

Chapter 3: Trigonometric Functions

NCERT · Class 11 · Maths

Exercise 3.1

Question 1

Find the radian measures corresponding to the following degree measures:

(i) 25°

Solution

Ans: As we all know that $180^\circ = \pi$ radian. So, $25^\circ = \frac{\pi}{180} \times 25$ radian = $\frac{5\pi}{36}$ radian.

(ii) $-47^\circ 30'$

Ans: As we all know that, $-47^\circ 30' = -47\frac{1}{2}$

$= \frac{-95}{2}$ degree

Because of the $180^\circ = \pi$ radian $\frac{-95}{2}$ degree = $\frac{\pi}{180} \times \left(\frac{-95}{2}\right)$ radian = $\left(\frac{-19}{36 \times 2}\right) \pi$ radian = $\frac{-19}{72} \pi$ radian.

So, $-47^\circ 30' = \frac{-19}{72} \pi$ radian.

(iii) 240°

Ans: As we all know that $180^\circ = \pi$ radian

So, $240^\circ = \frac{\pi}{180} \times 240$ radian = $\frac{4}{3} \pi$ radian

(iv) 520°

Ans: As we all know that $180^\circ = \pi$ radian. So, $520^\circ = \frac{\pi}{180} \times 520$ radian = $\frac{26\pi}{9}$ radian.

Question 2

Find the degree measures corresponding to the following radian measures. (Use $\pi = \frac{22}{7}$)

(i) $\frac{11}{16}$

Solution

Ans: As we all know that π radian = 180° . So, $\frac{11}{16}$ radian = $\frac{180}{\pi} \times \frac{11}{16}$ degree = $\frac{45 \times 11}{\pi \times 4}$ degree

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ degree} = \frac{315}{8} \text{ degree}$$

$$= 36\frac{3}{8} \text{ degree}$$

$$= 39^\circ + \frac{3 \times 60}{8} \text{ minutes} \quad [\text{We know that } 1^\circ = 60']$$

$$= 39^\circ + 22' + \frac{1}{2} \text{ minutes}$$

$$= 39^\circ 22' 30'' \quad [1' = 60'']$$

(ii) -4

Ans: As we all know that π radian = 180° . So, -4 radian = $\frac{180}{\pi} \times (-4)$ degree = $\frac{180 \times 7(-4)}{22}$ degree

$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$

$$= -229^\circ + \frac{1 \times 60}{11} \text{ minutes}$$

$$[\text{We know that } 1^\circ = 60']$$

$$= -229^\circ + 5' + \frac{5}{11} \text{ minutes}$$

$$= -229^\circ 5' 27'' \quad [\text{We know that } 1' = 60'']$$

(iii) $\frac{5\pi}{3}$

Ans: As we all know that π radian = 180° . So,

$$\frac{5\pi}{3}$$

$$\text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree} = 300^\circ$$

(iv) $\frac{7\pi}{6}$

Ans: As we all know that π radian = 180° . So, $\frac{7\pi}{6}$ radian = $\frac{180}{\pi} \times \frac{7\pi}{6} = 210^\circ$.

Question 3

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution

Ans: The number of revolutions made by the wheel in one minute = 360

As a result, the number of revolutions made by the wheel in one second = $\frac{360}{60} = 6$

The wheel rotates at a 2π radian angle in one complete revolution. As a result, it will turn an angle of $6 \times 2\pi$ radian in 6 complete revolutions, i.e., 12π radian

As a result, the wheel rotates at a 12π radian angle in one second.

Question 4

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. (Use $\pi = \frac{22}{7}$)

Solution

Ans: As we all know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

Hence, for $r = 100$ cm, $l = 22$ cm, we have $\theta = \frac{22}{100}$ radian = $\frac{180}{\pi} \times \frac{22}{100}$ degree = $\frac{180 \times 7 \times 22}{22 \times 100}$ degree

$$= \frac{126}{10} \text{ degree} = 12\frac{3}{5} \text{ degree} = 12^\circ 36' \quad [1^\circ = 60']$$

As a result, the needed angle is $12^\circ 36'$.

Question 5

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution

Ans: The diameter of the circle = 40 cm

So, Radius (r) of the circle = $\frac{40}{2}$ cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.

In $\triangle OAB$, $OA = OB = \text{Radius of circle} = 20 \text{ cm}$

Also, $AB = 20 \text{ cm}$

As a result, $\triangle OAB$ is an equilateral triangle.

So, $\theta = 60^\circ = \frac{\pi}{3} \text{ radian}$

We also know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre then $\theta = \frac{l}{r}$

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

As a result, the length of the minor arc of the chord is $\frac{20\pi}{3} \text{ cm}$.

