

## Chapter 9: Mechanical Properties of Fluids

NCERT · Class 11 · Physics

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### Textbook exercises

#### Question 1

Explain why

(a) The blood pressure in humans is greater at the feet than at the brain

(b) Atmospheric pressure at the height of about

$$6 \text{ km}$$

decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than

$$100 \text{ km}$$

.

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

#### Solution

Ans: The pressure of a liquid with density

$$\rho$$

, with the liquid column of

$$h$$

is given by the relation:

$$P = \rho hg$$

here

$$g$$

is the acceleration due to the gravity

It can be inferred that pressure is directly proportional to height. In the case of humans, the circulatory system can be considered as the liquid (blood)-column.

The height of the column is the least at the head level and the maximum at the feet; hence, the pressure (blood pressure) in human vessels depends on this height. Thus, the blood pressure at the feet is more than what it is at the brain.

Ans: The density of a fluid depends on how much it has been pressed by the amount of fluid over it. This means that the density of air is the maximum near the sea level.

The density of air increases with a decrease in the height from the sea surface. At the height of about 6 km, the total mass squeezes the layer of air over here to nearly half of the pressure value at sea level.

In the case of just small changes in altitude, the atmospheric pressure is proportional to density. However, when larger height scales are considered, the density itself depends on the height, and the pressure isn't linearly dependent anymore.

Moreover, the pressure increases faster than the linear dependence expected in the case of small changes in altitudes.

Ans: Mathematically, pressure is the perpendicular force per unit area.

So, only the force component that is perpendicular to the surface or along the area vector is used. The direction of this component and the direction of the area vector are the same.

Hence effectively, there is no direction involved anymore.

In other words, when force is applied to a liquid, the pressure in the liquid is transferred in all directions. Thus, the hydrostatic pressure does not have a fixed direction, and it is a scalar physical quantity.

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## Question 2

Explain why

A.

The angle of contact of mercury with glass is obtuse, while that of water with glass is acute

B.

Water on a clean glass surface tends to spread out, while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

**C.**

The surface tension of a liquid is independent of the area of the surface

**D.**

Water with detergent dissolved in it should have small angles of contact.

### Solution

Ans: The angle between the surface inside the liquid and the tangent to the liquid surface at the point of contact is the angle of contact

$$(\theta)$$

, as shown in the following figure

Let

$$S_{la}$$

be the interfacial tension at the liquid-air interface,

$$S_{sl}$$

be at the solid-liquid interface, and

$$S_{sa}$$

be at the solid-air interface. At all the points of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

In the case of a mercury drop on glass,

$$S_{sl} > S_{sa}$$

, the angle of contact

$$(\theta)$$

is obtuse.

In the case of water on the glass,

$$S_{sl} < S_{la}$$

, the angle of contact

$$(\theta)$$

is acute.

Ans: The Mercury molecules have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, the angle of contact tends to become obtuse, and the molecules come closer and tend to form a drop.

While in the case of water, the ratio of attraction between water molecules and the attraction between water molecules and the glass is lower. Hence the water molecules stay closer to glass molecules as well, keeping the glass surface wet.

Ans: Surface tension can be defined as the force acting per unit length at the interface between the liquid and any other material.

There is no dependency of this force on the area of the surface.

Hence, surface tension is independent of the area of the surface of the liquid.

Ans: Detergent water has small angles of contact

$$(\theta)$$

because the detergent molecules are sticky; in other words, they are significantly attracted to the water molecules and the solid which is in contact with the detergent water.

Therefore, the molecules of water can get closer to the solid.

In other words,

$$S_{sl} < S_{la}$$

and the value of

$$\cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

increases, leading to a decrease in the value of

$$\theta$$

making it acute or small.

(e) A drop of liquid under no external forces is always spherical in shape

Ans: Surface tension pulls the surface together as much as possible.

This means that in the absence of any other external force/pressure. The surface tension will try to shrink the area as much as possible.

Beyond a certain extent, the volume cannot be shrunk, and the shape with minimum surface area for the unchanged volume is a sphere.

Hence a droplet will tend to become spherical.

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### Question 3

Fill in the blanks using the word(s) from the list appended with each statement:

**A.**

Surface tension of liquids generally .....with temperatures (increases / decreases)

**B.**

Viscosity of gases..... with temperature, whereas viscosity of liquids ..... with temperature (increases / decreases)

**C.**

For solids with elastic modulus of rigidity, the shearing force is proportional to....., while for fluids it is proportional to .....(shear strain / rate of shear strain)

**D.**

For a fluid in a steady flow, the increase in flow speed at a constriction follows..... (conservation of mass / Bernoulli's principle)

## Solution

Ans: decreases.

The surface tension of a liquid surface is inversely proportional to the motion of the liquid molecules. An increase in the temperature leads to an increase in the random motion of the molecules, and hence the surface tension drops.

Ans: increases; decreases

All fluids experience resistance to their motion when they are made to flow. This resistance is known as viscosity.

The motion of the gas molecules becomes very random, and giving them a direction gets difficult.

The viscosity of gases increases with the increasing random motion of the molecules. The molecules of the liquid are more restricted than and not as free as those of gas. Increasing the temperature of liquids increases the motion of the molecules, making them more mobile and improves their ability to flow.

Hence the viscosity decreases with an increase in temperature.

