

Chapter 6: Permutations and Combinations

NCERT · Class 11 · Maths

Exercise 6.1

Question 1

How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

(i) Repetition of the digits is allowed?

(ii) Repetition of the digits is not allowed?

Solution

Ans: There will be however many ways as there are methods of filling 3 empty spots in progression by the given five digits. In this case, repetition of digits is allowed.

Therefore, the units spot can be filled in by any of the given five digits.

Similarity, tens and hundred digits can be filled in by any of the given five digits.

Hence, by the multiplication principle, the number of ways in which three-digit numbers can be formed from the given digits is $5 \times 5 \times 5 = 125$.

Ans: In this case, repetition of digits is not allowed. Here, on the off chance that unit place is filled in first, it very well may be filled by any of the given five digits.

Therefore, the number of ways of filling the units spot of the three-digit number is 5.

Then, at that point, the tens spot can be filled with any of the leftover four digits and the hundreds spot can be filled with any of the excess three digits. Thus, by the multiplication principle, the number of ways in which three-digit numbers can be formed without repeating the given digits is $5 \times 4 \times 3 = 60$.

Question 2

How many 3-digit even numbers can be formed the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Solution

Ans: There will be as many ways as there are ways of filling 3 vacant places in succession by the given six digits.

For this situation, the units spot can be filled by 2 or 4 or 6 just i.e., the units spot can be filled in 3 ways.

The tens spot can be filled by any of the 6 digits in 6 distinctive manners and furthermore the hundreds spot can be filled by any of the 6 digits in 6 diverse manners, as the digits can be repeated. Thus, by multiplication principle, the required number of three digit even numbers is $3 \times 6 \times 6 = 108$.

Question 3

How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Solution

Ans: There are as many codes as there are ways of filling 4 vacant places in succession by the first 10 letters of the English alphabet, keeping in mind that the repetition of letters is not allowed.

The primary spot can be filled in 10 distinctive manners by any of the initial 10 letters of the English letters in order following which, the runner up can be filled in by any of the leftover letters in 9 diverse manners. The third spot can be filled in by any of the excess 8 letters in 8 diverse manners and the fourth spot can be filled in by any of the leftover 7 letters in 7 distinctive manners.

Therefore, by multiplication principle, the required numbers of ways in which 4 empty places can be filled is $10 \times 9 \times 8 \times 7 = 5040$.

Hence, 5040 four-letter codes can be shaped utilizing the initial 10 letters of the English letters in order, if no letter is repeated.

Question 4

How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Solution

Ans: Since, it is given that the 5-digit telephone numbers always start with 67.

Therefore, there will be as many phone numbers as there are methods of filling 3 empty spots 6, 7, __, __, __ by the digits 0 – 9, remembering that the digits cannot be repeated.

In this way, the units spot can be filled in 8 diverse manners following which, the tens spot can be filled in by any of the leftover 7 digits in 7 distinctive manners, and the hundreds spot can be filled in by any of the excess 6 digits in 6 distinctive manners. Therefore, by multiplication principle, the required number of ways in which 5-digit telephone numbers can be constructed is $8 \times 7 \times 6 = 336$.

Question 5

A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Solution

Ans: When a coin is tossed once, the number of outcomes is 2 (Head and tail) i.e., in each throw, the number of ways of showing a different face is 2.

Thus, by multiplication principle, the required number of possible outcomes is $2 \times 2 \times 2 = 8$.

Question 6

Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Solution

Ans: Since, each signal requires the use of 2 flags.

There will be however many flags as there are methods of filling in 2 empty spots in progression by the given 5 flags of various colours.

The upper empty spot can be filled in 5 distinctive manners by any of the 5 flags following which, the lower empty spot can be filled in 4 diverse manners by any of the leftover 4 unique flags.

Thus, by multiplication principle, the number of different signals that can be generated is $5 \times 4 = 20$.

Exercise 6.2

Question 1

Evaluate.

(i) $8!$

(ii) $4! - 3!$

Solution

Ans: A number's factorial is the function that multiplies it by each natural number below it. The product of the first n natural numbers is n factorial, which is written as $n!$ and is given by, $n! = n \times (n - 1)!$.

Now, the given expression is $8!$.

