

Chapter 10: Thermal Properties of Matter

NCERT · Class 11 · Physics

Textbook exercises

Question 1

The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on Celsius and Fahrenheit scales.

Solution

Ans: The relation between Kelvin and Celsius scales:

$$T_C = T_K - 273.15$$

The relation between Celsius and Fahrenheit scales:

$$T_F = \frac{9}{5}T_C + 32$$

For neon:

$$T_K = 24.57K$$

$$\Rightarrow T_C = 24.57 - 273.15 = -248.58^\circ C$$

$$\text{As } T_F = \frac{9}{5}T_C + 32$$

$$\Rightarrow T_F = \frac{9}{5}(-248.58) + 32$$

$$\Rightarrow T_F = -415.44^\circ F$$

For carbon dioxide:

$$T_K = 216.55K$$

$$\Rightarrow T_C = 216.55 - 273.15 = -56.60^\circ C$$

$$\text{As } T_F = \frac{9}{5}T_C + 32$$

$$\Rightarrow T_F = \frac{9}{5}(-56.60) + 32$$

$$\Rightarrow T_F = -69.88^\circ F$$

Therefore, the temperatures on the Celsius scale and Fahrenheit scale are $-248.58^{\circ}C$ and $-415.44^{\circ}F$ respectively for neon and $-56.60^{\circ}C$ and $-69.88^{\circ}F$ respectively for carbon dioxide.

Question 2

Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

Solution

Ans: Given that,

Triple point of water on absolute scale A, $T_1 = 200A$

Triple point of water on absolute scale B, $T_2 = 350B$

Triple point of water on Kelvin scale, $T_K = 273.15K$

The temperature 273.15 K on Kelvin scale is equivalent to 200 A on an absolute scale A.

$$T_1 = T_K$$

$$\Rightarrow 200A = 273.15K$$

$$\Rightarrow A = \frac{273.15}{200}$$

The temperature 273.15 K on Kelvin scale is equivalent to 350 B on absolute scale B.

$$T_2 = T_K$$

$$\Rightarrow 350B = 273.15$$

$$\Rightarrow B = \frac{273.15}{350}$$

T_A is the triple point of water on scale A.

T_B is the triple point of water on scale B.

Therefore,

$$\frac{273.15}{200} \times T_A = \frac{273.15}{350} \times T_B$$

$$\Rightarrow T_A = \frac{200}{350} T_B$$

Therefore, the ratio $T_A : T_B$ is given as 4: 7.

Question 3

The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law $R = R_0 [1 + \alpha (T - T_0)]$

The resistance is 101.6Ω at the triple point of water $273.16K$, and 165.5Ω at the normal melting point of lead ($600.5K$). What is the temperature when the resistance is 123.4Ω ?

Solution

Ans: It is given that:

$$R = R_0 [1 + \alpha (T - T_0)]$$

...

Where, R_0 and T_0 are the initial resistance and temperature respectively;

R and T are the final resistance and temperature respectively;

α is constant.

At the triple point of water, $T_0 = 273.15K$

Resistance of lead, $R_0 = 101.6\Omega$

At normal melting point of lead, $T = 600.5K$

Resistance of lead, $R = 165.5\Omega$

Substituting these values in, we get:

$$R = R_0 [1 + \alpha (T - T_0)]$$

$$\Rightarrow 165.5 = 101.6 [1 + \alpha (600.5 - 273.15)]$$

$$\Rightarrow 1.629 = 1 + \alpha(327.35)$$

$$\Rightarrow \alpha = \frac{0.629}{327.35} = 1.92 \times 10^{-3}$$

For resistance, $R_1 = 123.4 \Omega$

$$R = R_0 [1 + \alpha (T - T_0)]$$

where, T is the temperature when the resistance of lead is 123.4Ω

$$\Rightarrow 123.4 = 101.6 [1 + 1.92 \times 10^{-3} (T - 273.15)]$$

$$\Rightarrow 1.214 = 1.192 \times 10^{-3} (T - 273.15)$$

$$\Rightarrow \frac{0.214}{1.92 \times 10^{-3}} = (T - 273.15)$$

$$\Rightarrow T = 384.61K$$

Clearly, the temperature when the resistance is 123.4Ω is $384.61K$.

Question 4

Answer the following:

1. The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

Solution

Ans: The triple point of water has a unique value of $273.16 K$.