Ans: Shear strain; Rate of shear strain

For elastic solids, shearing force is proportional to the shear strain.

However, the shearing force is proportional to the rate of shear strain in the case of fluids.

Ans: Conservation of mass/Bernoulli's principle

Bernoulli's principle works in the conservation of mass and energy, which includes the effects of the continuity equation. Hence, Conservation of mass and Bernoulli's Equation is the correct options

(e) For the model of a plane in a wind tunnel, turbulence occurs at a ..... speed for turbulence for an actual plane (greater / smaller)

Ans: Greater

As provided in the question, there is a model of a plane in a wind tunnel. Following Bernoulli's Principle and referring to the relevant Reynolds number it can be seen that the turbulence occurs at a greater speed.

## Question 4

### Explain why

**A.**

To keep a piece of paper horizontal, you should blow over, not under, it

**B.**

When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers

**C.**

The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection

**D.**

A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel

### Solution

Ans: As per Bernoulli's principle, the fluid pressure reduces if the velocity is increased.

The paper falls under gravity, and to make it stay up and horizontal, we need to reduce the pressure on the top of it. For this reason, one should blow on the upper side of the paper.

Ans: According to the continuity equation, for a flowing fluid, the rate of volume of fluid crossing any cross-section is constant. This means that the velocity of the flowing fluid is high if the cross-section of the passage is reduced.

Covering the tap opening reduces the cross-section, and hence the water gushes out with a lot of speed.

Ans: The liquid velocity at the needle is much higher than that inside the syringe. And higher velocity means much lower pressure.

Furthermore, the changes in the pressures at the different cross-sections are proportional to the pressures in the regions.

A human thumb cannot constantly provide constant pressure. So the significant change in the pressures is carried to the needle end as well. The pressure over here changes proportionally.

But as mentioned before, the changes in the pressure at the needle end are proportional to the pressure at the needle end, which is very small.

Hence the absolute change in the pressure at the needle end is also minimal. Thus it can be said that the consistency of the pressure is controlled by the cross-section of the needle.

Ans: As the fluid flows out of a small orifice, its velocity is significantly high according to the continuity equation.

This outgoing liquid generates momentum.

Going by the law of conservation of momentum, a counter momentum is generated on the vessel, making it experience a backward thrust.

(e) A spinning cricket ball in air does not follow a parabolic trajectory

Ans: A thrown spinning cricket ball has two motions – rotational and linear. The falling ball experiences viscosity in the upwards direction.

A rotating ball experiences viscosity due to the relative motion of the surface of the ball and the air in the vicinity. So at any instance, the face that is moving downwards faces an upwards viscosity force while the face going upwards faces a downwards viscosity force.

These viscous forces add up in such a way that the velocity of the air on one side of the ball is much higher than on their side, reducing the pressure on the side which has a higher velocity.

This pressure imbalance causes a force directed towards the low-pressure region. Hence the ball deviates from the expected parabolic trajectory.

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### Question 5

A

$50\text{ kg}$

girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter

$1.0\text{ cm}$

. What is the pressure exerted by the heel on the horizontal floor?

**Solution**

Ans: As provided, the mass of the girl,

$$m = 50 \text{ kg}$$

, the diameter of the heel

$$d = 1 \text{ cm} = 0.01 \text{ m}$$

, the radius of the heel,

$$r = \frac{d}{2} = 0.005 \text{ m}$$

Therefore, the area of the heel,

$$A = \pi r^2$$

$$\Rightarrow A = \pi \times 0.005^2$$

$$\Rightarrow A = 7.85 \times 10^{-5} \text{ m}^2$$

The force exerted by the heel on the floor is:

$$F = mg = 50 \times 9.8 = 490 \text{ N}$$

The pressure exerted on the floor by the heel:

$$P = \frac{F}{A} = \frac{490}{7.85 \times 10^{-5}} = 6.24 \times 10^6 \text{ Nm}^{-2}$$

Hence clearly, the pressure exerted by the heel on the horizontal floor is

$$6.24 \times 10^6 \text{ Nm}^{-2}$$

.

**Question 6**

Torricelli's barometer used mercury. Pascal duplicated it using French wine of density

$$984 \text{ kg/m}^3$$

. Determine the height of the wine column for normal atmospheric pressure.

## Solution

Ans: Provided that,

Density of mercury,

$$\rho_1 = 13.6 \times 10^3 \text{ kg/m}^3$$

The height of the mercury column,

$$h_1 = 0.76 \text{ m}$$

Provided density of French wine,

$$\rho_2 = 0.984 \times 10^3 \text{ kg/m}^3$$

Let the height of the column of wine

$$= h_2$$

The acceleration due to gravity

$$= g = 9.8 \text{ ms}^{-2}$$

The pressure in both the columns (given by

$$\rho h g$$

) is equal at equal altitudes of the same liquid, which means:

$$\begin{aligned} \rho_1 h_1 g_1 &= \rho_2 h_2 g_2 \\ \Rightarrow h_2 &= \frac{\rho_1 h_1 g}{\rho_2 g} = 10.5 \text{ m} \end{aligned}$$

Thus, the height of the column of wine at normal atmospheric pressure is

$$10.5 \text{ m}$$

.

