

Chapter 9: Sequences and Series Exercise 9.3

NCERT · Class 11 · Maths

Questions

Question 1

Find the 20th and n^{th} terms of the G.P. $5/2, 5/4, 5/8, \dots$

Solution

Given G.P. is $5/2, 5/4, 5/8, \dots$

Here, a = First term = $5/2$

r = Common ratio = $(5/4)/(5/2) = 1/2$

Thus, the 20th term and n^{th} term

$$a_{20} = ar^{20-1} = \frac{5}{2} \left(\frac{1}{2} \right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$
$$a_n = ar^{n-1} = \frac{5}{2} \left(\frac{1}{2} \right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

Question 2

Find the 12th term of a G.P. whose 8th term is 192, and the common ratio is 2.

Solution

Given,

The common ratio of the G.P., $r = 2$

And, let a be the first term of the G.P.

Now,

$$a_8 = ar^{8-1} = ar^7$$

$$ar^7 = 192$$

$$a(2)^7 = 192$$

$$a(2)^7 = (2)^6 (3)$$

So,

$$a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

Hence,

$$a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

Question 3

The 5th, 8th and 11th terms of a G.P. are p , q and s , respectively. Show that $q^2 = ps$.

Solution

Let's take a to be the first term and r to be the common ratio of the G.P.

Then according to the question, we have

$$a_5 = ar^{5-1} = ar^4 = p \dots (i)$$

$$a_8 = ar^{8-1} = ar^7 = q \dots (ii)$$

$$a_{11} = ar^{11-1} = ar^{10} = s \dots (iii)$$

Dividing equation (ii) by (i), we get

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \quad \dots (iv)$$

On dividing equation (iii) by (ii), we get

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$r^3 = \frac{s}{q} \quad \dots (v)$$

Equating the values of r^3 obtained in (iv) and (v), we get

$$\frac{q}{p} = \frac{s}{q}$$

$$q^2 = ps$$

Hence proved.

Question 4

The 4th term of a G.P. is the square of its second term, and the first term is -3 . Determine its 7th term.

Solution

Let's consider a to be the first term and r to be the common ratio of the G.P.

Given, $a = -3$

And we know that,

$$a_n = ar^{n-1}$$

$$\text{So, } a_4 = ar^3 = (-3) r^3$$

$$a_2 = ar^1 = (-3) r$$

Then from the question, we have

$$(-3) r^3 = [(-3) r]^2$$

$$\Rightarrow -3r^3 = 9r^2$$

$$\Rightarrow r = -3$$

$$a_7 = ar^{7-1} = ar^6 = (-3) (-3)^6 = -(3)^7 = -2187$$

Therefore, the seventh term of the G.P. is -2187 .

Question 5

Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \dots$ is 128 ? (b) $\sqrt{3}, 3, 3\sqrt{3}, \dots$ is 729 ?

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$ is $\frac{1}{19683}$?

Solution

(a) The given sequence, $2, 2\sqrt{2}, 4, \dots$

We have,

$$a = 2 \text{ and } r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Taking the n^{th} term of this sequence as 128, we have

$$\begin{aligned} a_n &= ar^{n-1} \\ (2)(\sqrt{2})^{n-1} &= 128 \\ (2)(2)^{\frac{n-1}{2}} &= (2)^7 \\ (2)^{\frac{n-1}{2}+1} &= (2)^7 \\ \frac{n-1}{2} + 1 &= 7 \\ \frac{n-1}{2} &= 6 \\ n-1 &= 12 \\ n &= 13 \end{aligned}$$

Therefore, the 13^{th} term of the given sequence is 128.