The melting point of ice and boiling point of water do not have particular values because these points depend on pressure and temperature.

1. There were two fixed points in the original Celsius scale as mentioned above which were assigned the numbers $0^\circ C$ and $100^\circ C$ respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number $273.16 K$. What is the other fixed point on this (Kelvin) scale?

Ans: The absolute zero or $0 K$ is itself the other fixed point on the Kelvin absolute scale.

1. The absolute temperature (Kelvin scale) T is related to the temperature t_C the Celsius scale by
- $$t_C = T - 273.15.$$

Why do we have 273.15 in this relation, and not 273.16?

Ans: The temperature 273.16 K is the triple point of water; not the melting point of ice.

The temperature 0°C on Celsius scale is the melting point of ice.

Its corresponding value on the Kelvin scale is 273.15 K.

Thus, absolute temperature (Kelvin scale) T , is related to temperature t_C on Celsius scale as:

$$t_C = T - 273.15.$$

1. What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Ans: Let T_F be the temperature on Fahrenheit scale and T_K be the temperature on absolute scale.

Both the temperatures have the relation:

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

...

Let T_{F1} be the temperature on Fahrenheit scale and T_{K1} be the temperature on absolute scale. Both the temperatures can be related as:

$$\frac{T_{F1} - 32}{180} = \frac{T_{K1} - 273.15}{100} \quad \dots$$

It is given that:

$$T_{K1} - T_K = 1K$$

Subtracting equation (a) from equation (b), we get:

$$\begin{aligned} \frac{T_{F1} - T_F}{180} &= \frac{T_{K1} - T_K}{100} = \frac{1}{100} \\ \Rightarrow T_{F1} - T_F &= \frac{1 \times 180}{100} = \frac{9}{5} \end{aligned}$$

$$\text{Triple point of water} = 273.16K$$

$$\text{Therefore, triple point of water on absolute scale} = 273.16 \times \frac{9}{5} = 491.69K.$$

Question 5

Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting point of Sulfur	$1.797 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

1. What is the absolute temperature of the normal melting point of Sulfur as read by thermometers A and B?

Solution

Ans: Given that,

Triple point of water, $T = 273.16 \text{ K}$

At this temperature, pressure in thermometer A,

$$P_A = 1.250 \times 10^5$$

Pa

Let T_1 be the normal melting point of Sulfur.

At this temperature, pressure in thermometer A, $P_1 = 1.797 \times 10^5 \text{ Pa}$

According to Charles' law, we have the relation:

$$\frac{P_A}{T} = \frac{P_1}{T_1}$$

$$\Rightarrow T_1 = \frac{P_1 T}{P_A} = \frac{1.797 \times 10^5 \times 273.16}{1.25 \times 10^5}$$

$$\Rightarrow T_1 = 392.69 \text{ K}$$

Therefore, the absolute temperature of the normal melting point of Sulfur as read by thermometer A is 392.69 K.

Now, at triple point 273.16 K, the pressure in thermometer B,

$$P_B = 0.200 \times 10^5$$

Pa

At temperature T_1 , the pressure in thermometer B,

$$P_2 = 0.287 \times 10^5$$

Pa

According to Charles' law, we can write the relation:

$$\frac{P_B}{T} = \frac{P_1}{T_1}$$

$$\Rightarrow T_1 = \frac{0.287 \times 10^5 \times 273.16}{0.200 \times 10^5} = 391.98 K$$

Therefore, the absolute temperature of the normal melting point of Sulfur as read by thermometer B is 391.98 K.

1. What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

Ans: Oxygen, as well as hydrogen gas contained in thermometers A and B respectively, are not perfect ideal gasses.

There is a minor difference between the readings of thermometers A and B.

To lessen the discrepancy between the two readings, the experiment must be carried under low pressure conditions so that gasses can behave as ideal gas.

Question 6

A steel tape 1m long is correctly calibrated for a temperature of $27.0^\circ C$. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is $45.0^\circ C$. What is

the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear expansion of steel $= 1.20 \times 10^{-5} \text{K}^{-1}$.

