

Chapter 5: Linear Inequalities

NCERT · Class 11 · Maths

Exercise 5.1

Question 1

Solve $24x < 100$ when

(i) x is a natural number

(ii) x is an integer.

Solution

Ans: We are given $24x < 100$.

$$24x < 100$$

$$\Rightarrow \frac{24x}{24} < \frac{100}{24}$$

$$\Rightarrow x < \frac{25}{6}$$

Therefore, we can say that only 1, 2, 3, 4 are only natural numbers less than $\frac{25}{6}$.

Therefore, the solution set is $\{1, 2, 3, 4\}$

Ans: We are given $24x < 100$.

$$24x < 100$$

$$\Rightarrow \frac{24x}{24} < \frac{100}{24}$$

$$\Rightarrow x < \frac{25}{6}$$

Therefore, we can say that only $-3, -2, -1, 0, 1, 2, 3, 4$. the integers less than $\frac{25}{6}$.

So, the solution set is $\{..... -3, -2, -1, 0, 1, 2, 3, 4\}$.

Question 2

Solve $-12x > 30$, when

(i) x is a natural number

Solution

Ans: We are given $-12x > 30$.

$$-12x > 30$$

$$\Rightarrow \frac{-12}{12}x > \frac{30}{12}$$

$$\Rightarrow -x > \frac{5}{2}$$

$$\Rightarrow x < -\frac{5}{2}$$

Therefore, we can say that there is no natural number less than $\left(-\frac{5}{2}\right)$

So, there is no solution set for the given condition.

(ii) x is an integer.

We are given $-12x > 30$.

$$-12x > 30$$

$$\Rightarrow \frac{-12}{12}x > \frac{30}{12}$$

$$\Rightarrow -x > -\frac{5}{2}$$

$$\Rightarrow x < \frac{5}{2}$$

Therefore, we can say that $\dots\dots\dots -5, -4, -3$ are the numbers that satisfy the given inequality.

Hence, the solution set is $\{\dots\dots -5, -4, -3\}$.

Question 3

Solve $5x - 3 < 7$, when

(i) x is an integer.

Solution

Ans: We are given $5x - 3 < 7$.

$$5x - 3 < 7$$

$$\Rightarrow 5x - 3 + 3 < 7 + 3$$

$$\Rightarrow 5x < 10$$

$$\Rightarrow \frac{5}{5}x < \frac{10}{5}$$

$$\Rightarrow x < 2$$

Therefore, we can say that $\dots\dots\dots -4, -3, -2, -1, 0, 1$ are the integers that satisfy the given inequality.

So, the solution set is $\{\dots\dots\dots -4, -3, -2, -1, 0, 1\}$

(ii) x is a real number.

We are given $5x - 3 < 7$.

$$5x - 3 < 7$$

$$\Rightarrow 5x - 3 + 3 < 7 + 3$$

$$\Rightarrow 5x < 10$$

$$\Rightarrow \frac{5}{5}x < \frac{10}{5}$$

$$\Rightarrow x < 2$$

Therefore, we can say that the real numbers which are less than 2 will satisfy the inequality.

Hence, the solution set is $x \in (-\infty, 2)$.

Question 4

Solve $3x + 8 > 2$, when

(i) x is an integer.

Solution

Ans: We are given $3x + 8 > 2$.

$$3x + 8 > 2$$

$$\Rightarrow 3x + 8 - 8 > 2 - 8$$

$$\Rightarrow 3x > -6$$

$$\Rightarrow \frac{3}{3}x > \frac{-6}{3}$$

$$\Rightarrow x > -2$$

Therefore, we can say that $-1, 0, 1, 2, 3, \dots$ are the integers that satisfy the given inequality.

So, the solution set is $\{-1, 0, 1, 2, 3, \dots\}$

(ii) x is a real number.

$$3x + 8 > 2$$

$$\Rightarrow 3x + 8 - 8 > 2 - 8$$

$$\Rightarrow 3x > -6$$

$$\Rightarrow \frac{3}{3}x > \frac{-6}{3}$$

$$\Rightarrow x > -2$$

Therefore, we can say that the real numbers which are greater than -2 will satisfy the inequality.

Hence, the solution set is $x \in (-2, \infty)$.