Question 6

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution

Ans: The radii of the two circles should be r_1 and r_2 . Whereas an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 , while let an arc of length/subtend an angle of 75° at the centre of the circle of radius r_2 .

Now, $60^\circ = \frac{\pi}{3} \text{ radian}$ and $75^\circ = \frac{5\pi}{12} \text{ radian}$. We also know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ . radian at the centre then $\theta = \frac{l}{r}$ or $l = r\theta$

$$\text{So, } l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

As a result, the ratio of the radii is 5: 4.

Question 7

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.

(i) 10cm

Solution

Ans: We also know that in a circle of radius r unit, if an arc of length l unit subtends An angle θ radian at the centre, then $\theta = \frac{l}{r}$ It is given that $r = 75\text{ cm}$.

Now, $l = 10\text{ cm}$

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) 15cm

Ans: We also know that in a circle of radius r unit, if an arc of length l unit subtends An angle θ radian at the centre, then $\theta = \frac{l}{r}$ It is given that $r = 75\text{ cm}$.

Now, $l = 15\text{ cm}$

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) 21cm

Ans: We also know that in a circle of radius r unit, if an arc of length l unit subtends An angle θ radian at the centre, then $\theta = \frac{l}{r}$ It is given that $r = 75\text{ cm}$.

Now, $l = 21\text{ cm}$

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

Exercise 3.2

Question 1

Find the values of the other five trigonometric functions if $\cos x = -\frac{1}{2}$, x lies in the third quadrant.

Solution

Ans:

Here given that, $\cos x = -\frac{1}{2}$

Therefore we have,

$$\sec x = \frac{1}{\cos x}$$

$$= \frac{1}{\left(-\frac{1}{2}\right)}$$

$$= -2$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore we have, $\sin^2 x = 1 - \cos^2 x$

Substituting $\cos x = -\frac{1}{2}$ in the formula, we obtain,

$$\sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\sin^2 x = 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

Since x lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$$\sin x = -\frac{\sqrt{3}}{2}$$

Therefore, $\operatorname{cosec} x = \frac{1}{\sin x}$

$$= \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{2}{\sqrt{3}}$$

Hence ,

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$$

$$=\sqrt{3}$$

And

$$\cot x = \frac{1}{\tan x}$$

$$=\frac{1}{\sqrt{3}}$$

Question 2

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, x lies in second quadrant.

Solution

Ans:

Here given that, $\sin x = \frac{3}{5}$

Therefore we have,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \frac{1}{\left(\frac{3}{5}\right)}$$

$$= \frac{5}{3}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore we have, $\cos^2 x = 1 - \sin^2 x$

Substituting $\sin x = \frac{3}{5}$ in the formula, we obtain,

$$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 x = 1 - \frac{9}{25}$$

$$= \frac{16}{25}$$

$$\cos x = \pm \frac{4}{5}$$

Since x lies in the 2^{nd} quadrant, the value of $\cos x$ will be negative.

$$\cos x = -\frac{4}{5}$$

Therefore, $\sec x = \frac{1}{\cos x}$

$$= \frac{1}{\left(-\frac{4}{5}\right)}$$

$$= -\frac{5}{4}$$

Hence ,

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)}$$

$$= -\frac{3}{4}$$

And

$$\cot x = \frac{1}{\tan x}$$

$$= -\frac{4}{3}$$

Question 3

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

Solution

Ans:

Here given that, $\cot x = \frac{3}{4}$

Therefore we have,

$$\tan x = \frac{1}{\cot x}$$

$$= \frac{1}{\left(\frac{3}{4}\right)}$$

$$= \frac{4}{3}$$

Now we know that ,

$$\sec^2 x - \tan^2 x = 1$$

Therefore we have, $\sec^2 x = 1 + \tan^2 x$

Substituting $\tan x = \frac{4}{3}$ in the formula, we obtain,

$$\sec^2 x = 1 + \left(\frac{4}{3}\right)^2$$

$$\sec^2 x = 1 + \frac{16}{9}$$

$$= \frac{25}{9}$$

$$\sec x = \pm \frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of $\sec x$ will be negative.

$$\sec x = -\frac{5}{3}$$

$$\text{Therefore, } \cos x = \frac{1}{\sec x}$$

$$= \frac{1}{\left(-\frac{5}{3}\right)}$$

$$= -\frac{3}{5}$$

$$\text{Now, } \tan x = \frac{\sin x}{\cos x}$$

$$\text{Therefore, } \sin x = \tan x \cos x$$

$$\text{Hence we have, } \sin x = \frac{4}{3} \times \left(-\frac{3}{5}\right)$$

$$= \left(-\frac{4}{5}\right)$$

And

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= -\frac{5}{4}$$

Question 4

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution

Ans:

