

Chapter 7: Gravitation

NCERT · Class 11 · Physics

Textbook exercises

Question 1

Answer the following:

A. You Can Shield a Charge from Electrical Forces by Putting it Inside a Hollow Conductor. Can You Shield a Body from the Gravitational Influence of Nearby Matter by Putting it Inside a Hollow Sphere or by Some Other Means?

B. An Astronaut Inside a Small Spaceship Orbiting Around the Earth Cannot Detect Gravity. If the Space Station Orbiting Around the Earth Has a Large Size, Can He Hope to Detect Gravity?

C. If You Compare the Gravitational Force on the Earth Due to the Sun to That Due to the Moon, You Would Find That the Sun's Pull Is Greater Than the Moon's Pull. (You Can Check This Yourself Using the Data Available in the Succeeding Exercises). However, the Tidal Effect of the Moon's Pull Is Greater Than the Tidal Effect of the Sun. Why?

Solution

Ans: No.

The gravitational force of matter on nearby objects cannot be eliminated by any means. This is because gravitational force is independent of the nature of the medium's material.

Ans: Yes.

If the space station's size is large enough, then the astronaut will identify the change in Earth's gravity.

Ans: The tidal effect depends on the inverse of distance's cube while gravitational force depends on the inverse of distance's square.

Since the distance between the Moon and the Earth is shorter than the distance between the Sun and the Earth, the tidal influence of the Moon's pull is higher than the tidal effect of the Sun's influence.

Question 2

Choose the Correct Alternative:

A.

Acceleration Due to Gravity Increases/decreases With Increasing Altitude.

B.

Acceleration Due to Gravity Increases/decreases With Increasing Depth. (assume the Earth to Be a Sphere of Uniform Density).

C.

Acceleration due to gravity is independent of mass of the earth/mass of the body.

D.

The Formula

$$-GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

is more/less accurate than the formula

$$mg(r_2 - r_1)$$

for the difference of Potential Energy Between Two Points and Distance Away from the Centre of the Earth.

Solution

Ans: Acceleration due to gravity decreases with increasing altitude.

Acceleration due to gravity at height

$$h$$

is given by:

$$g_h = \left(1 - \frac{2h}{R_e}\right) g$$

Where,

$$R_e =$$

Radius of the Earth

$$g =$$

Acceleration due to gravity on the surface of the Earth

It is clear from the relation that acceleration due to gravity lowers with a height increment.

Ans: Acceleration due to gravity decreases with increasing depth.

Acceleration due to gravity at depth

$$d$$

is given by:

$$g_d = \left(1 - \frac{d}{R_e}\right) g$$

It is clear from the relation that acceleration due to gravity lowers with a depth increment.

Ans: Acceleration due to gravity is independent of mass of the body.

Acceleration due to gravity of body having mass

$$m$$

is given by:

$$g = \frac{GM}{R^2}$$

Where,

$$G$$

= Universal gravitational constant

$$M$$

= Mass of the Earth

$$R$$

= Radius of the Earth

Hence, it is clear that acceleration due to gravity is not dependent on the body's mass.

Ans: Gravitational potential energy of two points at

$$r_2$$

and

$$r_1$$

distance away from the Earth centre is given by:

$$V(r_1) = -\frac{GmM}{r_1}$$

and

$$V(r_2) = -\frac{GmM}{r_2}$$

$$\therefore$$

Difference in potential energy,

$$\begin{aligned} V &= V(r_2) - V(r_1) \\ \Rightarrow V &= -GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \end{aligned}$$

Hence, this formula is more reliable than the formula

$$mg(r_2 - r_1)$$

.

Question 3

Suppose there Existed a Planet That Went Around the Sun Twice as Fast as the Earth. What would Its Orbital Size be Compared to that of the Earth?

Solution

Ans: Time taken by the Earth to complete one revolution,

$$T_e = 1 \text{ year}$$

.

Orbital radius of the Earth in its orbit,

$$R_e = 1AU$$

.

Time taken by the planet to complete one revolution,

$$T_p = \frac{1}{2}T_e = \frac{1}{2}year$$

.

Orbital radius of the planet

$$= R_p$$

.

From Kepler's third law of planetary motion, we can write:

$$\left(\frac{R_p}{R_e}\right)^3 = \left(\frac{T_p}{T_e}\right)^2$$

$$\Rightarrow \frac{R_p}{R_e} = \left(\frac{T_p}{T_e}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = \left(\frac{\frac{1}{2}}{1}\right)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = (0.5)^{\frac{2}{3}}$$

$$\Rightarrow \frac{R_p}{R_e} = 0.63$$

Hence, the orbital radius of the planet will be

$$0.63$$

times smaller than that of the Earth.

