

Chapter 9: Straight lines

NCERT · Class 11 · Maths

EXERCISE 10.1 · Textbook page 211

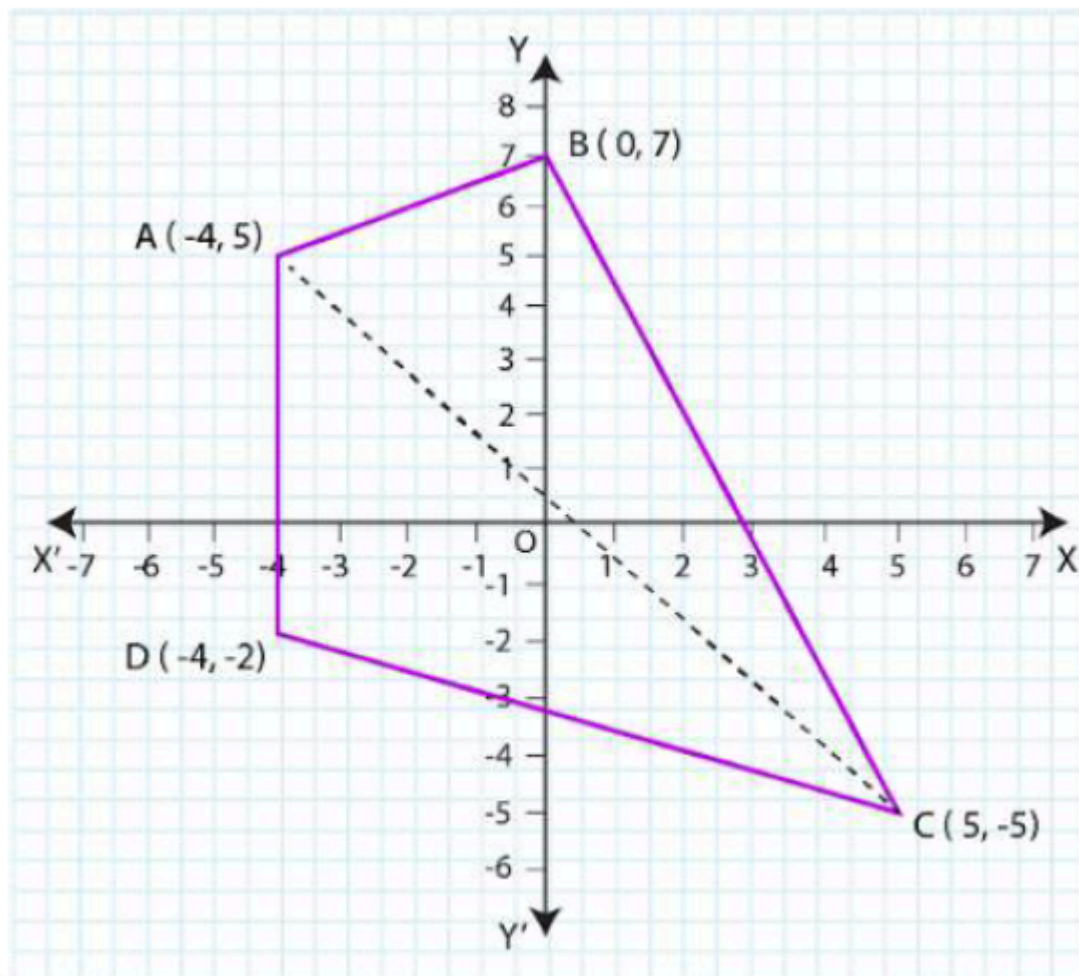
Question 1

Draw a quadrilateral in the Cartesian plane whose vertices are $(-4, 5)$, $(0, 7)$, $(5, -5)$ and $(-4, -2)$. Also, find its area.

Solution

Let ABCD be the given quadrilateral with vertices A $(-4, 5)$, B $(0, 7)$, C $(5, -5)$ and D $(-4, -2)$.

Now, let us plot the points on the Cartesian plane by joining the points AB, BC, CD, and AD, which give us the required quadrilateral.



To find the area, draw diagonal AC.

So, area (ABCD) = area (ΔABC) + area (ΔADC)

Then, area of triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\text{Area of } \Delta ABC = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{2} [-4 (7 + 5) + 0 (-5 - 5) + 5 (5 - 7)] \text{ unit}^2$$

$$= \frac{1}{2} [-4 (12) + 5 (-2)] \text{ unit}^2$$

$$= \frac{1}{2} (58) \text{ unit}^2$$

$$= 29 \text{ unit}^2$$

$$\text{Area of } \Delta ACD = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{2} [-4 (-5 + 2) + 5 (-2 - 5) + (-4) (5 - (-5))] \text{ unit}^2$$

$$= \frac{1}{2} [-4 (-3) + 5 (-7) - 4 (10)] \text{ unit}^2$$

$$= \frac{1}{2} (-63) \text{ unit}^2$$

$$= -63/2 \text{ unit}^2$$

Since area cannot be negative, area $\Delta ACD = 63/2 \text{ unit}^2$

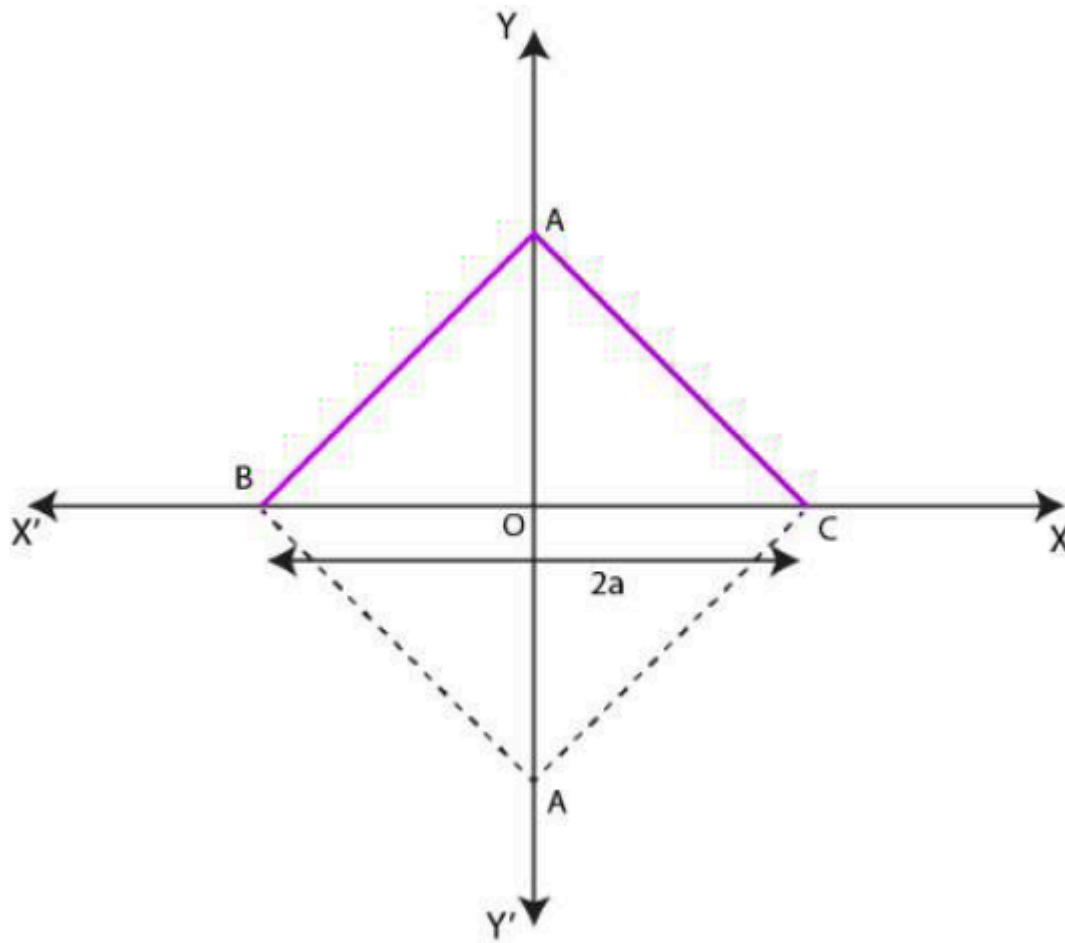
$$\text{Area (ABCD)} = 29 + 63/2$$

$$= 121/2 \text{ unit}^2$$

Question 2

The base of an equilateral triangle with side $2a$ lies along the y-axis such that the mid-point of the base is at the origin. Find the vertices of the triangle.

Solution



Let us consider ABC, the given equilateral triangle with side $2a$.

Where, $AB = BC = AC = 2a$

In the above figure, assuming that the base BC lies on the x-axis such that the mid-point of BC is at the origin, i.e., $BO = OC = a$, where O is the origin.

The coordinates of point C are $(a, 0)$ and that of B are $(-a, 0)$.

The line joining a vertex of an equilateral Δ with the mid-point of its opposite side is perpendicular.

So, vertex A lies on the y-axis.

By applying Pythagoras' theorem,

$$(AC)^2 = OA^2 + OC^2$$

$$(2a)^2 = a^2 + OC^2$$

$$4a^2 - a^2 = OC^2$$

$$3a^2 = OC^2$$

$$OC = \sqrt{3}a$$

Co-ordinates of point C = $\pm \sqrt{3}a, 0$

$\therefore$ The vertices of the given equilateral triangle are $(0, a), (0, -a), (\sqrt{3}a, 0)$

Or $(0, a), (0, -a)$ and $(-\sqrt{3}a, 0)$

Question 3

Find the distance between P (x_1, y_1) and Q (x_2, y_2) when: (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis.

Solution

Given:

Points P (x_1, y_1) and Q (x_2, y_2)

(i) When PQ is parallel to the y-axis, then $x_1 = x_2$

So, the distance between P and Q is given by

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(y_2 - y_1)^2}$$

$$= |y_2 - y_1|$$

(ii) When PQ is parallel to the x-axis, then $y_1 = y_2$

So, the distance between P and Q is given by =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2}$$

$$= |x_2 - x_1|$$

Question 4

Find a point on the x-axis which is equidistant from points (7, 6) and (3, 4).

Solution

Let us consider (a, 0) to be the point on the x-axis that is equidistant from the point (7, 6) and (3, 4).

So,

$$\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$$

$$\sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}$$

$$\sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$$

Now, let us square on both sides; we get,

$$a^2 - 14a + 85 = a^2 - 6a + 25$$

$$-8a = -60$$

$$a = 60/8$$

$$= 15/2$$

∴ The required point is (15/2, 0)

Question 5

Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points P (0, -4) and B (8, 0).

Solution

The co-ordinates of the mid-point of the line segment joining the points P (0, -4) and B (8, 0) are $(0+8)/2, (-4+0)/2 = (4, -2)$

The slope 'm' of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x_1$

The slope of the line passing through (0, 0) and (4, -2) is $(-2-0)/(4-0) = -1/2$

∴ The required slope is -1/2.

Question 6

Without using Pythagoras' theorem, show that the points (4, 4), (3, 5) and (−1, −1) are the vertices of a right-angled triangle.

Solution

The vertices of the given triangle are (4, 4), (3, 5) and (−1, −1).

The slope (m) of the line non-vertical line passing through the point (x_1, y_1) and

(x_2, y_2) is given by $m = (y_2 - y_1)/(x_2 - x_1)$ where, $x \neq x_1$

So, the slope of the line AB (m_1) = $(5-4)/(3-4) = 1/-1 = -1$

The slope of the line BC (m_2) = $(-1-5)/(-1-3) = -6/-4 = 3/2$

The slope of the line CA (m_3) = $(4+1)/(4+1) = 5/5 = 1$

It is observed that $m_1 \cdot m_3 = -1 \cdot 1 = -1$

Hence, the lines AB and CA are perpendicular to each other.

$\therefore$ given triangle is right-angled at A (4, 4)

And the vertices of the right-angled Δ are (4, 4), (3, 5) and (−1, −1)

Question 7

Find the slope of the line, which makes an angle of 30° with the positive direction of the y-axis measured anticlockwise.

Solution

We know that if a line makes an angle of 30° with the positive direction of the y-axis measured anti-clock-wise, then the angle made by the line with the positive direction of the x-axis measured anti-clock-wise is $90^\circ + 30^\circ = 120^\circ$

$\therefore$ The slope of the given line is $\tan 120^\circ = \tan (180^\circ - 60^\circ)$

$= -\tan 60^\circ$

$= -\sqrt{3}$

Question 8

Find the value of x for which the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear.

