

Chapter 7: Binomial Theorem

NCERT · Class 11 · Maths

Exercise 7.1

Question 1

Expand the expression $(1 - 2x)^5$.

Solution

Ans. By using Binomial Theorem, the expression $(1 - 2x)^5$ can be expanded as

$$\begin{aligned}(1 - 2x)^5 &= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 \\ &\quad - {}^5C_5(2x)^5 \\ &= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - 32x^5 \\ &= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5\end{aligned}$$

Question 2

Expand the expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$.

Solution

Ans. By using Binomial Theorem, the expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$ can be expanded as

$$\begin{aligned}\left(\frac{2}{x} - \frac{x}{2}\right)^5 &= {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)^1\left(\frac{x}{2}\right)^4 \\ &\quad - {}^5C_5\left(\frac{x}{2}\right)^5 \\ &= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32} \\ &= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}\end{aligned}$$

Question 3

Expand the expression $(2x - 3)^6$.

Solution

Ans. By using Binomial Theorem, the expression $(2x - 3)^6$ can be expanded as

$$\begin{aligned}
 (2x - 3)^6 &= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 + {}^6C_4(2x)^2(3)^4 \\
 &\quad - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6 \\
 &= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) + 15(4x^2)(81) \\
 &\quad - 6(2x)(243) + 729 \\
 &= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729
 \end{aligned}$$

Question 4

Expand the expression $\left(\frac{x}{3} + \frac{1}{x}\right)^5$.

Solution

Ans. By using Binomial Theorem, the expression $\left(\frac{x}{3} + \frac{1}{x}\right)^5$ can be expanded as

$$\begin{aligned}
 \left(\frac{x}{3} + \frac{1}{x}\right)^5 &= {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)^1\left(\frac{1}{x}\right)^4 \\
 &\quad + {}^5C_5\left(\frac{1}{x}\right)^5 \\
 &= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5} \\
 &= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}
 \end{aligned}$$

Question 5

Expand the expression $\left(x + \frac{1}{x}\right)^6$.

Solution

Ans. By using Binomial Theorem, the expression $\left(x + \frac{1}{x}\right)^6$ can be expanded as

$$\begin{aligned}
\left(x + \frac{1}{x}\right)^6 &= {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 \\
&\quad + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6 \\
&= x^6 + 6(x^5)\left(\frac{1}{x}\right) + 15(x^4)\left(\frac{1}{x^2}\right) + 20(x^3)\left(\frac{1}{x^3}\right) + 15(x^2)\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6} \\
&= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}
\end{aligned}$$

Question 6

Using Binomial Theorem, evaluate $(96)^3$.

Solution

Ans. 96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, $96 = 100 - 4$

$$\begin{aligned}
(96)^3 &= (100 - 4)^3 \\
&= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3 \\
&= 1000000 - 3(10000)(4) + 3(100)(16) - 64 \\
&= 1000000 - 120000 + 4800 - 64 \\
&= 884736
\end{aligned}$$

Question 7

Using Binomial Theorem, evaluate $(102)^5$.

Solution

Ans. 102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, $102 = 100 + 2$

$$\begin{aligned}
(102)^5 &= (100 + 2)^5 \\
&= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 \\
&\quad + {}^5C_5(2)^5 \\
&= 10000000000 + 5(100000000)(2) + 10(1000000)(4) + 10(10000)(8) \\
&\quad + 5(100)(16) + 32 \\
&= 10000000000 + 1000000000 + 40000000 + 80000 + 8000 + 32 \\
&= 11040808032
\end{aligned}$$

Question 8

Using Binomial Theorem, evaluate $(101)^4$.

Solution

Ans. 101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, $101 = 100 + 1$

$$\begin{aligned}(101)^4 &= (100 + 1)^4 \\&= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4 \\&= 100000000 + 4(1000000) + 6(10000) + 4(100) + 1 \\&= 100000000 + 4000000 + 60000 + 400 + 1 \\&= 104060401\end{aligned}$$

Question 9

Using Binomial Theorem, evaluate $(99)^5$.