According to the definition of the factorial,

$$8! = 8 \times (8 - 1)!$$

$$8! = 8 \times 7!$$

This is further evaluated as,

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$8! = 40320$$

Therefore, the value of $8!$ is found to be 40320.

Ans: The product of the first n natural numbers is n factorial, which is written as $n!$ and is given by, $n! = n \times (n - 1)!$.

Now, the given expression is $4! - 3!$.

First, consider and evaluate $4!$.

According to the definition of the factorial,

$$4! = 4 \times (4 - 1)!$$

$$4! = 4 \times 3!$$

This is further evaluated as,

$$4! = 4 \times 3 \times 2 \times 1$$

$$4! = 24$$

Thus, the value of $4!$ is found to be 24.

Now, consider and evaluate for $3!$.

According to the definition of the factorial,

$$3! = 3 \times (3 - 1)!$$

$$3! = 3 \times 2!$$

This is further evaluated as,

$$3! = 3 \times 2 \times 1$$

$$3! = 6$$

Thus, the value of $3!$ is found to be 6.

Finally, consider the expression, $4! - 3!$ and evaluate.

$$4! - 3! = 24 - 6$$

$$4! - 3! = 18$$

Therefore, the value of the expression on evaluation is found to be 18.

Question 2

Is $4! + 3! = 7!$?

Solution

Ans: The product of the first n natural numbers is n factorial, which is written as $n!$ and is given by,

$$n! = n \times (n - 1)!.$$

First, consider and evaluate $4!$.

According to the definition of the factorial,

$$4! = 4 \times (4 - 1)!$$

$$4! = 4 \times 3!$$

This is further evaluated as,

$$4! = 4 \times 3 \times 2 \times 1$$

$$4! = 24$$

Thus, the value of $4!$ is found to be 24.

Now, consider and evaluate for $3!$.

According to the definition of the factorial,

$$3! = 3 \times (3 - 1)!$$

$$3! = 3 \times 2!$$

This is further evaluated as,

$$3! = 3 \times 2 \times 1$$

$$3! = 6$$

Thus, the value of $3!$ is found to be 6.

Now, consider the expression, $4! + 3!$ and evaluate.

$$4! + 3! = 24 + 6$$

$$4! + 3! = 30$$

Further, consider $7!$.

According to the definition of the factorial,

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$7! = 5040$$

Therefore, it is observed that $4! + 3! \neq 7!$.

Question 3

Compute $\frac{8!}{6! \times 2!}$.

Solution

Ans: The product of the first n natural numbers is n factorial, which is written as $n!$ and is given by,
 $n! = n \times (n - 1)!$.

First consider the expression in the numerator, that is, $8!$.

According to the definition of the factorial,

$$8! = 8 \times (8 - 1)!$$

$$8! = 8 \times 7!$$

This is further evaluated as,

$$8! = 8 \times 7 \times 6!$$

Now, consider the expression in the denominator, that is, $6! \times 2!$.

This expression can also be written in a way that is, $6! \times 2 \times 1$.

Consider the expression that is to be determined and evaluated.

$$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$$

$$\frac{8!}{6! \times 2!} = \frac{8 \times 7}{2}$$

$$\frac{8!}{6! \times 2!} = 28$$

Therefore, the value of the expression given on evaluation is found to be 28.

Question 4

$$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$$

Solution

Ans: The product of the first n natural numbers is n factorial, which is written as $n!$ and is given by,
 $n! = n \times (n - 1)!$.

The given expression is $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$.

Consider making the denominator of the left-hand side of the given expression equal.

$$\frac{1}{6!} + \frac{1}{7 \times 6!} = \frac{x}{8!}$$

Take out the common terms aside,

$$\frac{1}{6!} \left(1 + \frac{1}{7} \right) = \frac{x}{8!}$$

$$\frac{1}{6!} \left(\frac{8}{7} \right) = \frac{x}{8!}$$

Now, rearrange the above equation and solve for x .

$$x = \frac{8!}{6!} \left(\frac{8}{7} \right)$$

$$x = \frac{8 \times 8 \times 7 \times 6!}{7 \times 6!}$$

$$x = 8 \times 8$$

$$x = 64$$

Therefore, the value of x of the expression on evaluation is found to be 64.

Question 5

Evaluate $\frac{n!}{(n-r)!}$, when

(i) $n = 6, r = 2$

(ii) $n = 9, r = 5$

Solution

Ans: The expression given is $\frac{n!}{(n-r)!}$.

