

Chapter 12: Linear Programming

NCERT · Class 12 · Maths

Exercise 12.1

Question 1

Maximise $Z = 3x + 4y$

Subject to the constraints: $x + y \geq 4$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints: $x + y \geq 4$, $x \geq 0$, and $y \geq 0$, determine the feasible region as shown below:

The values of Z at the corner points $O(0, 0)$, $A(4, 0)$, $B(0, 4)$ of the feasible region are:

Corner point	$Z = 3x + 4y$	
$O(0, 0)$	0	
$A(4, 0)$	12	
$B(0, 4)$	16	Maximum

Hence, the maximum value of Z is 16 at the point $B(0, 4)$.

Question 2**Minimise** $Z = 3x + 4y$.**Subject to constraints** $x + 2y \leq 8$, $3x + 3y \leq 12$, $x \geq 0$ and $y \geq 0$.**Solution**

Ans: The constraints $x + 2y \leq 8$, $3x + 3y \leq 12$, $x \geq 0$ and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $O(0, 0)$, $A(4, 0)$, $B(2, 3)$ and $C(0, 4)$ of the feasible region are:

Corner point	$Z = 3x + 4y$	
$O(0, 0)$	0	
$A(4, 0)$	-12	Minimum
$B(2, 3)$	6	
$C(0, 4)$	16	

Hence, the minimum value of Z is -12 at the point $A(4, 0)$.

Question 3**Maximise** $Z = 5x + 3y$ **Subject to the constraints:** $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints: $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $O(0, 0)$, $A(2, 0)$, $B(0, 3)$ and $C(\frac{20}{19}, \frac{45}{19})$ of the feasible region are:

Corner point	$Z = 5x + 3y$	
$O(0, 0)$	0	
$A(2, 0)$	10	
$B(0, 3)$	9	
$C(\frac{20}{19}, \frac{45}{19})$	$\frac{235}{19}$	Maximum

Hence, the maximum value of Z is $\frac{235}{19}$ at the point $C(\frac{20}{19}, \frac{45}{19})$.

Question 4

Minimise $Z = 3x + 5y$

Subject to the constraints: $x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints $x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

Therefore, the feasible region is unbounded.

The values of Z at the corner points $A(3, 0)$, $B(\frac{3}{2}, \frac{1}{2})$ and $C(0, 2)$ of the feasible region are:

Corner point	$Z = 3x + 5y$	
$A(3, 0)$	9	
$B(\frac{3}{2}, \frac{1}{2})$	7	Smallest
$C(0, 2)$	10	

Since, the feasible region is unbounded, 7 may or may not be the minimum value of Z .

So, we draw the graph of the inequality, $3x + 5y < 7$, and check if the resulting half-plane has common points with the feasible region.

As the feasible region has no common point with $3x + 5y < 7$, the minimum value of Z is 7 at the point $B(\frac{3}{2}, \frac{1}{2})$.

Question 5

Maximise $Z = 3x + 2y$

Subject to the constraints: $x + 2y \leq 10$, $3x + 5y \leq 15$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints, $x + 2y \leq 10$, $3x + 5y \leq 15$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $A(5, 0)$, $B(4, 3)$ and $C(0, 5)$ of the feasible region are:

Corner point	$Z = 3x + 2y$	
$A(5, 0)$	15	
$B(4, 3)$	18	Maximum
$C(0, 5)$	10	

Hence, the maximum value of Z is 18 at the point $B(4, 3)$.

Question 6

Minimise $Z = x + 2y$

Subject to the constraints: $2x + y \geq 8$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints $2x + y \geq 8$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $A(6, 0)$ and $B(0, 3)$ of the feasible region are:

Corner point	$Z = x + 2y$
$A(6, 0)$	6
$B(0, 3)$	6

As the value of Z at points $A(6, 0)$ and $B(0, 3)$ is same, we need to take any other point, for example, $(2, 2)$ on line $x + 2y = 6$

Now, $Z = 6$.

Thus, the minimum value of Z occurs at more than 2 points. Hence, we can say that the value of Z is minimum at every point on the line $x + 2y = 6$.

Question 7

Minimise and Maximise $Z = 5x + 10y$

Subject to the constraints: $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 60, x \geq 0, y \geq 0$

Solution

Ans: The constraints $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 60, x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $A(60, 0), B(120, 0), C(60, 30)$ and $D(40, 20)$ of the feasible region are:

Corner point	$Z = 5x + 10y$	
$A(60, 0)$	300	Minimum
$B(120, 0)$	600	Maximum
$C(60, 30)$	600	Maximum

$D(40, 20)$	400	
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Hence, the minimum value of Z is 300 at the point $A(60, 0)$ and the maximum value of Z is 600 at all the points lying on the line segment joining the points $B(120, 0)$ and $C(60, 30)$.

Question 8

Minimise and Maximise $Z = x + 2y$

Subject to the constraints: $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, $y \geq 0$

Solution

Ans: The constraints $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

The values of Z at the corner points $A(0, 50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$ of the feasible region are:

Corner point	$Z = x + 2y$	
$A(0, 50)$	100	Minimum
$B(20, 40)$	100	Minimum
$C(50, 100)$	250	
$D(0, 200)$	400	Maximum

Hence, the maximum value of Z is 400 at the point $D(0, 200)$ and the minimum value of Z is 100 at all the points lying on the line segment joining the points $A(0, 50)$ and $B(20, 40)$.

Question 9

Maximise $Z = -x + 2y$

Subject to the constraints: $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$

Solution

Ans: The constraints $x \geq 3, x + y \geq 5, x + 2y \geq 6$, and $y \geq 0$ determine the feasible region as shown below:

It is visible that the feasible region is unbounded.

The values of Z at corner points $A(6, 0)$, $B(4, 1)$ and $C(3, 2)$ are given as:

Corner point	$Z = -x + 2y$
$A(6, 0)$	-6
$B(4, 1)$	-2
$C(3, 2)$	1

Since, the feasible region is unbounded, $Z = 1$ may or may not be the maximum value.

So, we will draw the graph of the inequality, $-x + 2y > 1$, and check if the resulting half-plane has common points with the feasible region.

As the resulting feasible region has common points with the feasible region. Therefore, $Z = 1$ is not the maximum value. Also, Z has no maximum value.

Question 10

Maximise $Z = x + y$

Subject to the constraints: $x - y \leq -1$, $-x + y \leq 0$, $x \geq 0$, and $y \geq 0$

Solution

Ans: The constraints, $x - y \leq -1$, $-x + y \leq 0$, $x \geq 0$, and $y \geq 0$ determine the feasible region as shown below:

It can be seen that there is no feasible region. Thus, Z does not have any maximum value.