Here given that, $\sec x = \frac{13}{5}$

Therefore we have,

$$\cos x = \frac{1}{\sec x}$$

$$= \frac{1}{\left(\frac{13}{5}\right)}$$

$$= \frac{5}{13}$$

Now we know that, $\sec^2 x - \tan^2 x = 1$

Therefore we have, $\tan^2 x = \sec^2 x - 1$

Substituting

$$\sec x = \frac{13}{5}$$

in the formula, we obtain,

$$\tan^2 x = \left(\frac{13}{5}\right)^2 - 1$$

$$\tan^2 x = \frac{169}{25} - 1$$

$$= \frac{144}{25}$$

$$\tan x = \pm \frac{12}{5}$$

Since x lies in the 4th quadrant, the value of $\tan x$ will be negative.

$$\tan x = -\frac{12}{5}$$

Therefore,

$$\cot x = \frac{1}{\tan x}$$

$$= -\frac{5}{12}$$

Now , $\tan x = \frac{\sin x}{\cos x}$

Therefore, $\sin x = \tan x \cos x$

Hence we have, $\sin x = \frac{5}{13} \times \left(-\frac{12}{5}\right)$

$$= \left(-\frac{12}{13}\right)$$

And

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= -\frac{13}{12}$$

Question 5

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, x lies in second quadrant.

Solution

Ans:

Here given that, $\tan x = -\frac{5}{12}$

Therefore we have,

$$\cot x = \frac{1}{\tan x}$$

$$= \frac{1}{\left(-\frac{5}{12}\right)}$$

$$= -\frac{12}{5}$$

Now we know that , $\sec^2 x - \tan^2 x = 1$

Therefore we have, $\sec^2 x = 1 + \tan^2 x$

Substituting $\tan x = -\frac{5}{12}$ in the formula, we obtain,

$$\sec^2 x = 1 + \left(-\frac{5}{12}\right)^2$$

$$\sec^2 x = 1 + \frac{25}{144}$$

$$= \frac{169}{144}$$

$$\sec x = \pm \frac{13}{12}$$

Since x lies in the 2^{nd} quadrant, the value of $\sec x$ will be negative.

$$\sec x = -\frac{13}{12}$$

$$\text{Therefore, } \cos x = \frac{1}{\sec x}$$

$$= -\frac{12}{13}$$

$$\text{Now, } \tan x = \frac{\sin x}{\cos x}$$

$$\text{Therefore, } \sin x = \tan x \cos x$$

$$\text{Hence we have, } \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right)$$

$$= \left(\frac{5}{13}\right)$$

And

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \frac{13}{5}$$

Question 6

Find the value of the trigonometric function $\sin 765^\circ$.

Solution

Ans: We know that the values of $\sin x$ repeat after an interval of 2π or 360° .

Therefore we can write,

$$\sin 765^\circ = \sin (2 \times 360^\circ + 45^\circ)$$

$$= \sin 45^\circ$$

$$= \frac{1}{\sqrt{2}}.$$

Question 7

Find the value of the trigonometric function $\operatorname{cosec}(-1410^\circ)$

Solution

Ans: We know that the values of $\operatorname{cosec} x$ repeat after an interval of 2π or 360° .

Therefore we can write,

$$\operatorname{cosec}(-1410^\circ) = \operatorname{cosec}(-1410^\circ + 4 \times 360^\circ)$$

$$= \operatorname{cosec}(-1410^\circ + 1440^\circ)$$

$$= \operatorname{cosec}30^\circ$$

$$= 2$$

Question 8

Find the value of the trigonometric function $\tan \frac{19\pi}{3}$.

Solution

Ans: We know that the values of $\tan x$ repeat after an interval of π or

$$180^\circ$$

.

Therefore we can write,

$$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

$$= \tan \left(6\pi + \frac{\pi}{3} \right)$$

$$= \tan \frac{\pi}{3}$$

$$= \sqrt{3}$$

Question 9

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

Solution

Ans: We know that the values of $\sin x$ repeat after an interval of 2π or 360° .

Therefore we can write,

$$\begin{aligned}\sin\left(-\frac{11\pi}{3}\right) &= \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right) \\ &= \sin\left(\frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

Question 10

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Solution

Ans: We know that the values of $\cot x$ repeat after an interval of π or

$$180^\circ$$

.