Substitute 6 for n and 2 for r in the given expression, the expression obtained is,

$$\frac{6!}{(6-2)!}$$

On evaluating the denominator, the expression becomes, $\frac{6!}{4!}$.

Now, consider the expression in the numerator, that is $6!$.

According to the definition of the factorial,

$$6! = 6 \times (6 - 1)!$$

$$6! = 6 \times 5!$$

$$6! = 6 \times 5 \times 4!$$

Further, consider the expression $\frac{6!}{4!}$.

$$\frac{6!}{4!} = \frac{6 \times 5 \times 4!}{4!}$$

$$\frac{6!}{4!} = 6 \times 5$$

$$\frac{6!}{4!} = 30$$

Therefore, the value of the expression $\frac{n!}{(n-r)!}$, when $n = 6, r = 2$ is found to be 30.

Ans: The expression given is $\frac{n!}{(n-r)!}$.

Substitute 9 for n and 5 for r in the given expression, the expression obtained is,

$$\frac{9!}{(9-5)!}$$

On evaluating the denominator, the expression becomes, $\frac{9!}{4!}$.

Now, consider the expression in the numerator, that is $9!$.

According to the definition of the factorial,

$$9! = 9 \times (9 - 1)!$$

$$9! = 9 \times 8!$$

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times$$

Further, consider the expression $\frac{9!}{4!}$.

$$\frac{9!}{4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!}$$

$$\frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5$$

$$\frac{9!}{4!} = 15120$$

Therefore, the value of the expression $\frac{n!}{(n-r)!}$, when $n = 9, r = 5$ is found to be 15120.

Exercise 6.3

Question 1

How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution

Ans:

We have to form a 3-digit numbers using digits 1 – 9. That means, the order of the digits matter.

Hence, there will be as many 3-digit numbers as there are permutations of 9 digits taken 3 at a time.

So, required number of 3-digit numbers

$$= {}^9P_3$$

$$\Rightarrow \frac{9!}{(9-3)!} = \frac{9!}{6!}$$

$$\Rightarrow \frac{9 \times 8 \times 7 \times 6!}{6!}$$

$$\Rightarrow 9 \times 8 \times 7 = 504$$

Hence, there are 504 3-digit numbers formed.

Question 2

How many 4-digit numbers are there with no digit repeated?

Solution

Ans:

The thousands place of the 4-digit number is to be filled with any of the digits from 1 to 9 as the digit 0 cannot be included. Therefore, the number of ways in which thousands place can be filled is 9. The hundreds, tens, and units place can be filled by any of the digits from 0 to 9. However, the digits cannot be repeated in the 4-digit numbers and thousands place is already occupied with a digit. The hundreds, tens, and units place is to be filled by the remaining 9-digits. Therefore, there will be as many such 3-digit numbers as there are permutations of 9 different digits taken 3 at a time.

Number of such 3-digit numbers = ${}^9P_3$

$$\Rightarrow \frac{9!}{(9-3)!} = \frac{9!}{6!}$$

$$\Rightarrow \frac{9 \times 8 \times 7 \times 6!}{6!}$$

$$\Rightarrow 9 \times 8 \times 7 = 504$$

Thus, by multiplication principle, the required number of 4-digit numbers are $9 \times 504 = 4536$.

Question 3

How many 3-digit even numbers can be made using the digits

1, 2, 3, 4, 6, 7,

if no digit is repeated?

Solution

Ans:

We have to form 3-digit even numbers using the given six digits,

1, 2, 3, 4, 6,

and 7, without repeating the digits. Then, units can be filled in 3 ways by any of the digits, 2, 4 or 6. Since the digits cannot be repeated in the 3-digit numbers and units place is already occupied with a

digit (which is even), the hundreds and tens place is to be filled by the remaining 5-digits. Therefore, the number of ways in which hundreds and tens place can be filled with the remaining 5-digits is the permutation of 5 different digits taken 2 at a time.

So, the number of ways of filling hundreds and tens place

$$\Rightarrow {}^5P_2$$

$$\Rightarrow \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

$$\Rightarrow \frac{5 \times 4 \times 3!}{3!}$$

$$\Rightarrow 5 \times 4 = 20$$

Thus, by multiplication principle, the required number of 3-digit numbers are $3 \times 20 = 60$.