Solution

Ans. 99 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, $99 = 100 - 1$

$$\begin{aligned}(99)^5 &= (100 - 1)^5 \\&= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 + {}^5C_4(100)(1)^4 \\&\quad - {}^5C_5(1)^5 \\&= 10000000000 - 5(100000000) - 10(1000000) - 10(10000) + 5(100) - 1 \\&= 10000000000 - 500000000 - 10000000 - 100000 + 500 - 1 \\&= 9509900499\end{aligned}$$

Question 10

Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Solution

Ans. By splitting 1.1 and then applying Binomial Theorem, the first few terms of $(1.1)^{10000}$ be obtained as

$$\begin{aligned}
 (1.1)^{10000} &= (1 + 0.1)^{10000} \\
 &= {}^{10000}C_0 + {}^{10000}C_1 (1.1) + \text{Other positive terms} \\
 &= 1 + 10000 \times 1.1 + \text{Other positive terms} \\
 &= 1 + 11000 + \text{Other positive terms} \\
 &> 1000
 \end{aligned}$$

Hence,

$$(1.1)^{10000} > 1000$$

Question 11

Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$.

Solution

Ans. Using Binomial Theorem, the expressions, $(a + b)^4$ and $(a - b)^4$, can be expanded as

$$\begin{aligned}
 (a + b)^4 &= {}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 b^4 \\
 (a - b)^4 &= {}^4C_0 a^4 - {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 - {}^4C_3 a b^3 + {}^4C_4 b^4
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 (a + b)^4 - (a - b)^4 &= {}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 b^4 - \\
 &\quad [{}^4C_0 a^4 - {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 - {}^4C_3 a b^3 + {}^4C_4 b^4] \\
 &= 2 ({}^4C_1 a^3 b + {}^4C_3 a b^3) \\
 &= 2 (4a^3 b + 4ab^3) \\
 &= 8ab (a^2 + b^2)
 \end{aligned}$$

By putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\begin{aligned}
 (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8 (\sqrt{3}) (\sqrt{2}) [(\sqrt{3})^2 + (\sqrt{2})^2] \\
 &= 8\sqrt{6} (3 + 2) \\
 &= 40\sqrt{6}
 \end{aligned}$$

Question 12

Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$.

Solution

Ans. Using Binomial Theorem, the expressions, $(x + 1)^6$ and $(x - 1)^6$, can be expanded as

$$\begin{aligned}(x+1)^6 &= {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 \\(x-1)^6 &= {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6\end{aligned}$$

Therefore,

$$\begin{aligned}(x+1)^6 + (x-1)^6 &= {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 \\&\quad + [{}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6] \\&= 2({}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6) \\&= 2(x^6 + 15x^4 + 15x^2 + 1)\end{aligned}$$

By putting $x = \sqrt{2}$, we obtain

$$\begin{aligned}(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 &= 2\left[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1\right] \\&= 2[8 + 15 \cdot 4 + 15 \cdot 2 + 1] \\&= 2[8 + 60 + 30 + 1] \\&= 2 \times 99 \\&= 198\end{aligned}$$

Question 13

Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Solution

Ans. In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be prove that, $9^{n+1} - 8n - 9 = 64k$, where k is some natural number.

By Binomial Theorem,

$$(1+a)^m = {}^mC_0 + {}^mC_1a + {}^mC_2a^2 + \dots + {}^mC_ma^m$$

For $a = 8$ and $m = n + 1$, we obtain

$$\begin{aligned}(1+8)^{n+1} &= {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \dots + {}^{n+1}C_{n+1}(8)^{n+1} \\9^{n+1} &= 1 + (n+1)(8) + 8^2 \left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \right] \\9^{n+1} &= 9 + 8n + 64 \left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \right] \\9^{n+1} - 8n - 9 &= 64k, \text{ where } k = {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \text{ is a natural number}\end{aligned}$$

Thus, $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Question 14

Prove that $\sum_{r=0}^n 3^r {}^nC_r = 4^n$.

Solution

Ans. By Binomial Theorem,

$$\sum_{r=0}^n {}^nC_r a^{n-r} b^r = (a+b)^n$$

By putting $b = 3$ and $a = 1$ in the above equation, we obtain

$$\begin{aligned} \sum_{r=0}^n {}^nC_r (1)^{n-r} (3)^r &= (1+3)^n \\ \sum_{r=0}^n 3^r {}^nC_r &= 4^n \end{aligned}$$

Hence proved.