### Question 7

A vertical off-shore structure is built to withstand a maximum stress of

$$10^9 \text{ Pa}$$

. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly

$$3 \text{ km}$$

, and ignore ocean currents

### Solution

Ans: Yes

As provided, the maximum allowable stress for the structure,

$$P = 10^9 \text{ Pa}$$

Depth of the ocean,

$$d = 3 \times 10^3 \text{ m}$$

The density of water,

$$\rho = 10^3 \text{ kg/m}^3$$

The acceleration due to gravity,

$$g = 9.8 \text{ m/s}^2$$

The pressure exerted because of water at depth,

$$P = \rho dg = 3 \times 10^3 \times 10^3 \times 9.8 = 2.94 \times 10^7 \text{ Pa}$$

Hence it has been calculated that the maximum allowable stress for the structure is significantly greater than the pressure of the water and hence can be used safely.

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### Question 8

A hydraulic automobile lift is designed to lift cars with a maximum mass of

$$3000 \text{ kg}$$

. The area of cross-section of the piston carrying the load is

$$425 \text{ cm}^2$$

. What maximum pressure would the smaller piston have to bear?

### Solution

Ans: As provided, the mass limit on the mass of a car that can be lifted,

$$m = 3000 \text{ kg}$$

The area of cross-section of the load-carrying piston,

$$A = 0.0425 \text{ m}^2$$

The maximum force exerted by the load,

$$F = mg = 3000 \times 9.8 = 29400 \text{ N}$$

The maximum pressure experienced by the piston that is carrying the load is,

$$P = \frac{F}{A} = \frac{29400}{425 \times 10^{-4}} = 6.917 \times 10^{-4} \text{ Pa}$$

Since the pressure is transferred/transmitted equally in all directions in a liquid, the maximum pressure on the smaller piston would be

$$6.917 \times 10^{-4} \text{ Pa}$$

.

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### Question 9

A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with

$$10.0 \text{ cm}$$

of water in one arm and

$$12.5 \text{ cm}$$

of spirit in the other. What is the specific gravity of spirit?

### Solution

Ans: The provided system of water, mercury, and methylated spirit can be shown as follows:

The height of the spirit column,

$$h_1 = 12.5 \text{ cm} = 0.125 \text{ m}$$

The height of the water column is:

$$h_2 = 10 \text{ cm} = 0.1 \text{ m}$$

$$P_0 =$$

Atmospheric pressure,

$$\rho_1 =$$

Density of spirit,

$$\rho_2 =$$

Density of water

Pressure at point A,

$$P_0 + \rho_1 h_1 g$$

Pressure at point B,

$$P_0 + \rho_2 h_2 g$$

The values of pressures at points A and B are equal

$$P_0 + \rho_1 h_1 g = P_0 + \rho_2 h_2 g$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1} = 0.8$$

Hence, clearly, the specific gravity of spirit is

$$0.8$$

### Question 10

In problem 10.9, if

$$15.0 \text{ cm}$$

of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury =

$$13.6$$

)

### Solution

Ans: Provided that,

Height of the water column,

$$h_1 = 10 + 15 = 25 \text{ cm}$$

Height of the spirit column,

$$h_2 = 12.5 + 15 = 27.5 \text{ cm}$$

The density of water,

$$\rho_1 = 1 \text{ g cm}^{-3}$$

; that of spirit,

$$\rho_2 = 0.8 \text{ g cm}^{-3}$$

and that of mercury is

$$13.6 \text{ g cm}^{-3}$$

Now, let

$$h$$

be the difference between the levels of mercury in the two arms. The pressure exerted by the height

$$h$$

of the mercury column:

$$= \rho h g = h \times 13.6$$

The difference between the pressures exerted by water and spirit:

$$\rho_1 h_1 g - \rho_2 h_2 g = g (25 \times 1 - 27.5 \times 0.8) = 3g$$

Equating the pressures, we get:

$$13.6 h g = 3g$$

$$\Rightarrow h = 0.220588 \approx 0.221 \text{ cm}$$

The difference between the levels of mercury in the two arms is

$$0.221 \text{ cm}$$

.

### Question 11

Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

### Solution

Ans: Bernoulli's equation cannot describe the turbulent flow of fluids like that in a rapid river.

Water flow in a river is chaotic. Bernoulli's equation suggests that an increase in a fluid's velocity occurs concurrently with a decrease in static pressure or simultaneously with a reduction in potential energy throughout the water flow.

This theory is only applicable to isentropic flows, wherein turbulence and other disturbances are ignored.

There are various versions of Bernoulli's equation with respect to various flow kinds. For instance,

incompressible flows obey the simple form of Bernoulli's equation whereas compressible flows obey an advanced form of the equation.

Bernoulli's theory is in accordance with the principle of energy conservation. Clearly, at each point on the flowing water in the river, the sum of any and all sources of energy is equal to that as in a steady flow. The sum for kinetic energy, internal energy and potential energy should also maintain stability. Thus, Bernoulli's equation cannot describe the flow of water through a rapid in a river.

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### Question 12

**Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.**

### Solution

Ans: Gauge pressure gives us the pressure difference in the fluid under consideration and some standard pressure. Using gauge pressure will only tell us about the pressure difference.

Bernoulli's principle assumes that the pressures under consideration are linearly proportional to the square of the velocity and the height of the fluid.

However, the difference in pressure cannot be directly shown to have this proportionality. Hence, we need to convert the gauge pressure to absolute pressure before plugging it into Bernoulli's equation.

