

Chapter 13: Probability

NCERT · Class 12 · Maths

Exercise 13.1

Question 1

Given that E and F are events such that $P(E) = 0.6$, and $P(F) = 0.3$

$$P(E \cap F) = 0.2$$

, find $P(E|F)$ and $P(F|E)$.

Solution

Ans: It is given in the question that $P(E) = 0.6$, $P(F) = 0.3$, and $P(E \cap F) = 0.2$

Now $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0.2}{0.3}$$

Hence we found that

$$P(E|F) = \frac{2}{3}$$

With similar idea and the same formula we can proceed to find $P(F|E)$ as shown

$$P(F|E) = \frac{P(F \cap E)}{P(E)}$$

Now we know that

$$P(F \cap E)$$

and $P(E \cap F)$ is same

$$\therefore P(F|E) = \frac{0.2}{0.6}$$

Hence we found that

$$P(F|E) = \frac{1}{3}$$

Thus $P(E|F) = \frac{2}{3}$ and $P(F|E) = \frac{1}{3}$

Question 2

Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

Solution

Ans: Given in the question $P(B) = 0.5$ and $P(A \cap B) = 0.32$

So to find $P(A|B)$ we use the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{0.32}{0.5}$$

$$\frac{32}{50}$$

$$\frac{16}{25}$$

Hence we found that

$$P(A|B) = \frac{16}{25}$$

Question 3

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$ find

(i) $P(A \cap B)$

(iii) $P(A|B)$

(iii) $P(A \cup B)$

Solution

Ans: It is given that $P(A) = 0.8$, $P(B) = 0.5$, and

$$P(B|A) = 0.4$$

Put all the data in the following formula

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = 0.4 \times 0.8$$

Thus we found that $P(A \cap B) = 0.32$

Ans: Given in the question $P(A) = 0.8$, $P(B) = 0.5$, and

$$P(B|A) = 0.4$$

We know that

$$P(A|B)$$

is the probability of occurrence of A when B has already happened

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now put $P(A \cap B) = 0.32$, $P(B) = 0.5$ in the above equation as shown

$$\Rightarrow P(A|B) = \frac{0.32}{0.5}$$

$$= 0.64$$

Thus we found that $P(A|B) = 0.64$

Ans: Given in the question $P(A) = 0.8$, $P(B) = 0.5$, $P(B) = 0.5$ and

$$P(B|A) = 0.4$$

Now we have the formula as shown

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Put $P(A) = 0.8$,

$$P(B) = 0.5,$$

$$P(A \cap B) = 0.32 \text{ in the above as shown}$$

$$\therefore P(A \cup B) = 0.8 + 0.5 - 0.32$$

$$\Rightarrow P(A \cup B) = 0.98$$

Thus we found that $P(A \cup B) = 0.98$

Question 4

Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Solution

Ans: It is Given that

$$2P(A) = P(B) = \frac{5}{13} \text{ and}$$

$$P(A|B) = \frac{2}{5}$$

$$\Rightarrow P(A) = \frac{5}{26}$$

Now we know the formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{2}{5} = \frac{13P(A \cap B)}{5} \text{ (since } P(B) = \frac{5}{13}, P(A|B) = \frac{2}{5} \text{)}$$

$$\Rightarrow P(A \cap B) = \frac{2}{13}$$

Also $P(A \cup B)$ is given by the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$\Rightarrow P(A \cup B) = \frac{11}{26}$$

Thus we found that $P(A \cup B) = \frac{11}{26}$

Question 5

If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ find

(i) $P(A \cap B)$

(ii) $P(A|B)$

(iii) $P(B|A)$

Solution

Ans: Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

Also it is given that

$$P(A \cup B) = \frac{7}{11}$$

And we know that it is given by the formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{11}{11} - \frac{7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

Thus we found that $P(A \cap B) = \frac{4}{11}$

Ans: Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

Also we know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{4}{11} \times \frac{11}{5}$$

(since $P(A \cap B) = \frac{4}{11}$)

Thus we found that

$$P(A|B) = \frac{4}{5}$$

Ans: Given $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$

Also we know that $P(B|A)$ is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{4}{11} \times \frac{11}{6}$$

Thus we found that $P(B|A) = \frac{2}{3}$

Question 6

A coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail.

Solution

Ans: Sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$$E = \{HHH, HTH, THH, TTH\}$$

Therore $P(E) = \frac{4}{8}$

$$F = \{HHH, HHT\}$$

Therore $P(F) = \frac{2}{8}$

$$\therefore E \cap F = \{HHH\}$$

Therore $P(E \cap F) = \frac{1}{8}$

And hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{8}}{\frac{2}{8}}$$

Thus

$$P(E|F) = \frac{1}{2}$$

Ans: Sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$$E = \{HHH, HHT, THH, HTH\}$$

Therore $P(E) = \frac{4}{8}$

$$F = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Therore $P(F) = \frac{7}{8}$

$$E \cap F = \{HHT, HTH, THH\}$$

Therore $P(E \cap F) = \frac{3}{8}$

And hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{3}{8}}{\frac{7}{8}}$$

Thus

$$P(E|F) = \frac{3}{7}$$

Ans: Sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ and the events E and F and their probabilities are given by

$$E = \{HHH, HHT, THH, HTH, THH, THT, TTH\}$$

Therore $P(E) = \frac{7}{8}$

$$F = \{HHT, HTT, HTH, THH, THT, TTH, TTT\}$$

Therore $P(F) = \frac{7}{8}$

$$E \cap F = \{HHT, HTT, HTH, THH, THT, TTH\}$$

Therore $P(E \cap F) = \frac{6}{8}$

And hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{6}{8}}{\frac{7}{8}}$$

Thus

$$P(E|F) = \frac{6}{7}$$

Question 7

Two coins are tossed once, where

(i) E: tail appears on one coin, F: one coin shows head

(ii) E: not tail appears, F: no head appears

Solution

Ans: Sample Space is given by

$$S = \{HH, HT, TH, TT\}$$

The events E and F and their probabilities are given by

$$E = \{HT, TH\}$$

$$\text{Therefore } P(E) = \frac{2}{4}$$

$$F = \{HT, TH\}$$

$$\text{Therefore } P(F) = \frac{2}{4}$$

Also $E \cap F$ is given by

$$E \cap F = \{HT, TH\}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{2}{4}}{\frac{2}{4}}$$

Thus we found that $P(E|F) = 1$

That is, $(E|F)$ is a sure event

Ans: Sample Space is given by

$$S = \{HH, HT, TH, TT\}$$

The events E and F and their probabilities are given by

$$E = \{HH\}$$

$$\text{Therefore } P(E) = \frac{1}{4}$$

$$F = \{TT\}$$

$$\text{Therefore } P(F) = \frac{1}{4}$$

Therefore $E \cap F$ is given by

$$E \cap F = \phi$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{0}{\frac{1}{4}}$$