Question 4

I_O , One of the Satellites of Jupiter, Has an Orbital Period of

$$1.769 \text{ days}$$

and the radius of the orbit is

$$4.22 \times 10^8 m$$

. Show That the Mass of Jupiter is About One-Thousandth That of the Sun.

Solution

Ans: Given that,

Orbital period of

$$I_O = T_{I_O} = 1.769 \text{ days} = 1.769 \times 24 \times 60 \times 60 s$$

Orbital radius of

$$I_O = R_{I_O} = 4.22 \times 10^8 m$$

Satellite

$$I_O$$

is revolving around Jupiter.

Mass of the Jupiter is given by:

$$M_J = \frac{4\pi^2 R_{I_O}^3}{GT_{I_O}^2}$$

.....(i)

Where,

$$M_J$$

= Mass of Jupiter

$$G$$

is the Universal gravitational constant

Orbital period of the Earth,

$$T_e = 365.25 \text{ days} = 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Orbital radius of the Earth,

$$R_e = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

Mass of sun is given as:

$$M_s = \frac{4\pi^2 R_e^3}{GT_e^2}$$

.....(ii)

$$\begin{aligned} \Rightarrow \frac{M_S}{M_J} &= \frac{4\pi^2 R_e^3}{GT_e^2} \times \frac{GT_{Io}^2}{4\pi^2 R_{Io}^3} = \frac{R_e^3}{R_{Io}^3} \times \frac{T_{Io}^2}{T_e^2} \\ \Rightarrow \frac{M_S}{M_J} &= \left(\frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60} \right)^2 \times \left(\frac{1.496 \times 10^{11}}{4.22 \times 10^8} \right)^3 \\ \Rightarrow \frac{M_S}{M_J} &= 1045.04 \\ \therefore \frac{M_s}{M_J} &\sim 1000 \end{aligned}$$

We get,

$$M_s \sim 1000 \times M_J$$

Hence, it can be concluded that the mass of Jupiter is about one-thousandth that of the Sun.

Question 5

Let Us Assume That Our Galaxy Consists of

$$2.5 \times 10^{11}$$

stars Each of One Solar Mass. How Long Will a Star be at a Distance of

$$50,000 ly$$

From the Galactic Centre to Complete One Revolution? Take the Diameter of the Milky Way to be

$$10^5 ly$$

.

Solution

Ans: Mass of galaxy Milky way,

$$M = 2.5 \times 10^{11}$$

solar mass

Solar mass = Mass of sun

$$= 2.0 \times 10^{36} kg$$

Mass of our galaxy,

$$M = 2.5 \times 10^{11} \times 2 \times 10^{35}$$

$$M = 5 \times 10^{41} kg$$

Diameter of Milky Way,

$$d = 10^5 ly$$

Radius of Milky Way,

$$r = 5 \times 10^4 ly$$

$$1 ly = 9.46 \times 10^{15} m$$

$$r = 5 \times 10^4 \times 9.46 \times 10^{15}$$

$$\Rightarrow r = 4.73 \times 10^{20} m$$

Since a star rotates around the galactic centre of the Milky Way, its time period is given by:

$$T = \left(\frac{4\pi^2 r^3}{GM} \right)^{\frac{1}{2}} = \left(\frac{4 \times (3.14)^2 \times (4.73)^3 \times 10^{60}}{6.67 \times 10^{-11} \times 5 \times 10^{41}} \right)$$

$$\Rightarrow T = (125.27 \times 10^{30})^{\frac{1}{2}}$$

$$\Rightarrow T = 1.12 \times 10^{16} s$$

As we know,

$$1 \text{ year} = 365 \times 24 \times 60 \times 60 s$$

We get,

$$1 s = \frac{1}{365 \times 24 \times 60 \times 60} \text{ years}$$

$$\Rightarrow 1.12 \times 10^{16} s = \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60}$$

$$\therefore 1.12 \times 10^{16} s = 3.55 \times 10^8 \text{ years}$$

Star will take

$$3.55 \times 10^8 \text{ years}$$

to complete one revolution.

Question 6

Choose the Correct Alternative:

A. If the Zero of Potential Energy is at Infinity, the Total Energy of an Orbiting Satellite Is Negative of Its Kinetic/Potential Energy.

B. the Energy Required to Launch an Orbiting Satellite Out of Earth's Gravitational Influence is More/less Than the Energy Required to Project a Stationary Object at the Same Height (as the Satellite) Out of Earth's Influence.