So yes, it matters if we use gauge instead of absolute pressures in applying Bernoulli's equation.

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### Question 13

**Glycerin flows steadily through a horizontal tube of length**

$$1.5 \text{ m}$$

**and radius**

$$1.0 \text{ cm}$$

**. If the amount of glycerin collected per second at one end is**

$$4.0 \times 10^{-3} \text{ kg/s}$$

, what is the pressure difference between the two ends of the tube? (Density of glycerin

$$1.3 \times 10^3 \text{ kg/m}^3$$

and viscosity of glycerin

$$= 0.83 \text{ Pa s}$$

). (You may also like to check if the assumption of laminar flow in the tube is correct).

### Solution

Ans: Provided that,

Length of the horizontal tube,

$$l = 1.5 \text{ m}$$

The radius of the tube,

$$r = 1 \text{ cm} = 0.01 \text{ m}$$

Thus, the diameter of the tube,

$$\therefore d = 0.02 \text{ m}$$

As given in the question, Glycerine is flowing at a rate of

$$M = 4.0 \times 10^{-3} \text{ kg s}^{-1}$$

The density of Glycerine,

$$\rho = 1.3 \times 10^3 \text{ kg m}^{-3}$$

Viscosity of Glycerine

$$\eta = 0.83 \text{ Pa s}$$

The volume of Glycerine flowing per sec:

$$V = M/\rho = \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} = 3.08 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

According to Poiseuille's Law, we have the expression for the rate of flow:

$$V = \frac{\pi P r^4}{8 \eta l}$$

here, P is the pressure difference between the two ends of the tube

$$P = \frac{V 8 \eta l}{\pi r^4} = 9.8 \times 10^2 \text{ Pa}$$

Reynolds' number is given by the following expression:

$$\frac{4 \rho V}{\pi d \eta} = \frac{4(1.3 \times 10^3)(3.08 \times 10^{-6})}{\pi(0.02)0.83} = 0.3$$

Reynolds' number is about

$$0.3$$

; therefore, the flow is laminar.

### Question 14

In a test experiment on a model aero plane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are

$$70 \text{ m/s}$$

and

$$63 \text{ m/s}$$

respectively. What is the lift on the wing if its area is

$$2.5 \text{ m}^2$$

? Take the density of air to be

$$1.3 \text{ kg m}^{-3}$$

.

## Solution

Ans: The given speed of wind on the wing's upper surface, and the speed of wind relative to the wing's lower surface

$$v_2 = 63 \text{ m s}^{-1}$$

.

The area of the wing,

$$A = 2.5 \text{ m}^2$$

The density of air,

$$\rho = 1.3 \text{ kg/m}^3$$

Now, according to Bernoulli's theorem, we have the relation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

In the current case, let

$$P_2$$

be the pressure on the lower side of the wing and

$$P_1$$

be the pressure on the upper side, and the v-s are the velocities mentioned initially.

The required lift is the force that comes from the pressure difference in the lower and the upper surface.

This can be obtained as:

$$Lift = (P_1 - P_2)A = \left( \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \right) A$$

$$Lift = \frac{1}{2}\rho(v_2^2 - v_1^2)A = \frac{1}{2}1.3(70^2 - 63^2)2.5 = 1.51 \times 10^3 \text{ N}$$

Hence,

$$1.51 \times 10^3 \text{ N}$$

is the lift on the wing of the airplane.

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### Question 15

Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?

### Solution

Ans: Considering the second figure:

Let us say  $A_1$  and  $A_2$  are the cross-section areas of the pipe and the narrow region, respectively and  $V_1$  and  $V_2$  are the speeds of the fluid in the normal region of the pipe and at the narrow region, respectively

According to the law of continuity:

$$A_1 \times V_1 = A_2 \times V_2$$

Hence it can be noted that the velocity of the liquid at the narrow region is higher than that in the normal region.

Now using Bernoulli's theorem, it can be said that the pressure is high in a region where the velocity is low. Hence the pressure at the normally thick (at  $A_1$ ) will be higher than that at  $A_2$ .

Hence the column connected at this place will also have higher pressure at the point of contact. This is why the liquid column will also rise higher than the column connected at the narrow junction. Therefore, figure (b) is possible.

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### Question 16

The cylindrical tube of a spray pump has a cross-section of

$$8.0\text{cm}^2$$

one end of which has

$$40$$

fine holes each of diameter

$$1.0\text{mm}$$

. If the liquid flow inside the tube is

$$1.5\text{m min}^{-1}$$

, what is the speed of ejection of the liquid through the holes?

### Solution

Ans: Provided that,

Area of the cross-section of the spray pump

$$A_1 = 8 \times 10^{-4}\text{m}^2$$

Number of holes,

$$n = 40$$

Diameter of each hole,

$$d = 10^{-3}\text{m}$$

The radius of each hole,

$$r = 0.5 \times 10^{-3}\text{m}$$

Area of the cross-section of each hole,

$$a = \pi r^2 = \pi(0.5 \times 10^{-3})^2$$

The total area of 40 holes,

$$A = n a = n\pi r^2 = 31.41 \times 10^{-6}\text{m}^2$$

The speed of flow of liquid inside the tube,

$$v_1 = 1.5\text{m/min} = 0.025\text{m/s}$$

The liquid is ejected through the holes at the speed of

$$v_2$$

According to the continuity equation, we have

$$v_2 = \frac{A_1 v_1}{A_2} = 0.633\text{ m/s}$$

Therefore, it has been calculated that the speed of ejection of the liquid through the holes is

$$0.633 \text{ m/s}.$$

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### Question 17

A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of

$$1.5 \times 10^{-2} \text{ N}$$

. (Which includes the small weight of the slider). The length of the slider is

$$30 \text{ cm}$$

. What is the surface tension of the film?