Thus we found that $P(E|F) = 0$

Question 8

A die is thrown three times,

E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses

Solution

Ans: Number of elements in Sample space is given by 216

The events E and F and their probabilities are given by

$$E = \left\{ \begin{array}{l} (1, 1, 4), (1, 2, 4) \dots\dots (1, 6, 4) \\ (2, 1, 4), (2, 2, 4) \dots\dots (2, 6, 4) \\ (3, 1, 4), (3, 2, 4) \dots\dots (3, 6, 4) \\ (4, 1, 4), (4, 2, 4) \dots\dots (4, 6, 4) \\ (5, 1, 4), (5, 2, 4) \dots\dots (5, 6, 4) \\ (6, 1, 4), (6, 2, 4) \dots\dots (6, 6, 4) \end{array} \right\}$$

$$F = \{ (6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6) \}$$

$$\text{Therefore } P(F) = \frac{6}{216}$$

$$\text{And hence } E \cap F = \{ (6, 5, 4) \}$$

$$\text{Therefore } P(E \cap F) = \frac{1}{216}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{216}}{\frac{6}{216}}$$

$$\text{Thus } P(E|F) = \frac{1}{6}$$

Question 9

Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

Solution

Ans: Let mother (M), father (F), and son (S) line up for the family picture, then the sample space will be as shown

$$A = \{ MFS, MSF, FMS, FSM, SMF, SFM \}$$

The events E and F and their probabilities are

$$E = \{ MFS, FMS, SMF, SFM \}$$

$$F = \{ MFS, SFM \}$$

Therefore $P(F) = \frac{2}{6}$

Hence $E \cap F = \{MFS, SFM\}$

Therefore $P(E \cap F) = \frac{2}{6}$

We know that $P(E|F)$ is given by

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{2}{6}}{\frac{2}{6}}$$

Thus $P(E|F) = 1$

Question 10

A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

Solution

Ans: Let the first observation come from black die and second from red die respectively

In the case when two dices are rolled the elements in sample space is 36

The events A and B and their probabilities are given by

$$A = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (4, 6), (6, 6)\}$$

Where A is the event when sum is greater than 9

Similarly,

$$B = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$$

Where B is the event when black die resulted in a 5

Therefore $P(B) = \frac{6}{36}$

And hence $A \cap B = \{(5, 5), (5, 6)\}$

Therefore the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5 $P(A|B)$ is given by

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{\frac{2}{36}}{\frac{6}{36}}$$

$$\text{Thus } P(A|B) = \frac{1}{3}$$

1. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans: Let E and F be the events and their probabilities defined as

E: Sum of the observations is 8

$$E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

F: red die resulted in a number less than 4

$$F = \left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3) \\ (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3) \\ (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3) \end{array} \right\}$$

$$\text{Therefore } P(F) = \frac{18}{36}$$

$$\text{And hence } E \cap F = \{(5, 3), (6, 2)\}$$

$$\text{Therefore } P(E \cap F) = \frac{2}{36}$$

The conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4 is given $P(E|F)$ as shown

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{2}{36}}{\frac{18}{36}}$$

$$\text{Thus } P(E|F) = \frac{1}{9}$$

Question 11

A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$. Find

(i) $P(E|F)$ and $P(F|E)$

(iii) $P((E \cup G)|G)$ and $P((E \cap G)|G)$

Solution

Ans: The sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

It is given in the question

$$E = \{1, 3, 5\}$$

$$F = \{2, 3\}$$

$$\text{Therefore } E \cap F = \{3\}$$

$$\text{Hence } P(E \cap F) = \frac{1}{6}$$

$$\therefore P(E) = \frac{3}{6}$$

$$\therefore P(F) = \frac{2}{6}$$

Hence $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{6}}{\frac{2}{6}}$$

Similarly $P(F|E)$ is given by

$$P(F|E) = \frac{\frac{1}{6}}{\frac{3}{6}}$$

$$\text{Thus } P(E|F) = \frac{1}{2} \text{ and } P(F|E) = \frac{1}{3}$$

(ii). $P(E|G)$ and $P(G|E)$

It is given in the question

$$E = \{1, 3, 5\}$$

$$\therefore P(E) = \frac{3}{6}$$

$$G = \{2, 3, 4, 5\}$$

$$\therefore P(G) = \frac{4}{6}$$

$$\therefore E \cap G = \{3, 5\}$$

$$\therefore P(E \cap G) = \frac{2}{6}$$

$$\text{Therefore } P(E|G) = \frac{(E \cap G)}{P(G)}$$

$$\Rightarrow P(E|G) = \frac{\frac{2}{6}}{\frac{4}{6}}$$

$$\text{Thus } P(E|G) = \frac{1}{2}$$

Similarly

$$P(G|E) = \frac{\frac{2}{6}}{\frac{3}{6}}$$

$$\text{Thus } P(G|E) = \frac{2}{3}$$

$$\text{Therefore, } P(E|G) = \frac{1}{2} \text{ and } P(G|E) = \frac{2}{3}$$

Ans: The sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$G = \{2, 3, 4, 5\}$$

$$\text{We have } E \cup F = \{1, 2, 3, 5\}$$

Therefore

$$\begin{aligned}(E \cup F) \cap G &= \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} \\ &= \{2, 3, 5\}\end{aligned}$$

Also

$$\begin{aligned}(E \cap F) \cap G &= \{1, 2, 3, 5\} \cap \{3\} \\ &= \{3\}\end{aligned}$$

$$\therefore P(E \cup F) \cap G = \frac{3}{6}$$

$$\therefore P(E \cap F) \cap G = \frac{1}{6}$$

$$\therefore P(E \cup F | G) = \frac{P(E \cup F) \cap G}{P(G)}$$

$$\Rightarrow P(E \cup F | G) = \frac{\frac{3}{6}}{\frac{4}{6}}$$

$$\text{Thus } P(E \cup F | G) = \frac{3}{4}$$

Similarly

$$P(E \cap F | G) = \frac{\frac{1}{6}}{\frac{4}{6}}$$

$$\text{Thus, } P(E \cup F | G) = \frac{3}{4} \text{ and } P(E \cap F | G) = \frac{1}{4}$$

Question 12

Assume that each born child is equally like to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- i. The youngest is a girl
- ii. At least one is a girl

Solution

Ans: The sample space for a family having two children is given by

$$S = \{(B, B), (B, G), (G, G), (G, B)\}$$

Where B refers to boy child and G refers to girl child

Let an event be defined as

E: Both children are girls

$$E = \{(GG)\}$$

$$\therefore P(E) = \frac{2}{4}$$

Let F be an event defined as

F: youngest child is girl

$$F = \{(BG), (GG)\}$$

$$\therefore E \cap F = \{(GG)\}$$

$$\therefore P(E \cap F) = \frac{1}{4}$$

We know that $P(E|F)$ is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$\Rightarrow P(E|F) = \frac{\frac{1}{4}}{\frac{2}{4}}$$

$$\text{Thus } P(E|F) = \frac{1}{2}$$

Ans: The sample space for a family having two children is given by

$$S = \{(B, B), (B, G), (G, G), (G, B)\}$$

Where B refers to boy child and G refers to girl child

Let an event be defined as

E: Both children are girls

$$E = \{(GG)\}$$

$$\therefore P(E) = \frac{2}{4}$$

Let A be an event defined as

$$A = \{(B, G), (G, B), (G, G)\}$$

$$\therefore P(A) = \frac{3}{4}$$

$$\therefore E \cap A = \{(G, G)\}$$

$$\therefore P(E \cap A) = \frac{1}{4}$$

We know that

$P(E|A)$ is given by

$$P(E|A) = \frac{P(E \cap A)}{P(A)}$$

$$\Rightarrow P(E|A) = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$\text{Thus } P(E|A) = \frac{1}{3}$$

Question 13

An instructor has a bank consisting of 300 easy True/False s, 200 difficult True/False s, 500 easy multiple choice s and 400 difficult multiple choice s. If a is selected at random from the bank, what is the probability that it will be an easy given that it is a multiple choice ?