Therefore we can write,

$$\begin{aligned}\cot\left(-\frac{15\pi}{4}\right) &= \cot\left(-\frac{15\pi}{4} + 4\pi\right) \\ &= \cot\frac{\pi}{4} \\ &= 1\end{aligned}$$

Exercise 3.3

Question 1

Prove that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

Solution

Ans: Substituting the values of $\sin \frac{\pi}{6}, \cos \frac{\pi}{6}, \tan \frac{\pi}{4}$ on left hand side,

$$\begin{aligned}\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} &= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 \\&= \frac{1}{4} + \frac{1}{4} - 1 \\&= -\frac{1}{2} \\&= \text{R.H.S.}\end{aligned}$$

Hence proved.

Question 2

Prove that $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

Solution

Ans:

Substituting the values of $\sin \frac{\pi}{6}, \operatorname{cosec} \frac{7\pi}{6}, \cos \frac{\pi}{3}$ on left hand side,

$$\begin{aligned}\text{L.H.S.} &= 2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} \\&= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6}\right) \left(\frac{1}{2}\right)^2 \\&= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6}\right)^2 \left(\frac{1}{4}\right)\end{aligned}$$

$$= \frac{1}{2} + (-2)^2 \left(\frac{1}{4} \right)$$

Since cosec x repeat its value after an interval of 2π ,

$$\text{we have, } \operatorname{cosec} \frac{7\pi}{6} = -\operatorname{cosec} \frac{\pi}{6}$$

$$\text{L.H.S} = \frac{1}{2} + \frac{4}{4}$$

$$= \frac{3}{2}$$

$$= \text{R.H.S.}$$

Hence proved.

Question 3

$$\text{Prove that } \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$$

Solution

Ans:

Substituting the values of $\cot \frac{\pi}{6}, \operatorname{cosec} \frac{5\pi}{6}, \tan \frac{\pi}{6}$ on left hand side,

$$\begin{aligned} \text{L.H.S.} &= \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} \\ &= \left(\sqrt{3} \right)^2 + \operatorname{cosec} \left(\pi - \frac{\pi}{6} \right) + 3 \left(\frac{1}{\sqrt{3}} \right)^2 \\ &= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3} \end{aligned}$$

Since cosec x repeat its value after an interval of 2π ,

$$\text{we have, } \operatorname{cosec} \frac{5\pi}{6} = \operatorname{cosec} \frac{\pi}{6}$$

$$\text{L.H.S} = 3 + 2 + 1$$

$$= 6$$

= R.H.S.

Hence proved.

Question 4

Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Solution

Ans:

Substituting the values of $\sin\frac{3\pi}{4}, \cos\frac{\pi}{4}, \sec\frac{\pi}{3}$ on left hand side,

$$\begin{aligned}\text{L.H.S.} &= 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} \\ &= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2 \\ &= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8\end{aligned}$$

Since $\sin x$ repeats its value after an interval of 2π ,

we have, $\sin\frac{3\pi}{4} = \sin\frac{\pi}{4}$

$$\text{L.H.S.} = 1 + 1 + 8$$

$$= 10$$

= R.H.S.

Hence proved.

Question 5

Find the value of :

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$ **Solution**

Ans: We have,

$$\sin 75^\circ = \sin (45^\circ + 30^\circ)$$

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

Since we know that, $\sin (x+y) = \sin x \cos y + \cos x \sin y$

Therefore we have,

$$\sin 75^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Ans: We have,

$$\tan 15^\circ = \tan (45^\circ - 30^\circ)$$

$$= \frac{(\tan 45^\circ - \tan 30^\circ)}{1 + (\tan 45^\circ \tan 30^\circ)}$$

Since we know, $\tan (x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Therefore we have,

$$\tan 15^\circ = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

Further computing we have,

$$\tan 15^\circ = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1}$$

$$= 2-\sqrt{3}$$

Question 6

Prove that $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Solution

Ans: We know that, $\cos(x+y)=\cos x\cos y-\sin x\sin y$

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right]$$

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]$$

$$=\sin(x+y)$$

L.H.S = R.H.S.

Hence proved.

Question 7

Prove that $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Solution

Ans:

We know that, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

and $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

Using the above formula,

$$\text{L.H.S.} = \frac{\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}\right)}{\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}}$$

$$= \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)}$$

Substituting $\tan\frac{\pi}{4} = 1$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

$$= \text{R.H.S.}$$

Hence proved.

Question 8

Prove that $\frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

Solution

Ans: Observe that $\cos x$ repeats the same value after an interval 2π and $\sin x$ repeat the same value after an interval 2π .

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} \\ &= \frac{[-\cos x] [\cos x]}{(\sin x) (-\sin x)} \\ &= \frac{-\cos^2 x}{-\sin^2 x} \\ &= \cot^2 x \end{aligned}$$

Hence proved.