Solution

If the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear, then the Slope of AB = Slope of BC

Then, $(1+1)/(2-x) = (5-1)/(4-2)$

$$2/(2-x) = 4/2$$

$$2/(2-x) = 2$$

$$2 = 2(2-x)$$

$$2 = 4 - 2x$$

$$2x = 4 - 2$$

$$2x = 2$$

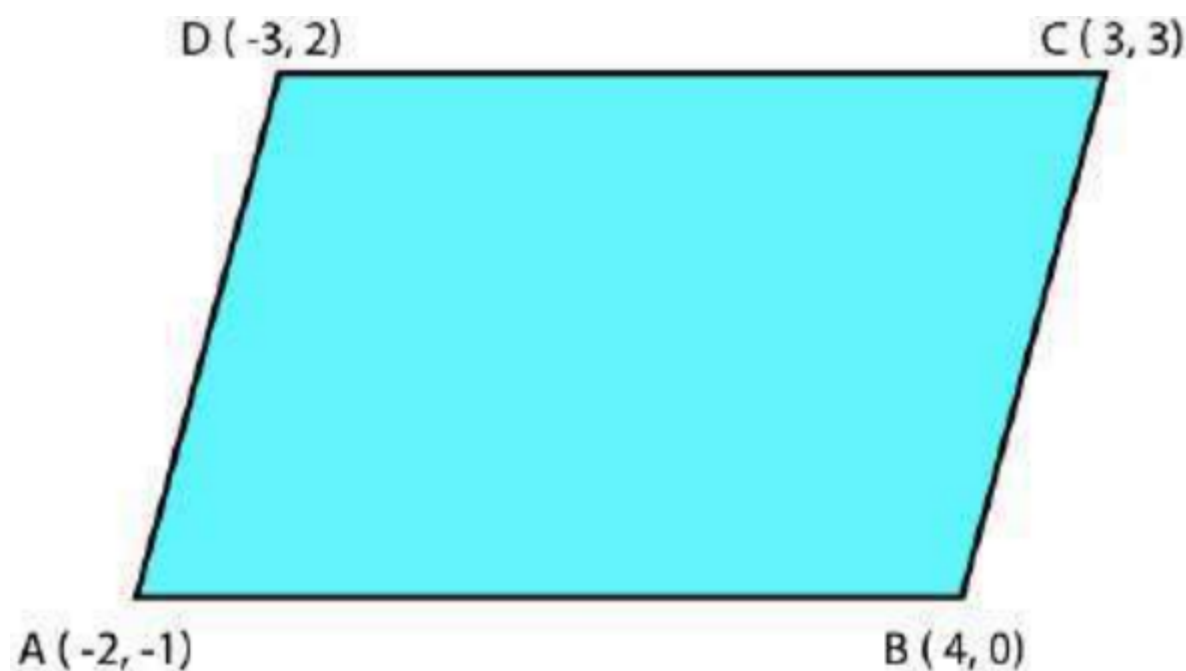
$$x = 2/2$$

$$= 1$$

∴ The required value of x is 1.

Question 9

Without using the distance formula, show that points $(-2, -1)$, $(4, 0)$, $(3, 3)$ and $(-3, 2)$ are the vertices of a parallelogram.

Solution

Let the given point be A (-2, -1) , B (4, 0) , C (3, 3) and D (-3, 2)

So now, the slope of AB = $(0+1)/(4+2) = 1/6$

The slope of CD = $(3-2)/(3+3) = 1/6$

Hence, the Slope of AB = Slope of CD

$\therefore AB \parallel CD$

Now,

The slope of BC = $(3-0)/(3-4) = 3/-1 = -3$

The slope of AD = $(2+1)/(-3+2) = 3/-1 = -3$

Hence, the Slope of BC = Slope of AD

$\therefore BC \parallel AD$

Thus, the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.

Hence, the given vertices, A (-2, -1), B (4, 0), C(3, 3) and D(-3, 2) are vertices of a parallelogram.

Question 10

Find the angle between the x-axis and the line joining the points (3, -1) and (4, -2).

Solution

The Slope of the line joining the points (3, -1) and (4, -2) is given by

$$m = (y_2 - y_1)/(x_2 - x_1) \text{ where, } x \neq x_1$$

$$m = (-2 - (-1))/(4-3)$$

$$= (-2+1)/(4-3)$$

$$= -1/1$$

$$= -1$$

The angle of inclination of the line joining the points (3, -1) and (4, -2) is given by

$$\tan \theta = -1$$

$$\theta = (90^\circ + 45^\circ) = 135^\circ$$

$\therefore$ The angle between the x-axis and the line joining the points (3, -1) and (4, -2) is 135° .

Question 11

The slope of a line is double the slope of another line. If the tangent of the angle between them is $1/3$, find the slopes of the lines.

Solution

Let us consider ' m_1 ' and ' m ' be the slope of the two given lines such that $m_1 = 2m$

We know that if θ is the angle between the lines l_1 and l_2 with slope m_1 and m_2 , then

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

Given here that the tangent of the angle between the two lines is $1/3$

So,

$$\frac{1}{3} = \left| \frac{m - 2m}{1 + 2m \times m} \right| = \left| \frac{-m}{1 + 2m^2} \right|$$

$$\frac{1}{3} = \frac{m}{1 + 2m^2}$$

Now, case 1:

$$\frac{1}{3} = \frac{-m}{1 + 2m^2}$$

$$1+2m^2 = -3m$$

$$2m^2 + 1 + 3m = 0$$

$$2m(m+1) + 1(m+1) = 0$$

$$(2m+1)(m+1) = 0$$

$$m = -1 \text{ or } -1/2$$

If $m = -1$, then the slope of the lines are -1 and -2

If $m = -1/2$, then the slope of the lines are $-1/2$ and -1

Case 2:

$$\frac{1}{3} = \frac{-m}{1+2m^2}$$

$$2m^2 - 3m + 1 = 0$$

$$2m^2 - 2m - m + 1 = 0$$

$$2m(m-1) - 1(m-1) = 0$$

$$m = 1 \text{ or } 1/2$$

If $m = 1$, then the slope of the lines are 1 and 2

If $m = 1/2$, then the slope of the lines are $1/2$ and 1

$\therefore$ The slope of the lines are $[-1 \text{ and } -2]$ or $[-1/2 \text{ and } -1]$ or $[1 \text{ and } 2]$ or $[1/2 \text{ and } 1]$

Question 12

A line passes through (x_1, y_1) and (h, k) . If the slope of the line is m , show that $k - y_1 = m(h - x_1)$.

Solution

Given: the slope of the line is ' m '.

The slope of the line passing through (x_1, y_1) and (h, k) is $(k - y_1)/(h - x_1)$

So,

$$(k - y_1)/(h - x_1) = m$$

$$(k - y_1) = m (h - x_1)$$

Hence, proved.

Question 13

If three points $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $a/h + b/k = 1$

Solution

Let us consider if the given points A $(h, 0)$, B (a, b) and C $(0, k)$ lie on a line.

Then, the slope of AB = slope of BC

$$(b - 0)/(a - h) = (k - b)/(0 - a)$$

By simplifying, we get

$$-ab = (k-b) (a-h)$$

$$-ab = ka - kh - ab + bh$$

$$ka + bh = kh$$

Divide both sides by kh ; we get

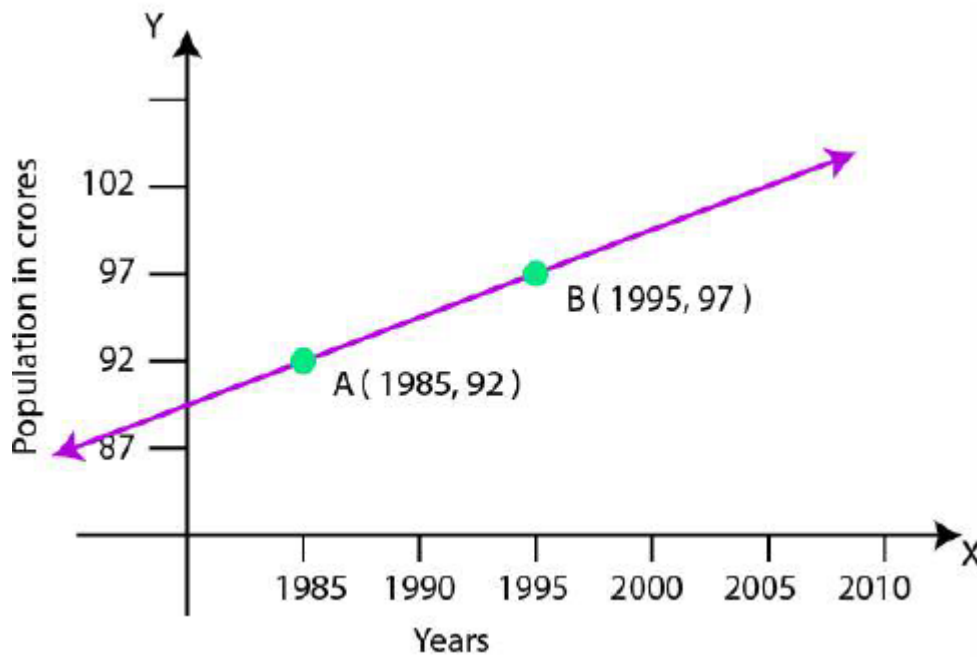
$$ka/kh + bh/kh = kh/kh$$

$$a/h + b/k = 1$$

Hence, proved.

Question 14

Consider the following population and year graph (Fig 10.10), find the slope of the line AB and using it, find what will be the population in the year 2010?



Solution

We know that line AB passes through points A (1985, 92) and B (1995, 97).

Its slope will be $(97 - 92)/(1995 - 1985) = 5/10 = 1/2$

Let 'y' be the population in the year 2010. Then, according to the given graph, AB must pass through point C (2010, y)

So now, slope of AB = slope of BC

$$\frac{1}{2} = \frac{y - 97}{2010 - 1995}$$

$$15/2 = y - 97$$

$$y = 7.5 + 97 = 104.5$$

∴ The slope of line AB is $1/2$, while in the year 2010, the population will be 104.5 crores.

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Question 1

Write the equations for the x-and y-axes.

Solution

The y-coordinate of every point on the x-axis is 0.

∴ The equation of the x-axis is $y = 0$.

The x-coordinate of every point on the y-axis is 0.

∴ The equation of the y-axis is $y = 0$.

Question 2

Passing through the point $(-4, 3)$ with slope $1/2$

Solution

Given:

Point $(-4, 3)$ and slope, $m = 1/2$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$$\text{So, } y - 3 = 1/2(x - (-4))$$

$$y - 3 = 1/2(x + 4)$$

$$2(y - 3) = x + 4$$

$$2y - 6 = x + 4$$

$$x + 4 - (2y - 6) = 0$$

$$x + 4 - 2y + 6 = 0$$

$$x - 2y + 10 = 0$$

∴ The equation of the line is $x - 2y + 10 = 0$

Question 3

Passing through $(0, 0)$ with slope m .

Solution

Given:

Point $(0, 0)$ and slope, $m = m$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$$\text{So, } y - 0 = m(x - 0)$$

$$y = mx$$

$$y - mx = 0$$

$\therefore$ The equation of the line is $y - mx = 0$

Question 4

Passing through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75° .

Solution

Given: point $(2, 2\sqrt{3})$ and $\theta = 75^\circ$

$$\text{Equation of line: } (y - y_1) = m(x - x_1)$$

where, $m = \text{slope of line} = \tan \theta$ and (x_1, y_1) are the points through which line passes

$$\therefore m = \tan 75^\circ$$

$$75^\circ = 45^\circ + 30^\circ$$

Applying the formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Let us rationalizing we get,

$$\tan 75^\circ = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_1, y_1) , only if its coordinates satisfy the equation $y - y_1 = m(x - x_1)$

$$\text{Then, } y - 2\sqrt{3} = (2 + \sqrt{3})(x - 2)$$

$$y - 2\sqrt{3} = 2x - 4 + \sqrt{3}x - 2\sqrt{3}$$

$$y = 2x - 4 + \sqrt{3}x$$

$$(2 + \sqrt{3})x - y - 4 = 0$$

$$\therefore \text{The equation of the line is } (2 + \sqrt{3})x - y - 4 = 0$$

Question 5

Intersecting the x-axis at a distance of 3 units to the left of origin with slope -2 .

