

Chapter 9: Sequences and Series Exercise 9.4

NCERT · Class 11 · Maths

Questions

Question 1

$$1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$$

Solution

Given series is $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

It's seen that,

$$n^{\text{th}} \text{ term, } a_n = n(n+1)$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned} S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n k(k+1) \\ &= \sum_{k=1}^n k^2 + \sum_{k=1}^n k \\ &= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{2} \left(\frac{2n+1}{3} + 1 \right) \\ &= \frac{n(n+1)}{2} \left(\frac{2n+4}{3} \right) \\ &= \frac{n(n+1)(n+2)}{3} \end{aligned}$$

Question 2

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots$$

Solution

Given series is $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots$

It's seen that,

$$n^{\text{th}} \text{ term, } a_n = n (n + 1) (n + 2)$$

$$= (n^2 + n) (n + 2)$$

$$= n^3 + 3n^2 + 2n$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned} S_n &= \sum_{k=1}^n a_k \\ &= \sum_{k=1}^n k^3 + 3 \sum_{k=1}^n k^2 + 2 \sum_{k=1}^n k \\ &= \left[\frac{n(n+1)}{2} \right]^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} \\ &= \left[\frac{n(n+1)}{2} \right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1) \\ &= \frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + 2n+1+2 \right] \\ &= \frac{n(n+1)}{2} \left[\frac{n^2 + n + 4n + 6}{2} \right] \\ &= \frac{n(n+1)}{4} (n^2 + 5n + 6) \\ &= \frac{n(n+1)}{4} (n^2 + 2n + 3n + 6) \\ &= \frac{n(n+1)[n(n+2) + 3(n+2)]}{4} \\ &= \frac{n(n+1)(n+2)(n+3)}{4} \end{aligned}$$

Question 3

$$3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \dots$$

Solution

Given series is $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \dots$

It's seen that,

$$n^{\text{th}} \text{ term, } a_n = (2n + 1) n^2 = 2n^3 + n^2$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned} S_n &= \sum_{k=1}^n a_k \\ &= \sum_{k=1}^n (2k^3 + k^2) = 2 \sum_{k=1}^n k^3 + \sum_{k=1}^n k^2 \\ &= 2 \left[\frac{n(n+1)}{2} \right]^2 + \frac{n(n+1)(2n+1)}{6} \\ &= \frac{n^2(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} \\ &= \frac{n(n+1)}{2} \left[n(n+1) + \frac{2n+1}{3} \right] \\ &= \frac{n(n+1)}{2} \left[\frac{3n^2 + 3n + 2n + 1}{3} \right] \\ &= \frac{n(n+1)}{2} \left[\frac{3n^2 + 5n + 1}{3} \right] \\ &= \frac{n(n+1)(3n^2 + 5n + 1)}{6} \end{aligned}$$

Question 4

Find the sum to n terms of the series

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$

Solution

Given series is, $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$

It's seen that,

$$n^{\text{th}} \text{ term, } a_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad (\text{By partial fractions})$$

$$a_1 = \frac{1}{1} - \frac{1}{2}$$

$$a_2 = \frac{1}{2} - \frac{1}{3}$$

$$a_3 = \frac{1}{3} - \frac{1}{4} \dots$$

$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

On adding the above terms column wise, we get

$$a_1 + a_2 + \dots + a_n = \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right] - \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n+1} \right]$$

$$\therefore S_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

Question 5

Find the sum to n terms of the series $5^2 + 6^2 + 7^2 + \dots + 20^2$

Solution

Given series is $5^2 + 6^2 + 7^2 + \dots + 20^2$

It's seen that,

$$n^{\text{th}} \text{ term, } a_n = (n+4)^2 = n^2 + 8n + 16$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned}
 S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n (k^2 + 8k + 16) \\
 &= \sum_{k=1}^n k^2 + 8 \sum_{k=1}^n k + \sum_{k=1}^n 16 \\
 &= \frac{n(n+1)(2n+1)}{6} + \frac{8n(n+1)}{2} + 16n
 \end{aligned}$$

Now, its found that

16th term is $(16 + 4)^2 = 20^2$

Thus,

$$\begin{aligned}
 S_{16} &= \frac{16(16+1)(2 \times 16+1)}{6} + \frac{8 \times 16 \times (16+1)}{2} + 16 \times 16 \\
 &= \frac{(16)(17)(33)}{6} + \frac{(8) \times 16 \times (16+1)}{2} + 16 \times 16 \\
 &= \frac{(16)(17)(33)}{6} + \frac{(8)(16)(17)}{2} + 256 \\
 &= 1496 + 1088 + 256 \\
 &= 2840
 \end{aligned}$$

Hence, $5^2 + 6^2 + 7^2 + \dots + 20^2 = 2840$

Question 6

Find the sum to n terms of the series $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$