Question 9

Prove that,

$$\cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right] = 1$$

Solution

Ans:

We know that $\cot x$ repeats the same value after an interval 2π .

$$\begin{aligned} \text{L.H.S} &= \cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right] \\ &= \sin x \cos x [\tan x + \cot x] \end{aligned}$$

Substituting $\tan x = \frac{\sin x}{\cos x}$ and

$$\cot x = \frac{\cos x}{\sin x},$$

$$\text{L.H.S.} = \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

Question 10

Prove that $\sin (n+1)x \sin (n+2)x + \cos (n+1)x \cos (n+2)x = \cos x$

Solution

Ans:

We know that, $\cos (x-y) = \cos x \cos y + \sin x \sin y$

$$\text{L.H.S.} = \sin (n+1)x \sin (n+2)x + \cos (n+1)x \cos (n+2)x$$

$$= \cos [(n+1)x - (n+2)x]$$

$$= \cos (-x)$$

$$= \cos x$$

$$= \text{R.H.S.}$$

Question 11

Prove that $\cos \left(\frac{3\pi}{4} + x \right) - \cos \left(\frac{3\pi}{4} - x \right) = -\sqrt{2} \sin x$

Solution

Ans: We know that, $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$$\therefore \text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\}$$

$$= -2\sin\left(\frac{3\pi}{4}\right) \sin x$$

Since $\sin x$ repeats the same value after an interval 2π ,

$$\text{we have, } \sin\left(\frac{3\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right)$$

$$\text{Therefore, L.H.S.} = -2\sin\frac{\pi}{4} \sin x$$

$$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\sqrt{2} \sin x$$

$$= \text{R.H.S}$$

Hence proved.

Question 12

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution

$$\text{Ans: We know that, } \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\text{And } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\begin{aligned}
\therefore \text{L.H.S} &= \sin^2 6x - \sin^2 4x \\
&= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x) \\
&= \left[2\sin \left(\frac{6x+4x}{2} \right) \cos \left(\frac{6x-4x}{2} \right) \right] \left[2\cos \left(\frac{6x+4x}{2} \right) \sin \left(\frac{6x-4x}{2} \right) \right] \\
&= (2\sin 5x \cos x) (2\cos 5x \sin x)
\end{aligned}$$

Now we know that, $\sin 2x = 2\sin x \cos x$,

Therefore we have,

$$\begin{aligned}
\text{L.H.S} &= (2\sin 5x \cos 5x) (2\sin x \cos x) \\
&= \sin 10x \sin 2x \\
&= \text{R.H.S.}
\end{aligned}$$

Question 13

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution

Ans: We know that,

$$\cos A + \cos B = 2\cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \cos A - \cos B = -2\sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\begin{aligned}
\text{L.H.S.} &= \cos^2 2x - \cos^2 6x \\
&= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x) \\
&= \left[2\cos \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] \left[-2\sin \left(\frac{2x+6x}{2} \right) \sin \left(\frac{2x-6x}{2} \right) \right]
\end{aligned}$$

Further computing, we have,

$$\begin{aligned}
\text{L.H.S} &= [2\cos 4x \cos (-2x)] [-2\sin 4x \sin (-2x)] \\
&= [2\cos 4x \cos 2x] [-2\sin 4x (-\sin 2x)]
\end{aligned}$$

$$= (2\sin 4x \cos 4x) (2\sin 2x \cos 2x)$$

Now we know that, $\sin 2x = 2\sin x \cos x$

Therefore we have,

$$\text{L.H.S} = \sin 8x \sin 4x$$

$$= \text{R.H.S}$$

Hence proved.

Question 14

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution

Ans: We know that, $\sin A + \sin B = 2\sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$

$$\text{L.H.S} = \sin 2x + 2\sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2\sin 4x$$

$$= \left[2\sin \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] + 2\sin 4x$$

$$= 2\sin 4x \cos (-2x) + 2\sin 4x$$

Further computing,

$$\text{We have, L.H.S} = 2\sin 4x \cos 2x + 2\sin 4x$$

$$= 2\sin 4x (\cos 2x + 1)$$

Now we know that, $\cos 2x + 1 = 2\cos^2 x$

Therefore we have,

$$\text{L.H.S} = 2\sin 4x (2\cos^2 x)$$

$$= 4\cos^2 x \sin 4x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 15

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Solution

Ans: We know that, $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$$\text{L.H.S.} = \cot 4x (\sin 5x + \sin 3x)$$

$$= \frac{\cot 4x}{\sin 4x} \left[2\sin\left(\frac{5x+3x}{2}\right) \cos\left(\frac{5x-3x}{2}\right) \right]$$

$$= \left(\frac{\cos 4x}{\sin 4x} \right) [2\sin 4x \cos x]$$

$$= 2\cos 4x \cos x$$

Now also, we know that, $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$$\text{R.H.S} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cos x}{\sin x} \left[2\cos\left(\frac{5x+3x}{2}\right) \sin\left(\frac{5x-3x}{2}\right) \right]$$

$$= \frac{\cos x}{\sin x} [2\cos 4x \sin x]$$

$$= 2\cos 4x \cos x$$

Therefore, we can conclude that,

$$\text{L.H.S} = \text{R.H.S}$$

Hence proved.