Solution

Ans: Given that,

Length of the steel tape at temperature 27.0°C , $l = 1\text{m} = 100\text{cm}$

At temperature $T_1 = 45^{\circ}\text{C}$, the length of the steel rod, $l_1 = 63\text{cm}$

Coefficient of linear expansion of steel, $\alpha = 1.20 \times 10^{-5} \text{K}^{-1}$

Let l_2 be the actual length of the steel rod and l' be the length of the steel tape at 45°C .

$$l' = l + \alpha l (T_1 - T)$$

$$\Rightarrow l' = 100 + 1.20 \times 10^{-5} \times 100 (45 - 27)$$

$$\Rightarrow l' = 100.0216\text{cm}$$

Hence, the actual length of the steel rod measured by the steel tape at 45°C can be calculated as:

$$l_2 = \frac{100.0216}{100} \times 63 = 63.0136\text{cm}.$$

Therefore, the actual length of the rod at 45°C is 63.0136 cm. Its length at 27.0°C is 63.0 cm.

Question 7

A large steel wheel is to be fitted onto a shaft of the same material. At 27°C , the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range

: $\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{K}^{-1}$.

Solution

Ans: Here,

The given temperature, $T = 27^\circ C$ can be written in Kelvin as

$$27 + 273 = 300 \text{ K}$$

.

Outer diameter of the steel shaft at T, $d_1 = 8.70 \text{ cm}$

Diameter of the central hole in the wheel at T, $d_2 = 8.69 \text{ cm}$

Coefficient of linear expansion of steel, : $\alpha_{steel} = 1.20 \times 10^{-5} K^{-1}$

After the shaft is cooled using 'dry ice', its temperature becomes T_1 .

The wheel will slip on the shaft, if the change in diameter,

$$\Delta d = 8.698.70 = 0.01$$

cm

Temperature T_1 , can be calculated from the relation:

$$\Delta d = d_1 \alpha_{steel} (T_1 - T)$$

$$\Rightarrow \Delta d = 8.70 \times 1.20 \times 10^{-5} (T_1 - 300)$$

$$\Rightarrow (T_1 - 300) = 95.78$$

$$\Rightarrow T_1 = 204.21 \text{ K}$$

$$\text{Or } T_1 = 204.21 - 273.16 = -68.95^\circ C$$

Clearly, the wheel will slip on the shaft when the temperature of the shaft is approximately $-69^\circ C$.

Question 8

A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at $27.0^\circ C$. What is the change in the diameter of the hole when the sheet is heated to $227^\circ C$? Coefficient of linear expansion of copper = $1.70 \times 10^{-5} K^{-1}$.

Solution

Ans: Given that,

Initial temperature, $T_1 = 27.0^\circ C$

Diameter of the hole at T_1 , $d_1 = 4.24cm$

Final temperature, $T_2 = 227^\circ C$

Diameter of the hole at $T_2 = d_2$

Coefficient of linear expansion of copper, $\alpha_{Cu} = 1.70 \times 10^{-5} K^{-1}$

For coefficient of superficial expansion β , and change in temperature ΔT , we have the relation:

$$\frac{\text{change in area}(\Delta A)}{\text{original area}(A)} = \beta \Delta T$$

$$\Rightarrow \frac{\left(\pi \frac{d_2^2}{4} - \pi \frac{d_1^2}{4}\right)}{\left(\pi \frac{d_1^2}{4}\right)} = \frac{\Delta A}{A}$$

$$\Rightarrow \frac{\Delta A}{A} = \frac{d_2^2 - d_1^2}{d_1^2}$$

However, $\beta = 2\alpha$

$$\Rightarrow \frac{d_2^2}{d_1^2} - 1 = 2\alpha (T_2 - T_1)$$

$$\Rightarrow \frac{d_2^2 - d_1^2}{d_1^2} = 2\alpha \Delta T$$

$$\Rightarrow \frac{d_2^2}{(4.24)^2} = 2 \times 1.7 \times 10^{-5} (227 - 27) + 1$$

$$\Rightarrow d_2^2 = 17.98 \times 1.0068 = 18.1$$

$$\Rightarrow d_2 = 4.2544cm$$

Change in diameter $= d_2 - d_1 = 4.2544 - 4.24 = 0.0144cm$.

Clearly, the diameter increases by $1.44 \times 10^{-2}cm$.

Question 9

A brass wire 1.8 m long at 27°C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of -39°C , what is the tension developed in the wire, if its diameter is 2.0 mm? Coefficient of linear expansion of brass $= 2.0 \times 10^{-5} \text{K}^{-1}$.