Solution

Ans: The given data can be represented as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us have following notations

E for easy questions, M for multiple questions, D for difficult questions and T for true/false questions

It is given that total number of questions is 1400, that of multiple questions is 900

Hence probability for selecting easy multiple choice questions is given by

$$P(E \cap M) = \frac{5}{14} \text{ (since } \frac{500}{1400} = \frac{5}{14} \text{)}$$

Probability for selecting multiple choice questions is given by

$$P(M) = \frac{9}{14} \text{ (since } \frac{90}{1400} = \frac{9}{14} \text{)}$$

$$\therefore P(E|M) = \frac{P(\cap M)}{P(M)}$$

$$\Rightarrow P(E|M) = \frac{\frac{5}{14}}{\frac{9}{14}}$$

$$\text{Thus } P(E|M) = \frac{5}{9}$$

Question 14

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Solution

Ans: Let A and B be events defined as

A: the sum of the numbers on the dice is 4

B: the two numbers appearing on throwing the two dice are different.

$$\therefore A = \{(1, 3), (2, 3), (3, 1)\}$$

$$\therefore B = \left\{ \begin{array}{l} (1, 2), (1, 3), \dots, (1, 6) \\ (2, 1), (2, 2), \dots, (2, 6) \\ (3, 1), (3, 2), \dots, (3, 6) \\ (4, 1), (4, 2), \dots, (4, 6) \\ (5, 1), (5, 2), \dots, (5, 6) \\ (6, 1), (6, 2), \dots, (6, 6) \end{array} \right\}$$

$$\text{Therefore } P(B) = \frac{30}{36}$$

$$\therefore A \cap B = \{(1, 3), (3, 1)\}$$

$$P(A \cap B) = \frac{2}{36}$$

We know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{\frac{2}{36}}{\frac{30}{36}}$$

Thus

$$P(A|B) = \frac{1}{15}$$

Question 15

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows as 3'.

Solution

Ans: For this case the sample space is given by

$$S = \left\{ (1, H), (2, H), (1, T), (1, T), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, H), (4, T), (5, H), (5, T), (6, 1), (6, 2), \dots, (6, 6) \right\}$$

Let A and B be events defined as

A: The coin shows tail

B: at least one die shows 3

$$\therefore A = \{(1, T), (2, T), (4, T), (5, T)\}$$

$$\therefore B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$$

$$\therefore P(B) = \frac{7}{36}$$

Also it is observable that $A \cap B = \phi$

We know that $P(A|B)$ given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{0}{\frac{7}{36}}$$

$$\text{Thus } P(A|B) = 0$$

Question 16

If $P(A) = \frac{1}{2}$ and $P(B) = 0$ then $P(A|B)$ is

1. 0
2. $\frac{1}{2}$
3. Not defined

4. 1

Solution

Ans: It is given that $P(A) = \frac{1}{2}$ and $P(B) = 0$

We know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{0}$$

Thus $P(A|B)$ is not defined

Question 17

If A and B are events such that $P(A|B) = P(B|A)$ then

(A) $A \subset B$ but $A \neq B$ (B) $A = B$ (C) $A \cap B = \phi$ (D) $P(A) = P(B)$

Solution

Ans: It is given in the question that

$$P(A|B) = P(B|A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \frac{P(A)}{1} = \frac{P(B)}{1}$$

Thus we found that $P(A) = P(B)$

Exercise 13.2**Question 1**

if $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events

Solution

Ans: It is given in the question that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

Also it is given that A and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \cap B) = \frac{3}{5} \times \frac{1}{5}$$

$$\text{Thus } P(A \cap B) = \frac{3}{25}$$

Question 2

Two cards are drawn at random and without replacement from a pack of 2 playing cards. Find the probability that both the cards are black.

Solution

Ans: Given a deck of 52 cards

We know that the number of black cards are 26

Let us have a notation $P(A)$ denote the probability of picking a black card at first chance

$$\therefore P(A) = \frac{26}{52}$$

Similarly, let us have a notation $P(B)$ denote the probability of picking a black card at second chance.

Since there is no replacement

$$\therefore P(B) = \frac{25}{51}$$

The probability that the both the cards are black is

$$\begin{aligned} P(A \cap B) &= \frac{1}{2} \times \frac{25}{51} \\ &= \frac{25}{102} \end{aligned}$$

The probability that the both the cards are black is $\frac{25}{102}$

Question 3

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Solution

Ans: Let A,B and C be the events and their probabilities defined as

A: the first orange is good

$$\therefore P(A) = \frac{12}{15}$$

B: the second orange is good

$$\therefore P(B) = \frac{11}{14}$$

C: the third orange is good

$$\therefore P(C) = \frac{10}{13}$$

It is given that the box is approved for sale only when all the oranges are good.

Probability that the box is approved for sale=probability of all oranges to be good

$$\text{Therefore probability of all oranges to be good} = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$

$$\text{Thus probability that the box is approved for sale} = \frac{44}{91} = 0.48$$

Question 4

A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event 3 on the die'. Check whether A and B are independent events or not.

Solution

Ans: The sample space is given by

$$S = \left\{ (H, 1), (H, 2), \dots, (H, 6) \right\} \\ \left\{ (T, 1), (T, 2), \dots, (T, 6) \right\}$$

According to the question

$$A = \{(H, 1), (H, 2), \dots, (H, 6)\}$$

$$\therefore P(A) = \frac{6}{12}$$

$$B = \{(H, 3), (T, 3)\}$$

$$\therefore P(B) = \frac{2}{12}$$

$$\text{Also } (A \cap B) = \{(H, 3)\}$$

$$\therefore P(A \cap B) = \frac{1}{12}$$

We know that for A and B to be independent events

$$P(A \cap B) = P(A) P(B)$$

$$\text{Now } P(A) P(B) = \frac{6}{12} \times \frac{2}{12}$$

$$\Rightarrow P(A) P(B) = \frac{1}{12}$$

Which is as same as $P(A \cap B)$

Thus A and B are independent events

Question 5

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the events, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

Solution

Ans: The sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

According to the question

$$A = \{2, 4, 6\}$$

$$\therefore P(A) = \frac{3}{6}$$

$$B = \{1, 2, 3\}$$

$$\therefore P(B) = \frac{3}{6}$$

$$\text{Also } (A \cap B) = \{2\}$$

$$\therefore P(A \cap B) = \frac{1}{6}$$

We know that for A and B to be independent events

$$P(A \cap B) = P(A) P(B)$$

$$\text{Now } P(A) P(B) = \frac{3}{6} \times \frac{3}{6}$$

$$\Rightarrow P(A) P(B) = \frac{1}{4}$$

Which is not as same as $P(A \cap B)$

Thus A and B are not independent events

Question 6

Let E and F be the vents with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$

Are E and F independent?

