A$$

and

$$B' = B$$

...(1)

Here,

$$(AB - BA)' = (AB)' - (BA)'$$

$$\Rightarrow (AB - BA)' = B'A' - A'B'$$

From (1),

$$\Rightarrow (AB - BA)' = BA - AB$$

$$\Rightarrow (AB - BA)' = -(AB - BA)$$

...(2)

From equation (2),

$$AB - BA$$

is a skew symmetric matrix.

Question 2

Show that the matrix

$$B'AB$$

is symmetric or skew symmetric accordingly as

$$A$$

is symmetric or skew symmetric.

Solution

Ans: Let

$$A$$

be a symmetric matrix, then

$$A' = A$$

...(1)

$$(B'AB)' = \{B'(AB)\}'$$

$$\Rightarrow (B'AB)' = (AB)'B'$$

$$\Rightarrow (B'AB)' = B'(A'B)$$

From (1),

$$\Rightarrow (B'AB)' = B'(AB)$$

Thus, if

$$A$$

is a symmetric matrix, then

$$B'AB$$

is a symmetric matrix.

Let

$$A$$

be a skew symmetric matrix, then

$$A' = -A$$

...(2)

$$(B'AB)' = \{B'(AB)\}'$$

$$\Rightarrow (B'AB)' = (AB)'B'$$

$$\Rightarrow (B'AB)' = B'(A'B)$$

From (2),

$$\Rightarrow (B'AB)' = B'(-AB)$$

$$\Rightarrow (B'AB)' = -B'AB$$

Thus, if

$$A$$

is a skew-symmetric matrix, then

$$B'AB$$

is a skew-symmetric matrix.

Therefore, the matrix

$$B'AB$$

is symmetric or skew symmetric accordingly as

$$A$$

is symmetric or skew symmetric.

Question 3

Find the values of x, y, z if the matrix $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfy the equation $A'A = I$

Solution

Ans: Given matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

Transpose of $A = A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Given $A'A = I$

$$= \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 + x^2 + x^2 & 0 + xy - xy & 0 - xz + xz \\ 0 + xy - xy & 4y^2 + y^2 + y^2 & 2yz - yz - yz \\ 0 - zx + zx & 2yz - yz - yz & z^2 + z^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To find the values of x, y, z we have to equate the entries with corresponding matrix

$$2x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$6y^2 = 1$$

$$\Rightarrow y^2 = \frac{1}{6}$$

$$\Rightarrow y = \pm \frac{1}{\sqrt{6}}$$

And

$$3z^2 = 1$$

$$\Rightarrow z^2 = \frac{1}{3}$$

$$\Rightarrow z = \pm \frac{1}{\sqrt{3}}$$

Question 4

For what values of

$$x$$

,

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

?

Solution

Ans: Given

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = O$$

$$\Rightarrow [4 + 4x] = [0]$$

$$\Rightarrow 4 + 4x = 0$$

$$\therefore x = -1$$

Thus, the required value of

$$x$$

is

$$-1$$

.

Question 5

If

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

, show that

$$A^2 - 5A + 7I = O$$

.

Solution

Ans: Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3(3) + 1(-1) & 3(1) + 1(2) \\ -1(3) + 2(-1) & -1(1) + 2(2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

LHS:

$$A^2 - 5A + 7I$$

$$\therefore \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow O$$

RHS:

$$\Rightarrow O$$

$$LHS = RHS$$

$$\therefore A^2 - 5A + 7I = O$$

hence proved.

Question 6

Find

$$X$$

, if

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

.

Solution

Ans: Given

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x(x-2) - 40 + 2x - 8] = O$$

$$\Rightarrow [x^2 - 48] = [0]$$

$$\Rightarrow x^2 - 48 = 0$$

$$\therefore x = \pm 4\sqrt{3}$$

Thus, the required value of

$$x$$

is

$$\pm 4\sqrt{3}$$

Question 7

A manufacture produces three products

X, Y, Z

which he sells in two markets.

Annual sales are indicated below:

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of

X, Y, Z

are Rs

2.50

, Rs

1.50

and Rs

1.00

, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs

2.00

, Rs

1.00

and

50

paise respectively. Find the gross profit.

