

Chapter 8: Binomial Theorem Exercise 8.2

NCERT · Class 11 · Maths

Questions

Question 1

 x^5 in $(x + 3)^8$

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^n C_r a^{n-r} b^r$

Here, x^5 is the T_{r+1} term, so $a = x$, $b = 3$ and $n = 8$

$$T_{r+1} = {}^8 C_r x^{8-r} 3^r \dots\dots\dots (i)$$

To find out x^5

We have to equate $x^5 = x^{8-r}$

$$\Rightarrow r = 3$$

Putting the value of r in (i), we get

$$T_{3+1} = {}^8 C_3 x^{8-3} 3^3$$

$$T_4 = \frac{8!}{3! 5!} \times x^5 \times 27$$

$$= 1512 x^5$$

Hence, the coefficient of $x^5 = 1512$

Question 2

 $a^5 b^7$ in $(a - 2b)^{12}$.

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^n C_r a^{n-r} b^r$

Here, $a = a$, $b = -2b$ & $n = 12$

Substituting the values, we get

$$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r \dots\dots\dots (i)$$

To find a^5

$$\text{Equate } a^{12-r} = a^5$$

$$r = 7$$

Putting $r = 7$ in (i)

$$T_8 = {}^{12}C_7 a^5 (-2b)^7$$

$$T_8 = \frac{12!}{7!5!} \times a^5 \times (-2)^7 b^7$$

$$= -101376 a^5 b^7$$

Hence, the coefficient of $a^5 b^7 = -101376$

Write the general term in the expansion of

Question 3

$$(x^2 - y)^6$$

Solution

The general term T_{r+1} in the binomial expansion is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r \dots\dots\dots (i)$$

Here, $a = x^2$, $n = 6$ and $b = -y$

Putting values in (i)

$$T_{r+1} = {}^6C_r x^{2(6-r)} (-1)^r y^r$$

$$\begin{aligned}
 &= \frac{6!}{r!(6-r)!} \times x^{12-2r} \times (-1)^r \times y^r \\
 &= -1^r \frac{6!}{r!(6-r)!} \times x^{12-2r} \times y^r \\
 &= -1^r {}^6C_r \cdot x^{12-2r} \cdot y^r
 \end{aligned}$$

Question 4

$(x^2 - yx)^{12}$, $x \neq 0$.

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here, $n = 12$, $a = x^2$ and $b = -yx$

Substituting the values, we get

$$\begin{aligned}
 T_{n+1} &= {}^{12}C_r \times x^{2(12-r)} (-1)^r y^r x^r \\
 &= \frac{12!}{r!(12-r)!} \times x^{24-2r} (-1)^r y^r x^r \\
 &= -1^r \frac{12!}{r!(12-r)!} x^{24-r} y^r \\
 &= -1^r {}^{12}C_r \cdot x^{24-2r} \cdot y^r
 \end{aligned}$$

Question 5

Find the 4th term in the expansion of $(x - 2y)^{12}$.

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here, $a = x$, $n = 12$, $r = 3$ and $b = -2y$

By substituting the values, we get

$$T_4 = {}^{12}C_3 x^9 (-2y)^3$$

$$\begin{aligned}
 &= \frac{12!}{3!9!} \times x^9 \times -8 \times y^3 \\
 &= -\frac{12 \times 11 \times 10 \times 8}{3 \times 2 \times 1} \times x^9 y^3 \\
 &= -1760 x^9 y^3
 \end{aligned}$$

Question 6

Find the 13th term in the expansion of

$$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$$

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here $a=9x$, $b = -\frac{1}{3\sqrt{x}}$ $n=18$ and $r = 12$

Putting values

$$\begin{aligned}
 T_{13} &= \frac{18!}{12!6!} 9x^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12} \\
 &= \frac{(18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12!)}{12! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3^{12} \times x^6 \times \frac{1}{x^6} \times \frac{1}{3^{12}} \\
 &= 18564
 \end{aligned}$$

Question 7

Find the middle terms in the expansions of

Find the middle terms in the expansions of

$$7. \left(3 - \frac{x^3}{6}\right)^7$$

Solution

Here $n = 7$ so there would be two middle terms given by

$$\left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = 4^{\text{th}} \text{ and } \left(\frac{n+1}{2} + 1\right)^{\text{th}} \text{ term} = 5^{\text{th}}$$