Solution

Given:

$$\text{Slope, } m = -2$$

We know that if a line L with slope m makes x-intercept d , then the equation of L is

$$y = m(x - d).$$

If the distance is 3 units to the left of the origin, then $d = -3$

$$\text{So, } y = (-2)(x - (-3))$$

$$y = (-2)(x + 3)$$

$$y = -2x - 6$$

$$2x + y + 6 = 0$$

∴ The equation of the line is $2x + y + 6 = 0$

Question 6

Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30° with the positive direction of the x-axis.

Solution

Given: $\theta = 30^\circ$

We know that slope, $m = \tan \theta$

$$m = \tan 30^\circ = (1/\sqrt{3})$$

We know that the point (x, y) on the line with slope m and y-intercept c lies on the line only if $y = mx + c$

If the distance is 2 units above the origin, $c = +2$

$$\text{So, } y = (1/\sqrt{3})x + 2$$

$$y = (x + 2\sqrt{3}) / \sqrt{3}$$

$$\sqrt{3} y = x + 2\sqrt{3}$$

$$x - \sqrt{3} y + 2\sqrt{3} = 0$$

∴ The equation of the line is $x - \sqrt{3} y + 2\sqrt{3} = 0$

Question 7

Passing through the points $(-1, 1)$ and $(2, -4)$.

Solution

Given:

Points $(-1, 1)$ and $(2, -4)$

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$
$$y - 1 = \frac{-4 - 1}{2 - (-1)} (x - (-1))$$

$$y - 1 = -5/3 (x + 1)$$

$$3(y - 1) = (-5)(x + 1)$$

$$3y - 3 = -5x - 5$$

$$3y - 3 + 5x + 5 = 0$$

$$5x + 3y + 2 = 0$$

∴ The equation of the line is $5x + 3y + 2 = 0$

Question 8

Perpendicular distance from the origin is 5 units, and the angle made by the perpendicular with the positive x-axis is 30° .

Solution

Given: $p = 5$ and $\omega = 30^\circ$

We know that the equation of the line having normal distance p from the origin and angle ω , which the normal makes with the positive direction of the x-axis, is given by $x \cos \omega + y \sin \omega = p$.

Substituting the values in the equation, we get

$$x \cos 30^\circ + y \sin 30^\circ = 5$$

$$x(\sqrt{3}/2) + y(1/2) = 5$$

$$\sqrt{3}x + y = 5(2) = 10$$

$$\sqrt{3}x + y - 10 = 0$$

∴ The equation of the line is $\sqrt{3}x + y - 10 = 0$

Question 9

The vertices of ΔPQR are P (2, 1), Q (-2, 3) and R (4, 5). Find the equation of the median through the vertex R.

Solution

Given:

Vertices of ΔPQR , i.e., P (2, 1), Q (-2, 3) and R (4, 5)

Let RL be the median of vertex R.

So, L is a midpoint of PQ.

We know that the midpoint formula is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

$$\therefore L = \left(\frac{2 + (-2)}{2}, \frac{1 + 3}{2} \right) = (0, 2)$$

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\therefore y - 5 = \frac{2 - 5}{0 - 4} (x - 4)$$

$$y - 5 = -3/4 (x - 4)$$

$$(-4) (y - 5) = (-3) (x - 4)$$

$$-4y + 20 = -3x + 12$$

$$-4y + 20 + 3x - 12 = 0$$

$$3x - 4y + 8 = 0$$

$\therefore$ The equation of median through the vertex R is $3x - 4y + 8 = 0$

Question 10

Find the equation of the line passing through (-3, 5) and perpendicular to the line through the points (2, 5) and (-3, 6).

Solution

Given:

Points are (2, 5) and (-3, 6).

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

$$= (6 - 5)/(-3 - 2)$$

$$= 1/-5 = -1/5$$

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, $m = (-1/m)$

$$= -1/(-1/5)$$

$$= 5$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x₀, y₀), only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$$\text{Then, } y - 5 = 5(x - (-3))$$

$$y - 5 = 5x + 15$$

$$5x + 15 - y + 5 = 0$$

$$5x - y + 20 = 0$$

∴ The equation of the line is $5x - y + 20 = 0$

Question 11

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.

Solution

We know that the coordinates of a point dividing the line segment joining the points (x₁, y₁) and (x₂, y₂) internally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\left(\frac{1(2) + n(1)}{1+n}, \frac{1(3) + n(0)}{1+n} \right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$$

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

$$= (3 - 0)/(2 - 1)$$

$$= 3/1$$

$$= 3$$

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

$$\text{Then, } m = (-1/m) = -1/3$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) , only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

Here, the point is

$$\left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$$

$$\left(y - \frac{3}{1+n} \right) = \frac{-1}{3} \left(x - \frac{2+n}{1+n} \right)$$

$$3((1+n)y - 3) = -(1+n)x + 2 + n$$

$$3(1+n)y - 9 = -(1+n)x + 2 + n$$

$$(1+n)x + 3(1+n)y - n - 9 - 2 = 0$$

$$(1+n)x + 3(1+n)y - n - 11 = 0$$

$$\therefore \text{The equation of the line is } (1+n)x + 3(1+n)y - n - 11 = 0$$

Question 12

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2, 3)$.

Solution

Given: the line cuts off equal intercepts on the coordinate axes, i.e., $a = b$

We know that equation of the line intercepts a and b on the x -and the y -axis, respectively, which is

$$x/a + y/b = 1$$

So, $x/a + y/a = 1$

$$x + y = a \dots (1)$$

Given: point $(2, 3)$

$$2 + 3 = a$$

$$a = 5$$

Substitute value of ' a ' in (1), we get

$$x + y = 5$$

$$x + y - 5 = 0$$

$\therefore$ The equation of the line is $x + y - 5 = 0$

Question 13

Find the equation of the line passing through the point $(2, 2)$ and cutting off intercepts on the axes whose sum is 9.

Solution

We know that equation of the line-making intercepts a and b on the x -and the y -axis, respectively, is $x/a + y/b = 1 \dots (1)$

Given: sum of intercepts = 9

$$a + b = 9$$

$$b = 9 - a$$

Now, substitute the value of b in the above equation, and we get

$$x/a + y/(9 - a) = 1$$

Given: the line passes through point $(2, 2)$

$$\text{So, } 2/a + 2/(9 - a) = 1$$

$$18/a(9 - a) = 1$$

$$18 = a(9 - a)$$

$$18 = 9a - a^2$$

$$a^2 - 9a + 18 = 0$$

Upon factorising, we get

$$a^2 - 3a - 6a + 18 = 0$$

$$a(a - 3) - 6(a - 3) = 0$$

$$(a - 3)(a - 6) = 0$$

$$a = 3 \text{ or } a = 6$$

Let us substitute in (1)

Case 1 ($a = 3$):

$$\text{Then } b = 9 - 3 = 6$$

$$x/3 + y/6 = 1$$

$$2x + y = 6$$

$$2x + y - 6 = 0$$

Case 2 ($a = 6$):

$$\text{Then } b = 9 - 6 = 3$$

$$x/6 + y/3 = 1$$

$$x + 2y = 6$$

$$x + 2y - 6 = 0$$

$\therefore$ The equation of the line is $2x + y - 6 = 0$ or $x + 2y - 6 = 0$

Question 14

Find the equation of the line through the point $(0, 2)$, making an angle $2\pi/3$ with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution

Given:

Point $(0, 2)$ and $\theta = 2\pi/3$

We know that $m = \tan \theta$

$$m = \tan (2\pi/3) = -\sqrt{3}$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) , only if its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

$$y - 2 = -\sqrt{3} (x - 0)$$

$$y - 2 = -\sqrt{3} x$$

$$\sqrt{3} x + y - 2 = 0$$

Given, the equation of the line parallel to the above-obtained equation crosses the y -axis at a distance of 2 units below the origin.

So, the point = $(0, -2)$ and $m = -\sqrt{3}$

From point slope form equation,

$$y - (-2) = -\sqrt{3} (x - 0)$$

$$y + 2 = -\sqrt{3} x$$

$$\sqrt{3} x + y + 2 = 0$$

$\therefore$ The equation of the line is $\sqrt{3} x + y - 2 = 0$, and the line parallel to it is $\sqrt{3} x + y + 2 = 0$

Question 15

The perpendicular from the origin to a line meets it at the point $(-2, 9)$. Find the equation of the line.

Solution

Given:

Points are origin $(0, 0)$ and $(-2, 9)$.

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

$$= (9 - 0)/(-2 - 0)$$

$$= -9/2$$

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

$$m = (-1/m) = -1/(-9/2) = 2/9$$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0) only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$$y - 9 = (2/9)(x - (-2))$$

$$9(y - 9) = 2(x + 2)$$

$$9y - 81 = 2x + 4$$

$$2x + 4 - 9y + 81 = 0$$

$$2x - 9y + 85 = 0$$

∴ The equation of the line is $2x - 9y + 85 = 0$

Question 16

The length L (in centimetres) of a copper rod is a linear function of its Celsius temperature C . In an experiment, if $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$, express L in terms of C .

Solution

Let us assume 'L' along X-axis and 'C' along Y-axis; we have two points $(124.942, 20)$ and $(125.134, 110)$ in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$C - 20 = \frac{110 - 20}{125.134 - 124.942} (L - 124.942)$$

$$C - 20 = \frac{90}{0.192} (L - 124.942)$$

$$0.192 (C - 20) = 90 (L - 124.942)$$

$$L = \frac{0.192}{90} (C - 20) + 124.942$$

$$\therefore \text{The required relation is } L = \frac{0.192}{90} (C - 20) + 124.942$$

Question 17

The owner of a milk store finds that he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs. 16/litre. Assuming a linear relationship between the selling price and demand, how many litres could he sell weekly at Rs. 17/litre?

Solution

Assuming the relationship between the selling price and demand is linear.

Let us assume the selling price per litre along X-axis and demand along Y-axis, we have two points (14, 980) and (16, 1220) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 980 = \frac{1220 - 980}{16 - 14} (x - 14)$$

$$y - 980 = \frac{240}{2} (x - 14)$$

$$y - 980 = 120 (x - 14)$$

$$y = 120 (x - 14) + 980$$

When $x = \text{Rs } 17/\text{litre}$,

$$y = 120 (17 - 14) + 980$$

$$y = 120(3) + 980$$

$$y = 360 + 980 = 1340$$

∴ The owner can sell 1340 litres weekly at Rs. 17/litre.

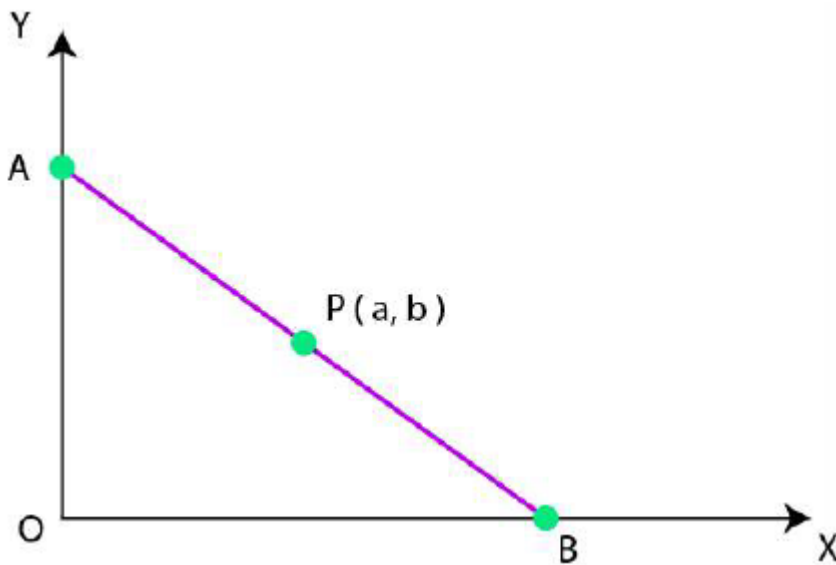
Question 18

$P(a, b)$ is the mid-point of a line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$

Solution

Let AB be a line segment whose midpoint is $P(a, b)$.

Let the coordinates of A and B be $(0, y)$ and $(x, 0)$, respectively.



