

Chapter 7: Permutations and Combinations Exercise 7.3

NCERT · Class 11 · Maths

Questions

Question 1

How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution

Total number of digits possible for choosing = 9

Number of places for which a digit has to be taken = 3

As there is no repetition allowed,

$$\Rightarrow \text{No. of permutations} = {}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 504.$$

Question 2

How many 4-digit numbers are there with no digit repeated?

Solution

To find the four-digit number (digits do not repeat)

Now, we will have 4 places where 4 digits are to be put.

So, at thousand's place = There are 9 ways, as 0 cannot be at thousand's place = 9 ways

At hundredth's place = There are 9 digits to be filled, as 1 digit is already taken = 9 ways

At ten's place = There are now 8 digits to be filled, as 2 digits are already taken = 8 ways

At unit's place = There are 7 digits that can be filled = 7 ways

The total number of ways to fill the four places = $9 \times 9 \times 8 \times 7 = 4536$ ways

So, a total of 4536 four-digit numbers can be there with no digits repeated.

Question 3

How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Solution

An even number means that the last digit should be even.

Number of possible digits at one's place = 3 (2, 4 and 6)

⇒ Number of permutations =

$${}^3_1P = \frac{3!}{(3-1)!} = 3$$

One of the digits is taken at one's place; the number of possible digits available = 5

⇒ Number of permutations =

$${}^5_2P = \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = 20.$$

Therefore, the total number of permutations = $3 \times 20 = 60$

Question 4

Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5 if no digit is repeated. How many of these will be even?

Solution

Total number of digits possible for choosing = 5

Number of places for which a digit has to be taken = 4

As there is no repetition allowed,

⇒ Number of permutations =

$${}^5_4P = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$$

The number will be even when 2 and 4 are in one's place.

The possibility of (2, 4) at one's place = $2/5 = 0.4$

The total number of even numbers = $120 \times 0.4 = 48$

Question 5

From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman, assuming one person cannot hold more than one position?

Solution

Total number of people in committee = 8

Number of positions to be filled = 2

⇒ Number of permutations =

$${}_8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 56.$$

Question 6

Find n if ${}^{n-1}P_3 : {}^nP_3 = 1 : 9$.

Solution

Given equation can be written as

$$\frac{{}^{n-1}P_3}{{}^nP_3} = \frac{1}{9}$$

By substituting the values we get

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

On simplification

$$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow n=9.$$

$$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow n=9.$$

Question 7

Find r if

$$(i) {}^5P_r = {}^2P_{r-1}$$

$$(ii) {}^5P_r = {}^6P_{r-1}$$

Solution

$$(i) {}^5P_r = 2 {}^{6P}_{r-1}$$

Substituting the values we get

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \frac{6!}{(7-r)!}$$

The above equation can be written as

$$\Rightarrow \frac{(7-r)!}{(5-r)!} = 2 \frac{6!}{5!}$$

On simplifying we get

$$\Rightarrow (7-r)(6-r) = 2(6)$$

$$\Rightarrow 42 - 13r + r^2 = 12$$

$$\Rightarrow r^2 - 13r + 30 = 0$$

$$\Rightarrow r^2 - 10r - 3r + 30 = 0$$

$$\Rightarrow \underline{r}(r-10) - 3(r-10) = 0$$

$$\Rightarrow (r-3)(r-10) = 0$$

$$r = 3 \text{ or } r = 10$$

But $r = 10$ is rejected, as in ${}^5P_r$, r cannot be greater than 5.

Therefore, $r = 3$.

$$(ii) {}^5P_r = {}^{6-r}_1P$$

The above equation can be written as

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$$

$$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$$

$$\Rightarrow (7-r)(6-r) = 6$$

$$\Rightarrow 42 - 13r + r^2 = 6$$

$$\Rightarrow r^2 - 13r + 36 = 0$$

$$\Rightarrow r^2 - 9r - 4r + 36 = 0$$

$$\Rightarrow r(r-9) - 4(r-9) = 0$$

$$\Rightarrow (r-4)(r-9) = 0$$

$$r = 4 \text{ or } r = 9$$

But $r=9$ is rejected, as in ${}^5P_r$, r cannot be greater than 5.

Therefore, $r=4$.

Question 8

How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution

Total number of different letters in EQUATION = 8

Number of letters to be used to form a word = 8

$\Rightarrow$ Number of permutations =

$${}^8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$$

Question 9

How many words, with or without meaning, can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

(i) 4 letters are used at a time

(ii) All letters are used at a time

(iii) all letters are used, but the first letter is a vowel

Solution

(i) Number of letters to be used = 4

⇒ Number of permutations =

$${}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$$

(ii) Number of letters to be used = 6

⇒ Number of permutations =

$${}^6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$$

(iii) Number of vowels in MONDAY = 2 (O and A)

⇒ Number of permutations in vowel =

$${}^2P_1 = 2$$

Now, the remaining places = 5

Remaining letters to be used = 5

⇒ Number of permutations =

$${}^5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$$

Therefore, the total number of permutations = $2 \times 120 = 240$

Question 10

In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution

Total number of letters in MISSISSIPPI = 11

Letter number of occurrence

M	1
I	4
S	4
P	2

⇒ Number of permutations =

$$\frac{11!}{1!4!4!2!} = 34650$$

We take that 4 I's come together, and they are treated as 1 letter.

∴ Total number of letters = $11 - 4 + 1 = 8$

⇒ Number of permutations =

$$\frac{8!}{1!4!2!} = 840$$

Therefore, the total number of permutations where four I's don't come together = $34650 - 840 = 33810$

Question 11

In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) Words start with P and end with S

(ii) Vowels are all together

(iii) There are always 4 letters between P and S

Solution

(i) Total number of letters in PERMUTATIONS = 12

The only repeated letter is T; 2 times

The first and last letters of the word are fixed as P and S, respectively.

Number of letters remaining = $12 - 2 = 10$

⇒ Number of permutations =

$$\frac{{}^{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$$

(ii) Number of vowels in PERMUTATIONS = 5 (E, U, A, I, O)

Now, we consider all the vowels together as one.

Number of permutations of vowels = 120

Now total number of letters = $12 - 5 + 1 = 8$

⇒ Number of permutations =

$$\frac{{}^8P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$$

Therefore, total number of permutations = $120 \times 20160 = 2419200$

(iii) Number of places is as 1 2 3 4 5 6 7 8 9 10 11 12

There should always be 4 letters between P and S.

Possible places of P and S are 1 and 6, 2 and 7, 3 and 8, 4 and 9, 5 and 10, 6 and 11, 7 and 12

Possible ways = 7

Also, P and S can be interchanged.

No. of permutations = $2 \times 7 = 14$

The remaining 10 places can be filled with 10 remaining letters,

∴ No. of permutations =

$$\frac{{}^{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$$

Therefore, the total number of permutations = $14 \times 1814400 = 25401600$

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