

Chapter 4: Complex Numbers and Quadratic Equations

NCERT · Class 11 · Maths

Exercise 4.1

Question 1

$$(5i) \left(-\frac{3}{5}i \right)$$

Solution

Ans:

$$\Rightarrow (5i) \left(-\frac{3}{5}i \right) = -5 \times \frac{3}{5} \times i \times i$$

$$= -3i^2$$

$$= -3(-1) \quad [i^2 = -1]$$

$$= 3$$

Question 2

$$i^9 + i^{19}$$

Solution

Ans:

$$\Rightarrow i^9 + i^{19} = i^{4 \times 2 + 1} + i^{4 \times 4 + 3}$$

$$= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3$$

$$= 1 \times i + 1 \times (-i) \quad [i^4 = 1, i^3 = -i]$$

$$= i + (-i)$$

$$= 0$$

Question 3

$$i^{-39}$$

Solution

Ans:

$$\begin{aligned}\Rightarrow i^{-39} &= i^{-4 \times 9 - 3} = (i^4)^{-9} \cdot i^{-3} \\ &= (1)^{-9} \cdot i^{-3} [i^4 = 1] \\ &= \frac{1}{i^3} = \frac{1}{-i} [i^3 = -i] \\ &= \frac{-1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i [i^2 = -1]\end{aligned}$$

Question 4

$$3(7 + i7) + i(7 + i7)$$

Solution

Ans:

$$\begin{aligned}\Rightarrow 3(7 + i7) + i(7 + i7) &= 21 + 21i + 7i + 7i^2 \\ &= 21 + 28i + 7 \times (-1) [\because i^2 = -1] \\ &= 14 + 28i\end{aligned}$$

Question 5

$$(1 - i) - (-1 + 6i)$$

Solution

Ans:

$$\begin{aligned}\Rightarrow (1 - i) - (-1 + i6) \\&= 1 - i + 1 - 6i \\&= 2 - 7i\end{aligned}$$

Question 6

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right).$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right) \\&= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i \\&= \left(\frac{1}{5} - 4\right) + \left(\frac{2}{5} - \frac{5}{2}\right)i \\&= \left(\frac{1 - (4 \times 5)}{5}\right) + \left(\frac{(2 \times 2) - (5 \times 5)}{5 \times 2}\right)i \\&= -\left(\frac{1 - 20}{5}\right) + \left(\frac{4 - 25}{10}\right)i \\&= -\frac{19}{5} + \left(\frac{-21}{10}\right)i \\&= -\frac{19}{5} - \frac{21}{10}i\end{aligned}$$

Question 7

$$\left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) - \left(-\frac{4}{3} + i \right) \right].$$

Solution

Ans:

$$\begin{aligned} &\Rightarrow \left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) - \left(-\frac{4}{3} + i \right) \right] \\ &\frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i \\ &= \left(\frac{1}{3} + 4 + \frac{4}{3} \right) + \left(\frac{7}{3} + \frac{1}{3} - 1 \right) i \\ &= \left(\frac{1 + (4 \times 3) + 4}{3} \right) + \left(\frac{7 + 1 - (1 \times 3)}{3} \right) i \\ &= \left(\frac{1 + 12 + 4}{3} \right) + \left(\frac{7 + 1 - 3}{3} \right) i \end{aligned}$$

 $a + ib :$

$$= \frac{17}{3} + i\frac{5}{3}$$

Question 8

$$(1 - i)^4$$

Solution

Ans:

 $\Rightarrow$

$$\begin{aligned} (1 - i)^4 &= \left[(1 - i)^2 \right]^2 \\ &= \left[1^2 + i^2 - 2i \right]^2 \end{aligned}$$

$$\begin{aligned}(1-i)^4 &= \left[(1-i)^2\right]^2 \\&= [1-1-2i]^2 \\&= (2i)^2 \\&= 4i^2 \\&= -4 [i^2 = -1] \\a+ib &= -4+0i\end{aligned}$$

Question 9

$$\left(\frac{1}{3} + 3i\right)^3.$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(\frac{1}{3} + 3i\right)^3 &= \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3} + 3i\right) \\&= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3} + 3i\right) \\&= \frac{1}{27} + 27(-i) + i + 9i^2 [i^3 = -i] \\&= \frac{1}{27} - 27i + i + 9i^2 [i^2 = -1] \\&= \left(\frac{1}{27} - 9\right) + i(-27 + 1) \\a+ib &= \frac{-242}{27} - 26i\end{aligned}$$