(ii) Given the sequence, $\sqrt{3}, 3, 3\sqrt{3}, \dots$

We have,

$$a = \sqrt{3} \text{ and } r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Taking the n^{th} term of this sequence to be 729, we have

$$\begin{aligned}
 a_n &= ar^{n-1} \\
 \therefore ar^{n-1} &= 729 \\
 (\sqrt{3})(\sqrt{3})^{n-1} &= 729 \\
 (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} &= (3)^6 \\
 (3)^{\frac{1}{2} + \frac{n-1}{2}} &= (3)^6
 \end{aligned}$$

Equating the exponents, we have

$$\begin{aligned}
 \frac{1}{2} + \frac{n-1}{2} &= 6 \\
 \frac{1+n-1}{2} &= 6 \\
 \therefore n &= 12
 \end{aligned}$$

Therefore, the 12th term of the given sequence is 729.

(iii) Given sequence, $1/3, 1/9, 1/27, \dots$

$$a = 1/3 \text{ and } r = (1/9)/(1/3) = 1/3$$

Taking the nth term of this sequence to be $1/19683$, we have

$$\begin{aligned}
 a_n &= ar^{n-1} \\
 \therefore ar^{n-1} &= \frac{1}{19683} \\
 \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} &= \frac{1}{19683} \\
 \left(\frac{1}{3}\right)^n &= \left(\frac{1}{3}\right)^9 \\
 n &= 9
 \end{aligned}$$

Therefore, the 9th term of the given sequence is $1/19683$.

Question 6

For what values of x , the numbers $-2/7, x, -7/2$ are in G.P.?

Solution

The given numbers are $-2/7, x, -7/2$.

$$\text{Common ratio} = x/(-2/7) = -7x/2$$

Also, common ratio = $(-7/2)/x = -7/2x$

$$\begin{aligned}\therefore \frac{-7x}{2} &= \frac{-7}{2x} \\ x^2 &= \frac{-2 \times 7}{-2 \times 7} = 1 \\ x &= \sqrt{1} \\ x &= \pm 1\end{aligned}$$

Therefore, for $x = \pm 1$, the given numbers will be in G.P.

Question 7

Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 ...

Solution

Given G.P., 0.15, 0.015, 0.00015, ...

Here, $a = 0.15$ and $r = 0.015/0.15 = 0.1$

We know that, $S_n = \frac{a(1-r^n)}{1-r}$

$$\begin{aligned}\therefore S_{20} &= \frac{0.15[1-(0.1)^{20}]}{1-0.1} \\ &= \frac{0.15}{0.9}[1-(0.1)^{20}] \\ &= \frac{15}{90}[1-(0.1)^{20}] \\ &= \frac{1}{6}[1-(0.1)^{20}]\end{aligned}$$

Question 8

Find the sum to n terms in the geometric progression $\sqrt{7}$, $\sqrt{21}$, $3\sqrt{7}$, ...

Solution

The given G.P is $\sqrt{7}$, $\sqrt{21}$, $3\sqrt{7}$, ...

Here,

$a = \sqrt{7}$ and

$$\begin{aligned}
 r &= \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3} \\
 S_n &= \frac{a(1-r^n)}{1-r} \\
 \therefore S_n &= \frac{\sqrt{7}[1-(\sqrt{3})^n]}{1-\sqrt{3}} \\
 &= \frac{\sqrt{7}[1-(\sqrt{3})^n]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad (\text{By rationalizing}) \\
 &= \frac{\sqrt{7}(1+\sqrt{3})[1-(\sqrt{3})^n]}{1-3} \\
 &= \frac{-\sqrt{7}(1+\sqrt{3})[1-(3)^{\frac{n}{2}}]}{2} \\
 &= \frac{\sqrt{7}(1+\sqrt{3})[(3)^{\frac{n}{2}}-1]}{2}
 \end{aligned}$$

Question 9

Find the sum to n terms in the geometric progression 1, $-a$, a^2 , $-a^3$ (if $a \neq -1$)