Solution

$$\text{Ans: Given } P(E) = \frac{3}{5}, P(F) = \frac{3}{10} \text{ and } P(E \cap F) = \frac{1}{5}$$

We know that for A and B to be independent events

$$P(A \cap B) = P(A) P(B)$$

$$\text{Now } P(A)P(B) = \frac{3}{5} \times \frac{3}{10}$$

$$\Rightarrow P(A)P(B) = \frac{9}{50}$$

$$\text{Also } P(E \cap F) = \frac{1}{5}$$

$$\text{Thus } P(A)P(B) \neq P(E \cap F)$$

And hence A and B are not independent events

Question 7

Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$ find p if they are

(i) Mutually Exclusive

(ii) Independent

Solution

Ans: It is given in the question that $P(A) = \frac{1}{2}$, $P(B) = p$ and $P(A \cup B) = \frac{3}{5}$

We know that if two events A and B are mutually exclusive then $P(A \cap B) = 0$

Also we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0$$

$$\Rightarrow p = \frac{3}{5} - \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{10}$$

Ans: It is given in the question that $P(A) = \frac{1}{2}$, $P(B) = p$ and $P(A \cup B) = \frac{3}{5}$

We know that if two events A and B are independent then $P(A \cap B) = P(A)P(B)$

Also we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{p}{2} \text{ (since } P(A)P(B) = \frac{p}{2}\text{)}$$

$$\Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2}$$

$$\Rightarrow p = \frac{2}{10}$$

$$= \frac{1}{5}$$

The value of p when A and B are independent event is $\frac{1}{5}$

Question 8

Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$ find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A|B)$

(iv) $P(B|A)$

Solution

Ans: It is given that A and B are independent events with given probabilities as shown

$$P(A) = 0.3$$

$$P(B) = 0.4$$

We know that if A and B are independent events then

$$P(A \cap B) = P(A)P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4$$

$$\Rightarrow P(A \cap B) = 0.12$$

Ans: It is given that A and B are independent events with given probabilities as shown

$$P(A) = 0.3$$

$$P(B) = 0.4$$

We know that if A and B are independent events then

$$P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4$$

$$\Rightarrow P(A \cap B) = 0.12$$

Also we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12$$

$$\Rightarrow P(A \cup B) = 0.58$$

Ans: It is given that A and B are independent events with given probabilities as shown

$$P(A) = 0.3$$

$$P(B) = 0.4$$

We know that if A and B are independent events then

$$P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4$$

$$\Rightarrow P(A \cap B) = 0.12$$

Also we know that $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.12}{0.4}$$

$$= 0.3$$

Thus we found that $P(A|B) = 0.3$

Ans: It is given that A and B are independent events with given probabilities as shown

$$P(A) = 0.3$$

$$P(B) = 0.4$$

We know that if A and B are independent events then

$$P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4$$

$$\Rightarrow P(A \cap B) = 0.12$$

Also we know that

$$P(B|A)$$

is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{0.12}{0.3}$$

$$= 0.4$$

Thus we found that $P(B|A) = 0.4$

Question 9

If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ find $P(\text{not A and not B})$

Solution

Ans: It is given that two events A and b have probabilities as shown

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{2} \text{ and}$$

$$P(A \cap B) = \frac{1}{8}$$

$$P(\text{not A and not B}) = P(A' \cap B')$$

And we know that

$$P(A' \cap B') = P(A \cup B)'$$

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$\text{Also } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$\Rightarrow P(A \cup B) = \frac{5}{8}$$

$$\therefore P(A' \cap B') = 1 - \frac{5}{8}$$

$$\text{Thus } P(\text{not A and not B}) = \frac{3}{8}$$

Question 10

Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not A or not B}) = \frac{1}{4}$. State whether A and B are independent

Solution

Ans: It is given that $P(A) = \frac{1}{2}$,

$$P(B) = \frac{7}{12}$$

and

$$P(A' \cup B') = \frac{1}{4}$$

According to the question

$$1 - P(A \cap B) = \frac{1}{4}$$

$$\Rightarrow P(A \cap B) = \frac{3}{4}$$

We know that A and B are independent events if

$$P(A)P(B) = P(A \cap B)$$

$$\therefore P(A)P(B) = \frac{1}{2} \times \frac{7}{12}$$

$$\text{Clearly } P(A)P(B) \neq P(A \cap B)$$

Thus A and B are not independent events

Question 11

Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find

- (i) $P(A \text{ and } B)$
- (ii) $P(A \text{ and not } B)$
- (iii) $P(A \text{ or } B)$
- (iv) $P(\text{neither } A \text{ and nor } B)$

Solution

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \text{ and } B) = 0.3 \times 0.6$$

Thus we found that $P(A \text{ and } B) = 0.18$

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

Now we know that

$$P(A \text{ and not } B) = P(A \cap B')$$

Also we know that

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A \cap B') = 0.3 - 0.18$$

Thus we found that $P(A \text{ and not } B) = 0.12$

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \text{ and } B) = 0.3 \times 0.6$$

Thus we found that $P(A \text{ and } B) = 0.18$

Also we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.3 + 0.6 - 0.18$$

Thus we know that $P(A \text{ or } B) = 0.72$

Ans: It is given that $P(A) = 0.3$, $P(B) = 0.6$

Where A and B are independent events

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \text{ and } B) = 0.3 \times 0.6$$

Thus we found that $P(A \text{ and } B) = 0.18$

Also we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.3 + 0.6 - 0.18$$

Thus we know that $P(A \text{ or } B) = 0.72$

$$P(\text{neither A nor B}) = P(A \cup B)'$$

$$\text{And } P(A \cup B)' = 1 - P(A \cup B)$$

$$P(A \cup B)' = 1 - 0.72$$

Thus $P(\text{neither A nor B}) = 0.28$

Question 12

A die tossed thrice. Find the probability of getting an odd number at least once.

Solution

Ans: It is given that a die is tossed thrice, the number of elements in sample space for each throw is 6

Now probability of getting atleast odd once = 1 - probability of getting even numbers in all three events

Now we know that

Probability of getting even numbers in all three events = $\left(\frac{1}{2}\right)^3$

Probability of getting at least odd once = $1 - \frac{1}{8}$

Thus probability of getting at least odd once = $\frac{7}{8}$

Question 13

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls.

Find the probability that

(i) Both balls are red.

(ii) First ball is black and the second is red.

(iii) One of them is black and the other is red.

Solution

Ans: It is given in the question that

Number of red balls = 8 and number of black balls = 10

Now probability of getting red ball at first draw, say $P(A) = \frac{8}{18} = \frac{4}{9}$

Also probability of getting red ball at second draw, say $(P) = \frac{8}{18} = \frac{4}{9}$

$\therefore P(A \cap B) = \frac{4}{9} \times \frac{4}{9}$ (since the events are independent)

$$= \frac{16}{81}$$

The probability of getting both balls red is $\frac{16}{81}$

Ans: It is given in the question that

Number of red balls = 8 and number of black balls = 10

Now probability of getting black ball at first draw = $\frac{10}{18} = \frac{5}{9}$

Also probability of getting red ball at second draw = $\frac{8}{18} = \frac{4}{9}$

Also above two events are independent therefore their intersection is given by

$$= \frac{5}{9} \times \frac{4}{9}$$
$$= \frac{20}{81}$$

probability of getting black ball in first draw

and red ball in second draw $= \frac{20}{81}$

Ans: Let us have following notations

A: first ball is black and the other is red

Now probability of getting a black ball in first draw, $P(A) = \frac{10}{18}$

$$= \frac{5}{9}$$

As the ball is replaced after first throw, probability of getting a red ball in second draw $= \frac{8}{18}$

$$= \frac{4}{9}$$

Probability of getting first ball as black and second ball as red $= \frac{5}{9} \times \frac{4}{9}$

$$= \frac{20}{81}$$

Probability of drawing red ball in first draw $= \frac{8}{18}$

$$= \frac{4}{9}$$

Probability of getting black ball in second draw $= \frac{10}{81}$

$$= \frac{5}{9}$$

Probability of drawing first ball as red and second as black $= \frac{5}{9} \times \frac{4}{9}$

$$= \frac{20}{81}$$

Probability of getting one of them is black and the other is red $= \frac{20}{81} + \frac{20}{81}$

$$= \frac{40}{81}$$

Probability of getting one of the ball is black and the other is red is $\frac{40}{81}$

Question 14

The probability of solving specific problems independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem

Solution

Ans: Given that the two events A and B are independent with probabilities given as shown

$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{3}$$

$$\therefore P(A \cap B) = P(A) P(B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{6}$$

Probability of problem being solved is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