Question 5

Solve the given inequality for real x : $4x + 3 < 5x + 7$

Solution

Ans: Let us rewrite the given inequality, we get

$$4x+3 < 5x+7$$

$$\Rightarrow 4x+3-7 < 5x+7-7$$

$$\Rightarrow 4x-4 < 5x$$

$$\Rightarrow 4x-4-4x < 5x-4x$$

$$\Rightarrow -4 < x$$

Therefore, we can say that the numbers greater than -4 will be the solutions that satisfy the given condition.

Therefore, we can say that the solution set for the given inequality is $(-4, \infty)$.

Question 6

Solve the given inequality for real x : $3x - 7 > 5x - 1$

Solution

Ans: Let us rewrite the given inequality, we get

$$3x - 7 > 5x - 1$$

$$\Rightarrow 3x - 7 + 7 > 5x - 1 + 7$$

$$\Rightarrow 3x > 5x + 6$$

$$\Rightarrow 3x - 5x > 5x - 5x + 6$$

$$\Rightarrow -2x > 6$$

$$\Rightarrow \frac{-2}{-2}x < -\frac{6}{-2}$$

$$\Rightarrow x < -3$$

Therefore, we can say that the numbers less than -3 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, -3)$.

Question 7

Solve the given inequality for real x : $3(x - 1) \leq 2(x - 3)$

Solution

Ans: Let us rewrite the given inequality, we get

$$3(x-1) \leq 2(x-3)$$

$$\Rightarrow 3x-3 \leq 2x-6$$

$$\Rightarrow 3x-3+3 \leq 2x-6+3$$

$$\Rightarrow 3x \leq 2x-3$$

$$\Rightarrow 3x-2x \leq 2x-2x-3$$

$$\Rightarrow x \leq -3$$

Therefore, we can say that the numbers less than or equal to -3 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, -3]$.

Question 8

Solve the given inequality for real x : $3(2 - x) \geq 2(1 - x)$

Solution

Ans: Let us rewrite the given inequality, we get

$$3(2-x) \geq 2(1-x)$$

$$\Rightarrow 6-3x \geq 2-2x$$

$$\Rightarrow 6-6-3x \geq 2-6-2x$$

$$\Rightarrow -3x \geq -4-2x$$

$$\Rightarrow -3x + 2x \geq -4 - 2x + 2x$$

$$\Rightarrow -x \geq -4$$

$$\Rightarrow x \leq 4$$

Therefore, we can say that the numbers less than or equal to 4 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 4]$.

Question 9

Solve the given inequality for real x : $x + \frac{x}{2} + \frac{x}{3} < 11$

Solution

Ans: Let us rewrite the given inequality, we get

$$x + \frac{x}{2} + \frac{x}{3} < 11$$

$$\Rightarrow x \left(1 + \frac{1}{2} + \frac{1}{3} \right) < 11$$

$$\Rightarrow x \left(\frac{6+3+2}{6} \right) < 11$$

$$\Rightarrow x \left(\frac{11}{6} \right) < 11$$

$$\Rightarrow x \left(\frac{11}{6} \right) \times \left(\frac{6}{11} \right) 11 \times \frac{6}{11}$$

$$\Rightarrow x < 6$$

Therefore, we can say that the numbers less than 6 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 6)$.

Question 10

Solve the given inequality for real x : $\frac{x}{3} > \frac{x}{2} + 1$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{x}{3} > \frac{x}{2} + 1$$

$$\Rightarrow \frac{x}{3} - \frac{x}{2} > \frac{x}{2} - \frac{x}{2} + 1$$

$$\Rightarrow x \left(\frac{1}{3} - \frac{1}{2} \right) > 1$$

$$\Rightarrow x \left(\frac{-1}{6} \right) > 1$$

$$\Rightarrow x \left(\frac{-1}{6} \right) \times \left(\frac{-6}{1} \right) < -6$$

$$\Rightarrow x < -6$$

Therefore, we can say that the numbers less than -6 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, -6)$.

Question 11

Solve the given inequality for real x : $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

$$\Rightarrow 9(x-2) \leq 25(2-x)$$

$$\Rightarrow 9x-18 \leq 50-25x$$

$$\Rightarrow 9x+25x-18 \leq 50-25x+25x$$

$$\Rightarrow 34x-18 \leq 50$$

$$\Rightarrow 34x-18+18 \leq 50+18$$

$$\Rightarrow 34x \leq 68$$

$$\Rightarrow x \leq \frac{68}{34}$$

$$\Rightarrow x \leq 2$$

Therefore, we can say that the numbers less than or equal to 2 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 2]$.