Solution

The given G.P. is 1, $-a$, a^2 , $-a^3$

Here, the first term = $a_1 = 1$

And the common ratio = $r = -a$

We know that,

$$\begin{aligned}
 S_n &= \frac{a_1(1-r^n)}{1-r} \\
 \therefore S_n &= \frac{1[1-(-a)^n]}{1-(-a)} = \frac{[1-(-a)^n]}{1+a}
 \end{aligned}$$

Question 10

Find the sum to n terms in the geometric progression x^3 , x^5 , x^7 , ... (if $x \neq \pm 1$)

Solution

Given G.P. is $x^3, x^5, x^7, \dots$

Here, we have $a = x^3$ and $r = x^5/x^3 = x^2$

We know that, $S_n = \frac{a(1-r^n)}{1-r}$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3[1-(x^2)^n]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

Question 11

Evaluate: $\sum_{k=1}^{11} (2+3^k)$

Solution

$$\sum_{k=1}^{11} (2+3^k) = \sum_{k=1}^{11} (2) + \sum_{k=1}^{11} 3^k = 2(11) + \sum_{k=1}^{11} 3^k = 22 + \sum_{k=1}^{11} 3^k \quad \dots(1)$$

$$\sum_{k=1}^{11} 3^k = 3^1 + 3^2 + 3^3 + \dots + 3^{11}$$

We can see that, the terms of this sequence $3, 3^2, 3^3, \dots$ forms a G.P

And, we know

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{11} = \frac{3[(3)^{11} - 1]}{3 - 1}$$

$$S_{11} = \frac{3}{2}(3^{11} - 1)$$

$$\therefore \sum_{k=1}^{11} 3^k = \frac{3}{2}(3^{11} - 1)$$

On substituting the above value in equation (1), we get

$$\sum_{k=1}^{11} (2+3^k) = 22 + \frac{3}{2}(3^{11} - 1)$$

Question 12

The sum of the first three terms of a G.P. is $39/10$, and their product is 1. Find the common ratio and the terms.

Solution

Let a/r , a , ar be the first three terms of the G.P.

$$a/r + a + ar = 39/10 \dots\dots (1)$$

$$(a/r) (a) (ar) = 1 \dots\dots\dots (2)$$

From (2), we have

$$a^3 = 1$$

Hence, $a = 1$ [Considering real roots only]

Substituting the value of a in (1), we get

$$1/r + 1 + r = 39/10$$

$$(1 + r + r^2)/r = 39/10$$

$$10 + 10r + 10r^2 = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r - 5) - 2(2r - 5) = 0$$

$$(5r - 2) (2r - 5) = 0$$

Thus,

$$r = 2/5 \text{ or } 5/2$$

Therefore, the three terms of the G.P. are $5/2$, 1 and $2/5$.

Question 13

How many terms of G.P. $3, 3^2, 3^3, \dots$ are needed to give the sum 120?

Solution

Given G.P. is $3, 3^2, 3^3, \dots$

Let's consider that n terms of this G.P. be required to obtain the sum 120.

We know that,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Here, $a = 3$ and $r = 3$

$$S_n = 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$120 = \frac{3(3^n - 1)}{2}$$

$$\frac{120 \times 2}{3} = 3^n - 1$$

$$3^n - 1 = 80$$

$$3^n = 81$$

$$3^n = 3^4$$

Equating the exponents, we get $n = 4$

Therefore, four terms of the given G.P. are required to obtain the sum 120.

Question 14

The sum of the first three terms of a G.P. is 16, and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Solution

Let's assume the G.P. to be $a, ar, ar^2, ar^3, \dots$

Then according to the question, we have

$$a + ar + ar^2 = 16 \text{ and } ar^3 + ar^4 + ar^5 = 128$$

$$a(1 + r + r^2) = 16 \dots (1) \text{ and,}$$

$$ar^3(1 + r + r^2) = 128 \dots (2)$$

Dividing equation (2) by (1), we get

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$$r^3 = 8$$

$$r = 2$$

Now, using $r = 2$ in (1), we get

$$a(1 + 2 + 4) = 16$$

$$a(7) = 16$$

$$a = 16/7$$

Now, the sum of terms is given as

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$\Rightarrow S_n = \frac{16}{7} \frac{(2^n - 1)}{2 - 1} = \frac{16}{7} (2^n - 1)$$

Question 15

Given a G.P. with $a = 729$ and 7th term 64, determine S_7 .