Solution

Ans: Kinetic energy.

The sum of kinetic energy and potential energy is the total mechanical energy. The satellite's gravitational potential energy is zero at infinity. As we know, the Earth-satellite system is a bound system; then, the satellite's total energy is negative.

Thus, at infinity, the orbiting satellite's total energy is equal to the negative of its kinetic energy.

Ans: Less.

An orbiting satellite gets a certain amount of energy that permits it to rotate around the Earth. Its orbit gives this energy. It requires comparatively lesser energy to go out of the influence of the Earth's gravitational field than a fixed object on the Earth's surface that initially holds no energy.

Question 7

Does the Escape Speed of a Body from the Earth Depend on

A.

The Mass of the Body?

B.

The Location from Where it is Projected?

C.

The Direction of Projection?

D.

The Height of the Location from Where the Body is Launched?

Solution

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the mass of the body.

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the location from where it is projected.

Ans: No.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does not depend on the direction of projection.

Ans: Yes.

Escape velocity of a body from the Earth is given by the relation:

$$v = \sqrt{2gR}$$

The escape speed of a body does slightly depend on the height of the location from where the body is launched. This is because escape velocity is slightly dependent on the gravitational potential at a certain height.

Question 8

A Comet Orbits the Sun in a Highly Elliptical Orbit. Does the Comet have a Constant

A.

Linear Speed?

B.

Angular Speed?

C.

Angular Momentum?

D.

Kinetic Energy?

Solution

Ans: No.

The expression for angular momentum is:

$$L = \frac{mv^2}{r}$$

As angular momentum is constant, linear speed varies as r varies.

Ans: No.

The expression for angular momentum is:

$$L = mw^2r$$

As angular momentum is constant, angular speed varies as r varies.

Ans: Yes.

There is no external torque and thus, the angular momentum is constant.

Ans: No.

The expression for kinetic energy is:

$$K = \frac{1}{2}mv^2$$

Kinetic energy is not constant because linear speed varies.

E. Potential Energy?

Ans: No.

By law of conservation of energy, total energy is constant and it is the sum of potential energy and kinetic energy.

As kinetic energy is not constant, potential energy is also not constant.

F. Total Energy Throughout Its Orbit? Neglect Any Mass Loss of the Comet When it Comes Very Close to the Sun.

Ans: Yes.

By law of conservation of energy, the total energy is constant.

Question 9

Which of the Following Symptoms is Likely to Afflict an Astronaut in Space

A.

Swollen Feet

B.

Swollen Face

C.

Headache

D.

Orientational Problem

Solution

Ans: Legs hold the whole mass in a standing place due to gravitational force.

In space, an astronaut appears weightless because of the absence of gravity. Therefore, swollen feet of a spaceman do not affect him/her in space.

Ans: A swollen face is usually caused because of seeming weightlessness in space.

Sense organs such as the nose, eyes, ears, and mouth establish a person's face. This symptom can affect a spacewalker in space.

Ans: Headaches are because of mental stress.

It can influence the working of an astronaut in space.

Ans: Space has diverse orientations.

Therefore, orientational difficulty can affect an astronaut in space.

Question 10

Choose the Correct Answer from Among the Given Ones:

The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12).

1. a,
2. b,
3. c,
4. O

Solution

Ans: Option (iii) is correct.

Gravitational potential (

V

) is fixed at all points in a spherical shell. Hence, the potential gravitational gradient

$$\left(\frac{dV}{dr} \right)$$

is zero everywhere inside the spherical surface.

The potential gravitational gradient is equivalent to the negative of gravitational intensity. Hence, intensity is also zero at all locations inside the spherical shell. This shows that gravitational forces operating at a point in a spherical shell are symmetric.

If the top half of a spherical shell is cut out, then the net gravitational force working on a particle located at centre O will be downward.

Since gravitational intensity is described as the gravitational force per unit mass at that location, it will also act downward. Thus, the gravitational intensity at the centre of the given hemispherical shell has the direction indicated by the arrow

c

.

Question 11

Choose the Correct Answer from Among the Given Ones:

For the problem 8.10, the direction of the gravitational intensity at an arbitrary point

P

is indicated by the arrow

1. d,
2. e,
3. f,
4. g.

Solution

Ans: Option (ii) is correct.

Gravitational potential (V)

$$V$$

) is fixed at all points in a spherical shell. Hence, the potential gravitational gradient

$$\left(\frac{dV}{dr} \right)$$

is zero everywhere inside the spherical surface.