Question 12

Solve the given inequality for real x : $\frac{1}{2} \left(\frac{3x}{5} + 4 \right) \geq \frac{1}{3}x - 2$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{1}{2} \left(\frac{3x}{5} + 4 \right) \geq \frac{1}{3}(x-6)$$

$$\Rightarrow 3 \left(\frac{3x}{5} + 4 \right) \geq 2(x-6)$$

$$\Rightarrow \frac{9}{5}x + 12 \geq 2x - 12$$

$$\Rightarrow \frac{9}{5}x + 12 - 12 \geq 2x - 12 - 12$$

$$\Rightarrow \frac{9}{5}x \geq 2x - 24$$

$$\Rightarrow 9x \geq 10x - 120$$

$$\Rightarrow 9x - 10x \geq 10x - 10x - 120$$

$$\Rightarrow -x \geq -120$$

$$\Rightarrow x \leq 120$$

Therefore, we can say that the numbers less than or equal to 120 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 120]$.

Question 13

Solve the given inequality for real x : $2(2x + 3) - 10 < 6(x - 2)$

Solution

Ans: Let us rewrite the given inequality, we get

$$2(2x+3) - 10 < 6(x-2)$$

$$\Rightarrow 4x+6-10 < 6x-12$$

$$\Rightarrow 4x-4 < 6x-12$$

$$\Rightarrow 4x-4+4 < 6x-12+4$$

$$\Rightarrow 4x < 6x-8$$

$$\Rightarrow 4x-6x < 6x-6x-8$$

$$\Rightarrow -2x < -8$$

$$\Rightarrow x > 4$$

Therefore, we can say that the numbers greater than 4 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(4, \infty)$.

Question 14

Solve the given inequality for real x : $37 - (3x + 5) \geq 9x - 8(x - 3)$

Solution

Ans: Let us rewrite the given inequality, we get

$$37 - (3x+5) \geq 9x-8(x-3)$$

$$\Rightarrow 37-3x-5 \geq 9x-8x+24$$

$$\Rightarrow -3x+32 \geq x+24$$

$$\Rightarrow -3x-x+32 \geq x-x+24$$

$$\Rightarrow -4x+32 \geq 24$$

$$\Rightarrow -4x + 32 - 32 \geq 24 - 32$$

$$\Rightarrow -4x \geq -8$$

$$\Rightarrow x \leq 2$$

Therefore, we can say that the numbers less than or equal to 2 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 2]$.

Question 15

Solve the given inequality for real x : $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\Rightarrow \frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$$

$$\Rightarrow \frac{x}{4} < \frac{25x-10-21x+9}{15}$$

$$\Rightarrow \frac{x}{4} < \frac{4x-1}{15}$$

$$\Rightarrow 15x < 4(4x-1)$$

$$\Rightarrow 15x - 16x < 16x - 16x - 4$$

$$\Rightarrow -x < -4$$

$$x > 4$$

Therefore, we can say that the numbers greater 4 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(4, \infty)$.

Question 16

Solve the given inequality for real x : $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$

$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$

$$\Rightarrow \frac{(2x-1)}{3} = \frac{15x-10-8+4x}{20}$$

$$\Rightarrow \frac{(2x-1)}{3} \geq \frac{19x-18}{20}$$

$$\Rightarrow 20(2x-1) \geq 3(19x-18)$$

$$\Rightarrow 40x-20 \geq 57x-54$$

$$\Rightarrow -20+54 \geq 57x-40x$$

$$\Rightarrow 34 \geq 17x$$

$$\Rightarrow 2 \geq x$$

Therefore, we can say that the numbers less than or equal to 2 will be in the solution set for satisfying the given condition.

Therefore, we can say that the solution set for the given inequality is $(-\infty, 2]$.