Question 10

$$\left(-2 - \frac{1}{3}i\right)^3.$$

Solution

Ans:

$$\begin{aligned}\Rightarrow \left(-2 - \frac{1}{3}i\right)^3 &= (-1)^3 \left(2 + \frac{1}{3}i\right)^3 \\&= - \left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2) \left(\frac{i}{3}\right) \left(2 + \frac{i}{3}\right)\right] \\&= - \left[8 + \frac{i^3}{27} + 2i \left(2 + \frac{i}{3}\right)\right] [i^3 = -i] \\&= - \left[8 - \frac{i}{27} + 4i - \frac{2i^2}{3}\right] [i^2 = -1] \\&= - \left[\frac{22}{3} + \frac{107i}{27}\right] \\a + ib &= -\frac{22}{3} - \frac{107}{27}i\end{aligned}$$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

Question 11

$$4 - 3i.$$

Solution

Ans:

Let

$$z = 4 - 3i$$

Then,

$$\bar{z} = 4 + 3i$$

and

$$|z| = 4^2 + (-3)^2 = 16 + 9 = 25$$

The multiplicative inverse of

$$4 - 3i$$

is given by

$$\begin{aligned}\Rightarrow z^{-1} &= \frac{\bar{z}}{|z|^2} \\ &= \frac{4 + 3i}{25} \\ &= \frac{4}{25} + \frac{3}{25}i\end{aligned}$$

Question 12

$$\sqrt{5} + 3i.$$

Solution

Ans:

$$\text{Let } z = \sqrt{5} + 3i$$

$$\text{Then, } \bar{z} = \sqrt{5} - 3i$$

$$|z| = (\sqrt{5})^2 + (3)^2 = 5 + 9 = 14$$

The multiplicative inverse of the complex number $\sqrt{5} + 3i$ is given by

$$\begin{aligned}z^{-1} &= \frac{\bar{z}}{|z|^2} \\ &= \frac{\sqrt{5} - 3i}{14}\end{aligned}$$

$$= \frac{\sqrt{5}}{14} - \frac{3}{14}i$$

Question 13

$-i$.

Solution

Ans:

Let $z = -i$

Then, $\bar{z} = i$

$$|z|^2 = 1$$

The multiplicative inverse of the complex number $-i$

$$\Rightarrow z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{i}{1}$$

$$= i$$

Question 14

Express the following expression in the form of $a + ib$.

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

Solution

Ans:

$$\begin{aligned}
&\Rightarrow \frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})} \\
&= \frac{(3^2 - (i\sqrt{5})^2)}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})} \quad \left[(a + b)(a - b) = a^2 - b^2 \right] \\
&= \frac{(9 - 5i^2)}{2\sqrt{2}i} \\
&= \frac{9 - 5(-1)}{2\sqrt{2}i} \quad [i^2 = -1] \\
&= \frac{14i}{2\sqrt{2}i^2} \\
&= \frac{14i}{2\sqrt{2}(-1)} \\
&= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{-7\sqrt{2}i}{2}
\end{aligned}$$

Miscellaneous Exercise

Question 1

Evaluate

$$\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$$

The expression

Solution

Ans: Expression

$$\begin{aligned}
\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3 &= \left[i^{44+2} + \frac{1}{i^{46+1}} \right]^3 \\
&= \left[(i^4)^4 \times i^2 + \frac{1}{(i^4)^6 \times i} \right]^3 \\
&= \left[i^2 + \frac{1}{i} \right]^3 \quad [i^4 = 1] \\
&= \left[-1 + \frac{1}{i} \times \frac{i}{i} \right]^3 \quad [i^2 = -1] \\
&= \left[-1 + \frac{i}{i^2} \right]^3 \\
&= (-1 + i)^3 \\
&= (-1)^3 [1 + i]^3 \\
&= - [1^3 + i^3 + 3 \times 1 \times i(1+i)] \\
&= - [1 + i^3 + 3i + 3i^2] \\
&= - [1 - i + 3i - 3] \\
&= - [-2 + 2i] \\
&= 2 - 2i
\end{aligned}$$

The expression is evaluated

Question 2

For any two complex numbers z_1 and z_2 , prove that $\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$

Solution

Ans: Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\begin{aligned}
z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\
&= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\
&= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2 \\
&= x_1 x_2 + ix_1 y_2 + iy_1 x_2 - y_1 y_2 \\
&= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2) \\
\operatorname{Re}(z_1 z_2) &= x_1 x_2 - y_1 y_2 \\
\operatorname{Re}(z_1 z_2) &= \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2
\end{aligned}$$