Young's modulus of brass $= 0.91 \times 10^{11} \text{Pa}$.

Solution

Ans: Given that,

Initial temperature, $T_1 = 27^{\circ}\text{C}$

Length of the brass wire at

$$T_1, l = 1.8\text{m}$$

Final temperature, $T_2 = -39^{\circ}\text{C}$

Diameter of the wire, $d = 2.0\text{mm} = 2 \times 10^{-3}\text{m}$

Tension developed in the wire = F

Coefficient of linear expansion of brass, $\alpha = 2.0 \times 10^{-5} \text{K}^{-1}$

Young's modulus of brass, $Y = 0.91 \times 10^{11} \text{Pa}$

Young's modulus is given by the relation:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\Rightarrow \Delta L = \frac{F \times L}{A \times Y} \quad \dots$$

Where, F = Tension developed in the wire

A = Area of cross-section of the wire

L = Change in the length, given by the relation:

$$L = \alpha (T_2 - T_1) \quad \dots$$

Equating equations (a) and (b), we get:

$$\alpha L (T_2 - T_1) = \frac{FL}{\pi \left(\frac{d}{2}\right)^2 \times Y}$$

$$\Rightarrow F = \alpha (T_2 - T_1) \pi Y \left(\frac{d}{2} \right)^2$$

$$\Rightarrow F = 2 \times 10^5 \times (-39 - 27) \times 3.14 \times 0.91 \times 10^{11} \times \left(\frac{2 \times 10^{-3}}{2} \right)^2$$

$$\Rightarrow F = -3.8 \times 10^2 N$$

(The negative sign indicates that the tension is directed inward.)

Clearly, the tension developed in the wire is $3.8 \times 10^2 N$.

Question 10

A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at $250^\circ C$, if the original lengths are at $40^\circ C$? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Coefficient of linear expansion of brass $= 2.0 \times 10^{-5} K^{-1}$, steel $= 1.2 \times 10^{-5} K^{-1}$).

Solution

Ans: Given that,

Initial Temperature, $T_1 = 40^\circ C$

Final temperature, $T_2 = 250^\circ C$

Change in temperature $= T_2 - T_1 = 210^\circ C$

Length of the brass rod at T_1 , $l_1 = 50 cm$

Diameter of the brass rod at T_1 , $d_1 = 3.0 mm$

Length of the steel rod at T_2 , $l_2 = 50 cm$

Diameter of the steel rod T_2 , $d_2 = 3.0 mm$

Coefficient of linear expansion of brass, $\alpha_1 = 2.0 \times 10^{-5} K^{-1}$

Coefficient of linear expansion of steel, $\alpha_2 = 1.2 \times 10^{-5} K^{-1}$

For the expansion in the brass rod, we have:

$$\frac{\text{change in length}(\Delta l_1)}{\text{original length}(l_1)} = \alpha_1 \Delta T$$

$$\Rightarrow \Delta l_1 = 50 \times (2.1 \times 10^{-5}) \times 210 = 0.2205 \text{ cm}$$

For the expansion in the steel rod, we have:

$$\frac{\text{change in length}(\Delta l_2)}{\text{original length}(l_2)} = \alpha_2 \Delta T$$

$$\Rightarrow \Delta l_2 = 50 \times (1.2 \times 10^{-5}) \times 210 = 0.126 \text{ cm}$$

Total change in the lengths of brass and steel, $\Delta l = \Delta l_1 + \Delta l_2$

$$\Rightarrow \Delta l = 0.2205 + 0.126 = 0.346 \text{ cm}$$

Clearly, the total change in the length of the combined rod = 0.346 cm.

As the rod expands freely from both ends, no thermal stress is developed at the junction.

Question 11

The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \text{ K}^{-1}$. What is the fractional change in its density for a 30°C rise in temperature?