Question 17

Solve the inequality and show the graph of the solution in number line:

$$3x - 2 < 2x + 1$$

Solution

Ans: Let us rewrite the given inequality, we get

$$3x-2 < 2x+1$$

$$\Rightarrow 3x-2x < 1+2$$

$$\Rightarrow x < 3$$

Therefore, the graphical representation for the inequality in number line is:

Question 18

Solve the inequality and show the graph of the solution in number line:

$$5x - 3 \geq 3x - 5$$

Solution

Ans: Let us rewrite the given inequality, we get

$$5x - 3 \geq 3x - 5$$

$$\Rightarrow 5x - 3x \geq -5 + 3$$

$$\Rightarrow 2x \geq -2$$

$$\Rightarrow \frac{2x}{2} \geq \frac{-2}{2}$$

$$\Rightarrow x \geq -1$$

Therefore, the graphical representation for the inequality in number line is:

Question 19

Solve the inequality and show the graph of the solution in number line:

$$3(1 - x) < 2(x + 4)$$

Solution

Ans: Let us rewrite the given inequality, we get

$$3(1 - x) < 2(x + 4)$$

$$\Rightarrow 3 - 3x < 2x + 8$$

$$\Rightarrow 3 - 8 < 2x + 3x$$

$$\Rightarrow -5 < 5x$$

$$\Rightarrow \frac{-5}{5} < \frac{5x}{5}$$

$$\Rightarrow -1 < x$$

Therefore, the graphical representation for the inequality in number line is:

Question 20

Solve the inequality and show the graph of the solution in number line:

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

Solution

Ans: Let us rewrite the given inequality, we get

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\Rightarrow \frac{x}{2} \geq \frac{5(5x-2)-3(7x-3)}{15}$$

$$\Rightarrow \frac{x}{2} \geq \frac{25x-10-21x+9}{15}$$

$$\Rightarrow \frac{x}{2} \geq \frac{4x-1}{15}$$

$$\Rightarrow 15x \geq 2(4x-1)$$

$$\Rightarrow 15x \geq 8x-2$$

$$\Rightarrow 15x-8x \geq 8x-2-8x$$

$$\Rightarrow 7x \geq -2$$

$$\Rightarrow x \geq -\frac{2}{7}$$

Therefore, the graphical representation for the inequality in number line is:

Question 21

Ravi obtained 70 and 75 marks in the first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Solution

Ans: Let us assume x as the marks obtained by Ravi in the third unit test.

As it is given that the student must have an average of 60 marks,

Therefore, the expression is:

$$\frac{70+75+x}{3} \geq 60$$

$$\Rightarrow 145+x \geq 180$$

$$\Rightarrow x \geq 180-145$$

$$\Rightarrow x \geq 35$$

Therefore, we can say that the student should obtain greater than or equal to 35 marks.

Question 22

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in the fifth examination to get grade 'A' in the course.

Solution

Ans: Let us assume x as the fifth examination marked by Sunita.

As it is given that to obtain 'A' one should obtain an average of 90 marks or more in five examinations.

So, the expression is:

$$\frac{87+92+94+95+x}{5} \geq 90$$

$$\Rightarrow \frac{368+x}{5} \geq 90$$

$$\Rightarrow 368 + x \geq 450$$

$$\Rightarrow x \geq 450 - 368$$

$$\Rightarrow x \geq 82$$

Therefore, we can say that sunita must obtain 82 or greater than it to obtain

‘A’ grade.

Question 23

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Solution

Ans: Let us assume x as the smallest number in two consecutive odd integers. Then, the other odd integer will be $x+2$.

As it is given that both the integers are smaller than 10, we get

$$x+2 < 10$$

$$\Rightarrow x < 10-2$$

$$\Rightarrow x < 8$$

Let us assume it as inequality (1), we get

$$x < 8 \dots \dots \dots (1)$$

Also, it is given that the sum of the two integers is more than 11.

So,

$$x + (x + 2) > 11$$

$$\Rightarrow 2x + 2 > 11$$

$$\Rightarrow 2x > 11 - 2$$

$$\Rightarrow 2x > 9$$

$$\Rightarrow x > \frac{9}{2}$$

$$\Rightarrow x > 4.5$$

Let us assume this as inequality (2)

$$x > 4.5 \dots \dots (2)$$

Therefore, from inequality (1) and (2), we get that

x can take the values of

5 and 7.

Therefore, the required pairs are (5, 7) and (7, 9).

Question 24

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Solution

Ans: Let us assume x as the smallest number in two consecutive even integers.