Hence, proved

Question 3

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$ to the standard form

Solution

Ans: Expression

$$\begin{aligned}
 & \left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i) - 2(1-4i)}{(1-4i)(1+i)}\right] \left[\frac{3-4i}{5+i}\right] \\
 & = \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right] \left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right] \left[\frac{3-4i}{5+i}\right] \\
 & = \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)} \\
 & = \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \quad \text{On multiplying numerator and denominator by } (14+5i) \\
 & = \frac{462+165i+434i+155i^2}{2[(14)^2-(5i)^2]} = \frac{307+599i}{2(196-25i^2)} \\
 & = \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}
 \end{aligned}$$

This is the required standard form

Question 4

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Solution

Ans:

Expression

$$\begin{aligned}
 x - iy &= \sqrt{\frac{a - ib}{c - id}} \\
 &= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \quad [\text{On multiplying numerator and denominator by } (c + id)] \\
 &= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}} \\
 (x - iy)^2 &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
 x^2 - y^2 - 2ixy &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}
 \end{aligned}$$

On comparing

$$\begin{aligned}
 x^2 - y^2 &= \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \dots (1) \\
 (x^2 + y^2)^2 &= (x^2 - y^2)^2 + 4x^2y^2 \\
 &= \left(\frac{ac + bd}{c^2 + d^2} \right)^2 + \left(\frac{ad - bc}{c^2 + d^2} \right)^2 \\
 &= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2} \\
 &= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2} \\
 &= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2} \\
 &= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2} \\
 &= \frac{a^2 + b^2}{c^2 + d^2}
 \end{aligned}$$

Hence, proved

Question 5

If $z_1 = 2 - i, z_2 = 1 + i$ Find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$

Evaluate

Solution

Ans: Complex numbers

$$z_1 = 2 - i, z_2 = 1 + i$$

$$\begin{aligned} \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| &= \left| \frac{(2 - i) + (1 + i) + 1}{(2 - i) - (1 + i) + 1} \right| \\ &= \left| \frac{4}{2 - 2i} \right| = \left| \frac{4}{2(1 - i)} \right| \\ &= \left| \frac{2}{1 - i} \times \frac{1 + i}{1 + i} \right| = \left| \frac{2(1 + i)}{(1^2 - i^2)} \right| \\ &= \left| \frac{2(1 + i)}{1 + 1} \right| \quad [i^2 = -1] \\ &= \left| \frac{2(1 + i)}{2} \right| \end{aligned}$$

$$= |1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$ is $\sqrt{2}$

Question 6

$$\text{If } a + ib = \frac{(x + i)^2}{2x^2 + 1}$$

$$\text{Prove that } a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

Solution

Ans: Expression

$$\begin{aligned}
 a + ib &= \frac{(x + i)^2}{2x^2 + 1} \\
 &= \frac{x^2 + i^2 + 2xi}{2x^2 + 1} \\
 &= \frac{x^2 - 1 + i2x}{2x^2 + 1} \\
 &= \frac{x^2 - 1}{2x^2 + 1} + i \left(\frac{2x}{2x^2 + 1} \right)
 \end{aligned}$$

On comparing

$$\begin{aligned}
 a^2 + b^2 &= \left(\frac{x^2 - 1}{2x^2 + 1} \right)^2 + \left(\frac{2x}{2x^2 + 1} \right)^2 \\
 &= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x^2 + 1)^2} \\
 &= \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2} \\
 &= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2} \\
 a^2 + b^2 &= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}
 \end{aligned}$$

Hence, proved

Question 7

Let $z_1 = 2 - i, z_2 = -2 + i$

Find

$$\begin{aligned}
 &\operatorname{Re} \left(\frac{z_1 z_2}{\bar{z}_1} \right) \\
 &\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)
 \end{aligned}$$

Solution

Ans: Complex numbers

$$z_1 = 2 - i, z_2 = -2 + i$$

$$z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$$

$$\bar{z}_1 = 2 + i$$

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{-3 + 4i}{2 + i}$$

On multiplying numerator and denominator by $(2 - i)$, we obtain

$$\begin{aligned} \frac{z_1 z_2}{\bar{z}_1} &= \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2} \\ &= \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5}i \end{aligned}$$

On comparing real parts, we obtain

$$\begin{aligned} \operatorname{Re} \left(\frac{z_1 z_2}{\bar{z}_1} \right) &= \frac{-2}{5} \\ \frac{1}{z_1 \bar{z}_1} &= \frac{1}{(2 - i)(2 + i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5} \end{aligned}$$