Thus probability of problem being solved is $\frac{2}{3}$

Ans: Given that the two events A and B are independent with probabilities given as shown

$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{3}$$

Either one of them solved the answer. So either A solve the answer but not B or B solved the answer but not A

$$P(A) \cdot P(B') + P(A') \cdot P(B)$$

$$= \frac{1}{2} \left(\frac{2}{3} \right) + \frac{1}{2} \left(\frac{1}{3} \right)$$

$$= \frac{2}{6} + \frac{1}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Probability of one of them solves the answer is $\frac{1}{2}$

Question 15

One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

Solution

Ans: Given a deck of 52 cards

We know that 13 cards are spades and 4 are aces

$$P(E) = \frac{13}{52} \text{ and } P(F) = \frac{4}{52}$$

Also in the deck only 1 card is ace of spade

$$\text{i.e. } P(E \cap F) = \frac{1}{52}$$

It is clearly visible that

$$P(E) P(F) = \frac{13}{52} \times \frac{4}{52} \text{ equals } P(E \cap F)$$

Hence the events are independent

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

Ans: We know that in the deck of cards 26 cards are black and 4 cards are kings

$$P(E) = \frac{26}{52} \text{ and } P(F) = \frac{4}{52}$$

Also there are 2 cards which are black as well as king

$$\text{i.e. } P(E \cap F) = \frac{2}{52}$$

$$\text{It is clearly visible that } \frac{26}{52} \times \frac{4}{52} = \frac{2}{52}$$

Hence these are independent events

(iii) E: 'the card drawn is a king and queen'

F: 'the card drawn is a queen or jack'

Ans: In a deck of 52 cards each king, queen and jack is 4

$$\therefore P(E) = P(F) = \frac{8}{52}$$

There are 4 cards which are king and queen or jack

Hence its probability is $\frac{4}{52}$

Clearly E and F are not independent

Question 16

In a hostel, 60 percent of the students read Hindi newspapers, 40 percent read English newspapers and 20 percent read both Hindi and English newspapers. A student is selected at random.

1. Find the probability that she reads neither Hindi nor English newspapers.

Solution

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$$\therefore P(H) = 0.6 \text{ and}$$

$$\therefore P(E) = 0.4$$

Also it is given that

$$P(H \cap E) = 0.2$$

Therefore the required probability is given by

$$P(H \cup E') = 1 - P(H \cup E)$$

$$P(H \cup E') = 1 - (P(H) + P(E) - P(H \cap E))$$

$$P(H \cup E') = 1 - 0.8$$

Hence the probability that she reads neither Hindi nor English is 0.2

1. If she reads Hindi news paper, find the probability that she reads English news paper.

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$$\therefore P(H) = 0.6 \text{ and}$$

$$\therefore P(E) = 0.4$$

Also it is given that

$$P(H \cap E) = 0.2$$

Now required probability is given by

$$P(E|H) = \frac{P(H \cap E)}{P(H)}$$
$$\Rightarrow P(E|H) = \frac{0.2}{0.6}$$

Thus the probability that she reads English newspapers if she reads Hindi news papers is $\frac{1}{3}$

1. If she reads English newspapers, find the probability that she reads Hindi newspapers.

Ans: Let us have following notations

H: Students who read Hindi newspapers

E: Students who read English newspapers

$$\therefore P(H) = 0.6 \text{ and}$$

$$\therefore P(E) = 0.4$$

Also it is given that

$$P(H \cap E) = 0.2$$

The required probability is given by

$$P(E|H) = \frac{P(H \cap E)}{P(H)}$$

$$\Rightarrow P(E|H) = \frac{0.2}{0.4}$$

Therefore the probability that she reads Hindi newspapers provided she reads English newspapers is $\frac{1}{2}$

Question 17

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

1. 0
2. $\frac{1}{3}$
3. $\frac{1}{12}$
4. $\frac{1}{36}$

Solution

Ans: Out of six numbers in a die, the only even prime number is 2

Therefore probability of getting an even prime number is given by

$$\text{Say } P(E) = \frac{1}{36}$$

Hence probability of getting an even prime number is $\frac{1}{36}$

Question 18

Two events A and B will be independently, if

1. A and B are mutually exclusive

Solution

Ans: Let $P(A) = p$ and $P(B) = q$ $0 < p, q < 1$

It is given that they are mutually exclusive

$$\therefore P(A \cap B) = 0$$

But $P(A)P(B) = pq$ which is not necessarily zero

Hence they are not independent

$$1. \quad P(A'B') = [1 - P(A)][1 - P(B)]$$

Ans: Two events are said to be independent if $P(AB) = P(A)P(B)$

Let us solve for option B first

$$P(A'B') = [1 - P(A)][1 - P(B)]$$

$$\Rightarrow P(A' \cap B') = 1 - P(A) - P(B) + P(A)P(B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$

Hence we found that $P(AB) = P(A)P(B)$

Hence these are independent events

$$1. \quad P(A) = P(B)$$

Ans: If $P(A) = P(B)$ then

Each of them has probability of $\frac{1}{2}$

Surely it can be sometime true but not a sufficient condition for two events to be independent

$$1. \quad P(A) + P(B) = 1$$

Ans: Similarly If $P(A) + P(B) = 1$ it is not necessary condition for two events to be independent

Exercise 13.3

Question 1

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution

Ans: It is given in the question that at random one ball is picked and its colour is noted then two balls of same colour is added, hence we have two cases here

Consider the first case

Let the first ball drawn be Red, its probability is $\frac{5}{10}$

Now two extra red balls are added,

$\therefore P(\text{drawing red ball at second attempt}) = \frac{7}{12}$

Since these two events are independent therefore required probability for this case is $\frac{5}{10} \times \frac{7}{12}$

Consider the second case

Let black ball be drawn at first attempt, its probability is $\frac{5}{10}$

Now two extra black balls will be added

$\therefore P(\text{drawing red ball at second attempt}) = \frac{5}{12}$

Since these two events are independent therefore required probability for this case is $\frac{5}{10} \times \frac{5}{12}$

Total probability is given by $\frac{5}{10} \times \frac{7}{12} + \frac{5}{10} \times \frac{5}{12} = \frac{5}{10}$

Thus the probability of drawing second ball red is $\frac{1}{2}$

Question 2

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution

Ans: Let E and F be the events of selecting first and second bag respectively

$\therefore P(E) = P(F) = \frac{1}{2}$

Let K be the event of getting a red ball

$\therefore P(K|E) = \frac{4}{8}$, where

$(K|E)$: drawing red ball from the first bag

Similarly, $P(K|F) = \frac{2}{8}$, where

$(K|F)$: drawing red ball from the first bag

Now from the Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$
$$\Rightarrow P(E|K) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}}$$

Thus the probability that the red ball is drawn from the first bag is $\frac{2}{3}$

Question 3

Of the students in a college, it is known that 60 percent reside in a hostel and 40 percent are day scholars (not residing in a hostel). Previous year results report that 30 percent of all students who reside in hostels attain A grade and 20 percent of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is hostile?