### Solution

Ans: The weight that the soap film supports,

$$W = 1.5 \times 10^{-2} \text{ N}$$

Length of the slider,

$$l = 0.3 \text{ m}$$

A soap film has two free surfaces.

Therefore, the total length

$$= 2l = 0.6 \text{ m}$$

Hence the surface tension is obtained as

$$S = \frac{\text{Force}}{2l} = \frac{1.5 \times 10^{-2} \text{ N}}{0.6 \text{ m}} = 2.5 \times 10^{-2} \text{ N/m}$$

Therefore, the surface tension of the film is obtained as

$$2.5 \times 10^{-2} \text{ N/m}$$

.

**Question 18**

Figure 10.24 (a) shows a thin liquid film supporting a small weight

$$4.5 \times 10^{-2} N$$

. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically

**Solution**

Ans: For case (a):

The length of the liquid film is supported by the weight

$$l = 40 \text{ cm} = 0.4 \text{ m}$$

.

The weight supported by the film,

$$W = 4.5 \times 10^{-2} N$$

Every liquid film has two free surfaces. Hence the surface tension can be calculated as

$$S = \frac{\text{Force}}{2l} = \frac{4.5 \times 10^{-2} N}{2 \times 0.4 m} = 5.625 \times 10^{-2} N/m$$

Since the properties of the liquid are the same in all the cases., the surface tension in all three cases is equal. And the weight supported is also the same in all three cases, i.e.,

$$= 5.625 \times 10^{-2} N/m$$

.

**Question 19**

What is the pressure inside the drop of mercury of radius

$$3.0 \text{ mm}$$

at room temperature? Surface tension of mercury at that temperature

$$(20^{\circ} C)$$

is

$$4.65 \times 10^{-1} Nm^{-1}$$

. The atmospheric pressure

$$1.01 \times 10^5 Pa$$

. Also give the excess pressure inside the drop.

### Solution

Ans: As provided, the radius of the mercury drop,

$$r = 3 \times 10^{-3} m$$

The surface tension of mercury,

$$S = 4.65 \times 10^{-1} N/m$$

The atmospheric pressure,

$$P_0 = 1.01 \times 10^5 Pa$$

Total pressure inside the mercury drop = Excess pressure inside mercury + Atmospheric pressure

Total pressure

$$= \frac{2S}{r} + P_0 = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} + 1.01 \times 10^5 = 1.013 \times 10^5 Pa$$

Hence the excess pressure is obtained as

$$= \frac{2S}{r} = 310 Pa$$

.

**Question 20**

What is the excess pressure inside a bubble of soap solution of radius

$$5.0\text{mm}$$

, given that the surface tension of soap solution at the temperature

$$(20^{\circ}\text{C})$$

is

$$4.65 \times 10^{-1}\text{Nm}^{-1}$$

. If an air bubble of the same dimension were formed at the depth of

$$40.0\text{cm}$$

inside a container containing the soap solution (of relative density

$$1.20$$

), what would be the pressure inside the bubble? (1 atmospheric pressure is

$$1.01 \times 10^5\text{Pa}$$

.

**Solution**

Ans: As provided in the question, the excess pressure inside the soap bubble is

$$20\text{Pa}$$

;

The pressure inside the air bubble is

$$= 1.06 \times 10^5\text{Pa}$$

A soap bubble is of radius,

$$r = 5 \times 10^{-3}\text{m}$$

The surface tension of the soap solution,

$$S = 2.5 \times 10^{-2}\text{N/m}$$

Also, the relative density of the soap solution

$$= 1.20$$

Thus, the density of the soap solution,

$$\rho = 1.2 \times \rho_{air} = 1.2 \times 10^3 kg/m^2$$

It is given that the air bubble is formed at a depth,

$$h = 40 \text{ cm} = 0.4 \text{ m}$$

The radius of the air bubble,

$$r = 5 \times 10^{-3} \text{ m}$$

Acceleration due to gravity,

$$g = 9.8 \text{ m/s}^2$$

Therefore, it can be noted that the excess pressure inside the soap bubble is given by:

$$P = \frac{4S}{r} = 20 \text{ Pa}$$

The excess pressure inside the air bubble can be written as:

$$P = \frac{2S}{r} = 10 \text{ Pa}$$

.

The total pressure inside the air bubble under consideration is the sum of the atmospheric pressure, the pressure due to the column of the liquid and the excess pressure:

$$P_{total} = P_0 + h g \rho + P = 1.057 \times 10^5 \text{ Pa}$$

Hence, the pressure inside the air bubble is

$$1.057 \times 10^5 \text{ Pa}$$

.