We know that the midpoint is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since P is the midpoint of (a, b),

$$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (a, b)$$

$$\left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$$

$$a = x/2 \text{ and } b = y/2$$

$$x = 2a \text{ and } y = 2b$$

$$A = (0, 2b) \text{ and } B = (2a, 0)$$

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2b = \frac{0 - 2b}{2a - 0}(x - 0)$$

$$y - 2b = \frac{-2b}{2a}(x)$$

$$y - 2b = \frac{-b}{a}(x)$$

$$a(y - 2b) = -bx$$

$$ay - 2ab = -bx$$

$$bx + ay = 2ab$$

Divide both sides with ab, then

$$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$$

$$\frac{x}{a} + \frac{y}{b} = 2$$

Hence, proved.

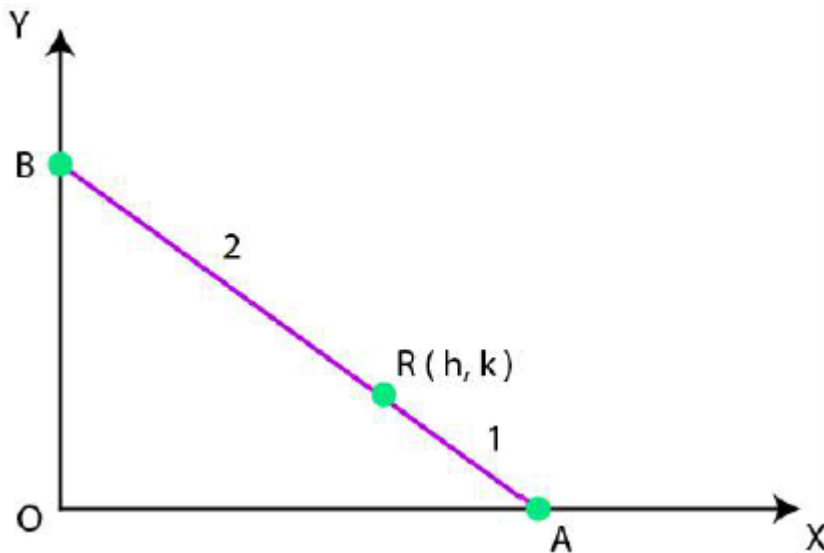
Question 19

Point R (h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the line.

Solution

Let us consider AB to be the line segment, such that r (h, k) divides it in the ratio 1: 2.

So, the coordinates of A and B be (0, y) and (x, 0), respectively.



We know that the coordinates of a point dividing the line segment join the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m: n$ is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\left(\frac{1(0) + 2(x)}{1+2}, \frac{1(y) + 2(0)}{1+2} \right) = (h, k)$$

$$\left(\frac{2x}{3}, \frac{y}{3} \right) = (h, k)$$

$$h = 2x/3 \text{ and } k = y/3$$

$$x = 3h/2 \text{ and } y = 3k$$

$$\therefore A = (0, 3k) \text{ and } B = (3h/2, 0)$$

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 3k = \frac{0 - 3k}{\frac{3h}{2} - 0} (x - 0)$$

$$3h(y - 3k) = -6kx$$

$$3hy - 9hk = -6kx$$

$$6kx + 3hy = 9hk$$

Let us divide both sides by $9hk$, and we get,

$$2x/3h + y/3k = 1$$

∴ The equation of the line is given by $2x/3h + y/3k = 1$

Question 20

By using the concept of the equation of a line, prove that the three points (3, 0), (− 2, − 2) and (8, 2) are collinear.

Solution

According to the question,

If we have to prove that the given three points (3, 0), (− 2, − 2) and (8, 2) are collinear, then we have to also prove that the line passing through the points (3, 0) and (− 2, − 2) also passes through the point (8, 2).

By using the formula,

The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\begin{aligned}y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\y - 0 &= \frac{-2 - 0}{-2 - 3}(x - 3) \\y &= \frac{-2}{-5}(x - 3)\end{aligned}$$

$$-5y = -2(x - 3)$$

$$-5y = -2x + 6$$

$$2x - 5y = 6$$

If $2x - 5y = 6$ passes through (8, 2),

$$2x - 5y = 2(8) - 5(2)$$

$$= 16 - 10$$

$$= 6$$

$$= \text{RHS}$$

The line passing through points (3, 0) and (− 2, − 2) also passes through the point (8, 2).

Hence, proved. The given three points are collinear.

EXERCISE 10.3 · Textbook page 227**Question 1**

Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.

(i) $x + 7y = 0$

(ii) $6x + 3y - 5 = 0$

(iii) $y = 0$

Solution

(i) $x + 7y = 0$

Given:

The equation is $x + 7y = 0$

The slope-intercept form is represented in the form ' $y = mx + c$ ', where m is the slope and c is the y-intercept.

So, the above equation can be expressed as

$$y = -1/7x + 0$$

∴ The above equation is of the form $y = mx + c$, where $m = -1/7$ and $c = 0$

(ii) $6x + 3y - 5 = 0$

Given:

The equation is $6x + 3y - 5 = 0$

The slope-intercept form is represented in the form ' $y = mx + c$ ', where m is the slope and c is the y-intercept.

So, the above equation can be expressed as

$$3y = -6x + 5$$

$$y = -6/3x + 5/3$$

$$= -2x + 5/3$$

∴ The above equation is of the form $y = mx + c$, where $m = -2$ and $c = 5/3$

(iii) $y = 0$

Given:

The equation is $y = 0$

The slope-intercept form is given by ' $y = mx + c$ ', where m is the slope and c is the y-intercept.

$$y = 0 \times x + 0$$

$\therefore$ The above equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$

Question 2

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$

(ii) $4x - 3y = 6$

(iii) $3y + 2 = 0$

Solution

(i) $3x + 2y - 12 = 0$

Given:

The equation is $3x + 2y - 12 = 0$

The equation of the line in intercept form is given by $x/a + y/b = 1$, where ' a ' and ' b ' are intercepted on the x-axis and the y-axis, respectively.

So, $3x + 2y = 12$

Now, let us divide both sides by 12; we get

$$3x/12 + 2y/12 = 12/12$$

$$x/4 + y/6 = 1$$

$\therefore$ The above equation is of the form $x/a + y/b = 1$, where $a = 4$, $b = 6$

The intercept on the x-axis is 4.

The intercept on the y-axis is 6.

(ii) $4x - 3y = 6$

Given:

The equation is $4x - 3y = 6$

The equation of the line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepted on the x-axis and the y-axis, respectively.

$$\text{So, } 4x - 3y = 6$$

Now, let us divide both sides by 6; we get

$$4x/6 - 3y/6 = 6/6$$

$$2x/3 - y/2 = 1$$

$$x/(3/2) + y/(-2) = 1$$

∴ The above equation is of the form $x/a + y/b = 1$, where $a = 3/2$, $b = -2$

The intercept on the x-axis is $3/2$.

The intercept on the y-axis is -2 .

(iii) $3y + 2 = 0$

Given:

The equation is $3y + 2 = 0$

The equation of the line in intercept form is given by $x/a + y/b = 1$, where 'a' and 'b' are intercepted on the x-axis and the y-axis, respectively.

$$\text{So, } 3y = -2$$

Now, let us divide both sides by -2; we get

$$3y/-2 = -2/-2$$

$$3y/-2 = 1$$

$$y/(-2/3) = 1$$

∴ The above equation is of the form $x/a + y/b = 1$, where $a = 0$, $b = -2/3$

The intercept on the x-axis is 0.

The intercept on the y-axis is $-2/3$.

Question 3

Reduce the following equations into normal form. Find their perpendicular distances from the origin and the angle between the perpendicular and the positive x-axis.

(i) $x - \sqrt{3}y + 8 = 0$

(ii) $y - 2 = 0$

(iii) $x - y = 4$

Solution

(i) $x - \sqrt{3}y + 8 = 0$

Given:

The equation is $x - \sqrt{3}y + 8 = 0$

The equation of the line in normal form is given by $x \cos \theta + y \sin \theta = p$ where ' θ ' is the angle between the perpendicular and the positive x-axis and ' p ' is the perpendicular distance from the origin.

So now, $x - \sqrt{3}y + 8 = 0$

$$x - \sqrt{3}y = -8$$

Divide both the sides by $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2$

$$x/2 - \sqrt{3}y/2 = -8/2$$

$$(-1/2)x + \sqrt{3}/2y = 4$$

This is in the form of: $x \cos 120^\circ + y \sin 120^\circ = 4$

$\therefore$ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 120^\circ$ and $p = 4$.

Perpendicular distance of the line from origin = 4

The angle between the perpendicular and positive x-axis = 120°

(ii) $y - 2 = 0$

Given:

The equation is $y - 2 = 0$

The equation of the line in normal form is given by $x \cos \theta + y \sin \theta = p$ where ' θ ' is the angle between the perpendicular and the positive x-axis and ' p ' is the perpendicular distance from the origin.

So now, $0 \times x + 1 \times y = 2$

Divide both sides by $\sqrt{0^2 + 1^2} = \sqrt{1} = 1$

$$0(x) + 1(y) = 2$$

This is in the form of: $x \cos 90^\circ + y \sin 90^\circ = 2$

$\therefore$ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 90^\circ$ and $p = 2$.

Perpendicular distance of the line from origin = 2

The angle between the perpendicular and positive x-axis = 90°

(iii) $x - y = 4$

Given:

The equation is $x - y + 4 = 0$

The equation of the line in normal form is given by $x \cos \theta + y \sin \theta = p$ where ' θ ' is the angle between the perpendicular and the positive x-axis and ' p ' is the perpendicular distance from the origin.

So now, $x - y = 4$

Divide both the sides by $\sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$

$$x/\sqrt{2} - y/\sqrt{2} = 4/\sqrt{2}$$

$$(1/\sqrt{2})x + (-1/\sqrt{2})y = 2\sqrt{2}$$

This is in the form: $x \cos 315^\circ + y \sin 315^\circ = 2\sqrt{2}$

$\therefore$ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 315^\circ$ and $p = 2\sqrt{2}$.

Perpendicular distance of the line from origin = $2\sqrt{2}$

The angle between the perpendicular and the positive x-axis = 315°

Question 4

Find the distance of the point $(-1, 1)$ from the line $12(x + 6) = 5(y - 2)$.

Solution

Given:

The equation of the line is $12(x + 6) = 5(y - 2)$.

$$12x + 72 = 5y - 10$$

$$12x - 5y + 82 = 0 \dots (1)$$

Now, compare equation (1) with the general equation of line $Ax + By + C = 0$, where $A = 12$, $B = -5$, and $C = 82$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Given point $(x_1, y_1) = (-1, 1)$

$\therefore$ Distance of point $(-1, 1)$ from the given line is

$$\begin{aligned} d &= \frac{|12 \times (-1) + (-5) \times 1 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{144 + 25}} = \frac{|65|}{\sqrt{169}} = \frac{65}{13} \text{ units} \\ &= 5 \text{ units} \end{aligned}$$

$\therefore$ The distance is 5 units.

Question 5

Find the points on the x-axis whose distances from the line $x/3 + y/4 = 1$ are 4 units.

Solution

Given:

The equation of the line is $x/3 + y/4 = 1$

$$4x + 3y = 12$$

$$4x + 3y - 12 = 0 \dots (1)$$

Now, compare equation (1) with the general equation of line $Ax + By + C = 0$, where $A = 4$, $B = 3$, and $C = -12$

Let $(a, 0)$ be the point on the x-axis whose distance from the given line is 4 units.

So, the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$
$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$
$$4 = \frac{|4a - 12|}{\sqrt{16 + 9}} = \frac{|4a - 12|}{5}$$

$$|4a - 12| = 4 \times 5$$

$$\pm (4a - 12) = 20$$

$$4a - 12 = 20 \text{ or } -(4a - 12) = 20$$

$$4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$a = 32/4 \text{ or } a = -8/4$$

$$a = 8 \text{ or } a = -2$$

$\therefore$ The required points on the x-axis are $(-2, 0)$ and $(8, 0)$

Question 6

Find the distance between parallel lines.

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Solution

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

Given:

The parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

By using the formula,

The distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, $A = 15$, $B = 8$, $C_1 = -34$, $C_2 = 31$

Distance between parallel lines is

$$\begin{aligned} d &= \frac{|-34 - 31|}{\sqrt{15^2 + 8^2}} \\ &= \frac{|-65|}{\sqrt{225 + 64}} \\ &= \frac{65}{\sqrt{289}} \\ &= 65/17 \end{aligned}$$

∴ The distance between parallel lines is 65/17

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Given:

The parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$

$lx + ly + p = 0$ and $lx + ly - r = 0$

By using the formula,

The distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, $A = l$, $B = l$, $C_1 = p$, $C_2 = -r$

Distance between parallel lines is

$$\begin{aligned} d &= \frac{|p - (-r)|}{\sqrt{l^2 + l^2}} \\ &= \frac{|p + r|}{\sqrt{2}l} \\ &= \frac{|p+r|}{l\sqrt{2}} \end{aligned}$$

∴ The distance between parallel lines is $|p+r|/l\sqrt{2}$

Question 7

Find the equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

Solution

Given:

The line is $3x - 4y + 2 = 0$

So, $y = 3x/4 + 2/4$

$= 3x/4 + 1/2$

Which is of the form $y = mx + c$, where m is the slope of the given line.