Solution

Ans: Here the total revenue in market

I

can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$\Rightarrow 1000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$$

$$\Rightarrow 46000$$

And, the total revenue in market

II

can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$\Rightarrow 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$$

$$\Rightarrow 53000$$

Thus, the total revenue in market

I

is Rs

46000

and the same in market

II

is Rs

53000

.

Ans: Here, the total cost prices of all the products in the market

I

can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$\Rightarrow 1000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50$$

$$\Rightarrow 31000$$

As the total revenue in market

I

is Rs

$$46000$$

, the gross profit in this market is

$$Rs46000 - Rs31000 = Rs15000$$

.

And, the total revenue in market

II

can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$\Rightarrow 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50$$

$$\Rightarrow 36000$$

As the total revenue in market

II

is Rs

$$53000$$

, the gross profit in this market is

$$Rs53000 - Rs36000 = Rs17000$$

.

Question 8

Find the matrix

$$X$$

so that

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

.

Solution

Ans: Given

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Here,

$$X$$

has to be a

$$2 \times 2$$

matrix.

Now, let

$$X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Thus, we have:

$$\begin{aligned} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements of two matrices, we have:

$$a + 4c = -7$$

$$2a + 5c = -8$$

$$3a + 6c = -9$$

$$b + 4d = 2$$

$$2b + 5d = 4$$

$$3b + 6d = 6$$

Now, solving the above equations we get,

$$\therefore a = 1, b = 2, c = -2, d = 0$$

Thus, the required matrix

$$X$$

is

$$\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

.

Question 9

Choose the correct answer in the following questions:

If

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

is such that

$$A^2 = I$$

then

- | | |
|----|----------------------------------|
| 1. | $1 + \alpha^2 + \beta\gamma = 0$ |
| 2. | $1 - \alpha^2 + \beta\gamma = 0$ |
| 3. | $1 - \alpha^2 - \beta\gamma = 0$ |
| 4. | $1 + \alpha^2 - \beta\gamma = 0$ |

Solution

Ans: Given

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\begin{aligned}\therefore A^2 &= A \cdot A \\ \Rightarrow A^2 &= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \\ \Rightarrow A^2 &= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}\end{aligned}$$

Now,

$$\begin{aligned}A^2 &= I \\ \Rightarrow \begin{bmatrix} \alpha^2 + \gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\end{aligned}$$

Equating the corresponding elements, we get:

$$\begin{aligned}\beta\gamma + \alpha^2 &= 1 \\ \Rightarrow \alpha^2 + \beta\gamma - 1 &= 0 \\ \Rightarrow 1 - \alpha^2 - \beta\gamma &= 0\end{aligned}$$

Question 10

If the matrix

$$A$$

is both symmetric and skew symmetric, then

1. A
is a diagonal matrix
2. A
is a zero matrix
3. A
is a square matrix
4. None of these

Solution

Ans: If a matrix

$$A$$

is both symmetric and skew symmetric, then

$$A' = A$$

and

$$A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = O$$

$$\Rightarrow A = O$$

Thus, option (B) is correct.

Question 11

If

$$A$$

is a square matrix such that

$$A^2 = A$$

, then

$$(I + A)^3 - 7A$$

is equal to

- | | |
|----|---------|
| 1. | A |
| 2. | $I - A$ |
| 3. | I |
| 4. | $3A$ |

Solution

Ans:

$$\begin{aligned}(I + A)^3 - 7A &= I^3 + A^3 + 3A + 3A^2 - 7A \\ \Rightarrow (I + A)^3 - 7A &= I + A^3 + 3A + 3A^2 - 7A\end{aligned}$$

Given that

$$A^2 = A$$

,

$$\Rightarrow (I + A)^3 - 7A = I + A^2 \cdot A + 3A + 3A^2 - 7A$$

$$\Rightarrow (I + A)^3 - 7A = I + A \cdot A - A$$

$$\Rightarrow (I + A)^3 - 7A = I$$

Thus, option (C) is correct.