Question 4

Find the number of 4-digit numbers that can be formed using the digits

1, 2, 3, 4, 5

if no digit is repeated. How many of these will be even?

Solution

Ans:

We have to form 4-digit numbers using the digits, 1, 2, 3, 4, and 5.

So, there will be as many 4-digit numbers as there are permutations of 5 different digits taken 4 at a time.

That is, required number of 4-digit numbers

$$\Rightarrow {}^5P_4$$

$$\Rightarrow \frac{5!}{(5-4)!} = \frac{5!}{1!}$$

$$\Rightarrow 1 \times 2 \times 3 \times 4 \times 5$$

$$\Rightarrow 120$$

Among the 4-digit numbers formed by using the digits, 1, 2, 3, 4, 5 even numbers end with either 2 or 4. The number of ways in which units place is filled with digits is 2. Since the digits are not repeated and the units place is already occupied with a digit (which is even), the remaining places are to be filled by the remaining 4-digits. Therefore, the number of ways in which the remaining places can be filled is the permutation of 4 different digits taken 3 at a time.

Number of ways of filling the remaining places

$$\Rightarrow {}^4P_3 = \frac{4!}{(4-3)!}$$

$$\Rightarrow 4!$$

$$\Rightarrow 4 \times 3 \times 2 \times 1 = 24$$

Thus, by multiplication principle, the required number of even numbers is $24 \times 2 = 48$.

Question 5

From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Solution

Ans:

We have to choose a chairman and a vice chairman from a committee of 8 persons in such a way that one person cannot hold more than one position. Here, the number of ways of choosing a chairman and a vice chairman is the permutation of 8 different objects taken 2 at a time.

Thus, required number of ways

$$\Rightarrow {}^8P_2 = \frac{8!}{(8-2)!}$$

$$\Rightarrow \frac{8 \times 7 \times 6!}{6!} = 8 \times 7$$

$$\Rightarrow 56$$

Question 6

Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.

Solution

Ans:

Here,

$${}^{n-1}P_3 : {}^nP_4 = 1 : 9$$

$$\Rightarrow \frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{9}$$

$$\Rightarrow \frac{\left[\frac{(n-1)!}{(n-1-3)!} \right]}{\left[\frac{n!}{(n-4)!} \right]} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

Hence, $n = 9$.**Question 7**Find r if:

$$(i). {}^5P_r = 2^6 P_{r-1}$$

$$(ii). {}^5P_r = {}^6P_{r-1}$$

Solution

Ans:

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{2 \times 6 \times 5!}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow (7-r)(6-r) = 12$$

$$\Rightarrow 42 - 6r - 7r + r^2 = 12$$

$$\Rightarrow r(r-3) - 10(r-3) = 0$$

$$\Rightarrow r = 3 \text{ or } r = 10$$

It is known that ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \leq r \leq n$.

$$\Rightarrow 0 \leq r \leq 5$$

Hence, $r = 3$.

Ans:

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6 \times 5!}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow (7-r)(6-r) = 6$$

$$\Rightarrow 42 - 6r - 7r + r^2 = 6$$

$$\Rightarrow r(r-4) - 9(r-4) = 0$$

It is known that ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \leq r \leq n$.

$$\Rightarrow 0 \leq r \leq 5$$

Hence, $r = 4$.

Question 8

How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution

Ans:

There are 8 different letters in the word EQUATION. Therefore, the number of words that can be formed using all the letters of the word EQUATION, using each letter exactly once, is the number of permutations of 8 different objects taken 8 at a time, which is ${}^8P_8 = 8!$.

Thus, required number of words that can be formed is

$$8! = 40320$$

Question 9

How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

- (i). 4 letters are used at a time,
- (ii). all letters are used at a time,
- (iii). all letters are used but first letter is a vowel.

Solution

Ans:

There are 6 different letters in the word MONDAY.

Number of 4-letter words that can be formed from the letters of the word MONDAY, without repetition of letters, is the number of permutations of 6 different objects taken 4 at a time, which is ${}^6P_4$.

Thus, required number of words that can be formed using 4 letters at a time are ${}^6P_4 = \frac{6!}{(6-4)!}$

$$\Rightarrow \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 6 \times 5 \times 4 \times 3$$

$$\Rightarrow 360$$

Ans:

Number of words that can be formed by using all the letters of the word MONDAY at a time is the number of permutations of 6 different objects taken 6 at a time, which is ${}^6P_6 = 6!$.