Question 16

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Solution

Ans: We know that,

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\text{And } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\begin{aligned} \text{L.H.S} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\ &= \frac{-2 \sin \left(\frac{9x+5x}{2} \right) \cdot \sin \left(\frac{9x-5x}{2} \right)}{2 \cos \left(\frac{17x+3x}{2} \right) \cdot \sin \left(\frac{17x-3x}{2} \right)} \end{aligned}$$

(Following the formula)

$$\begin{aligned} &= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x} \\ &= -\frac{\sin 2x}{\cos 10x} \end{aligned}$$

R.H.S.

Hence proved.

Question 17

Prove that: $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Solution

Ans: We know that

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right),$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{Now, L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

(Using the formula)

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x \cos x}{2\cos 4x \cos x}$$

Further computing we have,

$$\text{L.H.S.} = \tan 4x$$

$$= \text{R.H.S.}$$

Question 18

$$\text{Prove that } \frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

Solution

Ans: We know that,

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$\begin{aligned}
&= \frac{2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)} \\
&= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} \\
&= \tan\left(\frac{x-y}{2}\right)
\end{aligned}$$

Therefore L.H.S=R.H.S

Hence proved.

Question 19

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Solution

Ans: We know that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right),$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\text{Now, L.H.S} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$\begin{aligned}
&= \frac{2\sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}
\end{aligned}$$

(Using the formula)

$$= \frac{\sin 2x}{\cos 2x}$$

$$=\tan 2x$$

Therefore, L.H.S = R.H.S.

Hence proved.

Question 20

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Solution

Ans: We know that,

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\text{And } \cos^2 A - \sin^2 A = \cos 2A$$

$$\begin{aligned} \text{L.H.S} &= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \\ &= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x} \\ &= \frac{2\cos 2x \sin(-x)}{-\cos 2x} \\ &= -2 \times (-\sin x) \end{aligned}$$

Therefore, we have,

$$\text{L.H.S} = 2\sin x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 21

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Solution

Ans: We know that,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{And, } \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{Now, L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

(Using the Formulas)

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

Further computing, we obtain,

$$\text{L.H.S.} = \frac{\cos 3x (2\cos x + 1)}{\sin 3x (2\cos x + 1)}$$

$$= \cot 3x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 22

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Solution

Ans:

$$\text{We know that, } \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\text{Now, L.H.S.} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x+x) (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x)$$

Further computing we obtain,

$$\text{L.H.S.} = \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

Question 23

$$\text{Prove that } \tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

Solution

Ans:

$$\text{We know that } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\text{L.H.S.} = \tan 4x$$

$$= \tan 2(2x)$$

$$= \frac{2 \tan 2x}{1 - \tan^2 (2x)}$$

(Using the formula)

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

Further computing, we obtain,

$$\text{L.H.S} = \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

= R.H.S.

Hence proved.

Question 24

Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Solution

Ans:

We know that, $\cos 2x = 1 - 2 \sin^2 x$

And $\sin 2x = 2 \sin x \cos x$

L.H.S. = $\cos 4x$

= $\cos 2(2x)$

= $1 - 2 \sin^2 2x$

$$= 1 - 2(2\sin x \cos x)^2$$

Further computing we get,

$$\text{L.H.S} = 1 - 8\sin^2 x \cos^2 x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 25

Prove that $\cos 6x = 32x\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution

Ans:

We know that, $\cos 3A = 4\cos^3 A - 3\cos A$

and $\cos 2x = 1 - 2\sin^2 x$

$$\text{L.H.S} = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4\cos^3 2x - 3\cos 2x$$

$$= 4 \left[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1) \right]$$

Further computing,

$$\text{L.H.S} = 4 \left[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x) \right] - 6\cos^2 x + 3$$

$$= 4 \left[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x) \right] - 6\cos^2 x + 3$$

$$= 4 \left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x \right] - 6\cos^2 x + 3 = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

Therefore we have,

$$\text{L.H.S} = \text{R.H.S}$$

Hence proved.

