

Chapter 5: Complex Numbers and Quadratic Equations

Exercise 5.1

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Question 1

$$(5i) \left(-\frac{3}{5}i \right)$$

Solution

Ans:

$$\Rightarrow (5i) \left(-\frac{3}{5}i \right) = -5 \times \frac{3}{5} \times i \times i$$

$$= -3i^2$$

$$= -3(-1) \quad [i^2 = -1]$$

$$= 3$$

Question 2

$$i^9 + i^{19}$$

Solution

Ans:

$$\Rightarrow i^9 + i^{19} = i^{4 \times 2 + 1} + i^{4 \times 4 + 3}$$

$$= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3$$

$$= 1 \times i + 1 \times (-i) \quad [i^4 = 1, i^3 = -i]$$

$$= i + (-i)$$

$$= 0$$

Question 3

$$i^{-39}$$

Solution

Ans:

$$\begin{aligned}\Rightarrow i^{-39} &= i^{-4 \times 9 - 3} = (i^4)^{-9} \cdot i^{-3} \\ &= (1)^{-9} \cdot i^{-3} [i^4 = 1] \\ &= \frac{1}{i^3} = \frac{1}{-i} [i^3 = -i] \\ &= \frac{-1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i [i^2 = -1]\end{aligned}$$

Question 4

$$3(7 + i7) + i(7 + i7)$$

Solution

Ans:

$$\begin{aligned}\Rightarrow 3(7 + i7) + i(7 + i7) &= 21 + 21i + 7i + 7i^2 \\ &= 21 + 28i + 7 \times (-1) [\because i^2 = -1] \\ &= 14 + 28i\end{aligned}$$

Question 5

$$(1 - i) - (-1 + 6i)$$

Solution

Ans:

$$\begin{aligned}\Rightarrow (1 - i) - (-1 + i6) \\&= 1 - i + 1 - 6i \\&= 2 - 7i\end{aligned}$$

Question 6

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right).$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right) \\&= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i \\&= \left(\frac{1}{5} - 4\right) + \left(\frac{2}{5} - \frac{5}{2}\right)i \\&= \left(\frac{1 - (4 \times 5)}{5}\right) + \left(\frac{(2 \times 2) - (5 \times 5)}{5 \times 2}\right)i \\&= -\left(\frac{1 - 20}{5}\right) + \left(\frac{4 - 25}{10}\right)i \\&= -\frac{19}{5} + \left(\frac{-21}{10}\right)i \\&= -\frac{19}{5} - \frac{21}{10}i\end{aligned}$$

Question 7

$$\left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) - \left(-\frac{4}{3} + i \right) \right].$$

Solution

Ans:

$$\begin{aligned} &\Rightarrow \left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) - \left(-\frac{4}{3} + i \right) \right] \\ &\frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i \\ &= \left(\frac{1}{3} + 4 + \frac{4}{3} \right) + \left(\frac{7}{3} + \frac{1}{3} - 1 \right) i \\ &= \left(\frac{1 + (4 \times 3) + 4}{3} \right) + \left(\frac{7 + 1 - (1 \times 3)}{3} \right) i \\ &= \left(\frac{1 + 12 + 4}{3} \right) + \left(\frac{7 + 1 - 3}{3} \right) i \end{aligned}$$

 $a + ib :$

$$= \frac{17}{3} + i\frac{5}{3}$$

Question 8

$$(1 - i)^4$$

Solution

Ans:

 $\Rightarrow$

$$\begin{aligned} (1 - i)^4 &= \left[(1 - i)^2 \right]^2 \\ &= [1^2 + i^2 - 2i]^2 \end{aligned}$$

$$\begin{aligned}(1-i)^4 &= \left[(1-i)^2\right]^2 \\&= [1-1-2i]^2 \\&= (2i)^2 \\&= 4i^2 \\&= -4 [i^2 = -1] \\a+ib &= -4+0i\end{aligned}$$

Question 9

$$\left(\frac{1}{3} + 3i\right)^3.$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(\frac{1}{3} + 3i\right)^3 &= \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3} + 3i\right) \\&= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3} + 3i\right) \\&= \frac{1}{27} + 27(-i) + i + 9i^2 [i^3 = -i] \\&= \frac{1}{27} - 27i + i + 9i^2 [i^2 = -1] \\&= \left(\frac{1}{27} - 9\right) + i(-27 + 1) \\a+ib &= \frac{-242}{27} - 26i\end{aligned}$$

Question 10

$$\left(-2 - \frac{1}{3}i\right)^3.$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(-2 - \frac{1}{3}i\right)^3 &= (-1)^3 \left(2 + \frac{1}{3}i\right)^3 \\&= - \left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2) \left(\frac{i}{3}\right) \left(2 + \frac{i}{3}\right)\right] \\&= - \left[8 + \frac{i^3}{27} + 2i \left(2 + \frac{i}{3}\right)\right] [i^3 = -i] \\&= - \left[8 - \frac{i}{27} + 4i - \frac{2i^2}{3}\right] [i^2 = -1] \\&= - \left[\frac{22}{3} + \frac{107i}{27}\right] \\a + ib &= -\frac{22}{3} - \frac{107}{27}i\end{aligned}$$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

Question 11

$$4 - 3i.$$

Solution

Ans:

Let

$$z = 4 - 3i$$

Then,

$$\bar{z} = 4 + 3i$$

and

$$|z| = 4^2 + (-3)^2 = 16 + 9 = 25$$

The multiplicative inverse of

$$4 - 3i$$

is given by

$$\begin{aligned}\Rightarrow z^{-1} &= \frac{\bar{z}}{|z|^2} \\ &= \frac{4 + 3i}{25} \\ &= \frac{4}{25} + \frac{3}{25}i\end{aligned}$$

Question 12

$$\sqrt{5} + 3i.$$

Solution

Ans:

$$\text{Let } z = \sqrt{5} + 3i$$

$$\text{Then, } \bar{z} = \sqrt{5} - 3i$$

$$|z| = (\sqrt{5})^2 + (3)^2 = 5 + 9 = 14$$

The multiplicative inverse of the complex number $\sqrt{5} + 3i$ is given by

$$\begin{aligned}z^{-1} &= \frac{\bar{z}}{|z|^2} \\ &= \frac{\sqrt{5} - 3i}{14}\end{aligned}$$

$$= \frac{\sqrt{5}}{14} - \frac{3}{14}i$$

Question 13

$-i$.

Solution

Ans:

Let $z = -i$

Then, $\bar{z} = i$

$$|z|^2 = 1$$

The multiplicative inverse of the complex number $-i$

$$\Rightarrow z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{i}{1}$$

$$= i$$

Question 14

Express the following expression in the form of $a + ib$.

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

Solution

Ans:

$$\Rightarrow \frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

$$= \frac{(3^2 - (i\sqrt{5})^2)}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

$$[(a + b)(a - b) = a^2 - b^2]$$

$$= \frac{(9 - 5i^2)}{2\sqrt{2}i}$$

$$= \frac{9 - 5(-1)}{2\sqrt{2}i} [i^2 = -1]$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$