We have

$$a = 3, n = 7 \text{ and } b = -\frac{x^3}{6}$$

For T_4 , $r = 3$

The term will be

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$T_4 = \frac{7!}{3!} 3^4 \left(-\frac{x^3}{6}\right)^3$$

$$= -\frac{7 \times 6 \times 5 \times 4}{3 \times 2 \times 1} \times 3^4 \times \frac{x^9}{2^3 3^3}$$

$$= -\frac{105}{8} x^9$$

For T_5 term, $r = 4$

The term T_{r+1} in the binomial expansion is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$T_5 = \frac{7!}{4! 3!} 3^3 \left(-\frac{x^3}{6}\right)^4$$

$$= \frac{7 \times 6 \times 5 \times 4!}{4! 3!} \times \frac{3^3}{2^4 3^4} \times x^3 = \frac{35 x^{12}}{48}$$

Question 8

Find the middle terms in the expansions of

8. $\left(\frac{x}{3} + 9y\right)^{10}$

Solution

Here n is even so the middle term will be given by $\left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = 6^{\text{th}} \text{ term}$

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Now $a = \frac{x}{3}$, $b = 9y$, $n = 10$ and $r = 5$

Substituting the values

$$T_6 = \frac{10!}{5!5!} \times \left(\frac{x}{3}\right)^5 \times (9y)^5$$

$$T_6 = \frac{10!}{5!5!} \times \left(\frac{x}{3}\right)^5 \times (9y)^5$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{x^5}{3^5} \times 3^{10} \times y^5$$

$$= 61236 x^5 y^5$$

Question 9

In the expansion of $(1 + a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Solution

We know that the general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here, $n = m+n$, $a = 1$ and $b = a$

Substituting the values in the general form,

$$T_{r+1} = {}^{m+n}C_r 1^{m+n-r} a^r$$

$$= {}^{m+n}C_r a^r \dots \dots \dots (i)$$

Now, the general term for the expression is,

$$T_{r+1} = {}^{m+n}C_r a^r$$

Now, for the coefficient of a^m

$$T_{m+1} = {}^{m+n}C_m a^m$$

Hence, for the coefficient of a^m , the value of $r = m$

So, the coefficient is ${}^{m+n}C_m$

Similarly, the coefficient of a^n is ${}^{m+n}C_n$

$${}^{m+n}C_m = \frac{(m+n)!}{m!n!}$$

$$\text{And also, } {}^{m+n}C_n = \frac{(m+n)!}{m!n!}$$

The coefficient of a^m and a^n are same that is $\frac{(m+n)!}{m!n!}$

Question 10

The coefficients of the $(r-1)^{\text{th}}$, r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are in the ratio 1:3:5. Find n and r .

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here, the binomial is $(1+x)^n$ with $a = 1$, $b = x$ and $n = n$

The $(r+1)^{\text{th}}$ term is given by

$$T_{(r+1)} = {}^nC_r 1^{n-r} x^r$$

$$T_{(r+1)} = {}^nC_r x^r$$

The coefficient of $(r+1)^{\text{th}}$ term is ${}^nC_r$

The r^{th} term is given by $(r-1)^{\text{th}}$ term

$$T_{(r+1-1)} = {}^nC_{r-1} x^{r-1}$$

$$T_r = {}^nC_{r-1} x^{r-1}$$

∴ the coefficient of r^{th} term is ${}^nC_{r-1}$

For $(r-1)^{\text{th}}$ term, we will take $(r-2)^{\text{th}}$ term

$$T_{r-2+1} = {}^nC_{r-2} x^{r-2}$$

$$T_{r-1} = {}^nC_{r-2} x^{r-2}$$

∴ the coefficient of $(r-1)^{\text{th}}$ term is ${}^nC_{r-2}$

Given that the coefficient of $(r-1)^{\text{th}}$, r^{th} and $r+1^{\text{th}}$ term are in ratio 1:3:5

Therefore,

$$\frac{\text{the coefficient of } r-1^{\text{th}} \text{ term}}{\text{coefficient of } r^{\text{th}} \text{ term}} = \frac{1}{3}$$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3}$$

$$\Rightarrow \frac{\frac{n!}{(r-2)!(n-r+2)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{1}{3}$$