Miscellaneous Exercise

Question 1

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Solution

Ans: We know that $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$$\begin{aligned}\text{Now L.H.S.} &= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} \\ &= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13} - \frac{5\pi}{13}}{2}\right) \quad (\text{using the formula})\end{aligned}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{\pi}{13}\right)$$

Simplifying,

$$\begin{aligned}\text{L.H.S.} &= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right] \\ &= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2}\right)\cos\frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2}\right]\end{aligned}$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

Substituting $\cos\frac{\pi}{2}=0$, we get,

$$\text{L.H.S.} = 2\cos\frac{\pi}{13} \times 2 \times 0 \times \cos\frac{5\pi}{26}$$

$$= 0$$

= R.H.S

Hence proved.

Question 2

Prove that: $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

Solution

Ans: We know that, $\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$

And $\cos x - \cos y = -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$

Now,

$$\text{L.H.S.} = (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x \quad (\text{using the formula})$$

$$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$$

Simplifying we get,

$$\text{L.H.S.} = \cos (3x - x) - \cos 2x$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

$$= \text{R.H.S.}$$

Question 3

Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$

Solution

Ans: We know that, $\cos (x+y) = \cos x \cos y - \sin x \sin y$

and $L.H.S = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$$

Simplifying and using the formula,

$$L.H.S = 1 + 1 + 2\cos(x+y)$$

$$= 2[1 + \cos(x+y)]$$

$$= 2 \left[1 + 2\cos^2 \frac{(x+y)}{2} - 1 \right]$$

$$\text{since } 2\cos^2 \frac{(x+y)}{2} - 1 = \cos(x+y)$$

$$= 4\cos^2(x+y)$$

Therefore $L.H.S = R.H.S$

Hence proved.

Question 4

Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$

Solution

Ans: We know that, $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$L.H.S = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$$

Simplifying and using the formula we get,

$$L.H.S = 1 + 1 - 2[\cos(x-y)]$$

$$=2[1-\cos(x-y)]$$

$$=2\left[1-\left\{1-2\sin^2\left(\frac{x-y}{2}\right)\right\}\right]$$

$$\text{since } 1-2\sin^2\frac{(x-y)}{2}=\cos(x-y)$$

$$=4\sin^2\left(\frac{x-y}{2}\right)$$

Therefore L.H.S = R.H.S

Hence proved.

Question 5

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Solution

$$\text{Ans: We know that } \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$

Using the formula and simplifying,

$$= 2\sin\left(\frac{x+5x}{2}\right) \cdot \cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right) \cos\left(\frac{3x-7x}{2}\right)$$

$$= 2\cos 2x [\sin 3x + \sin 5x]$$

$$= 2\cos 2x \left[2\sin\left(\frac{3x+5x}{2}\right) \cdot \cos\left(\frac{3x-5x}{2}\right) \right]$$

$$= 2\cos 2x [2\sin 4x \cdot \cos(-x)]$$

Therefore we have,

$$\text{L.H.S} = 4 \cos 2x \sin 4x \cos x$$

$$= \text{R.H.S}$$

Question 6

Prove that:
$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Solution

Ans: We know that,

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

Using the formula and simplifying,

$$\begin{aligned} &= \frac{\left[2 \sin \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2 \sin \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]}{\left[2 \cos \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2 \cos \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]} \\ &= \frac{[2 \sin 6x \cdot \cos x] + [2 \sin 6x \cdot \cos 3x]}{[2 \cos 6x \cdot \cos x] + [2 \cos 6x \cdot \cos 3x]} \\ &= \frac{2 \sin 6x [\cos x + \cos 3x]}{2 \cos 6x [\cos x + \cos 3x]} \\ &= \tan 6x \end{aligned}$$

Therefore L.H.S = R.H.S

Hence proved.

Question 7

Prove that:

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

Solution

Ans: We know that,

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \sin A - \sin B = 2 \sin \left(\frac{A-B}{2} \right) \cdot \cos \left(\frac{A+B}{2} \right)$$

$$\text{L.H.S.} = \sin 3x + \sin 2x - \sin x$$

$$= \sin 3x + \left[2 \cos \left(\frac{2x+x}{2} \right) \sin \left(\frac{2x-x}{2} \right) \right]$$

$$= \sin 3x + \left[2 \cos \left(\frac{3x}{2} \right) \sin \left(\frac{x}{2} \right) \right]$$