Solution

Given,

$$a = 729 \text{ and } a_7 = 64$$

Let r be the common ratio of the G.P.

Then we know that, $a_n = a r^{n-1}$

$$a_7 = ar^{7-1} = (729)r^6$$

$$\Rightarrow 64 = 729 r^6$$

$$r^6 = 64/729$$

$$r^6 = (2/3)^6$$

$$r = 2/3$$

And we know that

$$\begin{aligned} S_n &= \frac{a(1-r^n)}{1-r} \\ \text{So,} \\ S_7 &= \frac{729 \left[1 - \left(\frac{2}{3} \right)^7 \right]}{1 - \frac{2}{3}} \\ &= 3 \times 729 \left[1 - \left(\frac{2}{3} \right)^7 \right] \\ &= (3)^7 \left[\frac{(3)^7 - (2)^7}{(3)^7} \right] \\ &= (3)^7 - (2)^7 \\ &= 2187 - 128 \\ &= 2059 \end{aligned}$$

Question 16

Find a G.P. for which the sum of the first two terms is -4 and the fifth term is 4 times the third term.

Solution

Consider a to be the first term and r to be the common ratio of the G.P.

$$\text{Given, } S_2 = -4$$

Then, from the question, we have

$$S_2 = -4 = \frac{a(1-r^2)}{1-r} \quad \dots(1)$$

And,

$$a_5 = 4 \times a_3$$

$$ar^4 = 4ar^2$$

$$r^2 = 4$$

$$r = \pm 2$$

Using the value of r in (1), we have

$$-4 = \frac{a[1-(2)^2]}{1-2} \text{ for } r = 2$$

$$-4 = \frac{a(1-4)}{-1}$$

$$-4 = a(3)$$

$$a = \frac{-4}{3}$$

$$\text{Also, } -4 = \frac{a[1-(-2)^2]}{1-(-2)} \text{ for } r = -2$$

$$-4 = \frac{a(1-4)}{1+2}$$

$$-4 = \frac{a(-3)}{3}$$

$$a = 4$$

Therefore, the required G.P is

$-4/3, -8/3, -16/3, \dots$ Or $4, -8, 16, -32, \dots$

Question 17

If the 4th, 10th and 16th terms of a G.P. are x, y and z , respectively. Prove that x, y , and z are in G.P.

Solution

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$$a_4 = a r^3 = x \dots (1)$$

$$a_{10} = a r^9 = y \dots (2)$$

$$a_{16} = a r^{15} = z \dots (3)$$

On dividing (2) by (1), we get

$$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$$

And, on dividing (3) by (2), we get

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$$

$$\frac{y}{x} = \frac{z}{y}$$

Therefore, x, y, z are in G. P.

Question 18

Find the sum to n terms of the sequence, 8, 88, 888, 8888...

Solution

Given sequence: 8, 88, 888, 8888...

This sequence is not a G.P.

But, it can be changed to G.P. by writing as

$$S_n = 8 + 88 + 888 + 8888 + \dots \text{to } n \text{ terms}$$

$$= \frac{8}{9} [9 + 99 + 999 + 9999 + \dots \text{to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \dots \text{to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 + 10^2 + \dots n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ terms})]$$

$$= \frac{8}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{8}{9} \left[\frac{10(10^n - 1)}{9} - n \right]$$

$$= \frac{80}{81} (10^n - 1) - \frac{8}{9} n$$

Question 19

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, 1/2.