Thus, required number of words that can be formed when all letters are used at a time

$$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6.$$

$$\Rightarrow 720$$

Ans:

In the given word, there are 2 different vowels, which have to occupy the rightmost place of the words formed. This can be done only in 2 ways. Since the letters cannot be repeated and the rightmost place is already occupied with a letter (which is a vowel), the remaining five places are to be filled by the remaining 5 letters. This can be done in

$$5!$$

ways.

Thus, in this case, required number of words that can be formed is

$$5! \times 2 = 120 \times 2$$

$$\Rightarrow 240$$

Question 10

In how many of the distinct permutations of the letters in MISSISSIPP do the four I's not come together?

Solution

Ans:

In the given word MISSISSIPPI, I appear 4 times, S appears 4 times, P appears 2 times, and M appears just once. Therefore, number of distinct permutations of the letters in the given word

$$\Rightarrow \frac{11!}{4!4!2!}$$

$$\Rightarrow \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$$\Rightarrow \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$$\Rightarrow 34650$$

There are 4 I's in the given word. When they occur together, they are treated as single object [IIII] for the time being. This single object together with the remaining 7 objects will account for 8 objects.

These 8 objects in which there are 4 S's, and 2 P's can be arranged in $\frac{8!}{4!2!}$ ways that means, 840 ways.

Number of arrangements where all I's occurred together is in 840 ways.

Thus, number of distinct permutations of the letters in MISSISSIPPI in which four IS do not come together

$$34650 - 840 = 33810$$

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Question 11

In how many ways can the letters of the word PERMUTATIONS be arranged if the

- (i). Words start with P and end with S,
- (ii). Vowels are all together,
- (iii). There are always 4 letters between P and S?

Solution

Ans:

In the word PERMUTATIONS, there are 2 T's, and all the other letters appear only once.

If P and S are fixed at the extreme ends (P at the left end and S at the right end), then 10 letters are left.

Hence, in this case, required number of arrangements

$$\frac{10!}{2!} = 1814400.$$

Ans:

There are 5 vowels in the given word, each appearing only once.

Since they have to always occur together, they are treated as a single object for the time being. This single object together with the remaining 7 objects will account for 8 objects, in total.

These 8 objects in which there are 2 T's can be arranged in $\frac{8!}{2!}$ ways.

Corresponding to each of these arrangements, the 5 different vowels can be arranged in $5!$ ways.

Therefore, by multiplication principle, required number of arrangements in this case

$$\frac{8!}{2!} \times 5! = 2419200.$$

Ans:

The letters have to be arranged in such a way that there are always 4 letters between P and S.

Therefore, in a way, the places of P and S are fixed. The remaining 10 letters in which there are 2 T's can be arranged in $\frac{10!}{2!}$ ways.

Since, Possible places of P & S are 1 & 6, 2 & 7, 3 & 8, 4 & 9, 5 & 10, 6 & 11, 7 & 12. Then there can be 4 letters between P & S. Also P & S can be interchanged as it won't effect the number of letters between them.

So, the letters P and S can be placed such that there are 4 letters between them in $2 \times 7 = 14$ ways. Therefore, by multiplication principle, required number of arrangements in this case

$$\frac{10!}{2!} \times 14 = 25401600.$$

Exercise 6.4

Question 1

If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$.

Solution

Ans: Since, we know that

$$\begin{aligned} {}^nC_a &= {}^nC_b \\ \Rightarrow a &= b \end{aligned}$$

Or,

$$n = a + b$$

Here $n=8+2=10$

Therefore,

$${}^{10}C_2 = \frac{10!}{2!(10-2)!}$$

$$\Rightarrow \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!}$$

Hence, ${}^nC_2 = 45$.

Question 2

Determine n if

(i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

(ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

Solution

$$\text{Ans: } \frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$$

$$\begin{aligned} &\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{12}{1} \\ &\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12 \end{aligned}$$

Therefore,

$$\begin{aligned} &\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12 \\ &\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12 \\ &\Rightarrow 2n-1 = 3(n-2) \\ &\Rightarrow 3n-2n = -1+6 \end{aligned}$$

Hence, $n = 5$.