Since we know that, $\sin 2x = 2 \sin x \cos x$

$$\text{L.H.S.} = 2 \sin \frac{3x}{2} \cdot \cos \frac{3x}{2} + 2 \cos \frac{3x}{2} \sin \frac{x}{2}$$

$$= 2 \cos \left(\frac{3x}{2} \right) \left[\sin \left(\frac{3x}{2} \right) + \sin \left(\frac{x}{2} \right) \right]$$

$$= 2 \cos \left(\frac{3x}{2} \right) \left[2 \sin \left\{ \frac{\left(\frac{3x}{2} \right) + \left(\frac{x}{2} \right)}{2} \right\} \cos \left\{ \frac{\left(\frac{3x}{2} \right) - \left(\frac{x}{2} \right)}{2} \right\} \right]$$

$$= 2 \cos \left(\frac{3x}{2} \right) \cdot 2 \sin x \cos \left(\frac{x}{2} \right)$$

Therefore

$$\text{L.H.S.} = 4 \sin x \cos \left(\frac{x}{2} \right) \cos \left(\frac{3x}{2} \right)$$

=R.H.S

Question 8

Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if $\tan x = -\frac{4}{3}$, x lies in 2nd quadrant.

Solution

Ans: Here, x is in 2nd quadrant.

Therefore,

$$\frac{\pi}{2} < x < \pi$$

$$\text{i.e., } \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

hence $\frac{x}{2}$ lies in 1st quadrant.

Therefore,

$$\sin \frac{x}{2}, \cos \frac{x}{2}$$

and $\tan \frac{x}{2}$ are positive.

$$\text{Given that } \tan x = -\frac{4}{3}$$

We know that, $\sec^2 x = 1 + \tan^2 x$

$$\sec^2 x = 1 + \tan^2 x$$

$$= 1 + \left(-\frac{4}{3}\right)^2$$

$$= \frac{25}{9}$$

As

x

is in 2nd quadrant, $\sec x$ is negative.

Therefore , $\sec x = -\frac{5}{3}$

Then $\cos x = -\frac{3}{5}$

Now we know that, $2\cos^2 \frac{x}{2} = \cos x + 1$

Computing we get, $2\cos^2 \frac{x}{2} = \frac{2}{5}$

Hence

$$\cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore substituting $\cos \frac{x}{2} = \frac{1}{\sqrt{5}}$ and computing ,

$$\sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

Hence,

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$= 2$$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$

are $\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 2$.

Question 9

Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if $\cos x = -\frac{1}{3}$, x lies in 3rd quadrant.

Solution

Ans: Here, x is in 3rd quadrant.

Therefore,

$$\pi x \frac{3\pi}{2}$$

$$\text{i.e., } \frac{\pi}{2} x \frac{3\pi}{4}$$

hence $\frac{x}{2}$ lies in 2nd quadrant.

Therefore, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are negative and $\sin \frac{x}{2}$ is positive.

$$\text{Given that } \cos x = -\frac{1}{3}$$

$$\text{Now we know that, } 2\cos^2 \frac{x}{2} = \cos x + 1$$

$$\text{Computing we get, } 2\cos^2 \frac{x}{2} = \frac{2}{3}$$

$$\text{Hence } \cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$$

$$\text{Now we know that, } \sin^2 x + \cos^2 x = 1$$

$$\text{Therefore substituting } \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \text{ and computing ,}$$

$$\sin \frac{x}{2} = \sqrt{\frac{2}{3}}$$

Hence,

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$= -\sqrt{2}$$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$

$$\text{are } \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -\sqrt{2}.$$

Question 10

Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if $\sin x = \frac{1}{4}$, x lies in 2nd quadrant.

Solution

Ans: Here, x lies in 2nd quadrant.

Therefore,

$$\frac{\pi}{2} < x < \pi$$

$$\text{i.e., } \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

hence $\frac{x}{2}$ lies in 1st quadrant.

Therefore, $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are positive.

$$\text{Given that } \sin x = \frac{1}{4}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore substituting $\sin x = \frac{1}{4}$ and computing ,

$$\cos x = -\frac{\sqrt{15}}{4}$$

since x lies in 2nd quadrant, $\cos x$ is negative.

Now we know that, $2\sin^2 \frac{x}{2} = 1 - \cos x$

Computing we get, $2\sin^2 \frac{x}{2} = 1 + \frac{\sqrt{15}}{4}$

$$\text{Hence } \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore substituting $\sin \frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}}$ and computing ,

$$\cos \frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

Hence ,

$$\begin{aligned} \tan \frac{x}{2} &= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \\ &= \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}} \\ &= 4+\sqrt{15} \end{aligned}$$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$

are $\sqrt{\frac{4+\sqrt{15}}{8}}, \sqrt{\frac{4-\sqrt{15}}{8}}, 4+\sqrt{15}$.