

Chapter 7: Permutations and Combinations Exercise 7.4

NCERT · Class 11 · Maths

Questions

Question 1

If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$.

Solution

Given ${}^nC_8 = {}^nC_2$

We know that if ${}^nC_r = {}^nC_p$ then either $r = p$ or $r = n - p$

Here ${}^nC_8 = {}^nC_2$

$$\Rightarrow 8 = n - 2$$

On rearranging we get

$$\Rightarrow n = 10$$

Now,

$$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!} \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right)$$

$$\Rightarrow {}^{10}C_2 = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = \frac{90}{2} = 45$$

Question 2

Determine n if

(i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

(ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

Solution

(i) Given: ${}^{2n}C_3 : {}^nC_3 = 12:1$

The above equation can be written as

$$\Rightarrow \frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$$

Substituting the formula we get

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

Expanding the factorial we get

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

On simplifying

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times 2 \times (n-1)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

On multiplying we get

$$\Rightarrow \frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

Simplifying and computing,

$$\Rightarrow 4 \times (2n - 1) = 12 \times (n - 2)$$

$$\Rightarrow 8n - 4 = 12n - 24$$

$$\Rightarrow 12n - 8n = 24 - 4$$

$$\Rightarrow 4n = 20$$

$$\therefore n = 5$$

(ii) Given: ${}^{2n}C_3 : {}^nC_3 = 11:1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^nC_3} = \frac{11}{1}$$

$$\frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$$

$$\Rightarrow {}^nC_3 = 12$$

$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n!}{3!(2n-3)!} = \frac{12}{1}$$

$$\frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times 2 \times (n-1)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

$$\Rightarrow 4 \times (2n-1) = 12 \times (n-2)$$

$$\Rightarrow 8n - 4 = 12n - 24$$

$$\Rightarrow 12n - 8n = 24 - 4$$

$$\Rightarrow 4n = 20$$

$$\Rightarrow 4n - 4 = 20 - 4$$

$$\Rightarrow 4n - 4 = 16$$

$$\Rightarrow 4n - 4 = 16$$

$$\Rightarrow 4n = 20$$

$$\Rightarrow 12n - 8n = 24 - 4$$

$$\Rightarrow 4n = 20$$

$$\Rightarrow 4n = 20$$

$$\Rightarrow n = 5$$

$$\therefore n = 5$$

Question 3

How many chords can be drawn through 21 points on a circle?

Solution

Given, 21 points on a circle.

We know that we require two points on the circle to draw a chord.

∴ The number of chords is

$$\Rightarrow {}^{21}C_2 =$$

$$\frac{21!}{2!(21-2)!} = \frac{21 \times 20 \times 19!}{2! \times 19!} = \frac{21 \times 20}{2 \times 1} = \frac{420}{2} = 210$$

∴ The total number of chords that can be drawn is 210.

Question 4

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution

Given, 5 boys and 4 girls are in total.

We can select 3 boys from 5 boys in ${}^5C_3$ ways.

Similarly, we can select 3 boys from 54 girls in ${}^4C_3$ ways.

∴ The number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3$

$$\Rightarrow {}^5C_3 \times {}^4C_3 =$$

$$\frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!} = \frac{5!}{3! \times 2!} \times \frac{4!}{3! \times 1!}$$

$$\Rightarrow {}^5C_3 \times {}^4C_3 = 10 \times 4 = 40$$

∴ The number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3 = 40$ ways

Question 5

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Solution

Given, 6 red balls, 5 white balls and 5 blue balls.

We can select 3 red balls from 6 red balls in ${}^6C_3$ ways.

Similarly, we can select 3 white balls from 5 white balls in ${}^5C_3$ ways.

Similarly, we can select 3 blue balls from 5 blue balls in ${}^5C_3$ ways.

∴ The number of ways of selecting 9 balls is ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!} = \frac{6!}{3! \times 3!} \times \frac{5!}{3! \times 2!} \times \frac{5!}{3! \times 2!}$$

$$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} \times \frac{5 \times 4 \times 3!}{3! \times 2!} \times \frac{5 \times 4 \times 3!}{3! \times 2!} = \frac{120}{3 \times 2 \times 1} \times \frac{20}{2 \times 1} \times \frac{20}{2 \times 1} = 20 \times 10 \times 10 = 2000$$

∴ The number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour is ${}^6C_3 \times {}^5C_3 \times {}^5C_3 = 2000$

Question 6

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution

Given, a deck of 52 cards.

There are 4 Ace cards in a deck of 52 cards.

According to the question, we need to select 1 Ace card out of the 4 Ace cards.

∴ The number of ways to select 1 Ace from 4 Ace cards is ${}^4C_1$

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ The number of ways to select 4 cards from 48 cards is ${}^{48}C_4$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!} = \frac{4!}{1! \times 3!} \times \frac{48!}{4! \times 44!}$$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4 \times 3!}{1! \times 3!} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4! \times 44!} = \frac{4}{1} \times \frac{4669920}{24} = 4 \times 194580 = 778320$$

∴ The number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination is 778320.

Question 7

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution

Given, 17 players, in which, only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers.

There are 5 players to bowl, and we can include 4 bowlers in a team of 11.

∴ The number of ways in which bowlers can be selected is ${}^5C_4$

Now, other players left are = $17 - 5(\text{bowlers}) = 12$

Since we need 11 players in a team and 4 bowlers have already been selected, we need to select 7 more players from 12.

∴ The number of ways we can select these players is ${}^{12}C_7$

∴ The total number of combinations possible is ${}^5C_4 \times {}^{12}C_7$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!} = \frac{5!}{4! \times 1!} \times \frac{12!}{7! \times 5!}$$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5 \times 4!}{1! \times 4!} \times \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{5! \times 7!} = \frac{5}{1} \times \frac{95040}{120} = 5 \times 792 = 3960$$

∴ The number of ways we can select a team of 11 players where 4 players are bowlers from 17 players is 3960.

Question 8

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution

Given, a bag contains 5 black and 6 red balls.

The number of ways we can select 2 black balls from 5 black balls is ${}^5C_2$

The number of ways we can select 3 red balls from 6 red balls is ${}^6C_3$

The number of ways 2 black and 3 red balls can be selected is ${}^5C_2 \times {}^6C_3$

$$\therefore {}^5C_2 \times {}^6C_3 = \frac{5!}{2!(5-2)!} \times \frac{6!}{3!(6-3)!} = \frac{5!}{2! \times 3!} \times \frac{6!}{3! \times 3!}$$

$$\Rightarrow {}^5C_2 \times {}^6C_3 = \frac{5 \times 4 \times 3!}{2! \times 3!} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{20}{2} \times \frac{120}{6} = 10 \times 20 = 200$$

$\therefore$ The number of ways in which 2 black and 3 red balls can be selected from 5 black and 6 red balls is 200.

Question 9

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution

Given, 9 courses are available, and 2 specific courses are compulsory for every student.

Here, 2 courses are compulsory out of 9 courses, so a student needs to select $5 - 2 = 3$ courses

$\therefore$ The number of ways in which 3 ways can be selected from $9 - 2$ (compulsory courses) = 7 is ${}^7C_3$

$$\therefore {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \times 4!}$$

$$\Rightarrow {}^7C_3 = \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!} = \frac{210}{6} = 35$$

$\therefore$ The number of ways a student selects 5 courses from 9 courses where 2 specific courses are compulsory is 35.

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