Then, the other odd integer will be $x+2$.

As it is given that both the integers are greater than 5, we get

$$x > 5 \dots \dots \dots (1)$$

Also, it is given that the sum of the two integers is less than 23.

So,

$$x + (x + 2) < 23$$

$$\Rightarrow 2x + 2 < 23$$

$$\Rightarrow 2x < 23 - 2$$

$$\Rightarrow 2x < 21$$

$$\Rightarrow x < \frac{21}{2}$$

$$\Rightarrow x < 10.5$$

Let us assume this as inequality (2)

$$x < 10.5 \dots \dots (2)$$

From (1) and (2), we obtain

$$5 < x < 10.5.$$

Therefore, we can say that x satisfies 6, 8 and 10 values.

Therefore, the pairs that are possible are (6, 8), (8, 10) and (10, 12).

Question 25

The longest side of a triangle is 3 times the shortest side and the third side is 2cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Solution

Ans: Let us assume the shortest side of the given triangle as x .

As it is given that the longest side is the three times the shortest side, we

get longest side = $3x$

Therefore, the length of the other side will be $(3x - 2)$

As it is given that the perimeter should be at least 61 cm, we can write it as:

$$x \text{ cm} + 3x \text{ cm} + (3x - 2) \text{ cm} = 61 \text{ cm}$$

$$\Rightarrow 7x - 2 \geq 61$$

$$\Rightarrow 7x \geq 61 + 2$$

$$\Rightarrow 7x \geq 63$$

$$\Rightarrow x \geq 9$$

Therefore, we can say that the shortest side of the given triangle should

satisfy the inequality $x \geq 9$.

Question 26

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

Solution

Ans: Let us assume the length of the shortest piece be x cm.

From the question we can write that the second and third piece will be $(x+3)$ cm and $2x$ respectively.

As it is given that the three lengths are single piece of board of length 91 cm,

$$x \text{ cm} + (x+3) \text{ cm} + 2x \text{ cm} = 91 \text{ cm}$$

$$\Rightarrow 4x + 3 \leq 91$$

$$\Rightarrow 4x \leq 91 - 3$$

$$\Rightarrow 4x \leq 88$$

$$\Rightarrow \frac{4}{4}x \leq \frac{88}{4}$$

$$\Rightarrow x \leq 22 \dots \dots (1)$$

Also, the third piece is at least 5 cm longer than the second piece.

$$\therefore 2x \geq (x+3) + 5$$

$$\Rightarrow 2x \geq x + 8$$

$$\Rightarrow x \geq 8 \dots \dots (2)$$

Therefore, from (1) and (2), we get

$$8 \leq x \leq 22$$

So, we can say that the shortest board should be greater than or equal to

8 cm but less than or equal to 22 cm.

Miscellaneous Exercise

Question 1

Solve the inequality $2 \leq 3x - 4 \leq 5$

Solution

$$\text{Ans: } 2 \leq 3x - 4 \leq 5$$

$$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$$

$$\Rightarrow 6 \leq 3x \leq 9$$

$$\Rightarrow 2 \leq x \leq 3$$

As a result, the solutions of the following inequality are all real values higher than or equal to 2 but less than or equal to 3. For the given inequality, the solution set is $[2, 3]$.

Question 2

Solve the inequality $6 \leq -3(2x - 4) < 12$

Solution

$$\text{Ans: } 6 \leq -3(2x - 4) < 12$$

$$\Rightarrow 2 \leq -(2x - 4) < 4$$

$$\Rightarrow -2 \geq 2x - 4 > -4$$

$$\Rightarrow 4 - 2 \geq 2x > 4 - 4$$

$$\Rightarrow 2 \geq 2x > 0$$

$$\Rightarrow 1 \geq x > 0$$

As a result, the set of solutions for the given inequality is $[1, 0)$.

Question 3

Solve the inequality $-3 \leq 4 - \frac{7x}{2} \leq 18$

Solution

$$\text{Ans: } -3 \leq 4 - \frac{7x}{2} \leq 18$$

$$\Rightarrow -3 - 4 \leq -\frac{7x}{2} \leq 18 - 4$$

$$\Rightarrow -7 \leq -\frac{7x}{2} \leq 14$$

$$\Rightarrow 7 \geq \frac{7x}{2} \geq -14$$

$$\Rightarrow 1 \geq \frac{x}{2} \geq -2$$

$$\Rightarrow 2 \geq x \geq -4$$

As a result, the set of solutions for the given inequality is $[-4, 2]$.