Solution

Ans: Given that,

Coefficient of volume expansion of glycerine, $\alpha_v = 49 \times 10^{-5} \text{ K}^{-1}$

Rise in temperature, $T = 30^\circ \text{C}$

Fractional change in its volume = $\frac{\Delta V}{V}$

This change is related with the change in temperature as:

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

$$\Rightarrow (V_{T_2} - V_{T_1}) = V_{T_1} \alpha_V \Delta T$$

$$\Rightarrow \frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}} \alpha_V \Delta T$$

Where,

m = Mass of glycerine

ρ_{T_1} = Initial density at T_1

ρ_{T_2} = Final density at T_2

$$\Rightarrow \frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \alpha_V \Delta T$$

Where,

$$\frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \text{Fractional change in density}$$

Therefore, fractional change in the density of glycerin = $49 \times 10^{-5} \times 30 = 1.47 \times 10^{-2}$.

Question 12

A 10kW drilling machine is used to drill a bore in a small aluminum block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings? Specific heat of aluminum = $0.91 J g^{-1} K^{-1}$.

Solution

Ans: Given that,

Power of the drilling machine, $P = 10 kW = 10 \times 10^3 W$

Mass of the aluminum block, $m = 8.0 kg = 8 \times 10^3 g$

Time for which the machine is used, $t = 2.5 \text{ min} = 2.5 \times 60 = 150 s$

Specific heat of aluminum, $c = 0.91 J g^{-1} K^{-1}$

Rise in the temperature of the block after drilling = δT

Total energy of the drilling machine = Pt

$$\Rightarrow Pt = 10 \times 10^3 \times 150 = 1.5 \times 10^6 J$$

It is given that only 50% of the power is useful.

$$\text{Useful energy, } \Delta Q = \frac{50}{100} \times 1.5 \times 10^6 J$$

$$\text{However, } \Delta Q = mc\Delta T$$

$$\Rightarrow \Delta T = \frac{\Delta Q}{mc}$$

$$\Rightarrow \Delta T = \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103^\circ C$$

Clearly, in 2.5 minutes of drilling, the rise in the temperature of the block is 103°C .

Question 13

A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper $= 0.39\text{Jg}^{-1}\text{K}^{-1}$ heat of fusion of water $= 335\text{Jg}^{-1}$).

Solution

Ans: Given that,

Mass of the copper block, $m = 2.5\text{kg} = 2500\text{g}$

Rise in the temperature of the copper block, $\Delta\theta = 500^{\circ}\text{C}$

Specific heat of copper, $C = \text{Jg}^{-1}\text{C}^{-1}$

Heat of fusion of water, $L = 335\text{Jg}^{-1}$

The maximum heat the copper block can lose, $Q = mC\Delta\theta$

$$\Rightarrow Q = 2500 \times 0.39 \times 500 = 487500\text{J}$$

Let m_1 be the amount of ice that melts when the copper block is placed on the ice block.

The heat gained by the melted ice, $Q = m_1L$

$$\Rightarrow m_1 = \frac{Q}{L} = \frac{487500}{335} = 1455.22\text{g}$$

Thus, the maximum amount of ice that can melt is 1.45 kg.

Question 14

In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C . The final temperature is 40°C . Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?

Solution

Ans: Given that,

Mass of the metal, $m = 0.20\text{kg} = 200\text{g}$

Initial temperature of the metal, $T_1 = 150^\circ\text{C}$

Final temperature of the metal, $T_2 = 40^\circ\text{C}$

Calorimeter has water equivalent of mass, $m = 0.025\text{kg} = 25\text{g}$

Volume of water, $V = 150\text{cm}^3$

Mass (M) of water at temperature $T = 27^\circ\text{C}$: $150 \times 1 = 150\text{g}$

Fall in the temperature of the metal:

$$\Delta T = T_1 - T_2 = 150 - 40 = 110^\circ\text{C}$$

Specific heat of water, $C_W = 4.186\text{J/g}/^\circ\text{K}$

Specific heat of the metal = C

Heat lost by the metal, $\theta = mc\Delta T$

Rise in the temperature of the water and calorimeter system:

$$\Delta T' = 40 - 27 = 13^\circ\text{C}$$

Heat gained by the water and calorimeter system:

$$\Delta\theta' = m_1 C_W \Delta T'$$

$$\Rightarrow \Delta\theta' = (M + m') C_W \Delta T'$$

Heat lost by the metal = Heat gained by the water and calorimeter system

$$mc\Delta T = (M + m') C_W \Delta T'$$

$$\Rightarrow 200 \times C \times 110 = (150 + 25) \times 4.186 \times 13$$

$$\Rightarrow C = \frac{175 \times 4.186 \times 13}{110 \times 200} = 0.43\text{Jg}^{-1}\text{K}^{-1}$$

Clearly, when some heat is lost to the surroundings, then the value of C would be smaller than the actual value.