Solution

Ans: Let us have following notations

E: The event when a student is a hostler

F: The event when a student is a day scholar

K: The event chosen student gets grade A

$$\therefore P(E) = 0.6$$

$$\therefore P(F) = 0.4$$

$$\therefore P(K|E) = 0.3$$

$$\therefore P(K|F) = 0.2$$

By Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2}$$

Thus the probability that the gets grade A provided he is a hostler is $\frac{9}{13}$

Question 4

In answering a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Solution

Ans: Let us have following notations

E: The event when a student knows the answer

F: The event when a student guesses the answer

K: The event that the answer is correct

$$\therefore P(E) = \frac{3}{4}$$

$$\therefore P(F) = \frac{1}{4}$$

$$\therefore P(K|E) = 1 \text{ (since if he knows the answer then he will answer it correctly, it's a trivial case)}$$

where $(K|E)$: the answer is correct provided he knows the answer

$$\therefore P(K|F) = \frac{1}{4}$$

$(K|F)$: the answer is correct provided he guesses the answer

By Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{\frac{3}{4} \times \frac{1}{1}}{\frac{3}{4} \times \frac{1}{1} + \frac{1}{4} \times \frac{1}{4}}$$

Thus the probability that he knows the answer provided he answered correctly is $\frac{12}{13}$

Question 5

A laboratory blood test is 99% effective in detecting a certain disease when it is, in fact, present.

However, the test also yields a false positive result for 0.5 percent of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Solution

Ans: Let us have following notations

E: The event when the person has disease

F: The event when the person does not have disease

K: The event that the test result is positive

$$\therefore P(E) = 0.001$$

It is clear that E and F are complementary events

$$\therefore P(E) + P(F) = 1$$

Hence

$$P(F) = 0.999$$

$$\therefore P(K|E) = 0.99$$

$(K|E)$: the result is correct provided he has the disease

$$\therefore P(K|F) = 0.005$$

$(K|F)$: the result is correct provided he doesn't have the disease

By Bay's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

Thus the probability that the person had the disease provided his test result is positive is $\frac{22}{133}$

Question 6

There are three coins. One is a two headed coin (having heads on both faces), another is a biased coin that comes up heads 75 percent of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Solution

Ans: Let us have following notations

E: The event when chosen coin is two-headed

F: The event when chosen coin is biased

G: The event when chosen coin is unbiased

K: The event that the coin shows heads

$$\therefore P(E) = \frac{1}{3}$$

$$P(F) = \frac{1}{3}$$

$$P(G) = \frac{1}{3}$$

$$\therefore P(K|E) = 1 \text{ (since it is a trivial case)}$$

$(K|E)$: the coins shows heads provided it is two headed

$$\therefore P(K|F) = \frac{3}{4}$$

$(K|F)$: the coins shows heads provided it is biased

$$\therefore P(K|G) = \frac{1}{2}$$

$(K|G)$: the coins shows heads provided it is unbiased

By Baye's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$P(E|K) = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}}$$

Thus the probability that the coin shows heads provided it is two headed coin is $\frac{4}{9}$

Question 7

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Solution

Ans: Let us have following notations

E: The event when the driver drives scooter

F: The event when the driver drives car

G: The event when the driver drives truck

K: The event that the driver meets accident

$$\therefore P(E) = \frac{2000}{12000}$$

$$P(F) = \frac{4000}{12000}$$

$$P(G) = \frac{6000}{12000}$$

$$\therefore P(K|E) = \frac{1}{100}$$

$(K|E)$: the driver meets accident provided he drives scooter

$$\therefore P(K|F) = \frac{3}{100}$$

$(K|F)$: the driver meets accident provided he drives car

$$\therefore P(K|G) = \frac{15}{100}$$

$(K|G)$: the driver meets accident provided he drives truck

By Baye's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$P(E|K) = \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

Thus the probability that the accidented person is a scooter driver is $\frac{1}{52}$

Question 8

A factory has two machines A and B. Past record shows that machine A produced 60 percent of the items of output and machine B produced 40 percent of the items. Future, 2 percent of the items produced by machine A and 1 percent produced by machine B were defective. All the items are put into one stockpile and then one item is chosen randomly from this and is found to be defective. What is the probability that it was produced by machine B?

Solution

Ans: Let us have following notations

E: The event when the production is from machine A

F: The event when the production is from machine B

K: The event that the item is defective

$$\therefore P(E) = 0.6$$

$$\therefore P(F) = 0.4$$

$$\therefore P(K|E) = 0.02$$

$(K|E)$: The event that the item is defective provided it from A

$$\therefore P(K|F) = 0.01$$

$(K|F)$: The event that the item is defective provided it from B

By Baye's theorem

$$P(F|K) = \frac{P(F)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(F|K) = \frac{0.4 \times 0.01}{0.6 \times 0.02 + 0.4 \times 0.01}$$

Thus the probability that that the item is defective provided it from B is $\frac{1}{4}$

Question 9

Two groups are competing for the position on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution

Ans: Let us have following notations

E: The event when the first group wins

F: The event when the second group wins

K: The event that the new item is produced

$$\therefore P(E) = 0.6$$

$$\therefore P(F) = 0.4$$

$$\therefore P(K|E) = 0.7$$

$(K|E)$: The event that the item is introduced by first group

$$\therefore P(K|F) = 0.3$$

$(K|F)$: The event that item is introduced by second group

By Baye's theorem

$$P(F|K) = \frac{P(F)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(F|K) = \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3}$$

Thus the probability that that the item is produced from second group is $\frac{2}{9}$

Question 10

Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die?

Solution

Ans: Let us have following notations

E: The event when 5 or 6 come

F: The event when 1,2,3 or 4 come

K: The event that the coin shows exactly one head

$$\therefore P(E) = \frac{2}{6}$$

$$\therefore P(F) = \frac{4}{6}$$

$$\therefore P(K|E) = \frac{3}{8}$$

$(K|E)$: The event that the coin shows exactly one head provided 5 or 6 come

$$\therefore P(K|F) = \frac{1}{2}$$

$(K|F)$: The event that the single throw of coin shows exactly one head provided 1,2,3 or 4 come

By Baye's theorem

$$P(F|K) = \frac{P(F)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(F|K) = \frac{\frac{4}{6} \times \frac{1}{2}}{\frac{2}{6} \times \frac{3}{8} + \frac{4}{6} \times \frac{1}{2}}$$

Thus the probability that that the item is produced from second group is $\frac{8}{11}$

Question 11

A manufacturer has three machine operators A, B and C. The first operator A produces 1 percent defective items, whereas the other two operators B and C produce 5 percent and 7 percent defective items respectively. A is on the job for 50 Percent of the time, B is on the job for 30 Percent of the time and C is on the job for 20 percent of the time. A defective item is produced, what is the probability that it was produced by A?

Solution

Ans: Let us have following notations

E: The event when A works

F: The event when B works

G: The event when C works

K: The event that the defective item is produced

$$\therefore P(E) = 0.5$$

$$\therefore P(F) = 0.3$$

$$\therefore P(G) = 0.2$$

$$\therefore P(K|E) = 0.01$$

$(K|E)$: The event that the defective item is produced from A

$$\therefore P(K|F) = 0.05$$

$(K|F)$: The event that the defective item is produced from B

$$\therefore P(K|G) = 0.07$$

The event that the defective item is produced from C

By Baye's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F) + P(G)P(K|G)}$$

$$\Rightarrow P(E|K) = \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.05 + 0.2 \times 0.07}$$

Thus the probability that the item is produced from A and found defective $\frac{5}{34}$

Question 12

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Solution

Ans: Let us have following notations

E: The event when a card is diamond

F: The event when the card is not diamond

K: The event to denote lost card

Clearly E and F are contemporary Events

$$\therefore P(E) = \frac{13}{52} \text{ (since 13 cards are diamonds)}$$

$$\therefore P(F) = \frac{39}{52}$$

$(K|E)$: The event of drawing two cards when one diamond card is lost

Since one diamond card is lost therefore left total cards are

51 and left diamond cards are 13

$$\therefore P(K|E) = \frac{{}^{12}C_2}{{}^{51}C_2}$$

$$\Rightarrow P(K|E) = \frac{22}{425}$$

$(K|F)$: The event of drawing two cards when one non diamond card is lost

Since one card is lost therefore left total cards are 51

$$\therefore P(K|F) = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$\Rightarrow P(K|F) = \frac{26}{425}$$