Miscellaneous Exercise**Question 1**

If a and b are distinct integers, prove that $a-b$ is a factor of $a^n - b^n$, whenever n is a positive integer

(hint: $a^n = (a-b+b)^n$)

Solution

Ans: To prove to prove that $(a-b)$ is a factor of $(a^n - b^n)$, it must be proved that

$$a^n - b^n =$$

$$k(a-b)$$

, where k is some natural number

It can be written that, $a = a - b + b$

$$\begin{aligned} ((a-b) + b)^n &= {}^nC_0 (a-b)^n + {}^nC_1 (a-b)^{n-1} b + {}^nC_2 (a-b)^{n-2} b^2 + \dots + {}^nC_{n-1} (a-b) b^{n-1} + {}^nC_n b^n \\ &= (a-b)^n + {}^nC_1 (a-b)^{n-1} b + {}^nC_2 (a-b)^{n-2} b^2 + \dots + {}^nC_{n-1} (a-b) b^{n-1} + b^n \\ &= a^n - b^n = (a-b) [(a-b)^{n-1} + {}^nC_1 (a-b)^{n-2} b + {}^nC_2 (a-b)^{n-3} b^2 + \dots + {}^nC_{n-1} b^{n-1}] \\ &\Rightarrow a^n - b^n = k(a-b) \end{aligned}$$

Where $k = [(a-b)^{n-1} + {}^nC_1 (a-b)^{n-2} b + {}^nC_2 (a-b)^{n-3} b^2 + \dots + {}^nC_{n-1} b^{n-1}]$ is a natural number this shows that

$$(a-b)$$

is a factor of $(a^n - b^n)$,

Where n is a positive integer.

Question 2

Evaluate

$$(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

Solution

Ans: Firstly, the expression

$(a + b)^6 - (a - b)^6$ is simplified by using Binomial Theorem. This can be done as

$$\begin{aligned} (a + b)^6 &= {}^6C_0(a)^6 + {}^6C_1(a)^5b + {}^6C_2(a)^4b^2 + {}^6C_3(a)^3b^3 + {}^6C_4(a)^2b^4 + {}^6C_5(a)b^5 + {}^6C_6b^6 \\ &= (a)^6 + 6(a)^5b + 15(a)^4b^2 + 20(a)^3b^3 + 15(a)^2b^4 + 6ab^5 + b^6 \end{aligned}$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\begin{aligned} &(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 \\ &= 2 \left(6 \cdot (\sqrt{3})^5(\sqrt{2}) + 20 \cdot (\sqrt{3})^3(\sqrt{2})^3 + 6 \cdot (\sqrt{3})(\sqrt{2})^5 \right) \end{aligned}$$

=

$$\begin{aligned} &2 \times 198\sqrt{6} \\ &= 396\sqrt{6} \end{aligned}$$

Question 3

Find the values of $\left(a^2 + \sqrt{a^2 - 1}\right)^4 + \left(a^2 - \sqrt{a^2 - 1}\right)^4$

Solution

Ans: Firstly, the expression is simplified by using Binomial Theorem.

$$(x + y)^4 + (x - y)^4$$

This can be done as

$$\begin{aligned} (x + y)^4 &= {}^4C_0(x)^4 + {}^4C_1(x)^3y + {}^4C_2(x)^2y^2 + {}^4C_3xy^3 + {}^4C_4y^4 \\ &= (x)^4 + 4(x)^3y + 6(x)^2y^2 + 4xy^3 + y^4 \\ (x - y)^4 &= {}^4C_0(x)^4 - {}^4C_1(x)^3y + {}^4C_2(x)^2y^2 - {}^4C_3xy^3 + {}^4C_4y^4 \end{aligned}$$

$$=(x)^4 - 4(x)^3y - 6(x)^2y^2 - 4xy^3 - y^4$$

Putting $x=a^2$ and $y = \sqrt{a^2 - 1}$, We obtain

$$\begin{aligned} & \left(a^2 + \sqrt{a^2 - 1}\right)^4 + \left(a^2 - \sqrt{a^2 - 1}\right)^4 \\ &= 2 \left[(a^2)^4 + 6(a^2)^2 (\sqrt{a^2 - 1})^2 + (\sqrt{a^2 - 1})^4 \right] \\ &= 2 \left[(a^8) + 6(a^4)(a^2 - 1) + (a^2 - 1)^2 \right] \\ &= 2 [a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1] \\ &= 2 [a^8 + 6a^6 - 5a^4 - 2a^2 + 1] \\ &= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2 \end{aligned}$$

Question 4

Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Solution

$$\text{Ans: } 0.99 = 1 - 0.01$$

$$(0.99)^5 = (1 - 0.01)^5$$

$${}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) - {}^5C_2(1)^3(0.01)^2$$

(Approximately)

$$= 1 - 0.05 + 0.001$$

$$= 1.001 - 0.05$$

$$= 0.951$$

Thus, the value of $(0.99)^5$ is approximately 0.951

Question 5

Expand using Binomial Theorem $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4$, $x \neq 0$