The potential gravitational gradient is equivalent to the negative of gravitational intensity. Hence, intensity is also zero at all locations inside the spherical shell. This shows that gravitational forces operating at a point in a spherical shell are symmetric.

If the top half of a spherical shell is cut out, then the net gravitational force working on a particle located at centre P will be downward.

Since gravitational intensity is described as the gravitational force per unit mass at that location, it will also act downward.

Thus, the gravitational intensity at the arbitrary point P of the given hemispherical shell has the direction indicated by the arrow

$$e$$

.

Question 12

A Rocket is Fired from the Earth Towards the Sun. At What Distance from the Earth's Centre is the Gravitational Force on the Rocket Zero?

Mass of the sun

$$= 2 \times 10^{30} kg$$

, mass of the earth

$$= 6 \times 10^{24} kg$$

. Neglect the effect of other planets etc. (orbital radius

$$= 1.5 \times 10^{11} m$$

).

Solution

Ans: Mass of the Sun,

$$M_s = 2 \times 10^{30} kg$$

Mass of the Earth,

$$M_e = 6 \times 10^{24} kg$$

Orbital radius,

$$r = 1.5 \times 10^{11} m$$

Mass of the rocket=

$$m$$

Let

$$x$$

be the distance from the Earth centre where the gravitational force working on the satellite

$$P$$

becomes zero.

From Newton's law of gravitation, we can equalize gravitational forces acting on satellite

$$P$$

under the effect of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

We get,

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\Rightarrow \frac{r-x}{x} = \left(\frac{2 \times 10^{30}}{60 \times 10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$\Rightarrow 1.5 \times 10^{11} - x = 577.35x$$

$$\Rightarrow 578.35x = 1.5 \times 10^{11}$$

$$\Rightarrow x = \frac{1.5 \times 10^{11}}{578.35} = 2.59 \times 10^8 m$$

Therefore,

$$2.59 \times 10^8 m$$

is the distance from the earth's centre at which the gravitational force on the rocket is zero.

Question 13

How will you 'weigh the sun', that is, estimate its mass? The mean orbital radius of the earth around the sun is

$$1.5 \times 10^8 km$$

.

Solution

Ans: Orbital radius of the Earth around the Sun,

$$r = 1.5 \times 10^{11} m$$

.

Time taken by the Earth to cover one revolution around the Sun,

$$T = 1 \text{ year}$$

$$\Rightarrow T = 365.25 \text{ days}$$

We get,

$$T = 365.25 \times 24 \times 60 \times 60 s$$

Universal gravitational constant,

$$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$$

.

Thus, we can use the relation given below for calculating the mass of the Sun,

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$\Rightarrow M = \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$\Rightarrow M = \frac{133.24 \times 10^{33}}{6.64 \times 10^4}$$

$$\Rightarrow M = 2.0 \times 10^{30} kg$$

Hence, the mass of the Sun is

$$2 \times 10^{30} kg$$

.

Question 14

A Saturn year is

$$29.5$$

times the earth year. How far is Saturn from the sun if the earth is

$$1.50 \times 10^8 km$$

away from the sun?

Solution

Ans: Distance of the Earth from the Sun,

$$r_e = 1.5 \times 10^8 km = 1.5 \times 10^{11} m$$

Time period of the Earth

$$= T_e$$

Time period of Saturn,

$$T_s = 29.5T_e$$

Distance of Saturn from the Sun

$$= r_s$$

From Kepler's third law of planetary motion, we have,

$$T = \left(\frac{4\pi^2 r^3}{GM} \right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write,

$$\frac{r_s^3}{r_e^3} = \frac{T_s^2}{T_e^2}$$

$$\Rightarrow r_s = r_e \left(\frac{T_s}{T_e} \right)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \left(\frac{29.5T_e}{T_e} \right)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} (29.5)^{\frac{2}{3}}$$

$$\Rightarrow r_s = 1.5 \times 10^{11} \times 9.55$$

$$\Rightarrow r_s = 14.32 \times 10^{11} m$$

Hence, the distance between the Sun and Saturn is

$$1.43 \times 10^{12} m$$

Question 15**A Body weighs**

$$63N$$

on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the Earth?

Solution

Ans: Weight of the body,

$$W = 63N$$

Acceleration due to gravity at height

$$h$$

from the Earth's surface

$$R_e =$$

Radius of the Earth

For

$$h = \frac{R_e}{2}$$

, gravity at h is given by:

$$g_h = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

$$\Rightarrow g_h = \frac{g}{\left(1 + \frac{1}{2}\right)^2}$$

$$\Rightarrow g_h = \frac{4}{9}g$$

Weight of a body of mass

$$m$$

at height

$$h$$

is given as:

$$W' = mg_h$$

$$\Rightarrow W' = m \times \frac{4}{9}g$$

$$\Rightarrow W' = \frac{4}{9} \times mg$$

$$\Rightarrow W' = \frac{4}{9}W$$

$$\Rightarrow W' = \frac{4}{9} \times 63$$

$$\Rightarrow W' = 28N$$

Thus, the gravitational force on the body due to the earth at a height equal to half the radius of the earth is $28N$.