Question 4

Solve the inequality $-15 < \frac{3(x-2)}{5} \leq 0$

Solution

$$\text{Ans: } -15 < \frac{3(x-2)}{5} \leq 0$$

$$\Rightarrow -75 < 3(x-2) \leq 0$$

$$\Rightarrow -25 < x-2 \leq 0$$

$$\Rightarrow -25 + 2 < x \leq 2$$

$$\Rightarrow -23 < x \leq 2$$

As a result, the set of solutions for the given inequality is $(-23, 2]$

Question 5

Solve the inequality $-12 < 4 - \frac{3x}{-5} \leq 2$

Solution

Ans:

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 - 4 < \frac{-3x}{-5} \leq 2 - 4$$

$$\Rightarrow -16 < \frac{3x}{5} \leq -2$$

$$\Rightarrow -80 < 3x \leq -10$$

$$\Rightarrow \frac{-80}{3} < x \leq \frac{-10}{3}$$

As a result, the set of solutions for the given inequality is $\left(\frac{-80}{3}, \frac{-10}{3}\right]$.

Question 6

Solve the inequality $7 \leq \frac{(3x + 11)}{2} \leq 11$

Solution

$$\text{Ans: } 7 \leq \frac{(3x + 11)}{2} \leq 11$$

$$\Rightarrow 14 \leq 3x + 11 \leq 22$$

$$\Rightarrow 14 - 11 \leq 3x \leq 22 - 11$$

$$\Rightarrow 3 \leq 3x \leq 11$$

$$\Rightarrow 1 \leq x \leq \frac{11}{3}$$

As a result, the set of solutions for the given inequality is $\left[1, \frac{11}{3}\right]$.

Question 7

Solve the inequalities and represent the solution graphically on number line:

$$5x + 1 > -24, 5x - 1 < 24$$

Solution

$$\text{Ans: } 5x + 1 > -24 \Rightarrow 5x > -25$$

$$\Rightarrow x > -5 \dots (1)$$

$$5x - 1 < 24 \Rightarrow 5x < 25$$

$$\Rightarrow x < 5$$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $(-5, 5)$.

On a number line, the solution to the above system of inequalities can be expressed as

Question 8

Solve the inequalities and represent the solution graphically on number line:

$$2(x - 1) < x + 5, 3(x + 2) > 2 - x$$

Solution

$$\text{Ans: } 2(x - 1) < x + 5 \Rightarrow 2x - 2 < x + 5 \Rightarrow 2x - x < 5 + 2$$

$$\Rightarrow x < 7$$

$$(1) 3(x + 2) > 2 - x \Rightarrow 3x + 6 > 2 - x \Rightarrow 3x + x > 2 - 6$$

$$\Rightarrow 4x > -4$$

$$\Rightarrow x > -1 \dots\dots (2)$$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $(-1, 7)$.

On a number line, the solution to the above system of inequalities can be expressed as

Question 9

Solve the following inequalities and represent the solution graphically on number line:

$$3x - 7 > 2(x - 6), 6 - x > 11 - 2x$$

Solution

$$\text{Ans: } 3x - 7 > 2(x - 6) \Rightarrow 3x - 7 > 2x - 12 \Rightarrow 3x - 2x > -12 + 7$$

$$\Rightarrow x > -5 \dots\dots\dots (1)$$

$$6 - x > 11 - 2x \Rightarrow -x + 2x > 11 - 6$$

$$\Rightarrow x > 5$$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be

$(5, \infty)$. On a number line, the solution to the above system of inequalities can be expressed as

Question 10

Solve the inequalities and represent the solution graphically on number line:

$$5(2x - 7) - 3(2x + 3) \leq 0, 2x + 19 \leq 6x + 47$$

Solution

$$\text{Ans: } 5(2x - 7) - 3(2x + 3) \leq 0 \Rightarrow 10x - 35 - 6x - 9 \leq 0 \Rightarrow 4x - 44 \leq 0 \Rightarrow 4x \leq 44$$

$$\Rightarrow x \leq 11$$

$$2x + 19 \leq 6x + 47 \Rightarrow 19 - 47 \leq 6x - 2x \Rightarrow -28 \leq 4x$$

$$\Rightarrow -7 \leq x$$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $[-7, 11]$.