Question 15

Given below are observations on molar specific heats at room temperature of some common gasses:

Gas	Molar specific Heat (C_v) $\text{cal mol}^{-1} \text{K}^{-1}$
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K . Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Solution

Ans: The gases listed in the provided table are diatomic.

Apart from the translational degree of freedom, they also possess other degrees of freedom (modes of motion).

Heat should be supplied to increase the temperature of these gases which would further increase the average energy of all the modes of motion.

Thus, the molar specific heat of diatomic gases turns out to be more than that of monatomic gases.

Moreover, when only the rotational mode of motion is considered, then the molar specific heat of a diatomic gas $= \frac{5}{2}R$

$$\Rightarrow \frac{5}{2}R = \frac{5}{2} \times 1.98 = 4.95 \text{ cal mol}^{-1} \text{ K}^{-1}$$

Excluding the data of chlorine, all the other observations in the given table agree with $\left(\frac{5}{2}R\right)$.

This is because at room temperature, chlorine also has vibrational modes of motion apart from rotational and translational modes of motion.

Question 16

Answer the following questions based on the P-T phase diagram of carbon dioxide:

1. At what temperature and pressure can the solid, liquid and vapor phases of CO_2 co-exist in equilibrium?

Solution

Ans: From the P-T phase diagram for CO_2 as shown in the figure above,

C is the triple point of the CO_2 phase diagram.

This suggests that at the temperature and pressure corresponding to this point (i.e., at $56.6^\circ C$ and 5.11 atm), the solid, liquid, and vaporous phases of CO_2 co-exist in equilibrium.

1. What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?

Ans: The fusion and boiling points of CO_2 decrease as pressure decreases.

1. What are the critical temperature and pressure for CO_2 ? What is their significance?

Ans: The critical temperature and critical pressure of CO_2 are $31.1^\circ C$ and 73 atm respectively.

Even when there is compression to a pressure greater than 73 atm, CO_2 would not liquify above the critical temperature.

1. Is CO_2 solid, liquid or gas at a) $-70^\circ C$ under 1 atm, b) $60^\circ C$ under 10 atm, c) $15^\circ C$ under 56 atm?

Ans: With reference to the P-T phase diagram of CO_2 ;

- a) CO_2 is gaseous at $-70^\circ C$, under 1 atm pressure.
b) CO_2 is solid at $60^\circ C$, under 10 atm pressure.
c) CO_2 is liquid at $15^\circ C$, under 56 atm pressure.

Question 17

Answer the following questions based on the P-T phase diagram of CO_2 :

1. CO_2 at 1 atm pressure and temperature $-60^\circ C$ is compressed isothermally. Does it go through a liquid phase?

Solution

Ans: The P-T phase diagram for CO_2 is considered as shown in the question.

At 1 atm pressure and at $-60^\circ C$, CO_2 positions itself to the left of $-56.6^\circ C$ (triple point C). Thus, it places itself in the region of vaporous and solid phases.

Clearly, CO_2 condenses into the solid state directly without going through the liquid state.

1. What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?

Ans: At 4 atm pressure, CO_2 places itself below 5.11 atm (triple point C).

Thus, it positions itself in the region of vaporous and solid phases.

Clearly, it condenses into the solid-state directly without passing through the liquid state.

1. Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature $-65^\circ C$ as it is heated up to room temperature at constant pressure.

Ans: If the temperature of a mass of solid CO_2 (at 10 atm pressure and at $-65^\circ C$) is increased, it changes to the liquid phase and then to the vaporous phase.

It projects a line parallel to the temperature axis at 10 atm.

Clearly, the fusion and boiling points are provided by the intersection point where this parallel line cuts the fusion and vaporization curves.

1. CO_2 is heated to a temperature $70^\circ C$ and compressed isothermally. What changes in its properties do you expect to observe?

Ans: When CO_2 is heated to $70^\circ C$ and compressed isothermally, it would not show any transition to the liquid state.

This is because $70^\circ C$ is higher than the critical temperature of CO_2 .

Clearly, it would remain in the vapor state but would depart from its ideal behavior as pressure is increased.