**Question 21**

A tank with a square base of area

$$1.0m^2$$

is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area

$$20.0\text{ cm}^2$$

. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of. compute the force necessary to keep the door closed.

$$4.0\text{ m}$$

**Solution**

Ans: Provided that,

Base area of the given tank,

$$A = 1.0m^2$$

Area of the hinged door,

$$a = 0.02m^2$$

The density of water,

$$\rho_1 = 10^3 kg\,m^{-3}$$

The density of acid,

$$\rho_2 = 1.7 \times 10^3 kg\,m^{-3}$$

Height of the water column,

$$h_1 = 4m$$

Height of the acid column,

$$h_2 = 4m$$

Acceleration due to gravity,

$$g = 9.8m/s^2$$

Now, the pressure due to water is given as:

$$P_1 = \rho_1 h_1 g = 4 \times 10^3 \times 9.8 = 3.92 \times 10^4 Pa$$

The pressure due to acid is given as:

$$P_2 = \rho_2 h_2 g = 4 \times 1.7 \times 10^3 \times 9.8 = 6.662 \times 10^4 Pa$$

The pressure difference between the two columns:

$$\Delta P = P_2 - P_1 = 2.744 \times 10^4 Pa$$

Thus, the force exerted on the door

$$F = P \times a = 54.88 N$$

It can be concluded that the force necessary to keep the door closed is

$$54.88 N$$

## Question 22

A manometer reads the pressure of a gas in an enclosure as shown in figure. When a pump removes some of the gas, the manometer reads as in figure (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).

## Solution

Ans: In the first case i.e., fig (a)

Atmospheric pressure,

$$P_0 = 76 \text{ cm Hg}$$

The pressure (or height) difference between the levels of mercury in the two limbs denotes the gauge pressure.

Hence, the gauge pressure is 20 cm of Hg.

$$P_{absolute} = P_0 + P_{Gauge} = 76 + 20 = 96 \text{ cm Hg}$$

In the second figure (figure (b)), the difference between the levels of mercury in the two limbs

$$= 18 \text{ cm}$$

Hence, gauge pressure is

$$= 18 \text{ cm}$$

Hg.

Again, the

$$P_{absolute} = P_0 + P_{Gauge} = 76 + (-18) = 58 \text{ cm Hg}$$

Ans: Now,

$$13.6 \text{ cm}$$

water is filled into the right limb of figure (b).

The relative density of mercury

$$13.6 \text{ cm}$$

In other words, a column of

$$13.6 \text{ cm}$$

water is equivalent to

$$1 \text{ cm}$$

mercury.

Consider

$$h$$

being the difference in the mercury levels in the two limbs.

The pressure experienced in the right limb is given as:

$$P_{right} = P_0 + 1 \text{ cm Hg} = 76 + 1 = 77 \text{ cm Hg}$$

The level of the mercury column will go up in the left limb.

Hence, pressure in the left limb,

$$P_L = 58 + h$$

Equating the pressures in both the limbs and simplifying for h, we get:

$$77 = 58 + h$$

$$\Rightarrow h = 19\text{cm}$$

It can be hence concluded that the mercury levels differ by

$$19\text{cm}$$

---

### Question 23

Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

### Solution

Ans: Yes,

Pressure in a liquid is given as:

$$P = \rho hg$$

The heights up to which the vessels are filled are equal, the densities are equal, and g is the same for both cases. Hence the pressures are also the same

Now, let the force that is experienced at the base of an area

$$A$$

be

$$F = P \times A$$

Therefore, the forces in both cases are equal.

---

### Question 24

During blood transfusion the needle is inserted in a vein where the gauge pressure is

$$2000 Pa$$

. At what height must the blood container be placed so that blood may just enter the vein? (Use the density of whole blood from Table 10.1).

### Solution

Ans: The given gauge pressure,

$$P = 2000 Pa$$

The density of whole blood,

$$\rho = 1.06 \times 10^3 kg m^{-3}$$

Let

$$h$$

be the height of the blood container

The pressure inside the blood container is

$$P = \rho hg$$

Here

$$g = 9.8 m/s^2$$

is the acceleration due to gravity,

$$\Rightarrow h = \frac{P}{g\rho} = \frac{2000}{9.8 \times 1.06 \times 10^3} = 0.1925m$$

This means that the blood would enter the vein only if the container is kept at a height of

$$0.1925m$$

higher than the height at which the needle is inserted.

### Question 25

In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy.

(a) What is the largest average velocity of blood flow in an artery of diameter

$$2.0 \times 10^{-3}m$$

if the flow must remain laminar?

(b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

### Solution

Ans: The diameter of the given artery,

$$d = 2 \times 10^{-3}m$$

The viscosity of blood,

$$\eta = 2.084 \times 10^{-3}Pa \cdot s$$

The density of blood,

$$\rho = 1.06 \times 10^3 kg/m^3$$

Reynolds' number for laminar flow,

$$N_R = 2000$$

The maximum average velocity of blood flowing through the d-diameter orifice is as follows:

$$V = \frac{N_R \eta}{\rho d} = \frac{2000 \times 2.084 \times 10^{-3}}{(1.06 \times 10^3)(2 \times 10^{-3})} = 1.966 \text{ m/s}$$

Therefore, the largest average velocity of blood is

$$1.966 \text{ m/s}$$

.

Ans: The dissipative forces become more prominent with an increase in the fluid velocity. As the velocity increases, the turbulence also increases. The turbulent nature of the flow causes dissipative loss in a fluid.

### Question 26

(a) What is the largest average velocity of blood flow in an artery of radius

$$2.0 \times 10^{-3} \text{ m}$$

if the flow must remain laminar?