Solution

The required sum = $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

$$= 64[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}]$$

Now, it's seen that

4, 2, 1, $\frac{1}{2}$, $\frac{1}{2^2}$ is a G.P.

With the first term, $a = 4$

Common ratio, $r = \frac{1}{2}$

We know,

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\therefore S_5 = \frac{4 \left[1 - \left(\frac{1}{2} \right)^5 \right]}{1 - \frac{1}{2}} = \frac{4 \left[1 - \frac{1}{32} \right]}{\frac{1}{2}} = 8 \left(\frac{32-1}{32} \right) = \frac{31}{4}$$

Therefore, the required sum = $64(31/4) = (16)(31) = 496$

Question 20

Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \dots, ar^{n-1}$ and $A, AR, AR^2, \dots, AR^{n-1}$ form a G.P, and find the common ratio.

Solution

To be proved: The sequence, $aA, arAR, ar^2AR^2, \dots, ar^{n-1}AR^{n-1}$, forms a G.P.

Now, we have

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$

$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Therefore, the above sequence forms a G.P., and the common ratio is rR .

Question 21

Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Solution

Consider a to be the first term and r to be the common ratio of the G.P.

Then,

$$a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

From the question, we have

$$a_3 = a_1 + 9$$

$$ar^2 = a + 9 \dots (i)$$

$$a_2 = a_4 + 18$$

$$ar = ar^3 + 18 \dots (ii)$$

So, from (1) and (2), we get

$$a(r^2 - 1) = 9 \dots (iii)$$

$$ar(1 - r^2) = 18 \dots (iv)$$

Now, dividing (4) by (3), we get

$$\frac{ar(1 - r^2)}{a(r^2 - 1)} = \frac{18}{9}$$

$$-r = 2$$

$$r = -2$$

On substituting the value of r in (i), we get

$$4a = a + 9$$

$$3a = 9$$

$$\therefore a = 3$$

Therefore, the first four numbers of the G.P. are 3, $3(-2)$, $3(-2)^2$, and $3(-2)^3$

i.e., 3, -6, 12, and -24.

Question 22

If the p^{th} , q^{th} and r^{th} terms of a G.P. are a , b and c , respectively. Prove that $a^{q-r} b^{r-p} c^{p-q} = 1$

Solution

Let's take A to be the first term and R to be the common ratio of the G.P.

Then according to the question, we have

$$AR^{p-1} = a$$

$$AR^{q-1} = b$$

$$AR^{r-1} = c$$

Then,

$$a^{q-r} b^{r-p} c^{p-q}$$

$$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$$

$$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r) + (rq-r+p-pq) + (pr-p-qr+q)}$$

$$= A^0 \times R^0$$

$$= 1$$

Hence proved.

Question 23

If the first and the n^{th} term of a G.P. are a and b , respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Solution

Given, the first term of the G.P. is a , and the last term is b .

Thus,

The G.P. is $a, ar, ar^2, ar^3, \dots, ar^{n-1}$, where r is the common ratio.

Then,

$$b = ar^{n-1} \dots (1)$$

P = Product of n terms

$$= (a) (ar) (ar^2) \dots (ar^{n-1})$$

$$= (a \times a \times \dots a) (r \times r^2 \times \dots r^{n-1})$$

$$= a^n r^{1+2+\dots+(n-1)} \dots (2)$$

Here, $1, 2, \dots, (n-1)$ is an A.P.