Question 16

Assuming the Earth to Be a Sphere of Uniform Mass Density, How Much Would a Body Weigh Half Way Down to the Centre of the Earth If it Weighed

$$250N$$

on the Surface?

Solution

Ans: Weight of a body of mass

$$m$$

at the Earth's surface is given by,

$$W = mg = 250N$$

Body of mass

$$m$$

is located at depth,

$$d = \frac{1}{2}R_e$$

Where,

$$R_e$$

is the radius of the Earth.

Acceleration due to gravity at depth

$$g_d$$

is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right) g$$

$$\Rightarrow \left(1 - \frac{R_e}{2 \times R_e}\right) g = \frac{1}{2}g$$

Weight of the body at depth

$$d$$

,

$$W' = mg$$

$$\Rightarrow W' = m \times \frac{1}{2}g = \frac{1}{2}mg = \frac{1}{2}W$$

$$\Rightarrow W' = \frac{1}{2} \times 250 = 125N$$

The weight of body half way down to the centre of the earth is $125N$.

Question 17

A Rocket is fired vertically with a speed of

$$5kms^{-1}$$

from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass

of the earth

$$= 6.0 \times 10^{24} kg$$

; mean radius of the earth

$$= 6.4 \times 10^6 m; G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$$

.

Solution

Ans: Distance from the centre of the Earth

$$= 8 \times 10^6 m$$

Velocity of the rocket,

$$v = 5 km s^{-1} = 5 \times 10^3 m s^{-1}$$

Mass of the Earth,

$$M_e = 6.0 \times 10^{24} kg$$

Radius of the Earth,

$$R_e = 6.4 \times 10^6 m$$

Height reached by rocket mass m , is

$$h$$

.

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_em}{R_e} \right)$$

At highest point h ,

$$v = 0$$

And, Potential energy

$$= -\frac{GM_em}{R_e + h}$$

Total energy of the rocket

$$= 0 + \left(\frac{GM_em}{R_e + h} \right) = -\frac{GM_em}{R_e + h}$$

From the law of energy conservation, we have

Total energy of the rocket at the Earth's surface = Total energy of rocket at height

$$h$$

.

We have,

$$\frac{1}{2}mv^2 + \left(-\frac{GM_em}{R_e} \right) = -\frac{GM_em}{R_e + h}$$

$$\Rightarrow \frac{1}{2}v^2 = GM_e \left(\frac{1}{R_e} - \frac{1}{R_e + h} \right)$$

$$\Rightarrow \frac{1}{2}v^2 = GM_e \left(\frac{R_e + h - R_e}{R_e(R_e + h)} \right)$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{GM_e h}{R_e(R_e + h)} \times \frac{R_e}{R_e}$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{gR_e h}{R_e + h}$$

Where,

$$g = \frac{GM}{R_e^2} = 9.8ms^{-2}$$

, is the acceleration due to gravity on the Earth's surface.

Clearly,

$$v^2(R_e + h) = 2gR_e h$$

$$\Rightarrow v^2 R_e = h(2gR_e - v^2)$$

$$\Rightarrow h = \frac{R_e v^2}{2gR_e - v^2}$$

$$\Rightarrow h = \frac{6.4 \times 25 \times 10^{12}}{100.44 \times 10^6}$$

$$\Rightarrow h = 1.6 \times 10^6 m$$

Height achieved by the rocket with respect to the centre of the Earth, H is given by:

$$H = R_e + h$$

$$\Rightarrow H = 6.4 \times 10^6 + 1.6 \times 10^6$$

We get,

$$H = 8.0 \times 10^6 m$$

The distance from the earth is

$$8.0 \times 10^6 m$$

where the rocket goes before returning to the earth.

Question 18

The escape speed of a projectile on the earth's surface is

$$11.2 km s^{-1}$$

. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Solution

Ans: Projectile's escape velocity from the Earth,

$$v_{esc} = 11.2 km s^{-1}$$

Projection velocity of the projectile,

$$v_p = 3v_{esc}$$

Mass of the projectile

$$= m$$

Projectile's velocity far away from the Earth

$$= v_f$$

Projectile's total energy on the Earth

$$= \frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$$

The projectile's gravitational potential energy far away from the Earth is zero.