Question 18

A child running a temperature of $101^\circ F$ is given an antipyrene (i.e., a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to $98^\circ F$ in 20 min, what is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of the human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about $580 \text{ cal } g^{-1}$.

Solution

Ans: Given that,

Initial temperature of the body of the child, $T_1 = 101^\circ F$

Final temperature of the body of the child, $T_2 = 98^\circ F$

Change in temperature, $\Delta T = \left[(101 - 98) \times \frac{5}{9} \right] ^\circ C$

Time taken to reduce the temperature, $t = 20 \text{ min}$

Mass of the child, $m = 30 \text{ kg} = 30 \times 10^3 \text{ g}$

Specific heat of the human body = Specific heat of water = $c = 100 \text{ cal/kg/}^\circ C$

Latent heat of evaporation of water, $L = 580 \text{ cal g}^{-1}$

The heat lost by the child is given as:

$$\Delta \theta = mc\Delta T$$

$$\Rightarrow \Delta \theta = 30 \times 1000 \times (101 - 98) \times \frac{5}{9} = 50000 \text{ cal}$$

Let m_1 be the mass of the water evaporated from the child's body in 20 min.

Loss of heat through water is given by:

$$\Delta \theta = m_1 L$$

$$\Rightarrow m_1 = \frac{\Delta \theta}{L}$$

$$\Rightarrow m_1 = \frac{50000}{580} = 86.2 \text{ g}$$

Therefore, average rate of extra evaporation caused by the drug = $\frac{m_1}{t}$

$$\Rightarrow \frac{m_1}{t} = \frac{86.2}{200} = 4.3 \text{ g/min}$$

Question 19

A 'thermocool' ice box is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is $45^\circ C$, and

coefficient of thermal conductivity of thermocol is $0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$. [Heat of fusion of water $= 335 \times 10^3 \text{ J kg}^{-1}$]

Solution

Ans: Given that,

Side of the given cubical ice box, $s = 30 \text{ cm} = 0.3 \text{ m}$

Thickness of the ice box, $l = 5.0 \text{ cm} = 0.05 \text{ m}$

Mass of ice kept in the ice box, $m = 4 \text{ kg}$

Time gap, $t = 6 \text{ h} = 6 \times 60 \times 60 \text{ s}$

Outside temperature, $T = 45^\circ \text{ C}$

Coefficient of thermal conductivity of thermacole, $K = 0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$

Heat of fusion of water, $L = 335 \times 10^3 \text{ J kg}^{-1}$

Assume m' be the total amount of ice that melts in 6 h.

The amount of heat lost by the food:

$$\theta = \frac{KA(T-0)t}{l}$$

where,

$$A = \text{Surface of the box} = 6s^2 = 6 \times (0.3)^2 = 0.54 \text{ m}^2$$

$$\Rightarrow \theta = \frac{0.01 \times 0.54 \times 45 \times 6 \times 60 \times 60}{0.05} = 104976 \text{ J}$$

$$\text{However, } \theta = m' L$$

$$\Rightarrow m' = \frac{\theta}{L}$$

$$\Rightarrow m' = \frac{104976}{335 \times 10^3} = 0.313 \text{ g}$$

$$\text{Mass of ice left } 4 - 0.313 = 3.687 \text{ kg}$$

Clearly, the amount of ice remaining after 6h is 3.687kg.

Question 20

A brass boiler has a base area of 0.15m^2 and thickness 1.0cm . It boils water at the rate of $6.0\text{kg}/\text{min}$ when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler.

Thermal conductivity of brass $= 109\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$;

Heat of vaporization of water $= 2256 \times 10^3\text{Jkg}^{-1}$.

Solution

Ans: Given that,

Base area of the boiler, $A = 0.15\text{m}^2$

Thickness of the boiler,

$$l = 1.0\text{ cm} = 0.01\text{ m}$$

Boiling rate of water, $R = 6.0\text{kg}/\text{min}$

Mass, $m = 6\text{ kg}$

Time, $t = 1\text{ min} = 60\text{ s}$

Thermal conductivity of brass, $K = 109\text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$

Heat of vaporization, $L = 2256 \times 10^3\text{Jkg}^{-1}$.