Ans:

$$\begin{aligned} &\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1} \\ &\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11 \\ &\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11 \end{aligned}$$

Therefore,

$$\begin{aligned} &\Rightarrow \frac{(2n-1)}{(n-2)} = \frac{11}{4} \\ &\Rightarrow 8n-4 = 11n-22 \\ &\Rightarrow 3n = 18 \end{aligned}$$

Hence, $n = 6$.

Question 3

How many chords can be drawn through 21 points on a circle?

Solution

Ans: For drawing one chord a circle, only 2 points are required.

To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.

Therefore, the chords can be drawn through 21 points taken 2 as equal to each chord.

Thus, required number of chords

$$\begin{aligned} {}^{21}C_2 &= \frac{21!}{2!(21-2)!} \\ \Rightarrow \frac{21!}{2!19!} &= \frac{21 \times 20}{2} \\ \Rightarrow 210 \end{aligned}$$

Question 4

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution

Ans: The team which is selected is of 3 boys and 3 girls from 5 boys and 4 girls.

The team of 3 boys can be selected from 5 boys in ${}^5C_3$ different ways.

The team of 3 girls can be selected from 4 girls in ${}^4C_3$ different ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected

$$\Rightarrow {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$$

$$\Rightarrow \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{4 \times 3!}{3!} = 10 \times 4$$

$$\Rightarrow 40$$

Question 5

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls each colour.

Solution

Ans: The number of balls from which we have to select 6 red balls, 5 white balls, and 4 blue balls is 9 balls. These balls have to be selected in such a manner that each selection of balls consists of 9 balls from each of the colour.

Here, given we have :

3 balls which can be opted from the 6 red balls in ${}^6C_3$ different ways.

3 balls which can be opted from the 5 white balls in ${}^5C_3$ different ways.

3 balls which can be opted from the 5 blue balls in ${}^5C_3$ different ways.

Thus, after applying the multiplication principle, required number of ways of selecting the 9 balls will be –

$$\begin{aligned}\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 &= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!} \\ \Rightarrow \frac{6 \times 5 \times 4 \times 3!}{3!3 \times 2} \times \frac{5 \times 4 \times 3!}{3!2 \times 1} \times \frac{5 \times 4 \times 3!}{3!2 \times 1} \\ \Rightarrow 20 \times 10 \times 10 &= 2000\end{aligned}$$

Question 6

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution

Ans: We have a deck of 52 cards, which contains 4 aces. When we make a combination, 5 cards should be made in such a manner that there is exactly one ace.

Now, 1 ace can be opted in ${}^4C_1$ different ways and the cards which are left, out of them 4 cards can be opted out of the 48 cards in ${}^{48}C_4$ different ways.

Therefore, after applying multiplication principle, the required number of 5 card combinations will be –

$$\begin{aligned}\Rightarrow {}^{48}C_4 \times {}^4C_1 &= \frac{48!}{4!44!} \times \frac{4!}{1!3!} \\ \Rightarrow \frac{48 \times 47 \times 46 \times 45 \times 44!}{44!4! \times 3 \times 2 \times 1} \times 4! \\ \Rightarrow \frac{48 \times 47 \times 46 \times 45}{3 \times 2 \times 1} \\ \Rightarrow 778320\end{aligned}$$

Question 7

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution

Ans: The number of players out of which we have to select is 17 players and only 5 players from them are bowlers.

A cricket team of 11 players need to be opted in such a way that there remains exactly 4 players which are bowlers.

4 bowlers can be opted in ${}^5C_4$ different ways whereas out of the remaining 7 players can be opted out from the 12 players in ${}^{12}C_7$ different ways.

Therefore, after applying multiplication principle, the required number of ways for selecting the cricket team will be –

$${}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!}$$

$$\Rightarrow 5! \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5! \times 4 \times 3 \times 2 \times 1} = 3960$$

Question 8

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution

Ans: There are 5 black and 6 red balls in the bag. 2 black balls can be selected out of 5 black balls in ${}^5C_2$ ways and 3 red balls can be selected out of 6 red balls in ${}^6C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls

$$\begin{aligned} {}^5C_2 \times {}^6C_3 &= \frac{5!}{2!3!} \times \frac{6!}{3!3!} \\ &\Rightarrow \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 \\ &\Rightarrow 200 \end{aligned}$$