Total energy of the projectile far away from the Earth

$$= \frac{1}{2}mv_f^2$$

From the law of energy conservation, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$

We get,

$$\begin{aligned} v_f &= \sqrt{v_p^2 - v_{esc}^2} \\ \Rightarrow v_f &= \sqrt{(3v_{esc})^2 - (v_{esc})^2} \\ \Rightarrow v_f &= \sqrt{8}v_{esc} \\ \Rightarrow v_f &= \sqrt{8} \times 11.2 \\ \Rightarrow v_f &= 31.68 \text{ km s}^{-1} \end{aligned}$$

Thus, the speed of the body far away from the earth is

$$31.68 \text{ km s}^{-1}$$

.

Question 19

A Satellite Orbits the Earth at a height of

$$400km$$

above the surface. How much Energy must be expended to Rocket the Satellite out of the earth's gravitational influence?

Mass of the satellite

$$= 200kg$$

;

mass of the earth

$$= 6.0 \times 10^{24}kg$$

;

radius of the earth

$$= 6.4 \times 10^6$$

;

$$G = 6.67 \times 10^{-11}Nm^2kg^{-2}$$

.

Solution

Ans: Given that,

Mass of the Earth,

$$M_e = 6.0 \times 10^{24}kg$$

Mass of the satellite,

$$m = 200kg$$

Radius of the Earth,

$$R_e = 6.4 \times 10^6m$$

Universal gravitational constant,

$$G = 6.67 \times 10^{-11}Nm^2kg^{-2}$$

Height of the satellite,

$$h = 400\text{km} = 4 \times 10^5\text{m} = 0.4 \times 10^6\text{m}$$

Total energy of the satellite at height h ,

$$TE = \frac{1}{2}mv^2 + \left(\frac{-GM_em}{R_e + h} \right)$$

Orbital velocity of the satellite,

$$v = \sqrt{\frac{GM_e}{R_e + h}}$$

Total energy at height h ,

$$TE = \frac{1}{2}m \left(\frac{GM_e}{R_e + h} \right) - \frac{GM_em}{R_e + h} = -\frac{1}{2} \left(\frac{GM_em}{R_e + h} \right)$$

The negative sign explains that the satellite is attached to the Earth. This is called the bound energy of the satellite.

The energy needed to send the satellite out of its orbit is equal to the negative of Bound energy.

Bound energy is,

$$BE = \frac{1}{2} \left(\frac{GM_em}{R_e + h} \right)$$

Where, M_e is the mass of the Earth.

R_e is the radius of the Earth.

h is the height.

m is the mass of the satellite.

$$\begin{aligned} \Rightarrow BE &= \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{(6.4 \times 10^6 + 0.4 \times 10^6)} \\ \Rightarrow BE &= \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.4 \times 10^6} \end{aligned}$$

$$\Rightarrow BE = 5.9 \times 10^9 J$$

Clearly,

$$5.9 \times 10^9 J$$

energy must be expended to rocket the satellite out of the earth's gravitational influence.

Question 20

Two stars each of one solar mass (

$$= 2 \times 10^{30} kg$$

) are approaching each other for a head on collision. When they are a distance

$$10^9 km$$

, their speeds are negligible. What is the speed with which they collide? The radius of each star is

$$10^4 km$$

. Assume the stars to remain undistorted until they collide. (Use the known value of

$$G$$

).

Solution

Ans: Mass of each star,

$$M = 2 \times 10^{30} kg$$

Radius of each star,

$$R = 10^4 km = 10^7 m$$

Distance between the stars,

$$r = 10^9 km = 10^{12} m$$

For negligible speeds,

$$v = 0$$

Total energy of two stars separated at distance

$$r$$

is given by

$$TE = \frac{-GMM}{r} + \frac{1}{2}mv^2$$

$$\Rightarrow TE = \frac{GMM}{r} + 0.$$

.....(i)

Now, consider the case when the stars are about to collide:

Velocity of the stars

$$= v$$

Distance between the centres of the stars

$$= 2R$$

Total kinetic energy of both stars

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

Total potential energy of both stars

$$= \frac{-GMM}{2R}$$

Total energy of the two stars

$$= Mv^2 - \frac{GMM}{2R}$$

.....(ii)