By Baye's theorem

$$P(E|K) = \frac{P(E)P(K|E)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{3}{4} \times \frac{26}{425}}$$

Thus the probability of the lost card being a diamond is $\frac{11}{50}$

Question 13

Probability that A speaks the truth is $\frac{4}{5}$. A coin is tossed. A report that a head appears. The probability that actually there was head is

1. $\frac{4}{5}$
2. $\frac{1}{2}$
3. $\frac{1}{5}$
4. $\frac{2}{5}$

Solution

Ans: Let us have following notations

E: The event when A speaks truth

F: The event when A speak false

K: The event it is head

Clearly E and F are contemporary Events

$$\therefore P(E) = \frac{4}{5} \text{ (since 13 cards are diamonds)}$$

$$\therefore P(F) = \frac{1}{5}$$

$(K|E)$: The event it is head and he speaks truth

$$\therefore P(K|E) = \frac{1}{2}$$

$(K|F)$: The event it is head and he speaks false

$$\therefore P(K|F) = \frac{1}{2}$$

By Baye's theorem

$$P(E|K) = \frac{P(E)P(K|F)}{P(E)P(K|E) + P(F)P(K|F)}$$

$$\Rightarrow P(E|K) = \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}}$$

Thus the probability of the lost card being a diamond is $\frac{4}{5}$

Question 14

If A and B are two events such that $A \subset B$ and $P(B) \neq 0$ then which of the following is correct

1. $P(A|B) = \frac{P(B)}{P(A)}$
2. $P(A|B) < P(A)$
3. $P(A|B) \geq P(A)$
4. None of these

Solution

Ans: Given $A \subset B$

$$\therefore A \cap B = A \text{ and } P(A) < P(B)$$

$$\text{Now } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{P(A)}{P(B)}$$

$$\text{Now } P(B) \leq 1$$

$$\therefore \frac{1}{P(B)} \geq 1$$

Hence option C is correct, i.e. $P(A|B) \geq P(A)$

Miscellaneous Exercise

Question 1

If A and B are two events such that $P(A) \neq 0$. Find $P(B|A)$ if

(i) A is a subset of B

(ii) $A \cap B = \phi$

Solution

Ans: It is given in the question that $P(A) \neq 0$ and A is a subset of B

$$\therefore P(A \cap B) = P(A)$$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(B|A) = \frac{P(A)}{P(A)} = 1$$

Ans: since it is given that $A \cap B = \phi$

Therefore $P(A \cap B) = 0$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

Question 2

A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Solution

Ans: The sample space for a family to have two children is given by

$$S = \{BB, GG, BG, GB\}$$

Let us have following notations and their probabilities given as shown

N: Both children are males

$$P(N) = \frac{1}{4}$$

K: at least one of the children is male

$$P(K) = \frac{3}{4}$$

$$\therefore N \cap K = BB$$

$$\therefore P(N \cap K) = \frac{1}{4}$$

Now probability that both of the children are males provided at least one of the child is male is given by

$$P(N|K) = \frac{P(N \cap K)}{P(K)}$$

$$P(N|K) = \frac{\frac{1}{4}}{\frac{3}{4}}$$

Hence probability that both of the children are males provided at least one of the child is male is $\frac{1}{3}$

Ans: The sample space for a family to have two children is given by

$$S = \{BB, GG, BG, GB\}$$

Let us have following notations and their probabilities given as shown

N: Both children are females

$$P(N) = \frac{1}{4}$$

K: elder child is a female

$$P(K) = \frac{2}{4}$$

$$\therefore N \cap K = GG$$

$$\therefore P(N \cap K) = \frac{1}{4}$$

Now probability that both of the children are males provided atleast one of the child is male is given by

$$P(N|K) = \frac{P(N \cap K)}{P(K)}$$

$$P(N|K) = \frac{\frac{1}{4}}{\frac{2}{4}}$$

Hence probability that both of the children are males provided atleast one of the child is male is $\frac{1}{2}$

Question 3

Suppose that 5% of men and 0.25% of women have grey hair. A haired person is selected at random. What is the probability of this person being male? Assume that there are equal numbers of males and females.

Solution

Ans: It is given that 5percent of males and 0.25 percent of females have grey hair

Therefore total people having grey hair is $(5 + 0.25) = 5.25$ percent

Hence probability of male being haired is $\frac{5}{5.25}$

Question 4

Suppose that 90 % of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Solution

Ans: Given probability of a person being right-handed is $p = 0.9$

Since a person can only be right-handed or left handed

Therefore it follows binomial distribution with

$$n = 10, p = \frac{9}{10}, q = \frac{1}{10}$$

Probability that at least 6 people are right handed is given by

$$\sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

Therefore probability that at most 6 people are right handed is given by

$$1 - \sum_{k=0}^{k=7} {}^{10}C_k \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{10-k}$$

Question 5

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- (i) all will bear 'X' mark.
- (ii) not more than 2 will bear the 'Y' mark.
- (iii) At least one ball will bear the 'Y' mark.
- (iv) The number of balls with 'X' mark and 'Y' mark will be equal.

Solution

Ans: from the given data we have

Probability of balls bearing X mark is $p = \frac{10}{25}$

Probability of balls bearing Y mark is $q = \frac{15}{25}$

Hence we can say that random variable K which measures the number of Y marked balls follows binomial distribution with

$$n = 6, p = \frac{2}{5}, q = \frac{3}{5}$$

Probability that all ball bear X is same as none of balls bear Y which is given by

$$P(K = 0) = {}^6C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^6$$

Hence Probability that all ball bear X is $\left(\frac{2}{5}\right)^6$

Ans: Probability of not more than 2MY balls are selected is given by

$$P(k \leq 2) = P(K = 0) + P(K = 1) + P(K = 2)$$

$$\Rightarrow P(k \leq 2) = {}^6C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^6 + {}^6C_1 \left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right)^5 + {}^6C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4$$

$$\Rightarrow P(k \leq 2) = \left(\frac{2}{5}\right)^4 \left(\frac{4}{25} + 6\left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right) + 15\left(\frac{3}{5}\right)^2\right)$$

Hence Probability of not more than 2 balls are selected is $7\left(\frac{2}{5}\right)^4$

Ans: Probability of at least one ball bear Y is given by

$$P(K \geq 1) = 1 - P(k = 0)$$

$$\Rightarrow P(K \geq 1) = 1 - \left(\frac{2}{5}\right)^6$$

Hence Probability of at least one ball bear Y is $1 - \left(\frac{2}{5}\right)^6$

Ans: Since there were 6 balls therefore each are of 3 if equal

Probability that number of X and Y is equal is given by

$$P(K = 3) = {}^6C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3$$

Hence Probability that number of X and Y is equal is $\frac{834}{3125}$

Question 6

In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$.

What is the probability that he will knock down fewer than 2 hurdles?