Question 9

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution

Ans: There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme is

$$\begin{aligned} {}^7C_3 &= \frac{7!}{3!4!} \\ &\Rightarrow \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35 \end{aligned}$$

NCERT Solution Class 11 Maths of Chapter 1 All Exercises

Exercise	Number of Questions
Exercise 6.1	6 Questions and Solutions
Exercise 6.2	5 Questions and Solutions
Exercise 6.3	11 Questions and Solutions
Miscellaneous Exercise	11 Questions and Solutions

Miscellaneous Exercise

Question 1

How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Solution

Ans: There are 3 vowels i.e. A, U and E and 5 consonants i.e. D, G, H, T and R in the given word.

Therefore, the number of ways of selecting 2 vowels out of 3 = ${}^3C_2 = 3$

Number of ways of selecting 3 consonants out of 5 = ${}^5C_3 = 10$

Thus, by multiplication principle, the number of combinations = $3 \times 10 = 30$.

Each of these combinations can be arranged in 5! ways.

Hence, the number of different words = $30 \times 5! = 3600$.

Question 2

How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that vowels and consonants occur together?

Solution

Ans: There are 5 vowels i.e. A, E, I, O and U and 3 consonants i.e. Q, T and N.

Since, vowels and consonants occur together, both (AEIOU) and (QTN) can be considered as single objects.

Thus, there are 5! Permutations of 5 vowels taken all at a time and 3! permutations of 3 consonants taken all at a time.

Therefore, by multiplication principle, the number of words = $2! \times 5! \times 3! = 1440$.

Question 3

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

(i) exactly 3 girls? (ii) at least 3 girls? (iii) at most 3 girls?

Solution

Ans: (i) Out of 9 boys and 4 girls, a committee of 7 has to be formed.

Given: exactly 3 girls should be there in a committee, hence, the number of boys = $(7 - 3) = 4$ boys only.

Therefore, many ways

$$\begin{aligned} {}^4C_3 \times {}^9C_4 &= \frac{4!}{3!1!} \times \frac{9!}{4!5!} \\ &= 4 \times \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 5!} \\ &= 504 \end{aligned}$$

(ii) Given: at least 3 girls are required in each committee. This can be done in 2 ways

(a) 3 girls and 4 boys or (b) 4 girls and 3 boys

3 girls and 4 boys can be selected in ${}^4C_3 \times {}^9C_4$ ways.

4 girls and 3 boys can be selected in ${}^4C_4 \times {}^9C_3$ ways.

Thus, the number of ways =

$$\begin{aligned} &{}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3 \\ &= 504 + 84 \\ &= 588 \end{aligned}$$

(iii) Given: at most 3 girls in every committee. This can be done in 4 ways

(a) 3 girls and 4 boys (b) 2 girls and 5 boys.

(c) 1 girl and 6 boys (d) No girl and 7 boys

3 girls and 4 boys can be selected in ${}^4C_3 \times {}^9C_4$ ways

2 girls and 5 boys can be selected in ${}^4C_2 \times {}^9C_5$ ways

1 girl and 6 boys can be selected in ${}^4C_1 \times {}^9C_6$ ways

No girl and 7 boys can be selected in ${}^4C_0 \times {}^9C_7$ ways.

Thus, many ways

$$\begin{aligned}
 &= {}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_5 + {}^4C_1 \times {}^9C_6 + {}^4C_0 \times {}^9C_7 \\
 &= \frac{4!}{3!1!} \times \frac{9!}{4!5!} + \frac{4!}{2!2!} \times \frac{9!}{5!4!} + \frac{4!}{1!3!} \times \frac{9!}{6!3!} + \frac{4!}{0!4!} \times \frac{9!}{7!2!} \\
 &= 504 + 756 + 336 + 36 \\
 &= 1632
 \end{aligned}$$

Question 4

If the different permutations of all the letters of the word EXAMINATION are listed as in a dictionary, how many words are there in the list before the first word starting with E?

Solution

Ans: There are a total of 11 letters out of which A, I and N appear 2 times and other letters appear only once.

The words starting with A will be the words listed before the words starting with E.

Thus, words starting with the letter A will have the letter A fixed at its extreme left end.

And remaining 10 letters are rearranged all at a time.