The slope of the given line is $3/4$

We know that parallel lines have the same slope.

$\therefore$ Slope of other line $= m = 3/4$

The equation of line having slope m and passing through (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

$\therefore$ The equation of the line having slope $3/4$ and passing through $(-2, 3)$ is

$$y - 3 = \frac{3}{4}(x - (-2))$$

$$4y - 3 \times 4 = 3x + 3 \times 2$$

$$3x - 4y = 18$$

$\therefore$ The equation is $3x - 4y = 18$

Question 8

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Solution

Given:

The equation of line is $x - 7y + 5 = 0$

So, $y = 1/7x + 5/7$ [which is of the form $y = mx + c$, where m is the slope of the given line.]

The slope of the given line is $1/7$

The slope of the line perpendicular to the line having slope m is $-1/m$

The slope of the line perpendicular to the line having a slope of $1/7$ is $-1/(1/7) = -7$

So, the equation of the line with slope -7 and the x -intercept 3 is given by $y = m(x - d)$

$$y = -7(x - 3)$$

$$y = -7x + 21$$

$$7x + y = 21$$

∴ The equation is $7x + y = 21$

Question 9

Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

Solution

Given:

The lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

So, $y = -\sqrt{3}x + 1$... (1) and

$$y = -1/\sqrt{3}x + 1/\sqrt{3} \dots (2)$$

The slope of the line (1) is $m_1 = -\sqrt{3}$, while the slope of the line (2) is $m_2 = -1/\sqrt{3}$

Let θ be the angle between two lines.

So,

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{-\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)} \right| = \left| \frac{\frac{-3 + 1}{\sqrt{3}}}{1 + 1} \right| = \left| \frac{-2}{2 \times \sqrt{3}} \right| \\ &= 1/\sqrt{3} \end{aligned}$$

$$\theta = 30^\circ$$

∴ The angle between the given lines is either 30° or $180^\circ - 30^\circ = 150^\circ$

Question 10

The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$. At the right angle. Find the value of h .

Solution

Let the slope of the line passing through $(h, 3)$ and $(4, 1)$ be m_1

$$\text{Then, } m_1 = (1-3)/(4-h) = -2/(4-h)$$

Let the slope of line $7x - 9y - 19 = 0$ be m_2

$$7x - 9y - 19 = 0$$

$$\text{So, } y = 7/9x - 19/9$$

$$m_2 = 7/9$$

Since the given lines are perpendicular,

$$m_1 \times m_2 = -1$$

$$-2/(4-h) \times 7/9 = -1$$

$$-14/(36-9h) = -1$$

$$-14 = -1 \times (36 - 9h)$$

$$36 - 9h = 14$$

$$9h = 36 - 14$$

$$h = 22/9$$

$\therefore$ The value of h is $22/9$

Question 11

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$.

Solution

Let the slope of line $Ax + By + C = 0$ be m

$$Ax + By + C = 0$$

$$\text{So, } y = -A/Bx - C/B$$

$$m = -A/B$$

By using the formula,

Equation of the line passing through point (x_1, y_1) and having slope $m = -A/B$ is

$$y - y_1 = m (x - x_1)$$

$$y - y_1 = -A/B (x - x_1)$$

$$B (y - y_1) = -A (x - x_1)$$

$$\therefore A(x - x_1) + B(y - y_1) = 0$$

So, the line through point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A (x - x_1) + B (y - y_1) = 0$

Hence, proved.

Question 12

Two lines passing through point $(2, 3)$ intersects each other at an angle of 60° . If the slope of one line is 2, find the equation of the other line.

Solution

Given: $m_1 = 2$

Let the slope of the first line be m_1

And let the slope of the other line be m_2 .

The angle between the two lines is 60° .

So,

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan 60^\circ = \left| \frac{2 - m_2}{1 + 2m_2} \right|$$

$$\sqrt{3} = \pm \left(\frac{2 - m_2}{1 + 2m_2} \right)$$

i.e.,

$$\sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = - \left(\frac{2 - m_2}{1 + 2m_2} \right)$$

$$\sqrt{3}(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2)$$

$$\sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$m_2(2\sqrt{3} + 1) = 2 - \sqrt{3} \text{ or } m_2(2\sqrt{3} - 1) = -(2 + \sqrt{3})$$

$$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

So now let us consider

Case 1: When

$$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1} \right) (x - 2)$$

$$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$$

$\therefore$ Equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$

Case 2: When

$$m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)} (x - 2)$$

$$y - 3 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}(x - 2)$$

$$(2\sqrt{3} - 1)y - 3(2\sqrt{3} - 1) = -(2 + \sqrt{3})x + 2(2 + \sqrt{3})$$

$$(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 8\sqrt{3} + 1$$

$$\therefore \text{Equation of the other line is } (2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 8\sqrt{3} + 1$$

Question 13

Find the equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

Solution

Given:

The right bisector of a line segment bisects the line segment at 90° .

End-points of the line segment AB are given as A (3, 4) and B (-1, 2).

Let the mid-point of AB be (x, y).

$$x = (3-1)/2 = 2/2 = 1$$

$$y = (4+2)/2 = 6/2 = 3$$

$$(x, y) = (1, 3)$$

Let the slope of line AB be m_1

$$m_1 = (2 - 4)/(-1 - 3)$$

$$= -2/(-4)$$

$$= 1/2$$

And let the slope of the line perpendicular to AB be m_2

$$m_2 = -1/(1/2)$$

$$= -2$$

The equation of the line passing through (1, 3) and having a slope of -2 is

$$(y - 3) = -2(x - 1)$$

$$y - 3 = -2x + 2$$

$$2x + y = 5$$

∴ The required equation of the line is $2x + y = 5$

Question 14

Find the coordinates of the foot of the perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$.

Solution

Let us consider the coordinates of the foot of the perpendicular from $(-1, 3)$ to the line $3x - 4y - 16 = 0$ be (a, b)

So, let the slope of the line joining $(-1, 3)$ and (a, b) be m_1

$$m_1 = (b-3)/(a+1)$$

And let the slope of the line $3x - 4y - 16 = 0$ be m_2

$$y = 3/4x - 4$$

$$m_2 = 3/4$$

Since these two lines are perpendicular, $m_1 \times m_2 = -1$

$$(b-3)/(a+1) \times (3/4) = -1$$

$$(3b-9)/(4a+4) = -1$$

$$3b - 9 = -4a - 4$$

$$4a + 3b = 5 \dots\dots(1)$$

Point (a, b) lies on the line $3x - 4y = 16$

$$3a - 4b = 16 \dots\dots(2)$$

Solving equations (1) and (2), we get

$$a = 68/25 \text{ and } b = -49/25$$

∴ The coordinates of the foot of perpendicular are $(68/25, -49/25)$

Question 15

The perpendicular from the origin to the line $y = mx + c$ meets it at the point $(-1, 2)$. Find the values of m and c .

Solution

Given:

The perpendicular from the origin meets the given line at $(-1, 2)$.

The equation of the line is $y = mx + c$

The line joining the points $(0, 0)$ and $(-1, 2)$ is perpendicular to the given line.

So, the slope of the line joining $(0, 0)$ and $(-1, 2) = 2/(-1) = -2$

The slope of the given line is m .

$$m \times (-2) = -1$$

$$m = 1/2$$

Since point $(-1, 2)$ lies on the given line,

$$y = mx + c$$

$$2 = 1/2 \times (-1) + c$$

$$c = 2 + 1/2 = 5/2$$

∴ The values of m and c are $1/2$ and $5/2$, respectively.

Question 16

If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \operatorname{cosec} \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Solution

Given:

The equations of the given lines are

$$x \cos \theta - y \sin \theta = k \cos 2\theta \dots\dots\dots (1)$$

$$x \sec \theta + y \operatorname{cosec} \theta = k \dots\dots\dots (2)$$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

So now, compare equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we get

$$A = \cos \theta, B = -\sin \theta, \text{ and } C = -k \cos 2\theta$$

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$p = \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = k \cos 2\theta$$

$$p = k \cos 2\theta$$

Let us square on both sides we get,

$$p^2 = k^2 \cos^2 2\theta \dots\dots\dots (3)$$

Now, compare equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we get

$$A = \sec \theta, B = \operatorname{cosec} \theta, \text{ and } C = -k$$

It is given that q is the length of the perpendicular from (0, 0) to line (2)

$$\begin{aligned} q &= \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \\ &= \frac{k}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \\ &= \frac{k}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}} = \frac{k \cos \theta \sin \theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = k \cos \theta \sin \theta \end{aligned}$$

$$q = k \cos \theta \sin \theta$$

Multiply both sides by 2, and we get

$$2q = 2k \cos \theta \sin \theta = k \times 2 \sin \theta \cos \theta$$

$$2q = k \sin 2\theta$$

Squaring both sides, we get

$$4q^2 = k^2 \sin^2 2\theta \dots\dots\dots(4)$$

Now add (3) and (4); we get

$$p^2 + 4q^2 = k^2 \cos^2 2\theta + k^2 \sin^2 2\theta$$

$$p^2 + 4q^2 = k^2 (\cos^2 2\theta + \sin^2 2\theta) \text{ [Since, } \cos^2 2\theta + \sin^2 2\theta = 1]$$

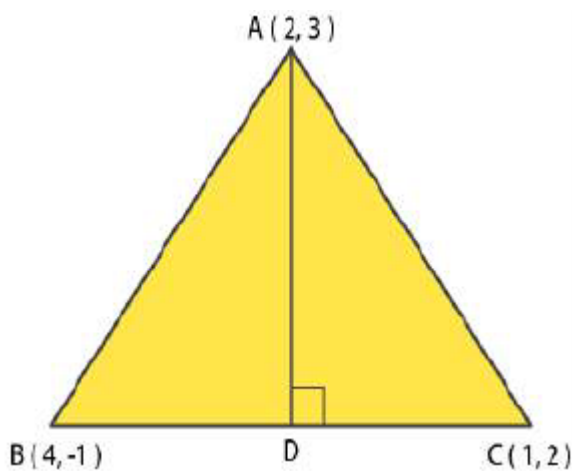
$$\therefore p^2 + 4q^2 = k^2$$

Hence proved.

Question 17

In the triangle ABC with vertices A (2, 3), B (4, -1) and C (1, 2), find the equation and length of altitude from vertex A.

Solution



Let AD be the altitude of triangle ABC from vertex A.

So, AD is perpendicular to BC.

Given:

Vertices A (2, 3), B (4, -1) and C (1, 2)

Let the slope of the line BC = m_1

$$m_1 = (-1 - 2)/(4 - 1)$$

$$m_1 = -1$$

Let the slope of the line AD be m_2

AD is perpendicular to BC.

$$m_1 \times m_2 = -1$$

$$-1 \times m_2 = -1$$

$$m_2 = 1$$

The equation of the line passing through the point (2, 3) and having a slope of 1 is

$$y - 3 = 1 \times (x - 2)$$

$$y - 3 = x - 2$$

$$y - x = 1$$

Equation of the altitude from vertex A = $y - x = 1$

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$y + 1 = -1 \times (x - 4)$$

$$y + 1 = -x + 4$$

$$x + y - 3 = 0 \dots\dots\dots(1)$$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now, compare equation (1) to the general equation of the line, i.e., $Ax + By + C = 0$; we get

Length of AD =

$$\frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} = \frac{|2|}{\sqrt{2}} = \sqrt{2} \text{ units}$$

$\therefore$ The equation and the length of the altitude from vertex A are $y - x = 1$ and

$\sqrt{2}$ units, respectively.