Solution

Given series is $3 \times 8 + 6 \times 11 + 9 \times 14 + \dots$

It's found out that,

$$a_n = (n^{\text{th}} \text{ term of } 3, 6, 9 \dots) \times (n^{\text{th}} \text{ term of } 8, 11, 14, \dots)$$

$$= (3n) (3n + 5)$$

$$= 9n^2 + 15n$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned}
 S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n (9k^2 + 15k) \\
 &= 9 \sum_{k=1}^n k^2 + 15 \sum_{k=1}^n k \\
 &= 9 \times \frac{n(n+1)(2n+1)}{6} + 15 \times \frac{n(n+1)}{2} \\
 &= \frac{3n(n+1)(2n+1)}{2} + \frac{15n(n+1)}{2} \\
 &= \frac{3n(n+1)}{2} (2n+1+5) \\
 &= \frac{3n(n+1)}{2} (2n+6) \\
 &= 3n(n+1)(n+3)
 \end{aligned}$$

Question 7

Find the sum to n terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

Solution

Given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

Finding the n^{th} term, we have

$$a_n = (1^2 + 2^2 + 3^2 + \dots + n^2)$$

$$\begin{aligned}
 &= \frac{n(n+1)(2n+1)}{6} \\
 &= \frac{n(2n^2+3n+1)}{6} \\
 &= \frac{2n^3+3n^2+n}{6} \\
 &= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n
 \end{aligned}$$

Now, the sum of n terms of the series can be expressed as

$$\begin{aligned}
S_n &= \sum_{k=1}^n a_k \\
&= \sum_{k=1}^n \left(\frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k \right) \\
&= \frac{1}{3} \sum_{k=1}^n k^3 + \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{6} \sum_{k=1}^n k \\
&= \frac{1}{3} \frac{n^2(n+1)^2}{(2)^2} + \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \times \frac{n(n+1)}{2} \\
&= \frac{n(n+1)}{6} \left[\frac{n(n+1)}{2} + \frac{(2n+1)}{2} + \frac{1}{2} \right] \\
&= \frac{n(n+1)}{6} \left[\frac{n^2 + n + 2n + 1 + 1}{2} \right] \\
&= \frac{n(n+1)}{6} \left[\frac{n^2 + n + 2n + 2}{2} \right] \\
&= \frac{n(n+1)}{6} \left[\frac{n(n+1) + 2(n+1)}{2} \right] \\
&= \frac{n(n+1)}{6} \left[\frac{(n+1)(n+2)}{2} \right] \\
&= \frac{n(n+1)^2(n+2)}{12}
\end{aligned}$$

Question 8

Find the sum to n terms of the series whose n^{th} term is given by $n(n+1)(n+4)$.

Solution

Given,

$$a_n = n(n+1)(n+4) = n(n^2 + 5n + 4) = n^3 + 5n^2 + 4n$$

Now, the sum of n terms of the series can be expressed as

$$\begin{aligned}
 S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k \\
 &= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \\
 &= \frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4 \right] \\
 &= \frac{n(n+1)}{2} \left[\frac{3n^2 + 3n + 20n + 10 + 24}{6} \right] \\
 &= \frac{n(n+1)}{2} \left[\frac{3n^2 + 23n + 34}{6} \right] \\
 &= \frac{n(n+1)(3n^2 + 23n + 34)}{12}
 \end{aligned}$$

Question 9

Find the sum to n terms of the series whose n^{th} term is given by $n^2 + 2^n$

Solution

Given,

n^{th} term of the series as:

$$a_n = n^2 + 2^n$$

Then, the sum of n terms of the series can be expressed as

$$S_n = \sum_{k=1}^n k^2 + 2^k = \sum_{k=1}^n k^2 + \sum_{k=1}^n 2^k \quad (1)$$

$$\text{Consider } \sum_{k=1}^n 2^k = 2^1 + 2^2 + 2^3 + \dots$$

The above series $2, 2^2, 2^3, \dots$ is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^n 2^k = \frac{(2)[(2)^n - 1]}{2 - 1} = 2(2^n - 1) \quad (2)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^n k^2 + 2(2^n - 1) = \frac{n(n+1)(2n+1)}{6} + 2(2^n - 1)$$

Question 10

Find the sum to n terms of the series whose n^{th} term is given by $(2n - 1)^2$

Solution

Given,

n^{th} term of the series as:

$$a_n = (2n - 1)^2 = 4n^2 - 4n + 1$$

Then, the sum of n terms of the series can be expressed as

$$\begin{aligned} S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n (4k^2 - 4k + 1) \\ &= 4 \sum_{k=1}^n k^2 - 4 \sum_{k=1}^n k + \sum_{k=1}^n 1 \\ &= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n \\ &= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n \\ &= n \left[\frac{2(2n^2 + 3n + 1)}{3} - 2(n+1) + 1 \right] \\ &= n \left[\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3} \right] \\ &= n \left[\frac{4n^2 - 1}{3} \right] \\ &= \frac{n(2n+1)(2n-1)}{3} \end{aligned}$$