(b) What is the corresponding flow rate? (Take viscosity of blood to be

$$2.084 \times 10^{-3} \text{ Pa s}$$

).

### Solution

Ans: Provided that,

Radius of the artery,

$$r = 2 \times 10^{-3} \text{ m}$$

.

Diameter of the artery,

$$d = 2(2 \times 10^{-3} \text{ m})$$

The viscosity of blood,

$$\eta = 2.084 \times 10^{-3} \text{ Pa s}$$

The density of blood,

$$\rho = 1.06 \times 10^3 \text{ kg/m}^3$$

The Reynolds number of Laminar flow,

$$N_R = 2000$$

The largest value of average velocity can be given as:

$$V = \frac{N_R \eta}{\rho d} = \frac{2000 \times 2.084 \times 10^{-3}}{(1.06 \times 10^3)(4 \times 10^{-3})} = 0.983 \text{ m/s}$$

Hence the largest average velocity of blood flow in an artery of the radius

$$2 \times 10^{-3} \text{ m}$$

is

$$0.983 \text{ m/s}$$

.

Ans: The flow rate is written as the time derivative of the volume that flowed past a cross-section:

$$R = \pi r^2 V_{Avg} = 3.14 (2 \times 10^{-3})^2 (9.83) = 1.235 \times 10^{-5} \text{ m}^3/\text{s}$$

Hence the corresponding flow rate is

$$1.235 \times 10^{-5} \text{ m}^3/\text{s}$$

.

### Question 27

A plane is in level flight at a constant speed and each of its two wings has an area of

$$25 \text{ m}^2$$

. If the speed of the air is

$$180 \text{ km/h}$$

over the lower wing and

$$234 \text{ km/h}$$

over the upper wing surface, determine the plane's mass. (Take air density to be

$$1 \text{ kg/m}^3$$

).

### Solution

Ans: As provided in the question, the area of the wings is

$$A = 2 \times 25 = 50 \text{ m}^2$$

Speed of air over the lower wing,

$$v_1 = 180 \text{ km/h} = 50 \text{ m/s}$$

Speed of air over the upper wing,

$$v_2 = 234 \text{ km/h} = 65 \text{ m/s}$$

The density of air,

$$\rho = 1 \text{ kg/m}^3$$

The pressure of air over the lower wing

$$= P_1$$

The pressure of air over the upper wing

$$= P_2$$

The upward force on the plane can be obtained using Bernoulli's equation as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Thus, the upward force is obtained as:

$$Lift = (P_1 - P_2)A = \left( \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \right) A = 43125N$$

The mass of the plane can be obtained as:

$$m = F/g = 4400.5kg$$

Hence, the mass of the plane is nearly

$$4400.5kg$$

.

### Question 28

In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius

$$2.0 \times 10^{-5} m$$

and density

$$1.2 \times 10^3 kg/m^3$$

. Take the viscosity of air at the temperature of the experiment to be

$$1.8 \times 10^{-5} Pa s$$

. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

### Solution

Ans: The given terminal speed

$$= 5.8cm/s$$

, the viscous force

$$= 3.9 \times 10^{-10} N$$

The radius of the given uncharged drop,

$$r = 2.0 \times 10^{-5} m$$

The density of the uncharged drop,

$$\rho = 1.2 \times 10^3 \text{ kg m}^{-3}$$

It is known that the viscosity of air,

$$\eta = 1.8 \times 10^{-5} \text{ Pa s}$$

In this case, the density of air (

$$\rho_0$$

) can be taken as zero to neglect buoyancy of air. Acceleration due to gravity is

$$g = 9.8 \text{ m/s}^2$$

Terminal velocity (v) is given as:

$$v = \frac{2r^2(\rho - \rho_0)g}{9\eta} = \frac{2(2.0 \times 10^{-5})^2(1.2 \times 10^3 - 0)9.8}{91.8 \times 10^{-5}} = 5.8 \times 10^{-2} \text{ m/s}$$

Hence, the terminal velocity of the drop is

$$5.8 \text{ cm/s}$$

The viscous force experienced by the drop is given by:

$$F = 6\pi\eta rv$$

$$\Rightarrow F = 3.9 \times 10^{-10} \text{ N}$$

Hence the viscous force is calculated to be

$$3.9 \times 10^{-10} \text{ N}$$

.

### Question 29

Mercury has an angle of contact equal to

$$140^\circ$$

with soda lime glass. A narrow tube of radius

$$1.00 \text{ mm}$$

made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is

$$4.65 \times 10^{-1} \text{ Nm}^{-1}$$

. Density of mercury is

$$13.6 \times 10^3 \text{ kg/m}^3$$

### Solution

Ans: As provided in the question, the angle of contact between mercury and soda-lime glass,

$$\theta = 140^\circ$$

The radius of the narrow tube

$$r = 1.0 \times 10^{-3} \text{ m}$$

The given surface tension of mercury,

$$S = 0.465 \text{ N/m}$$

The density of mercury,

$$\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$$

Dip in the height of the mercury

$$= h$$

Acceleration due to gravity,

$$g = 9.8 \text{ m/s}^2$$

The surface tension can be written in terms of the angle of contact and the dip in height:

$$S = \frac{h\rho gr}{2 \cos \theta}$$

$$\Rightarrow h = \frac{2 \cos \theta}{S \rho g r} = -0.00534m = -5.34mm$$

Notice the negative sign; it shows the dropping level of mercury.