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{R}$$

$$\begin{aligned}\Rightarrow v^2 &= \frac{-GM}{r} + \frac{GM}{2r} = GM \left(-\frac{1}{r} + \frac{1}{2R} \right) \\ \Rightarrow v^2 &= 6.67 \times 10^{-10} \times 2 \times 10^{30} \left[-\frac{1}{10^{12}} + \frac{1}{2 \times 10^7} \right] \\ \Rightarrow v^2 &= 13.34 \times 10^{19} [-10^{-12} + 5 \times 10^{-8}] \\ \Rightarrow v^2 &\sim 13.34 \times 10^{19} \times 5 \times 10^{-8} \\ \Rightarrow v^2 &\sim 6.67 \times 10^{12} \\ \Rightarrow v &= \sqrt{6.67 \times 10^{12}} \\ \Rightarrow v &= 2.58 \times 10^6 m/s\end{aligned}$$

The speed with which the two stars collide is

$$2.58 \times 10^6 m/s$$

Question 21

Two heavy spheres each of mass

$$100kg$$

and radius

$$0.10m$$

are placed

$$1.0m$$

apart on a horizontal table. What is the gravitational force and potential at the midpoint of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Solution

Ans: The situation is represented in the given figure:

Mass of each sphere,

$$M = 100kg$$

Separation between the spheres,

$$r = 1m$$

X

is the midpoint between the spheres. Gravitational force at point

X

will be zero. This is because the gravitational force applied by each sphere will act in opposite directions.

Gravitational potential at point

X

$$= \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)}$$

$$\Rightarrow PE = -4\frac{GM}{r}$$

$$\Rightarrow PE = \frac{4 \times 6.67 \times 10^{-11} \times 100}{r}$$

$$\Rightarrow PE = -2.67 \times 10^{-8} J/kg$$

Any object placed at point

X

will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will vary the effective force in that direction.

Question 22

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly

$$36,000km$$

from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero).

Mass of the Earth =

$$6.0 \times 10^{24} kg$$

,

Radius =

$$6400 km.$$

Solution

Ans: Mass of the Earth,

$$M = 6.0 \times 10^{24} kg$$

Radius of the Earth,

$$R = 6400 km = 6.4 \times 10^6 m$$

Height of a geostationary satellite from the surface of the Earth,

$$h = 36000 km = 3.6 \times 10^7 m$$

Gravitational potential energy due to Earth's gravity at height

$$h$$

,

$$PE = \frac{-GM}{(R + h)}$$

$$\Rightarrow PE = - \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.6 \times 10^7 + 0.64 \times 10^7}$$

$$\Rightarrow PE = - \frac{6.67 \times 6}{4.24} \times 10^{13-7}$$

$$\Rightarrow PE = -9.4 \times 10^6 J kg^{-1}$$

The potential energy due to earth's gravity at the site of this satellite is

$$-9.4 \times 10^6 J kg^{-1}$$

.

Question 23

A star 2.5 times the mass of the sun and collapsed to a size of

$$12km$$

rotates with a speed of

$$1.2$$

evolutions per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun

$$= 2 \times 10^{30} kg$$

).

Solution

Ans: Yes.

A body gets held to the star's surface if the inward gravitational force is larger than the outward centrifugal force generated by the star's rotation.

Gravitational force,

$$f_g = \frac{GMm}{R^2}$$

Where, M is mass of the star,

$$M = 2.5 \times 2 \times 10^{30} = 5 \times 10^{30} kg$$

m is the mass of the body

R is the radius of star,

$$R = 12km = 1.2 \times 10^4 m$$

$$\therefore f_g = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30} \times m}{(1.2 \times 10^4)^2}$$

$$f_g = 2.31 \times 10^{11} mN$$

Centrifugal force,

$$f_c = mr\omega^2$$

$$\omega = \text{Angular speed} = 2\pi v$$

$$v = \text{Angular frequency} = 1.2 rev\ s^{-1}$$

$$f_c = mR(2\pi v)^2$$

$$\Rightarrow f_c = m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2$$

$$\Rightarrow f_c = 1.7 \times 10^5 mN$$

Since

$$f_g > f_c$$

, the body will remain held to the surface of the star.

Question 24

A Spaceship is Stationed on Mars. How Much Energy Must Be Expended on the Spaceship to Launch it Out of the Solar System?

Mass of the spaceship

$$= 1000 kg$$

; mass of the Sun

$$= 2 \times 10^{30} kg$$

;

mass of mars

$$= 6.4 \times 10^{23} kg$$

; radius of mars

$$= 3395 km$$

;

radius of the orbit of mars

$$= 2.28 \times 10^8 kg; G = 6.67 \times 10^{-11} m^2 kg^{-2}$$

.