Solution

$$\begin{aligned}
\text{Ans: } & \left(1 + \frac{x}{2} - \frac{2}{x}\right)^4 \\
& = {}^nC_0 \left(1 + \frac{x}{2}\right)^4 - {}^nC_1 \left(1 + \frac{x}{2}\right)^3 \left(\frac{2}{x}\right) - {}^nC_2 \left(1 + \frac{x}{2}\right)^2 \left(\frac{2}{x}\right)^2 - {}^nC_3 \left(1 + \frac{x}{2}\right) \left(\frac{2}{x}\right)^3 - {}^nC_4 \left(\frac{2}{x}\right)^4 \\
& = \left(1 + \frac{x}{2}\right)^4 - 4 \left(1 + \frac{x}{2}\right)^3 \left(\frac{2}{x}\right) + 6 \left(1 + \frac{x}{2}\right)^2 \left(\frac{4}{x^2}\right) - 4 \left(1 + \frac{x}{2}\right) \left(\frac{8}{x^3}\right) + \left(\frac{16}{x^4}\right) \\
& = \left(1 + \frac{x}{2}\right)^4 - \left(1 + \frac{x}{2}\right)^3 \left(\frac{8}{x}\right) + \left(\frac{8}{x^2}\right) + \frac{24}{x} + 6 - \left(\frac{32}{x^3}\right) + \left(\frac{16}{x^4}\right) \\
& \dots(1)
\end{aligned}$$

Again, by using the Binomial Theorem, we obtain

$$\begin{aligned}
& \left(1 + \frac{x}{2}\right)^4 = {}^4C_0(1)^4 + {}^4C_1(1)^3 \left(\frac{x}{2}\right) + {}^4C_2(1)^2 \left(\frac{x}{2}\right)^2 + {}^4C_3(1) \left(\frac{x}{2}\right)^3 + {}^4C_4 \left(\frac{x}{2}\right)^4 \\
& = 1 + 4 \times \frac{x}{2} + 6 \times \frac{x^2}{4} + 4 \times \frac{x^3}{8} + \frac{x^4}{16} \\
& = 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} \dots(2) \\
& \left(1 + \frac{x}{2}\right)^3 = {}^3C_0(1)^3 + {}^3C_1(1)^2 \left(\frac{x}{2}\right) + {}^3C_2(1) \left(\frac{x}{2}\right)^2 + {}^3C_3 \left(\frac{x}{2}\right)^3 \\
& = 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8} + \frac{x^3}{8} \dots\dots (3)
\end{aligned}$$

From (1), (2), and (3) we obtain

$$\begin{aligned}
& \left(\left(1 + \frac{x}{2}\right) - \frac{2}{x}\right)^4 \\
& = 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - \left(\frac{8}{x}\right) \left(1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8}\right) + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \\
& = 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - \frac{8}{x} - 12 - 6x - x^2 - \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \\
& = \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5.
\end{aligned}$$

Question 6

Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

Solution

Ans: Using the Binomial Theorem, the given expression

$(3x^2 - 2ax + 3a^2)^3$ Can be expanded as

$$\begin{aligned}
 & (3x^2 - 2ax + 3a^2)^3 \\
 &= {}^3C_0(3x^2 - 2ax)^3 - {}^3C_1(3x^2 - 2ax)^2(3a^2) + {}^3C_2(3x^2 - 2ax)(3a^2)^2 - {}^3C_3(3a^2)^3 \\
 &= (3x^2 - 2ax)^3 + 3(9x^4 - 12ax^3 + 4a^2x^2)(3a^2) + 3(3x^2 - 2ax)(9a^4) + (2a^6) \\
 &= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6 \\
 &= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \dots (1)
 \end{aligned}$$

Again, by using the Binomial Theorem, we obtain

$$\begin{aligned}
 & (3x^2 - 2ax)^3 \\
 &= {}^3C_0(3x^2)^3 - {}^3C_1(3x^2)^2(2ax) + {}^3C_2(3x^2)(2ax)^2 - {}^3C_3(2ax)^3 \\
 &= (27x^6) - 3(9x^4)(2ax) + 3(3x^2)(4a^2x^2) - 8a^3x^3 \\
 &= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3
 \end{aligned}$$

..... (2)

From (1) and (2), we obtain

$$\begin{aligned}
 & (3x^2 - 2ax + 3a^2)^3 \\
 &= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \\
 &= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6
 \end{aligned}$$

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