In the remaining 10 letters, there are 2 I's and 2 N's.

$$\text{Number of words starting with A} = \frac{10!}{2!2!} = 907200$$

Therefore, the required number of words is 907200.

Question 5

How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Solution

Ans: A number can be divisible by 10 only if its unit digit is 0.

Hence, 0 is fixed at the unit place.

Therefore, the 5 vacant places can be filled by the remaining 5 digits.

These 5 empty places can be filled in $5!$ ways.

Therefore, number of 6-digit numbers = $5! = 120$.

Question 6

The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Solution

Ans: Given: 2 vowels and 2 consonants should be selected from the English alphabet.

We know that there are 5 vowels.

Thus, the number of ways of selecting 2 vowels out of 5 =

$${}^5C_2 = \frac{5!}{2!3!} = 10$$

Also, we know that there are 21 consonants.

Thus, the number of ways of selecting 2 consonants out of 21 =

$${}^{21}C_2 = \frac{21!}{2!19!} = 210$$

Thus, the total number of combinations of selecting 2 vowels and 2 consonants is

$$= 10 \times 210 = 2100.$$

Every 2100 combination consists of 4 letters, which can be arranged in $4!$ ways.

$$\text{Therefore, number of words} = 2100 \times 4! = 50400$$

Question 7

In an examination, a paper consists of 12 Questions divided into 2 parts i.e., Part I and Part II, containing 5 and 7 Questions, respectively. A student is required to attempt 8 Questions in all, selecting at least 3 from each part. In how many ways can a student select the Questions?

Solution

Ans: Given: 12 Qs are divided into 2 parts – Part I and Part II consisting of 5 and 7 Qs respectively. A student must attempt 8 Qs with at least 3 from each part. This can be done as:

1. 3 Qs from part I and 5 Qs from part II.
2. 4 Qs from part I and 4 Qs from part II.
3. 5 Qs from part I and 3 Qs from part II.

The first case can be selected in ${}^5C_3 \times {}^7C_5$ ways.

The second case can be selected in ${}^5C_4 \times {}^7C_4$ ways.

The third case can be selected in ${}^5C_5 \times {}^7C_3$ ways.

Thus, number of ways of selecting Q's

$$\begin{aligned} &= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3 \\ &= \frac{5!}{2!3!} \times \frac{7!}{2!5!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{3!4!} \\ &= 210 + 175 + 35 \\ &= 420 \end{aligned}$$

Question 8

Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Solution

Ans: Out of 52 cards, 5-card combinations have to be made in such a way that in each selection of 5 cards, there is exactly one king. We know that in 52 cards, there are 4 kings.

Out of 4 kings, 1 king can be selected in ${}^4C_1$ ways.

Out of the remaining 48 cards, 4 cards can be selected in ${}^{48}C_4$ ways.

Therefore, number of 5-card combinations = ${}^4C_1 \times {}^{48}C_4$ ways.

Question 9

It is required to seat 5 men and 4 women in a row so that the women can occupy the even places.

How many such arrangements are possible?

Solution

Ans: Given 5 men and 4 women should be seated so that women always occupy the even places.

Thus, the men can be seated in $5!$ ways.

For each arrangement, the women can be seated only in even places.

Thus, the women can be seated in $4!$ ways.

Therefore, possible number of arrangements = $4! \times 5! = 24 \times 120 = 2880$

Question 10

From the class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

Solution

Ans: Given: 10 are chosen for an excursion party out of 25 students.

There are 2 cases where 3 students decided either all or one of them would join.

Case 1: All the 3 students join.

The remaining 7 students can be chosen out of 22 students in ${}^{22}C_7$ ways.

Case 2: None of the 3 students join.

Thus, 10 students can be chosen in ${}^{22}C_{10}$ ways.

Hence, number of ways of choosing excursion party = ${}^{22}C_7 + {}^{22}C_{10}$

Question 11

In how many words can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Solution

Ans: There are 3 A's, 4 S's, 2 I's and all other letters appear only once in the word ASSASSINATION.

The given word should be arranged such that all the S's are together.

The 4 S's can be treated as a single object for the time being. This single object with the remaining objects will be 10 objects together.

These 10 objects with 3 A's, 2 I's and 2 N's can be arranged in $\frac{10!}{2!3!2!}$ ways.

Therefore, number of ways of arranging the given word = $\frac{10!}{2!3!2!} = 15120$.