So,

$$1 + 2 + \dots + (n-1) = \frac{n-1}{2} [2 + (n-1-1) \times 1] = \frac{n-1}{2} [2 + n - 2] = \frac{n(n-1)}{2}$$

And, the product of n terms P is given by,

$$\begin{aligned} P &= a^n r^{\frac{n(n-1)}{2}} \\ \therefore P^2 &= a^{2n} r^{n(n-1)} \\ &= [a^2 r^{(n-1)}]^n \\ &= [a \times ar^{n-1}]^n \\ &= (ab)^n \quad [\text{Using (1)}] \end{aligned}$$

Question 24

Show the ratio of the sum of the first n terms of a G.P. to the sum of terms from

$$(n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term is } \frac{1}{r^n}.$$

Solution

Let a be the first term and r be the common ratio of the G.P.

$$\text{Sum of first } n \text{ terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are n terms from $(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ term,

Sum of terms from $(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ term

$$= \frac{a_{n+1}(1-r^n)}{(1-r)}$$

$$a^{n+1} = ar^{n+1-1} = ar^n$$

Thus, the required ratio =

$$\frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)}$$

$$= \frac{1}{r^n}$$

Thus, the ratio of the sum of the first n terms of a G.P. to the sum of terms from $(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ term

$$\text{is } \frac{1}{r^n}.$$

Question 25

If a, b, c and d are in G.P., show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

Solution

Given, a, b, c, d are in G.P.

So, we have

$$bc = ad \quad \dots (1)$$

$$b^2 = ac \quad \dots (2)$$

$$c^2 = bd \quad \dots (3)$$

Taking the R.H.S., we have

R.H.S.

$$\begin{aligned}
&= (ab + bc + cd)^2 \\
&= (ab + ad + cd)^2 \text{ [Using (1)]} \\
&= [ab + d(a + c)]^2 \\
&= a^2b^2 + 2abd(a + c) + d^2(a + c)^2 \\
&= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2) \\
&= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2 \text{ [Using (1) and (2)]} \\
&= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2 \\
&= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2 \\
&= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2) \\
&= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \\
&= \text{L.H.S.}
\end{aligned}$$

Thus, L.H.S. = R.H.S.

Therefore,

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

Question 26

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Solution

Let's assume G_1 and G_2 to be two numbers between 3 and 81 such that the series 3, G_1 , G_2 , 81 forms a G.P.

And let a be the first term and r be the common ratio of the G.P.

Now, we have the 1st term as 3 and the 4th term as 81.

$$81 = (3)(r)^3$$

$$r^3 = 27$$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$$G_1 = ar = (3)(3) = 9$$

$$G_2 = ar^2 = (3)(3)^2 = 27$$

Therefore, the two numbers which can be inserted between 3 and 81 so that the resulting sequence becomes a G.P. are 9 and 27.

Question 27

Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b .

Solution

We know that,

The G. M. of a and b is given by $\sqrt{ab}$.

Then, from the question, we have

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

By squaring both sides, we get

$$\frac{(a^{n+1} + b^{n+1})^2}{(a^n + b^n)^2} = ab$$

Performing cross multiplication after expanding, we get,

$$\begin{aligned}
 a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= (ab)(a^{2n} + 2a^n b^n + b^{2n}) \\
 a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= a^{2n+1}b + 2a^{n+1}b^{n+1} + ab^{2n+1} \\
 a^{2n+2} + b^{2n+2} &= a^{2n+1}b + ab^{2n+1} \\
 a^{2n+2} - a^{2n+1}b &= ab^{2n+1} - b^{2n+2} \\
 a^{2n+1}(a-b) &= b^{2n+1}(a-b) \\
 \left(\frac{a}{b}\right)^{2n+1} &= 1 = \left(\frac{a}{b}\right)^0 \\
 2n+1 &= 0 \quad (\text{Equating the exponents}) \\
 n &= \frac{-1}{2}
 \end{aligned}$$

Question 28

The sum of two numbers is 6 times their geometric mean; show that numbers are in the ratio

$$(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$$

Solution

Consider the two numbers to be a and b .

Then, G.M. = $\sqrt{ab}$.