Question 18

If p is the length of the perpendicular from the origin to the line whose intercepts on the axes are a and b , then show that $1/p^2 = 1/a^2 + 1/b^2$

Solution

The equation of a line whose intercepts on the axes are a and b is $x/a + y/b = 1$

$$bx + ay = ab$$

$$bx + ay - ab = 0 \dots\dots\dots(1)$$

Perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now compare equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we get

$$A = b, B = a \text{ and } C = -ab$$

If p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we get

$$\begin{aligned} p &= \frac{|A \times 0 + B \times 0 - ab|}{\sqrt{a^2 + b^2}} \\ &= \frac{|-ab|}{\sqrt{a^2 + b^2}} \end{aligned}$$

Now, square on both sides; we get

$$\begin{aligned} p^2 &= \frac{(-ab)^2}{a^2 + b^2} \\ \frac{1}{p^2} &= \frac{a^2 + b^2}{a^2 b^2} \\ \frac{1}{p^2} &= \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2} \\ \therefore \frac{1}{p^2} &= \frac{1}{a^2} + \frac{1}{b^2} \end{aligned}$$

$$\therefore 1/p^2 = 1/a^2 + 1/b^2$$

Hence, proved.

Miscellaneous EXERCISE · Textbook page 233**Question 1**

Find the values of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is

(a) Parallel to the x-axis

(b) Parallel to the y-axis

(c) Passing through the origin

Solution

It is given that

$$(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0 \dots (1)$$

(a) Here, if the line is parallel to the x-axis

Slope of the line = Slope of the x-axis

It can be written as

$$(4 - k^2)y = (k - 3)x + k^2 - 7k + 6 = 0$$

We get

$$y = \frac{(k - 3)}{(4 - k^2)}x + \frac{k^2 - 7k + 6}{(4 - k^2)}$$

Which is of the form $y = mx + c$

Here the slope of the given line

$$= \frac{(k - 3)}{(4 - k^2)}$$

Consider the slope of x-axis = 0

$$\frac{(k - 3)}{(4 - k^2)} = 0$$

By further calculation,

$$k - 3 = 0$$

$$k = 3$$

Hence, if the given line is parallel to the x-axis, then the value of k is 3.

(b) Here, if the line is parallel to the y-axis, it is vertical, and the slope will be undefined.

So, the slope of the given line

$$= \frac{(k-3)}{(4-k^2)}$$

Here,

$$\frac{(k-3)}{(4-k^2)} \text{ is undefined at } k^2 = 4$$

$$k^2 = 4$$

$$k = \pm 2$$

Hence, if the given line is parallel to the y-axis, then the value of k is ± 2 .

(c) Here, if the line is passing through (0, 0), which is the origin satisfies the given equation of the line.

$$(k-3)(0) - (4-k^2)(0) + k^2 - 7k + 6 = 0$$

By further calculation,

$$k^2 - 7k + 6 = 0$$

Separating the terms,

$$k^2 - 6k - k + 6 = 0$$

We get

$$(k-6)(k-1) = 0$$

$$k = 1 \text{ or } 6$$

Hence, if the given line is passing through the origin, then the value of k is either 1 or 6.

Question 2

Find the values of θ and p , if the equation $x \cos \theta + y \sin \theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Solution

It is given that

$$\sqrt{3}x + y + 2 = 0$$

It can be reduced as

$$\sqrt{3}x + y + 2 = 0$$

$$-\sqrt{3}x - y = 2$$

By dividing both sides by $\sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$, we get

$$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = \frac{2}{2}$$

It can be written as

$$\left(-\frac{\sqrt{3}}{2}\right)x + \left(-\frac{1}{2}\right)y = 1 \quad \dots(1)$$

By comparing equation (1) to $x \cos \theta + y \sin \theta = p$, we get

$$\cos \theta = -\frac{\sqrt{3}}{2}, \quad \sin \theta = -\frac{1}{2}, \quad \text{and } p = 1$$

Here the values of $\sin \theta$ and $\cos \theta$ are negative

$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Hence, the respective values of θ and p are $7\pi/6$ and 1.

Question 3

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6, respectively.

Solution

Consider the intercepts cut by the given lines on the a and b axes.

$$a + b = 1 \quad \dots\dots (1)$$

$$ab = -6 \quad \dots\dots (2)$$

By solving both equations, we get

$$a = 3 \text{ and } b = -2 \text{ or } a = -2 \text{ and } b = 3$$

We know that the equation of the line whose intercepts on the a and b axes is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ or } bx + ay - ab = 0$$

Case I – a = 3 and b = – 2

So, the equation of the line is – 2x + 3y + 6 = 0, i.e. 2x – 3y = 6

Case II – a = -2 and b = 3

So, the equation of the line is 3x – 2y + 6 = 0, i.e. -3x + 2y = 6

Hence, the required equation of the lines are 2x – 3y = 6 and -3x + 2y = 6

Question 4

What are the points on the y-axis whose distance from the line $x/3 + y/4 = 1$ is 4 units?

Solution

Consider (0, b) as the point on the y-axis whose distance from line $x/3 + y/4 = 1$ is 4 units.

It can be written as $4x + 3y - 12 = 0$ (1)

By comparing equation (1) to the general equation of line $Ax + By + C = 0$, we get

A = 4, B = 3 and C = – 12

We know that the perpendicular distance (d) of a line $Ax + By + C = 0$ from (x_1, y_1) is written as

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

If (0, b) is the point on the y-axis whose distance from line $x/3 + y/4 = 1$ is 4 units, then

$$4 = \frac{|4(0) + 3(b) - 12|}{\sqrt{4^2 + 3^2}}$$

By further calculation

$$4 = \frac{|3b - 12|}{5}$$

By cross multiplication,

$$20 = |3b - 12|$$

We get

$$20 = \pm (3b - 12)$$

$$\text{Here, } 20 = (3b - 12) \text{ or } 20 = -(3b - 12)$$

It can be written as

$$3b = 20 + 12 \text{ or } 3b = -20 + 12$$

So, we get

$$b = 32/3 \text{ or } b = -8/3$$

Hence, the required points are $(0, 32/3)$ and $(0, -8/3)$.

Question 5

Find the perpendicular distance from the origin to the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$.

Solution

Here the equation of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is written as

$$y - \sin \theta = \frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} (x - \cos \theta)$$

By cross multiplication

$$y(\cos \phi - \cos \theta) - \sin \theta(\cos \phi - \cos \theta) = x(\sin \phi - \sin \theta) - \cos \theta(\sin \phi - \sin \theta)$$

By multiplying the terms we get

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \cos \theta \sin \phi - \cos \theta \sin \theta - \sin \theta \cos \phi + \sin \theta \cos \theta = 0$$

On further simplification

$$x(\sin \theta - \sin \phi) + y(\cos \phi - \cos \theta) + \sin(\phi - \theta) = 0$$

So we get

$$Ax + By + C = 0, \text{ where } A = \sin \theta - \sin \phi, B = \cos \phi - \cos \theta, \text{ and } C = \sin(\phi - \theta)$$

We know that the perpendicular distance (d) of a line $Ax + By + C = 0$ from (x_1, y_1) is written as

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

So the perpendicular distance (d) of the given line from $(x_1, y_1) = (0, 0)$ is

$$d = \frac{|(\sin \theta - \sin \phi)(0) + (\cos \phi - \cos \theta)(0) + \sin(\phi - \theta)|}{\sqrt{(\sin \theta - \sin \phi)^2 + (\cos \phi - \cos \theta)^2}}$$

By expanding using formula

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{\sin^2 \theta + \sin^2 \phi - 2 \sin \theta \sin \phi + \cos^2 \phi + \cos^2 \theta - 2 \cos \phi \cos \theta}}$$

Grouping of terms

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{(\sin^2 \theta + \cos^2 \theta) + (\sin^2 \phi + \cos^2 \phi) - 2(\sin \theta \sin \phi + \cos \theta \cos \phi)}}$$

By further simplification

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{1 + 1 - 2(\cos(\phi - \theta))}}$$

Taking out 2 as common

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2(1 - \cos(\phi - \theta))}}$$

Using the formula

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2\left(2\sin^2\left(\frac{\phi - \theta}{2}\right)\right)}}$$

We get

$$= \frac{|\sin(\phi - \theta)|}{\left|2\sin\left(\frac{\phi - \theta}{2}\right)\right|}$$

Question 6

Find the equation of the line parallel to the y-axis and draw through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Solution

Here, the equation of any line parallel to the y-axis is of the form

$$x = a \dots\dots (1)$$

Two given lines are

$$x - 7y + 5 = 0 \dots\dots (2)$$

$$3x + y = 0 \dots\dots (3)$$

By solving equations (2) and (3), we get

$$x = -5/22 \text{ and } y = 15/22$$

$(-5/22, 15/22)$ is the point of intersection of lines (2) and (3)

If the line $x = a$ passes through point $(-5/22, 15/22)$, we get $a = -5/22$

Hence, the required equation of the line is $x = -5/22$

Question 7

Find the equation of a line drawn perpendicular to the line $x/4 + y/6 = 1$ through the point where it meets the y-axis.

Solution

It is given that

$$x/4 + y/6 = 1$$

We can write it as

$$3x + 2y - 12 = 0$$

So, we get

$$y = -3/2 x + 6, \text{ which is of the form } y = mx + c$$

Here, the slope of the given line = $-3/2$

So, the slope of line perpendicular to the given line = $-1/(-3/2) = 2/3$

Consider the given line intersects, the y-axis at $(0, y)$

By substituting x as zero in the equation of the given line,

$$y/6 = 1$$

$$y = 6$$

Hence, the given line intersects the y-axis at $(0, 6)$.

We know that the equation of the line that has a slope of $2/3$ and passes through the point $(0, 6)$ is

$$(y - 6) = 2/3 (x - 0)$$

By further calculation,

$$3y - 18 = 2x$$

So, we get

$$2x - 3y + 18 = 0$$

Hence, the required equation of the line is $2x - 3y + 18 = 0$

Question 8

Find the area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $x - k = 0$.

Solution

It is given that

$$y - x = 0 \dots\dots (1)$$

$$x + y = 0 \dots\dots (2)$$

$$x - k = 0 \dots\dots (3)$$

Here, the point of intersection of

Lines (1) and (2) is

$$x = 0 \text{ and } y = 0$$

Lines (2) and (3) is

$$x = k \text{ and } y = -k$$

Lines (3) and (1) is

$$x = k \text{ and } y = k$$

So, the vertices of the triangle formed by the three given lines are $(0, 0)$, $(k, -k)$ and (k, k) .

Here, the area of triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$$

So, the area of the triangle formed by the three given lines is

$$= \frac{1}{2} |0 (-k - k) + k (k - 0) + k (0 + k)| \text{ square units}$$

By further calculation,

$$= \frac{1}{2} |k^2 + k^2| \text{ square units}$$

So, we get

$$= \frac{1}{2} |2k^2|$$

$$= k^2 \text{ square units}$$

Question 9

Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Solution

It is given that

$$3x + y - 2 = 0 \dots\dots (1)$$

$$px + 2y - 3 = 0 \dots\dots (2)$$

$$2x - y - 3 = 0 \dots\dots (3)$$

By solving equations (1) and (3), we get

$$x = 1 \text{ and } y = -1$$

Here, the three lines intersect at one point, and the point of intersection of lines (1) and (3) will also satisfy line (2)

$$p(1) + 2(-1) - 3 = 0$$

By further calculation,

$$p - 2 - 3 = 0$$

So we get

$$p = 5$$

Hence, the required value of p is 5.

Question 10

If three lines whose equations are $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent, then show that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

Solution

It is given that

$$y = m_1x + c_1 \dots\dots (1)$$

$$y = m_2x + c_2 \dots\dots (2)$$

$$y = m_3x + c_3 \dots (3)$$

By subtracting equation (1) from (2), we get

$$0 = (m_2 - m_1)x + (c_2 - c_1)$$

$$(m_1 - m_2)x = c_2 - c_1$$

So we get

$$x = \frac{c_2 - c_1}{m_1 - m_2}$$

By substituting this value in equation (1) we get

$$y = m_1 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_1$$

By multiplying the terms

$$y = \frac{m_1c_2 - m_1c_1}{m_1 - m_2} + c_1$$

Taking LCM

$$y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$$

On further simplification

$$y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

Here

$$\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right) \text{ is the point of intersection of lines (1) and (2)}$$

Lines (1), (2) and (3) are concurrent. So the point of intersection of lines (1) and (2) will satisfy equation (3)

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3 \left(\frac{c_2 - c_1}{m_1 - m_2} \right) + c_3$$

By multiplying the terms and taking LCM

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = \frac{m_3c_2 - m_3c_1 + c_3m_1 - c_3m_2}{m_1 - m_2}$$

By cross multiplication

$$m_1c_2 - m_2c_1 - m_3c_2 + m_3c_1 - c_3m_1 + c_3m_2 = 0$$

Taking out the common terms,

$$m_1 (c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2) = 0$$

$$\text{Therefore, } m_1 (c_2 - c_3) + m_2 (c_3 - c_1) + m_3 (c_1 - c_2) = 0$$

Question 11

Find the equation of the lines through the point (3, 2), which makes an angle of 45° with the line $x - 2y = 3$.