On a number line, the solution to the above system of inequalities can be expressed as

Question 11

A solution is to be kept between 68°F and 77°F . What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?

Solution

Ans: Because the solution must be preserved somewhere in the middle, 68°F and

77°F , $68 < F < 77$ Putting $F = \frac{9}{5}C + 32$, we obtain $68 < \frac{9}{5}C + 32 < 77$

$$\Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

As a result, the required temperature range in degrees Celsius is between 20°C and 25°C .

Question 12

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution

Ans: Let 2% of x litres of boric acid solution is required to be added. Then, total mixture = $(x + 640)$ litres.

This resulting mixture is to be more than 4% but less than 6% boric acid.

$$\therefore 2\% \text{ of } x + 8\% \text{ of } 640 > 4\% \text{ of } (x + 640)$$

$$\text{And, } 2\% \text{ of } x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)$$

$$\Rightarrow 2x + 5120 > 4x + 2560$$

$$\Rightarrow 5120 - 2560 > 4x - 2x$$

$$\Rightarrow 5120 - 2560 > 2x$$

$$\Rightarrow 2560 > 2x$$

$$\Rightarrow 1280 > x$$

$$2\%x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640) \quad \frac{2}{100}x + \frac{8}{100}(640) < \frac{6}{100}(x + 640)$$

$$\Rightarrow 2x + 5120 < 6x + 3840$$

$$\Rightarrow 5120 - 3840 < 6x - 2x$$

$$\Rightarrow 5120 - 3840 < 4x$$

$$\Rightarrow 1280 < 4x$$

$$\Rightarrow 320 < x$$

$$\therefore 320 < x < 1280$$

As a result, the total amount of boric acid solution to be added must be greater than 320 litres but less than 1280 litres.

Question 13

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Solution

Ans: Allow for the addition of x litres of water. The entire mixture is then calculated = $(x + 1125)$ litres

It is clear that the amount of acid in the final mixture is excessive. 45% of 1125 litres. The resulting mixture will have a higher concentration of 25% but less than 30% acid content.

$$\therefore 30\% \text{ of } (1125 + x) > 45\% \text{ of } 1125$$

$$\text{And, } 25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125$$

$$\Rightarrow \frac{30}{100}(1125 + x) > \frac{45}{100} \times 1125$$

$$\Rightarrow 30(1125 + x) > 45 \times 1125$$

$$\Rightarrow 30 \times 1125 + 30x > 45 \times 1125$$

$$\Rightarrow 30 > 45 \times 1125 - 30 \times 1125$$

$$\Rightarrow 30x > (45 - 30) \times 1125$$

$$\Rightarrow x > \frac{15 \times 1125}{30} = 562.5$$

$$25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125 \Rightarrow \frac{25}{100}(1125 + x) < \frac{45}{100} \times 1125$$

$$\Rightarrow 25(1125 + x) < 45 \times 1125$$

$$\Rightarrow 25 \times 1125 + 25x < 45 \times 1125$$

$$\Rightarrow 25x < 45 \times 1125 - 25 \times 1125$$

$$\Rightarrow 25x < (45 - 25) \times 1125$$

$$\Rightarrow x < \frac{20 \times 1125}{25} = 900$$

$$\therefore 562.5 < x < 900$$

As a result, the required number of litres of water must be greater than 562.5 but less than 900.

Question 14

IQ of a person is given by the formula $IQ = \frac{MA}{CA} \times 100$, Where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years old children, find the range of their mental age.

Solution

Ans: It is reported that for a group of twelve-year-olds $80 \leq IQ \leq 140 \dots\dots (i)$

For a group of 12 years old children, $CA = 12$ years $IQ = \frac{MA}{12} \times 100$

Putting this value of IQ in (i), we obtain $80 \leq \frac{MA}{12} \times 100 \leq 140$

$$\Rightarrow 80 \times \frac{12}{100} \leq MA \leq 140 \times \frac{12}{100}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

As a result, the mental age range of the 12-year-olds has widened. $\Rightarrow 9.6 \leq MA \leq 16.8$.