From the question, we have

$$a + b = 6\sqrt{ab} \quad \dots(1)$$

$$\Rightarrow (a + b)^2 = 36(ab)$$

Also,

$$(a - b)^2 = (a + b)^2 - 4ab = 36ab - 4ab = 32ab$$

$$\Rightarrow a - b = \sqrt{32}\sqrt{ab}$$

$$= 4\sqrt{2}\sqrt{ab} \quad \dots(2)$$

On adding (1) and (2), we get

$$2a = (6 + 4\sqrt{2})\sqrt{ab}$$

$$a = (3 + 2\sqrt{2})\sqrt{ab}$$

Substituting the value of a in (1), we get

$$b = 6\sqrt{ab} - (3 + 2\sqrt{2})\sqrt{ab}$$

$$b = (3 - 2\sqrt{2})\sqrt{ab}$$

$$\frac{a}{b} = \frac{(3 + 2\sqrt{2})\sqrt{ab}}{(3 - 2\sqrt{2})\sqrt{ab}} = \frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}$$

Therefore, the required ratio is $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$.

Question 29

If A and G be A.M. and G.M., respectively, between two positive numbers, prove that the

$$A \pm \sqrt{(A + G)(A - G)}$$

numbers are.

Solution

Given that A and G are A.M. and G.M. between two positive numbers.

And, let these two positive numbers be a and b.

$$\text{So, } AM = A = \frac{a+b}{2} \quad \dots(1)$$

$$GM = G = \sqrt{ab} \quad \dots(2)$$

From (1) and (2), we get

$$a + b = 2A \quad \dots (3)$$

$$ab = G^2 \quad \dots (4)$$

Substituting the value of a and b from (3) and (4) in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we have

$$(a - b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$$

$$(a - b)^2 = 4(A + G)(A - G)$$

$$(a - b) = 2\sqrt{(A + G)(A - G)} \quad \dots(5)$$

From (3) and (5), we get

$$2a = 2A + 2\sqrt{(A + G)(A - G)}$$

$$\Rightarrow a = A + \sqrt{(A + G)(A - G)}$$

Substituting the value of a in (3), we have

$$b = 2A - a = A - \sqrt{(A + G)(A - G)}$$

Therefore, the two numbers are $A \pm \sqrt{(A + G)(A - G)}$.

Question 30

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and n^{th} hour?

Solution

Given, the number of bacteria doubles every hour. Hence, the number of bacteria after every hour will form a G.P.

Here, we have $a = 30$ and $r = 2$

$$\text{So, } a_3 = ar^2 = (30)(2)^2 = 120$$

Thus, the number of bacteria at the end of 2nd hour will be 120.

$$\text{And, } a_5 = ar^4 = (30)(2)^4 = 480$$

The number of bacteria at the end of 4th hour will be 480.

$$a_{n+1} = ar^n = (30)2^n$$

Therefore, the number of bacteria at the end of n^{th} hour will be $30(2)^n$.

Question 31

What will Rs 500 amount to in 10 years after its deposit in a bank which pays an annual interest rate of 10% compounded annually?

Solution

Given,

The amount deposited in the bank is Rs 500.

At the end of first year, amount = Rs $500(1 + 1/10) = \text{Rs } 500 (1.1)$

At the end of 2nd year, amount = Rs $500 (1.1) (1.1)$

At the end of 3rd year, amount = Rs $500 (1.1) (1.1) (1.1)$ and so on....

Therefore,

The amount at the end of 10 years = Rs $500 (1.1) (1.1) \dots (10 \text{ times})$

$$= \text{Rs } 500(1.1)^{10}$$

Question 32

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Solution

Let's consider the roots of the quadratic equation to be a and b .

Then, we have

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \quad \dots(1)$$

$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \quad \dots(2)$$

We know that,

A quadratic equation can be formed as,

$$x^2 - x (\text{Sum of roots}) + (\text{Product of roots}) = 0$$

$$x^2 - x(a + b) + (ab) = 0$$

$$x^2 - 16x + 25 = 0 \text{ [Using (1) and (2)]}$$

Therefore, the required quadratic equation is $x^2 - 16x + 25 = 0$