On comparing imaginary parts, we obtain

$$\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right) = 0$$

Hence, solved

Question 8

Find the real numbers x, y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$

Solution

Ans: Let $z = (x - iy)(3 + 5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$

$$\bar{z} = (3x + 5y) - i(5x - 3y)$$

It is given that, $\bar{z} = -6 - 24i$

$$3x + 5y - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \quad \dots (i)$$

$$5x - 3y = 24 \dots (ii)$$

On solving we will get

$$3(3) + 5y = -6$$

$$5y = -6 - 9 = -15$$

$$y = -3$$

Thus, the values of x and y are 3 and -3 respectively

Question 9

Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Evaluate

Solution

Ans: Expression

$$\begin{aligned} \frac{1+i}{1-i} - \frac{1-i}{1+i} &= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\ &= \frac{1+i^2+2i - 1-i^2+2i}{1^2+1^2} \\ &= \frac{4i}{2} = 2i \\ \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| &= |2i| = \sqrt{2^2} = 2 \end{aligned}$$

Here we get the answer

Question 10

Find the modulus of $(x + iy)^3 = u + iv$

Than show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Solution

$$(x + iy)^3 = u + iv$$

$$x^3 + (iy)^3 + 3 \times x \times iy(x + iy) = u + iv$$

$$\text{Ans: } x^3 + i^3 y^3 + 3x^2 yi + 3xy^2 i^2 = u + iv$$

$$x^3 - iy^3 + 3x^2 yi - 3xy^2 = u + iv$$

$$(x^3 - 3xy^2) + i(3x^2 y - y^3) = u + iv$$

On equating real and imaginary

$$u = x^3 - 3xy^2, v = 3x^2 y - y^3$$

$$\frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2 y - y^3}{y}$$

$$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4(x^2 - y^2)$$

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Hence, proved

Question 11

If α and β are different complex numbers with $|\beta| = 1$, then find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = 1$

Solution

Ans:

$$\text{Let } \alpha = a + ib \text{ and } \beta = x + iy$$

It is given that, $|\beta| = 1$

$$\begin{aligned}
\sqrt{x^2+y^2} &= 1 \\
x^2+y^2 &= 1 \Rightarrow \left| \frac{\beta-\alpha}{1-\bar{\alpha}} \right| = \left| \frac{(x+iy) - (a+ib)}{1 - (a-ib)(x+iy)} \right| \\
&= \left| \frac{(x-a) + i(y-b)}{1 - (ax + aiy - ibx + by)} \right| \\
&= \left| \frac{(x-a) + i(y-b)}{(1-ax-by) + i(bx-ay)} \right| \\
&= \left| \frac{(x-a) + i(y-b)}{(1-ax-by) + i(bx-ay)} \right| \\
&= \frac{\sqrt{(x-a)^2 + (y-b)^2}}{\sqrt{(1-ax-by)^2 + (bx-ay)^2}} \\
&= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}} \\
&= \frac{\sqrt{(x^2+y^2) + a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2) + b^2(y^2+x^2) - 2ax-2by}} \\
\left| \frac{\beta-\alpha}{1-\bar{\alpha}} \right| &= 1
\end{aligned}$$

Question 12

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Solution

Ans: Equation

$$\begin{aligned}
 1 - i^x &= 2^x \\
 \left(\sqrt{1^2 + (-1)^2} \right)^x &= 2^x \\
 (\sqrt{2})^x &= 2^x \\
 2^{x/2} &= 2^x \\
 \frac{x}{2} &= x \\
 x &= 2x \\
 x &= 0
 \end{aligned}$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of nonzero integral solutions of the given equation is 0.

Question 13

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$ Then show that

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Solution

Ans: Expression

$$\begin{aligned}
 (a + ib)(c + id)(e + if)(g + ih) &= A + iB \\
 |(a + ib)(c + id)(e + if)(g + ih)| &= |A + iB| \\
 |a + ib| \times |c + id| \times |e + if| \times |g + ih| &= |A + iB| \quad \text{Q } [|z_1 z_2| = |z_1| |z_2|] \\
 \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2} \times \sqrt{e^2 + f^2} \times \sqrt{g^2 + h^2} &= \sqrt{A^2 + B^2}
 \end{aligned}$$

By squaring

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Hence, proved

Question 14

If $\left(\frac{1 + i}{1 - i} \right)^m = 1$

Then find the least positive integral value of m

Solution

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

Ans: $\left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$

$$\left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\left(\frac{2i}{2}\right)^m = 1$$

$$i^m = 1$$

$$i^m = i^{4k}$$

$m = 4k$, where k is some integer

Therefore, the least positive is one

Thus, the least positive integral value of m is $4 = (4 \times 1)$