Hence, the mercury level drops by 5.34 mm.

### Question 30

Two narrow bores of diameters

$$3.0 \text{ mm}$$

and

$$6.0 \text{ mm}$$

are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is

$$7.31 \times 10^{-2} Nm^{-1}$$

. Let the angle of contact to be zero and density of water be

$$1.0 \times 10^3 kg/m^3$$

.

### Solution

Ans: Provided that,

Diameter of the first bore,

$$d_1 = 3.0 \times 10^{-3}m$$

Hence the radius of the first bore,

$$r_1 = d_1/2 = 1.5 \times 10^{-3}m$$

Diameter of the second bore,

$$d_2 = 6.0 \times 10^{-3}m$$

Hence, the radius of the second bore,

$$r_2 = d_2/2 = 3.0 \times 10^{-3}m$$

The surface tension of water,

$$S = 7.3 \times 10^{-2}N/m$$

The angle of contact between the water surface and the bore surface,

$$\theta = 0^\circ$$

The density of water,

$$\rho = 1.0 \times 10^3 kg m^{-3}$$

Acceleration due to gravity,

$$g = 9.8m/s^2$$

Let

$$h_1$$

and

$$h_2$$

be the heights to which water rises in the first and second tubes, respectively. These are given as:

$$h_1 = \frac{2S \cos \theta}{\rho g r_1}$$

and

$$h_2 = \frac{2S \cos \theta}{\rho g r_2}$$

The difference in the water levels in the two limbs of the tube can be calculated as:

$$\Delta h = \frac{2S \cos \theta}{\rho g r_1} - \frac{2S \cos \theta}{\rho g r_2} = \frac{2 \times 7.3 \times 10^{-2} \times \cos 0}{1.0 \times 10^3 \times 9.8} = 4.97mm$$

Hence, the difference between the levels is obtained as

$$= 4.97mm$$

.

**Question 31**

(a) It is known that density

$$\rho$$

of air decreases with height

$$y$$

as

$$\rho_0 e^{y/y_0}$$

. Where

$$\rho_0 = 1.25 \text{ kg m}^{-3}$$

the density at sea is level, and

$$y_0$$

is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of the atmosphere remains a constant (isothermal conditions). Also assume that the value of

$$g$$

remains constant.

**Solution**

Ans: The given volume of the balloon is,

$$V = 1425 \text{ m}^3$$

Mass of the payload,

$$m = 400 \text{ kg}$$

And the acceleration due to gravity is,

$$g = 9.8 \text{ m/s}^2$$

$$y_0 = 8000 \text{ m}$$

The density of helium is

$$\rho_{He} = 0.18 \text{ kg m}^{-3}$$

, and that of air at sea level is

$$\rho_0 = 1.25 \text{ kg m}^{-3}$$

Let the density of the balloon be  $\rho$  and the height to which the balloon can rise be  $y$ .

The air density (

$$\rho$$

) of decreases with increasing height (

$$y$$

) as:

$$\rho/\rho_0 = e^{-y/y_0}$$

This relation is also called the law of atmospherics.

It can be inferred from the above law-equation that the rate of decrease of density with height is directly proportional to

$$\rho$$

,

$$-\frac{d\rho}{dy} \propto \rho$$

$$\Rightarrow -\frac{d\rho}{dy} = k\rho$$

$$\Rightarrow -\frac{d\rho}{\rho} = k dy$$

Here,  $k$  is a proportionality constant.

We set the limits such that the height changes from

$$0$$

to

$$y$$

, and the density varies from

$$\rho_0$$

to

$$\rho$$

.

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^y k dy$$

Next, integrating the sides between these limits, we obtain:

$$[\ln \rho]_{\rho_0}^{\rho} = -ky$$

$$\Rightarrow \rho/\rho_0 = e^{-ky}$$

Comparing the above equation with the law of atmospherics, we get

$$\Rightarrow k = \frac{1}{y_0}$$

In conclusion, we have been able to derive the expression for the law of atmospherics.

(b) A large He balloon of volume

$$1425m^3$$

is used to lift a payload of

$$400 \text{ kg}$$

. Assume that the balloon maintains a constant radius as it rises. How high does it rise?

(Take

$$y_0 = 8000m$$

and

$$\rho_{He} = 0.18 \text{ kg m}^{-3}$$

)

Ans: The generic expression for mass density is:

$$\rho = \frac{M}{V}$$

In the current case, it becomes the total mass (including the payload and the helium) per unit volume of the balloon:

$$\begin{aligned}\rho &= \frac{(M_{\text{payload}} + M_{He})}{V} = \frac{m + V\rho_{He}}{V} \\ \Rightarrow \rho &= 0.46 \text{ kg/m}^3\end{aligned}$$

Using the law of atmospherics, we can obtain the value of

$$y$$

:

$$\begin{aligned}\rho/\rho_0 &= e^{-y/y_0} \\ \Rightarrow \ln \rho/\rho_0 &= -y/y_0 \\ \Rightarrow y &= -8000 \times \ln(0.46/1.25) = 8000 \text{ m} = 8 \text{ km}\end{aligned}$$

Therefore, the balloon can rise to a height of 8 km.