Solution

Consider m_1 as the slope of the required line

It can be written as

$$y = \frac{1}{2}x - \frac{3}{2} \text{ which is of the form } y = mx + c$$

So, the slope of the given line $m_2 = \frac{1}{2}$

We know that the angle between the required line and line $x - 2y = 3$ is 45°

If θ is the acute angle between lines l_1 and l_2 with slopes m_1 and m_2 ,

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

We get

$$\tan 45^\circ = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Substituting the values

$$1 = \left| \frac{\frac{1}{2} - m_1}{1 + \frac{m_1}{2}} \right|$$

By taking LCM

$$1 = \left| \frac{\frac{(1 - 2m_1)}{2}}{\frac{2 + m_1}{2}} \right|$$

On further calculation

$$1 = \left| \frac{1 - 2m_1}{2 + m_1} \right|$$

We get

$$1 = \pm \left(\frac{1 - 2m_1}{2 + m_1} \right)$$

Here

$$1 = \frac{1 - 2m_1}{2 + m_1} \text{ or } 1 = - \left(\frac{1 - 2m_1}{2 + m_1} \right)$$

It can be written as

$$2 + m_1 = 1 - 2m_1 \text{ or } 2 + m_1 = -1 + 2m_1$$

$$m_1 = -1/3 \text{ or } m_1 = 3$$

Case I – $m_1 = 3$

Here, the equation of the line passing through (3, 2) and having a slope 3 is

$$y - 2 = 3(x - 3)$$

By further calculation,

$$y - 2 = 3x - 9$$

So, we get

$$3x - y = 7$$

Case II – $m_1 = -1/3$

Here, the equation of the line passing through (3, 2) and having a slope -1/3 is

$$y - 2 = -1/3(x - 3)$$

By further calculation,

$$3y - 6 = -x + 3$$

So, we get

$$x + 3y = 9$$

Hence, the equations of the lines are $3x - y = 7$ and $x + 3y = 9$

Question 12

Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Solution

Consider the equation of the line having equal intercepts on the axes as

$$x/a + y/a = 1$$

It can be written as

$$x + y = a \dots\dots (1)$$

By solving equations $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$, we get

$$x = 1/13 \text{ and } y = 5/13$$

$(1/13, 5/13)$ is the point of intersection of two given lines.

We know that equation (1) passes through the point $(1/13, 5/13)$.

$$1/13 + 5/13 = a$$

$$a = 6/13$$

So, equation (1) passes through $(1/13, 5/13)$.

$$1/13 + 5/13 = a$$

We get

$$a = 6/13$$

Her, equation (1) becomes

$$x + y = 6/13$$

$$13x + 13y = 6$$

Hence, the required equation of the line is $13x + 13y = 6$

Question 13

Show that the equation of the line passing through the origin and making an angle θ with the line $y =$

$$mx + c \text{ is } \frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}.$$

Solution

Consider $y = m_1x$ as the equation of the line passing through the origin

It is given that the line makes an angle θ with line $y = mx + c$, then angle θ is written as

$$\tan \theta = \left| \frac{m_1 - m}{1 + m_1 m} \right|$$

By substituting the values

$$\tan \theta = \left| \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right|$$

We get

$$\tan \theta = \pm \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

Here

$$\tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \quad \text{or} \quad \tan \theta = - \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

Case I –

$$\tan \theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x} m}$$

We can write it as

$$\tan \theta + \frac{y}{x} m \tan \theta = \frac{y}{x} - m$$

By further simplification

$$m + \tan \theta = \frac{y}{x} (1 - m \tan \theta)$$

So we get

$$\frac{y}{x} = \frac{m + \tan \theta}{1 - m \tan \theta}$$

Case II –

$$\tan \theta = - \left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x} m} \right)$$

We can write it as

$$\tan \theta + \frac{y}{x} m \tan \theta = -\frac{y}{x} + m$$

By further simplification

$$\frac{y}{x} (1 + m \tan \theta) = m - \tan \theta$$

So we get

$$\frac{y}{x} = \frac{m - \tan \theta}{1 + m \tan \theta}$$

Hence, the required line is given by

$$\frac{y}{x} = \frac{m \pm \tan \theta}{1 \mp m \tan \theta}$$

Question 14

In what ratio, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$?

Solution

We know that the equation of the line joining the points $(-1, 1)$ and $(5, 7)$ is given by

$$y - 1 = \frac{7-1}{5+1}(x+1)$$

By further calculation

$$y - 1 = \frac{6}{6}(x+1)$$

So we get

$$x - y + 2 = 0 \dots\dots (1)$$

So the equation of the given line is

$$x + y - 4 = 0 \dots\dots (2)$$

Here the point of intersection of lines (1) and (2) is given by

$$x = 1 \text{ and } y = 3$$

Consider $(1, 3)$ divide the line segment joining $(-1, 1)$ and $(5, 7)$ in the ratio $1: k$.

Using the section formula

$$(1, 3) = \left(\frac{k(-1) + 1(5)}{1+k}, \frac{k(1) + 1(7)}{1+k} \right)$$

By further calculation

$$(1, 3) = \left(\frac{-k+5}{1+k}, \frac{k+7}{1+k} \right)$$

So we get

$$\frac{-k+5}{1+k} = 1, \frac{k+7}{1+k} = 3$$

We can write it as

$$\frac{-k+5}{1+k} = 1$$

By cross multiplication,

$$-k + 5 = 1 + k$$

We get

$$2k = 4$$

$$k = 2$$

Hence, the line joining the points $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$ in the ratio 1: 2.

Question 15

Find the distance of the line $4x + 7y + 5 = 0$ from the point $(1, 2)$ along the line $2x - y = 0$.

Solution

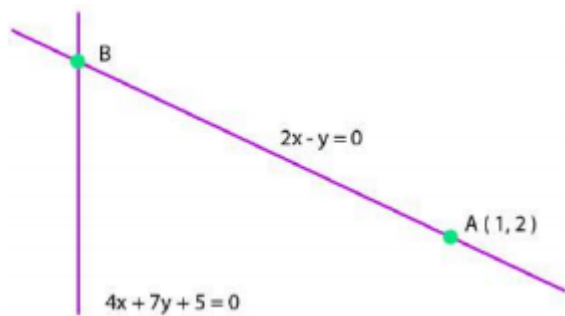
It is given that

$$2x - y = 0 \dots (1)$$

$$4x + 7y + 5 = 0 \dots (2)$$

Here, $A(1, 2)$ is a point on the line (1).

Consider B as the point of intersection of lines (1) and (2).



By solving equations (1) and (2), we get $x = -5/18$ and $y = -5/9$

So, the coordinates of point B are $(-5/18, -5/9)$.

From the distance formula, the distance between A and B

$$AB = \sqrt{\left(1 + \frac{5}{18}\right)^2 + \left(2 + \frac{5}{9}\right)^2} \text{ units}$$

By taking LCM

$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

It can be written as

$$= \sqrt{\left(\frac{23}{2 \times 9}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

So we get

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2} \text{ units}$$

By taking the common terms out

$$= \sqrt{\left(\frac{23}{9}\right)^2 \left(\frac{1}{4} + 1\right)} \text{ units}$$

We get

$$= \frac{23}{9} \sqrt{\frac{5}{4}} \text{ units}$$

$$= \frac{23}{9} \times \frac{\sqrt{5}}{2} \text{ units}$$

So we get

$$= \frac{23\sqrt{5}}{18} \text{ units}$$

Hence, the required distance is

$$\frac{23\sqrt{5}}{18} \text{ units}.$$

Question 16

Find the direction in which a straight line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

Solution

Consider $y = mx + c$ as the line passing through the point $(-1, 2)$.

So, we get

$$2 = m(-1) + c$$

By further calculation,

$$2 = -m + c$$

$$c = m + 2$$

Substituting the value of c

$$y = mx + m + 2 \dots\dots (1)$$

So the given line is

$$x + y = 4 \dots\dots (2)$$

By solving both equations, we get

$$x = \frac{2-m}{m+1} \text{ and } y = \frac{5m+2}{m+1}$$

$\left(\frac{2-m}{m+1}, \frac{5m+2}{m+1}\right)$ is the point of intersection of lines (1) and (2)

Here the point is at a distance of 3 units from $(-1, 2)$

From distance formula

$$\sqrt{\left(\frac{2-m}{m+1} + 1\right)^2 + \left(\frac{5m+2}{m+1} - 2\right)^2} = 3$$

Squaring on both sides

$$\left(\frac{2-m+m+1}{m+1}\right)^2 + \left(\frac{5m+2-2m-2}{m+1}\right)^2 = 3^2$$

By further calculation

$$\frac{9}{(m+1)^2} + \frac{9m^2}{(m+1)^2} = 9$$

Dividing the equation by 9

$$\frac{1+m^2}{(m+1)^2} = 1$$

By cross multiplication,

$$1 + m^2 = m^2 + 1 + 2m$$

So, we get

$$2m = 0$$

$$m = 0$$

Hence, the slope of the required line must be zero, i.e., the line must be parallel to the x-axis.

Question 17

The hypotenuse of a right-angled triangle has its ends at points (1, 3) and (-4, 1). Find the equation of the legs (perpendicular sides) of the triangle.

Solution

Consider ABC as the right angles triangle where $\angle C = 90^\circ$

Here, infinity such lines are present.

m is the slope of AC

So, the slope of BC = $-1/m$

Equation of AC –

$$y - 3 = m(x - 1)$$

By cross multiplication,

$$x - 1 = 1/m(y - 3)$$

Equation of BC –

$$y - 1 = -1/m(x + 4)$$

By cross multiplication,

$$x + 4 = -m(y - 1)$$

By considering the values of m, we get

If $m = 0$,

So, we get

$$y - 3 = 0, x + 4 = 0$$

If $m = \infty$,

So, we get

$$x - 1 = 0, y - 1 = 0 \text{ we get } x = 1, y = 1$$

Question 18

Find the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$, assuming the line to be a plane mirror.

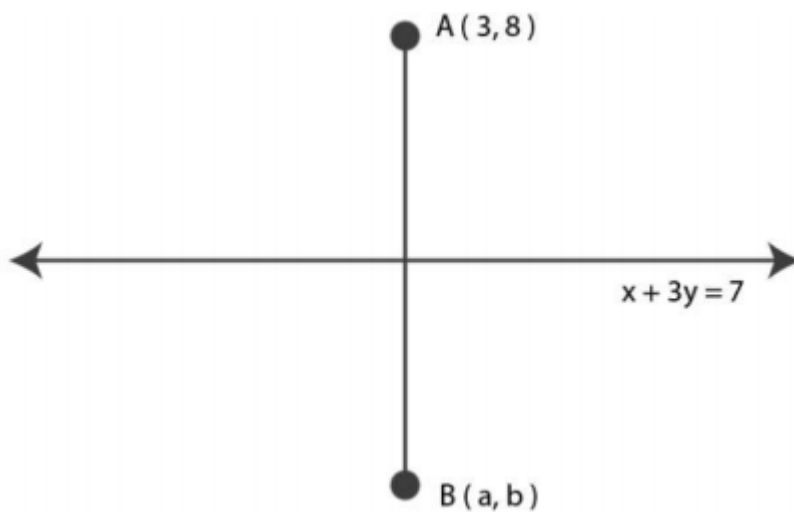
Solution

It is given that

$$x + 3y = 7 \dots\dots (1)$$

Consider $B(a, b)$ as the image of point $A(3, 8)$.

So line (1) is the perpendicular bisector of AB .



Here

$$\text{Slope of AB} = \frac{b-8}{a-3}$$

$$\text{slope of line (1)} = -\frac{1}{3}$$

Line (1) is perpendicular to AB

$$\left(\frac{b-8}{a-3}\right) \times \left(-\frac{1}{3}\right) = -1$$

By further calculation

$$\frac{b-8}{3a-9} = 1$$

By cross multiplication

$$b-8 = 3a-9$$

$$3a-b = 1 \dots\dots (2)$$

We know that

$$\text{Mid-point of AB} = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$$

So the mid-point of line segment AB will satisfy line (1)

From equation (1)

$$\left(\frac{a+3}{2}\right) + 3\left(\frac{b+8}{2}\right) = 7$$

By further calculation

$$a+3+3b+24 = 14$$

On further simplification,

$$a+3b = -13 \dots\dots (3)$$

By solving equations (2) and (3), we get

$$a = -1 \text{ and } b = -4$$

Hence, the image of the given point with respect to the given line is (-1, -4).