Solution

Ans: Given probability that he will clear each hurdle is $p = \frac{5}{6}$

And hence probability that he knocks down is given as $q = \frac{1}{6}$

Let k be the random variable that represents the hurdles he knocks down

Clearly K follows binomial distribution with

$$n = 10, p = \frac{5}{6}, q = \frac{1}{6}$$

Probability that he knocks down less than 2 hurdles is given by

$$P(K = 0) + P(K = 1)$$

$${}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 = \frac{5^{10}}{6^9 \times 2}$$

Hence Probability that he knocks down less than 2 hurdles is $\frac{5^{10}}{6^9 \times 2}$

Question 7

A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Solution

Ans: From the question we have

Probability of getting a six is $p = \frac{1}{6}$

Probability of getting no six is $q = \frac{5}{6}$

Let K be the random variable that represents the number of sixes

Clearly K follows binomial distribution with

$$n = 5, p = \frac{1}{6}, q = \frac{5}{6}$$

Probability that two sixes come in first two throws is given by

$$P(K = 2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

Also Probability that third six come is given by

$$\frac{1}{6} P(K = 2)$$

Hence probability of getting three sixes is $\frac{10 \times 125}{6^6}$

Question 8

If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Solution

Ans: In a leap year, we have 366 days i.e., 52 weeks and 2 days.

In 52 weeks, we have 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be any of the following

Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday and Sunday and Monday

Total number of cases = 7

$$\therefore \text{Probability that a leap year will have 53 Tuesdays} = \frac{2}{7}$$

Question 9

An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

Solution

Ans: Given that the probability of success is twice that of failure

$$\therefore p = 2q$$

Also we know that sum of probabilities of exhaustive events equals 1

$$\therefore p = \frac{2}{3}, q = \frac{1}{3}$$

Let K be the random variable that represents successes

$$\therefore P(K \geq 4) = P(K = 4) + P(K = 5) + P(K = 6)$$

$$\Rightarrow P(K \geq 4) = {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^1 + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0$$

Thus on solving we found the probability of getting at least 4 successes in six trials is $\frac{31}{9} \left(\frac{2}{3}\right)^4$

Question 10

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Solution

Ans: Let a person tosses the coin n times, n being Bernoulli Trials

Here probability of head equals that of tails and each equal to $\frac{1}{2}$

Numbers of trials for which at least one head is more than 0.9 is given by

$$1 - P(K = 0) > 0.9$$

$$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{2}\right)^n > \frac{9}{10}$$

$$\Rightarrow 2^n > 10$$

Hence the person must toss more than 4 times

Question 11

In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

Solution

Ans: We have

Probability of getting a six is $p = \frac{1}{6}$

Probability of getting no six is $q = \frac{5}{6}$

There are three possible ways to get a six in three trials:

1. He gets a six only at first throw

$$\text{probability} = \frac{1}{6}$$

$$\text{Rupee won} = 1$$

1. He gets a six at second throw

$$\text{probability} = \frac{1}{6} \times \frac{5}{6}$$

$$\text{Rupee won} = -1 + 1 = 0$$

1. He gets a six at third trial

$$\text{probability} = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$\text{Rupee won} = -1 - 1 + 1 = -1$$

Thus Expected value can be calculated as shown

$$1 \times \frac{1}{6} + 0 \times \frac{5}{36} + -1 \times \frac{25}{216} = \frac{11}{216}$$

Question 12

Suppose we have four boxes. A, B, C and D containing coloured marbles as given below.

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Solution

Ans: Let us have following notations

R be the event of drawing red marble

E be the event of selecting the box A

F be the event of selecting the box B

G be the event of selecting the box C

Total numbers of marbles is 40

Number of red marbles is 15

$$\therefore P(R) = \frac{15}{40}$$

Number of red marbles in box A i.e $n(R \cap E) = 1$

Number of red marbles in box B i.e $n(R \cap F) = 6$

Number of red marbles in box C i.e $n(R \cap G) = 8$

Now the probability that red marble is picked from box A is given by

$$P(R|E) = \frac{P(R \cap E)}{P(E)}$$

$$\Rightarrow P(R|E) = \frac{\frac{1}{40}}{\frac{15}{40}}$$

Hence E be the event of selecting the box A is $\frac{1}{15}$

Similarly, probability that red marble is picked from box B is given by

$$P(R|F) = \frac{\frac{6}{40}}{\frac{15}{40}}$$

Hence probability that red marble is picked from box B is $\frac{2}{5}$

Similarly, probability that red marble is picked from box C is given by

$$P(R|G) = \frac{\frac{8}{40}}{\frac{15}{40}}$$

Hence probability that red marble is picked from box C is $\frac{8}{15}$

Question 13

Assume that the chances of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drugs reduces its changes by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Solution

Ans: Let us have following notations

E: The events when the person took yoga and meditation courses

$$\therefore P(E) = \frac{1}{2}$$

F: The events when the person took drugs

$$\therefore P(F) = \frac{1}{2}$$

G: the person suffered heart attack

$$\therefore P(G) = 0.4$$

From the question also we have

$$P(G|E) = 0.4 \times 0.7 = 0.28$$

$$P(G|F) = 0.4 \times 0.75 = 0.30$$

Now probability that found person has heart attack despite of having yoga and meditation courses is given by

$$P(E|G) = \frac{P(E) \times P(G|E)}{P(E) \times P(G|E) + P(F) \times P(G|F)}$$

$$P(E|G) = \frac{0.5 \times 0.28}{0.5 \times 0.28 + 0.5 \times 0.3}$$

Hence probability that found person has heart attack despite of having yoga and meditation courses is $\frac{14}{29}$

Question 14

If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$)

Solution

Ans: It is clear that total number of determinants of second order entries being 0's or 1's is 2^4

The value of determinant is positive for cases as shown

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

i.e favourable case is 3

Hence probability that value of determinant is positive is $\frac{3}{16}$.

Question 15

An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

$$P(A \text{ fails}) = 0.2$$

$$P(B \text{ fails alone}) = 0.15$$

$$P(A \text{ and } B \text{ fails}) = 0.15$$

Evaluate the following probabilities

(i) $P(A \text{ fails} \mid B \text{ has failed})$

(ii) $P(A \text{ fails alone})$

Solution

Ans: Let us have following notations

E: A fails

F: B fails

Given in the question

$$P(E) = 0.2$$

$$P(E \cap F) = 0.15$$

$$P(E' \cap F) = 0.15$$

We know that

$$P(E' \cap F) = P(E) - P(E \cap F)$$

$$\Rightarrow P(E) = 0.3$$

Now the probability that A fails given B has failed is given by

$$P(E \mid F) = \frac{0.15}{0.3}$$

Hence the probability that A fails given B has failed is 0.5

Ans: Probability that A fails alone is given by

$$P(E \cap F') = P(F) - P(E \cap F)$$

$$P(E \cap F') = 0.05$$

Hence Probability that A fails alone is 0.05

Question 16

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Solution

Ans: Let us have following notations that

E: red ball is transferred

$$P(E) = \frac{3}{7}$$

F: black ball is transferred

$$P(F) = \frac{4}{7}$$

G: red ball is drawn

When a red ball is transferred

$$P(G|E) = \frac{5}{10}$$

Similarly, When a black ball is transferred

$$P(G|F) = \frac{4}{10}$$

$$P(F|G) = \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{4}{7} \times \frac{4}{10} + \frac{3}{7} \times \frac{5}{10}}$$

Hence probability that the transferred ball is black is $\frac{16}{31}$

Question 17

If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then what result can follow?

Solution

Ans: Given in the question $P(A) \neq 0$ and $P(B|A) = 1$

And we know that

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A) = P(A \cap B)$$

Hence $A \subset B$

Question 18

If $P(A|B) > P(A)$, then what is correct?

Solution

Ans: Given $P(A|B) > P(A)$

And we know that

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$$

Hence we found that $P(B|A) > P(B)$

Question 19

If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$ then what result can follow?

Solution

Ans: Given in the question we have

$$P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow P(B) = P(A \cap B)$$

Hence

$$P(A|B) = 1$$