The amount of heat flowing into water through the brass base of the boiler is given by:

$$\theta = \frac{KA(T_1 - T_2)t}{l}$$

Where,

T_1 = Temperature of the flame in contact with the boiler

T_2 = Boiling point of water $= 100^\circ\text{C}$

Heat required for boiling the water:

$$\theta = mL$$

Equating equations (a) and (b), we get:

$$mL = \frac{KA(T_1 - T_2)t}{l}$$

$$\Rightarrow (T_1 - T_2) = \frac{mLl}{KA\tau}$$

$$\Rightarrow (T_1 - T_2) = \frac{6 \times 2256 \times 10^3 \times 0.001}{109 \times 0.15 \times 60} = 137.98^\circ C$$

Clearly, the temperature of the part of the flame in contact with the boiler is $137.98^\circ C$.

Question 21

Explain why:

1. a body with large reflectivity is a poor emitter

Solution

Ans: It is known that a body possessing a large reflectivity turns out to be a poor absorber of light radiations.

A poor absorber would further be a poor emitter of radiation.

Clearly, a body possessing large reflectivity is a poor emitter.

1. a brass tumbler feels much colder than a wooden tray on a chilly day

Ans: It is known that brass is a good conductor of heat.

The moment at which one touches a brass tumbler, heat gets conducted from the body to the brass tumbler easily.

Thus, the temperature of the body lessens to a lower value and one feels cooler.

On the other hand, the moment at which one touches a wooden tray, very little heat is conducted from the body to the wooden tray because wood is a poor conductor of heat.

Thus, there is only a negligible drop in the temperature of the body and one does not feel cool.

Clearly, a brass tumbler feels colder than a wooden tray on a chilly day.

1. An optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red-hot iron piece in the open, but gives a

correct value for the temperature when the same piece is in the furnace.

Ans: An optical pyrometer calibrated for an ideal black body radiation provided too low a value for temperature of a red-hot iron piece kept in the open.

Black body radiation equation is given by:

$$E = \sigma (T^4 - T_0^4)$$

Where,

E = Energy radiation

T = Temperature of optical pyrometer

T_0 = Temperature of open space

σ = Constant

Thus, an increase in the temperature of open space lessens the radiation energy. If the same piece of iron is placed in a furnace, the radiation energy,

$$E = \sigma T^4$$

.

1. the earth without its atmosphere would be inhospitably cold

Ans: It is known that excluding its atmosphere, earth would be inhospitably cold.

When atmospheric gases are absent, no extra heat would be trapped.

Clearly, all the heat would be radiated back from the earth's surface.

1. heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Ans: A heating system on the basis of the circulation of steam would be more efficient in warming a building than that on the basis of the circulation of hot water.

This can be reasoned as steam contains surplus heat in the form of latent heat (540cal/g).

Question 22

A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C . The temperature of the surroundings is 20°C .

Solution

Ans: With respect to Newton's law of cooling;

$$-\frac{dT}{dt} = K(T - T_0)$$

$$\Rightarrow \frac{dT}{K(T - T_0)} = -K dt$$

Where,

Temperature of the body = T

Temperature of the surroundings = $T_s = 20^{\circ}\text{C}$

K is a constant

Temperature of the body falls from 80°C to 50°C in time, $t = 5 \text{ min} = 300 \text{ s}$

Integrating equation (a), we get:

$$\int_{50}^{80} \frac{dt}{K(T - T_0)} = - \int_0^{300} K dt$$

$$\Rightarrow [\log_e (T - T_0)]_{50}^{80} = -K[t]_0^{300}$$

$$\Rightarrow \frac{2.3026}{K} \log_{10} \frac{80-20}{50-20} = -300$$

$$\Rightarrow \frac{2.3026}{K} \log_{10} 2 = -300$$

$$\Rightarrow -\frac{2.3026}{300} \log_{10} 2 = K$$

The temperature of the body falls from 60°C to 30°C in time = t

Thus, we get:

$$\Rightarrow \frac{2.3026}{K} \log_{10} \frac{60-20}{30-20} = -t$$

$$\Rightarrow \frac{2.3026}{K} \log_{10} 4 = K$$

Equating equations (b) and (c), we get:

$$\frac{-2.3026}{K} \log_{10} 4 = \frac{2.3026}{K} \log_{10} 2$$

$$\Rightarrow t = 300 \times 2 = 600s = 10 \text{ min}$$

Clearly, the time taken to cool the body from $60^\circ C$ to $30^\circ C$ is 10 minutes.