Solution

Ans: Mass of the spaceship,

$$m_s = 1000 kg$$

Mass of the Sun,

$$M = 2 \times 10^{30} kg$$

Mass of Mars,

$$M_m = 6.4 \times 10^{23} kg$$

Orbital radius of Mars,

$$R = 2.28 \times 10^8 kg = 2.28 \times 10^{11} m$$

Radius of Mars,

$$r = 3395 km = 3.395 \times 10^6 m$$

Universal gravitational constant,

$$G = 6.67 \times 10^{-11} m^2 kg^{-2}$$

Potential energy of the spaceship due to the gravitational attraction of the Sun

$$= \frac{-GMm_s}{R}$$

Potential energy of the spaceship due to the gravitational attraction of Mars

$$= \frac{-GM_m m_s}{r}$$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship

$$= \frac{-GMm_s}{R} - \frac{GM_m m_s}{r}$$

$$TE = -Gm_s \left(\frac{M}{R} + \frac{M_m}{r} \right)$$

The negative sign indicates that the system is in a bound state.

Energy required for launching the spaceship out of the solar system = – (Total energy of the spaceship)

We have,

$$E = Gm_s \left(\frac{M}{R} + \frac{m_m}{r} \right)$$

$$\Rightarrow E = 6.67 \times 10^{-11} \times 10^3 \times \left(\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3.395 \times 10^5} \right)$$

$$\Rightarrow E = 6.67 \times 10^{-8} (87.72 \times 10^{17} + 1.88 \times 10^{17})$$

$$\Rightarrow E = 6.67 \times 10^{-8} \times 1.88 \times 10^{17}$$

$$\Rightarrow E = 596.97 \times 10^9$$

$$\Rightarrow E = 6 \times 10^{11} J$$

The required energy for the spaceship to launch it out of the solar system is

$$6 \times 10^{11} J$$

.

Question 25

A rocket is fired 'vertically' from the surface of mars with a speed of

$$2km/s$$

. If

of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars

$$= 6.4 \times 10^{23} kg$$

; radius of mars

$$= 3395 km$$

;

$$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$$

.

Solution

Ans: Initial velocity of the rocket,

$$v = 2 km/s = 2 \times 10^3 m/s$$

Mass of Mars,

$$M = 6.4 \times 10^{23} kg$$

Radius of Mars,

$$R = 3395 km = 3.395 \times 10^6 m$$

Universal gravitational constant,

$$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$$

Mass of the rocket

$$= m$$

Initial kinetic energy of the rocket

$$= \frac{1}{2}mv^2$$

Initial potential energy of the rocket

$$= \frac{-GMm}{R}$$

Total initial energy

$$= \frac{1}{2}mv^2 - \frac{GMm}{R}$$

If

$$20$$

of initial kinetic energy is lost due to Martian atmospheric resistance, then it shows only

$$80$$

of its kinetic energy helps in reaching a height.

Total initial energy available,

$$TE = \frac{80}{100} \times \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$TE = 0.4mv^2 - \frac{GMm}{R}$$

Maximum height reached by the rocket

$$= h$$

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h ,

$$= -\frac{GMm}{(R+h)}$$

Applying the law of energy conservation for the rocket, we can write:

$$0.4mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$\Rightarrow 0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$\Rightarrow 0.4v^2 = GM \left(\frac{1}{R} - \frac{1}{R+h} \right)$$

$$\Rightarrow 0.4v^2 = GM \left(\frac{R+h-R}{R(R+h)} \right)$$

After solving further, we get

$$\begin{aligned} 0.4v^2 &= \frac{GMh}{R(R+h)} \\ \Rightarrow \frac{R+h}{h} &= \frac{GM}{0.4v^2 R} \\ \Rightarrow \frac{R}{h} + 1 &= \frac{GM}{0.4v^2 R} \\ \Rightarrow \frac{R}{h} &= \frac{GM}{0.4v^2 R} - 1 \\ \Rightarrow h &= \frac{R}{\frac{GM}{0.4v^2 R} - 1} \\ \Rightarrow h &= \frac{0.4R^2v^2}{GM - 0.4v^2 R} \end{aligned}$$

Putting all the values, we get,

$$\begin{aligned} h &= \frac{0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)} \\ \Rightarrow h &= \frac{18.442 \times 10^{18}}{42.688 \times 10^{12} - 53432 \times 10^{12}} \\ \Rightarrow h &= \frac{18.442}{37.256} \times 10^6 \\ \Rightarrow h &= 495 \times 10^3 m \\ \Rightarrow h &= 495 km \end{aligned}$$

So, the height attained by the rocket from the surface will be $495 km$.