On rearranging we get

$$\frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{1}{3}$$

By multiplying

$$\Rightarrow \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)!} = \frac{1}{3}$$

$$\Rightarrow \frac{(r-1)(n-r+1)!}{(n-r+2)(n-r+1)!} = \frac{1}{3}$$

On simplifying we get

$$\Rightarrow \frac{(r-1)}{(n-r+2)} = \frac{1}{3}$$

$$\Rightarrow 3r - 3 = n - r + 2$$

$$\Rightarrow n - 4r + 5 = 0 \dots\dots\dots 1$$

Also

$$\frac{\text{the coefficient of } r^{\text{th}} \text{ term}}{\text{coefficient of } r + 1^{\text{th}} \text{ term}} = \frac{3}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{3}{5}$$

On rearranging we get

$$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{3}{5}$$

By multiplying

$$\Rightarrow \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r(n-r)!}{(n-r+1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r(n-r)!}{(n-r+1)(n-r)!} = \frac{3}{5}$$

On simplifying we get

$$\Rightarrow \frac{r}{(n-r+1)} = \frac{3}{5}$$

By cross multiplication,

$$\Rightarrow 5r = 3n - 3r + 3$$

$$\Rightarrow 8r - 3n - 3 = 0 \dots\dots\dots 2$$

We have 1 and 2 as

$$n - 4r + 5 = 0 \dots\dots\dots 1$$

$$8r - 3n - 3 = 0 \dots\dots\dots 2$$

Multiplying equation 1 by number 2,

$$2n - 8r + 10 = 0 \dots\dots\dots 3$$

Adding equations 2 and 3,

$$2n - 8r + 10 = 0$$

$$-3n - 8r - 3 = 0$$

$$\Rightarrow -n = -7$$

$$n = 7 \text{ and } r = 3$$

Question 11

Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

The general term for binomial $(1+x)^{2n}$ is

$$T_{r+1} = {}^{2n}C_r x^r \dots\dots\dots 1$$

To find the coefficient of x^n

$$r = n$$

$$T_{n+1} = {}^{2n}C_n x^n$$

The coefficient of $x^n = {}^{2n}C_n$

The general term for binomial $(1+x)^{2n-1}$ is

$$T_{r+1} = {}^{2n-1}C_r x^r$$

To find the coefficient of x^n

Putting $n = r$

$$T_{r+1} = {}^{2n-1}C_r x^n$$

The coefficient of $x^n = {}^{2n-1}C_n$

We have to prove

Coefficient of x^n in $(1+x)^{2n} = 2$ coefficient of x^n in $(1+x)^{2n-1}$

Consider LHS = ${}^{2n}C_n$

$$= \frac{2n!}{n! (2n - n)!}$$

$$= \frac{2n!}{n! (n)!}$$

Again consider RHS = $2 \times {}^{2n-1}C_n$

$$= 2 \times \frac{(2n - 1)!}{n! (2n - 1 - n)!}$$

$$= 2 \times \frac{(2n - 1)!}{n! (n - 1)!}$$

Now multiplying and dividing by n we get

$$= 2 \times \frac{(2n - 1)!}{n! (n - 1)!} \times \frac{n}{n}$$

$$= \frac{2n(2n - 1)!}{n! n(n - 1)!}$$

$$= \frac{2n!}{n! n!}$$

From above equations LHS = RHS

Hence the proof.

Question 12

Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

Solution

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

Here, $a = 1$, $b = x$ and $n = m$

Putting the value,

$$T_{r+1} = {}^m C_r 1^{m-r} x^r$$

$$= {}^m C_r x^r$$

We need the coefficient of x^2

$\therefore$ putting $r = 2$

$$T_{2+1} = {}^m C_2 x^2$$

The coefficient of $x^2 = {}^m C_2$

Given that coefficient of $x^2 = {}^m C_2 = 6$

$$\Rightarrow \frac{m!}{2!(m-2)!} = 6$$

$$\Rightarrow \frac{m(m-1)(m-2)!}{2 \times 1 \times (m-2)!} = 6$$

$$\Rightarrow m(m-1) = 12$$

$$\Rightarrow m^2 - m - 12 = 0$$

$$\Rightarrow m^2 - 4m + 3m - 12 = 0$$

$$\Rightarrow m(m-4) + 3(m-4) = 0$$

$$\Rightarrow (m+3)(m-4) = 0$$

$$\Rightarrow m = -3, 4$$

We need the positive value of m , so $m = 4$

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