Question 19

If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m .

Solution

It is given that

$$y = 3x + 1 \dots\dots (1)$$

$$2y = x + 3 \dots\dots (2)$$

$$y = mx + 4 \dots\dots (3)$$

Here, the slopes of

$$\text{Line (1), } m_1 = 3$$

$$\text{Line (2), } m_2 = \frac{1}{2}$$

$$\text{Line (3), } m_3 = m$$

We know that lines (1) and (2) are equally inclined to line (3), which means that the angle between lines (1) and (3) equals the angle between lines (2) and (3).

$$\left| \frac{m_1 - m_3}{1 + m_1 m_3} \right| = \left| \frac{m_2 - m_3}{1 + m_2 m_3} \right|$$

Substituting the values we get

$$\left| \frac{3 - m}{1 + 3m} \right| = \left| \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m} \right|$$

By taking LCM

$$\left| \frac{3 - m}{1 + 3m} \right| = \left| \frac{1 - 2m}{m + 2} \right|$$

It can be written as

$$\frac{3 - m}{1 + 3m} = \pm \left(\frac{1 - 2m}{m + 2} \right)$$

Here

$$\frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2} \text{ or } \frac{3 - m}{1 + 3m} = - \left(\frac{1 - 2m}{m + 2} \right)$$

If

$$\frac{3 - m}{1 + 3m} = \frac{1 - 2m}{m + 2}$$

By cross multiplication

$$(3 - m)(m + 2) = (1 - 2m)(1 + 3m)$$

On further calculation,

$$-m^2 + m + 6 = 1 + m - 6m^2$$

So, we get

$$5m^2 + 5 = 0$$

Dividing the equation by 5,

$$m^2 + 1 = 0$$

$m = \sqrt{-1}$, which is not real.

Therefore, this case is not possible.

If

$$\frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$$

By cross multiplication

$$(3-m)(m+2) = -(1-2m)(1+3m)$$

On further calculation

$$-m^2 + m + 6 = -(1 + m - 6m^2)$$

So we get

$$7m^2 - 2m - 7 = 0$$

Here we get

$$m = \frac{2 \pm \sqrt{4 - 4(7)(-7)}}{2(7)}$$

By further simplification

$$m = \frac{2 \pm 2\sqrt{1+49}}{14}$$

We can write it as

$$m = \frac{1 \pm 5\sqrt{2}}{7}$$

Hence, the required value of m is

$$\frac{1 \pm 5\sqrt{2}}{7}$$

Question 20

If the sum of the perpendicular distances of a variable point P (x, y) from the lines $x + y - 5 = 0$ and $3x - 2y + 7 = 0$ is always 10. Show that P must move on a line.

Solution

It is given that

$$x + y - 5 = 0 \dots (1)$$

$$3x - 2y + 7 = 0 \dots (2)$$

Here the perpendicular distances of P (x, y) from lines (1) and (2) are written as

$$d_1 = \frac{|x + y - 5|}{\sqrt{(1)^2 + (1)^2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{(3)^2 + (-2)^2}}$$

So we get

$$d_1 = \frac{|x + y - 5|}{\sqrt{2}} \text{ and } d_2 = \frac{|3x - 2y + 7|}{\sqrt{13}}$$

We know that $d_1 + d_2 = 10$

Substituting the values

$$\frac{|x + y - 5|}{\sqrt{2}} + \frac{|3x - 2y + 7|}{\sqrt{13}} = 10$$

By further calculation

$$\sqrt{13}|x + y - 5| + \sqrt{2}|3x - 2y + 7| - 10\sqrt{26} = 0$$

It can be written as

$$\sqrt{13}(x + y - 5) + \sqrt{2}(3x - 2y + 7) - 10\sqrt{26} = 0$$

Now by assuming $(x + y - 5)$ and $(3x - 2y + 7)$ are positive

$$\sqrt{13}x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} - 10\sqrt{26} = 0$$

Taking out the common terms

$$x(\sqrt{13} + 3\sqrt{2}) + y(\sqrt{13} - 2\sqrt{2}) + (7\sqrt{2} - 5\sqrt{13} - 10\sqrt{26}) = 0, \text{ which is the equation of a line.}$$

In the same way, we can find the equation of the line for any signs of $(x + y - 5)$ and $(3x - 2y + 7)$

Hence, point P must move on a line.

Question 21

Find the equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Solution

It is given that

$$9x + 6y - 7 = 0 \dots\dots (1)$$

$$3x + 2y + 6 = 0 \dots\dots (2)$$

Consider P (h, k) be the arbitrary point that is equidistant from lines (1) and (2)

Here the perpendicular distance of P (h, k) from line (1) is written as

$$d_1 = \frac{|9h + 6k - 7|}{\sqrt{(9)^2 + (6)^2}} = \frac{|9h + 6k - 7|}{\sqrt{117}} = \frac{|9h + 6k - 7|}{3\sqrt{13}}$$

Similarly the perpendicular distance of P (h, k) from line (2) is written as

$$d_2 = \frac{|3h + 2k + 6|}{\sqrt{(3)^2 + (2)^2}} = \frac{|3h + 2k + 6|}{\sqrt{13}}$$

We know that P (h, k) is equidistant from lines (1) and (2) $d_1 = d_2$

Substituting the values

$$\frac{|9h + 6k - 7|}{3\sqrt{13}} = \frac{|3h + 2k + 6|}{\sqrt{13}}$$

By further calculation

$$|9h + 6k - 7| = 3|3h + 2k + 6|$$

It can be written as

$$|9h + 6k - 7| = \pm 3(3h + 2k + 6)$$

Here,

$$9h + 6k - 7 = 3(3h + 2k + 6) \text{ or } 9h + 6k - 7 = -3(3h + 2k + 6)$$

$$9h + 6k - 7 = 3(3h + 2k + 6) \text{ is not possible as}$$

$$9h + 6k - 7 = 3(3h + 2k + 6)$$

By further calculation,

$$-7 = 18 \text{ (which is wrong)}$$

We know that

$$9h + 6k - 7 = -3(3h + 2k + 6)$$

By multiplication,

$$9h + 6k - 7 = -9h - 6k - 18$$

We get

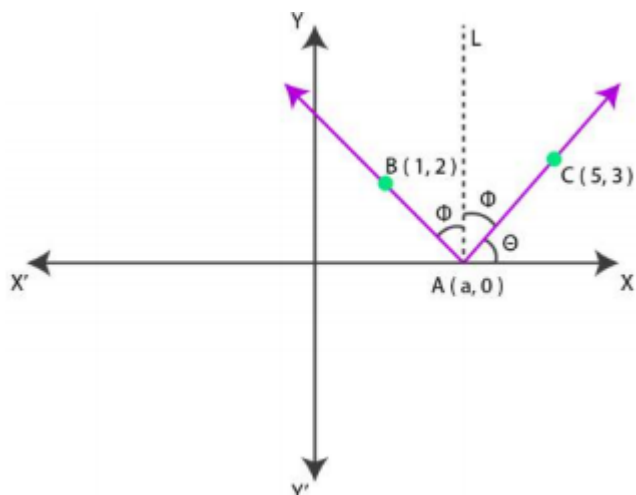
$$18h + 12k + 11 = 0$$

Hence, the required equation of the line is $18x + 12y + 11 = 0$

Question 22

A ray of light passing through the point (1, 2) reflects on the x-axis at point A, and the reflected ray passes through the point (5, 3). Find the coordinates of A.

Solution



Consider the coordinates of point A as (a, 0).

Construct a line (AL) which is perpendicular to the x-axis.

Here, the angle of incidence is equal to the angle of reflection

$$\angle BAL = \angle CAL = \phi$$

$$\angle CAX = \theta$$

It can be written as

$$\angle OAB = 180^\circ - (\theta + 2\phi) = 180^\circ - [\theta + 2(90^\circ - \theta)]$$

On further calculation,

$$= 180^\circ - \theta - 180^\circ + 2\theta$$

$$= \theta$$

So, we get

$$\angle BAX = 180^\circ - \theta$$

$$\text{slope of line AC} = \frac{3-0}{5-a}$$

$$\tan \theta = \frac{3}{5-a} \quad \dots(1)$$

$$\text{Slope of line AB} = \frac{2-0}{1-a}$$

We get

$$\tan(180^\circ - \theta) = \frac{2}{1-a}$$

By further calculation

$$-\tan \theta = \frac{2}{1-a}$$

$$\tan \theta = \frac{2}{a-1} \quad \dots(2)$$

From equations (1) and (2) we get

$$\frac{3}{5-a} = \frac{2}{a-1}$$

By cross multiplication,

$$3a - 3 = 10 - 2a$$

We get

$$a = 13/5$$

Hence, the coordinates of point A are (13/5, 0).

Question 23

Prove that the product of the lengths of the perpendiculars drawn from points

$$\left(\sqrt{a^2 - b^2}, 0\right) \text{ and } \left(-\sqrt{a^2 - b^2}, 0\right) \text{ to the line } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \text{ is } b^2.$$

Solution

It is given that

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

We can write it as

$$bx \cos \theta + ay \sin \theta - ab = 0 \dots (1)$$

Here the length of the perpendicular from point $(\sqrt{a^2 - b^2}, 0)$ to line (1)

$$p_1 = \frac{|b \cos \theta (\sqrt{a^2 - b^2}) + a \sin \theta (0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{|b \cos \theta \sqrt{a^2 - b^2} - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \dots (2)$$

Similarly the length of the perpendicular from point $(-\sqrt{a^2 - b^2}, 0)$ to line (2)

$$p_2 = \frac{|b \cos \theta (-\sqrt{a^2 - b^2}) + a \sin \theta (0) - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} = \frac{|b \cos \theta \sqrt{a^2 - b^2} + ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \dots (3)$$

By multiplying equations (2) and (3) we get

$$p_1 p_2 = \frac{|b \cos \theta \sqrt{a^2 - b^2} - ab| |b \cos \theta \sqrt{a^2 - b^2} + ab|}{(\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta})^2}$$

We get

$$= \frac{|(b \cos \theta \sqrt{a^2 - b^2} - ab)(b \cos \theta \sqrt{a^2 - b^2} + ab)|}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

From the formula

$$= \frac{|(b \cos \theta \sqrt{a^2 - b^2})^2 - (ab)^2|}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

By squaring the numerator we get

$$= \frac{b^2 \cos^2 \theta (a^2 - b^2) - a^2 b^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

By expanding using formula

$$= \frac{a^2 b^2 \cos^2 \theta - b^4 \cos^2 \theta - a^2 b^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

Taking out the common terms

$$= \frac{b^2 [a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

We get

$$= \frac{b^2 [a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2 \sin^2 \theta - a^2 \cos^2 \theta]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

Here $\sin^2 \theta + \cos^2 \theta = 1$

$$= \frac{b^2 [- (b^2 \cos^2 \theta + a^2 \sin^2 \theta)]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

So we get

$$= \frac{b^2 (b^2 \cos^2 \theta + a^2 \sin^2 \theta)}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$$

$$= b^2$$

Therefore, it is proved.

Question 24

A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the equation of the path that he should follow.

Solution

It is given that

$$2x - 3y + 4 = 0 \dots\dots (1)$$

$$3x + 4y - 5 = 0 \dots\dots (2)$$

$$6x - 7y + 8 = 0 \dots\dots (3)$$

Here, the person is standing at the junction of the paths represented by lines (1) and (2).

By solving equations (1) and (2), we get

$$x = -1/17 \text{ and } y = 22/17$$

Hence, the person is standing at point $(-1/17, 22/17)$.

We know that the person can reach path (3) in the least time if they walk along the perpendicular line to (3) from point $(-1/17, 22/17)$

Here, the slope of line (3) = $6/7$

We get the slope of the line perpendicular to the line (3) = $-1/(6/7) = -7/6$

So, the equation of the line passing through $(-1/17, 22/17)$ and having a slope of $-7/6$ is written as

$$\left(y - \frac{22}{17}\right) = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$

By further calculation,

$$6(17y - 22) = -7(17x + 1)$$

By multiplication,

$$102y - 132 = -119x - 7$$

We get

$$119x + 102y = 125$$

Therefore, the path that the person should follow is $119x + 102y = 125$
