

Chapter 3: Trigonometric Functions Miscellaneous Exercise

NCERT · Class 11 · Maths

Miscellaneous Exercise

Question 1

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Solution

Ans: We know that $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$$\text{Now L.H.S.} = 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13} - \frac{5\pi}{13}}{2}\right) \quad (\text{using the formula})$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{\pi}{13}\right)$$

Simplifying,

$$\text{L.H.S.} = 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2}\right)\cos\frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

Substituting $\cos\frac{\pi}{2}=0$, we get,

$$\text{L.H.S.} = 2 \cos \frac{\pi}{13} \times 2 \times 0 \times \cos \frac{5\pi}{26}$$

$$= 0$$

$$= \text{R.H.S.}$$

Hence proved.

Question 2

Prove that: $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

Solution

Ans: We know that, $\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$

And $\cos x - \cos y = -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$

Now,

$$\text{L.H.S.} = (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x \quad (\text{using the formula})$$

$$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$$

Simplifying we get,

$$\text{L.H.S.} = \cos (3x - x) - \cos 2x$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

$$= \text{R.H.S.}$$

Question 3

Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$

Solution

Ans: We know that, $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$\text{and L.H.S} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$$

Simplifying and using the formula,

$$\text{L.H.S} = 1 + 1 + 2\cos(x+y)$$

$$= 2[1 + \cos(x+y)]$$

$$= 2 \left[1 + 2\cos^2 \frac{(x+y)}{2} - 1 \right]$$

$$\text{since } 2\cos^2 \frac{(x+y)}{2} - 1 = \cos(x+y)$$

$$= 4\cos^2(x+y)$$

Therefore L.H.S = R.H.S

Hence proved.

Question 4

$$\text{Prove that: } (\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$$

Solution

Ans: We know that, $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$\text{L.H.S} = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2 [\cos x \cos y + \sin x \sin y]$$

Simplifying and using the formula we get,

$$\text{L.H.S} = 1 + 1 - 2 [\cos (x-y)]$$

$$= 2 [1 - \cos (x-y)]$$

$$= 2 \left[1 - \left\{ 1 - 2\sin^2 \left(\frac{x-y}{2} \right) \right\} \right]$$

$$\text{since } 1 - 2\sin^2 \frac{(x-y)}{2} = \cos (x-y)$$

$$= 4\sin^2 \left(\frac{x-y}{2} \right)$$

Therefore L.H.S = R.H.S

Hence proved.

Question 5

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Solution

$$\text{Ans: We know that } \sin A + \sin B = 2\sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{L.H.S.} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$

Using the formula and simplifying,

$$= 2\sin \left(\frac{x+5x}{2} \right) \cdot \cos \left(\frac{x-5x}{2} \right) + 2\sin \left(\frac{3x+7x}{2} \right) \cos \left(\frac{3x-7x}{2} \right)$$

$$= 2\cos 2x [\sin 3x + \sin 5x]$$

$$= 2\cos 2x \left[2\sin \left(\frac{3x+5x}{2} \right) \cdot \cos \left(\frac{3x-5x}{2} \right) \right]$$

$$= 2\cos 2x [2\sin 4x \cdot \cos (-x)]$$

Therefore we have,

$$\text{L.H.S} = 4\cos 2x \sin 4x \cos x$$

$$= \text{R.H.S}$$

Question 6

Prove that: $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Solution

Ans: We know that,

$$\sin A + \sin B = 2\sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \cos A + \cos B = 2\cos \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

Using the formula and simplifying,

$$\begin{aligned} &= \frac{\left[2\sin \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2\sin \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]}{\left[2\cos \left(\frac{7x+5x}{2} \right) \cdot \cos \left(\frac{7x-5x}{2} \right) \right] + \left[2\cos \left(\frac{9x+3x}{2} \right) \cdot \cos \left(\frac{9x-3x}{2} \right) \right]} \\ &= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 6x]} \end{aligned}$$

$$\begin{aligned}
 &= \frac{2\sin 6x [\cos x + \cos 3x]}{2\cos 6x [\cos x + \cos 3x]} \\
 &= \tan 6x
 \end{aligned}$$

Therefore L.H.S = R.H.S

Hence proved.

Question 7

Prove that:

$$\sin 3x + \sin 2x - \sin x = 4\sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

Solution

Ans: We know that,

$$\sin A + \sin B = 2\sin \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \sin A - \sin B = 2\sin \left(\frac{A-B}{2} \right) \cdot \cos \left(\frac{A+B}{2} \right)$$

$$\text{L.H.S.} = \sin 3x + \sin 2x - \sin x$$

$$= \sin 3x + \left[2\cos \left(\frac{2x+x}{2} \right) \sin \left(\frac{2x-x}{2} \right) \right]$$

$$= \sin 3x + \left[2\cos \left(\frac{3x}{2} \right) \sin \left(\frac{x}{2} \right) \right]$$

Since we know that, $\sin 2x = 2\sin x \cos x$

$$\text{L.H.S} = 2\sin \frac{3x}{2} \cdot \cos \frac{3x}{2} + 2\cos \frac{3x}{2} \sin \frac{x}{2}$$

$$= 2\cos \left(\frac{3x}{2} \right) \left[\sin \left(\frac{3x}{2} \right) + \sin \left(\frac{x}{2} \right) \right]$$

$$\begin{aligned}
 &= 2\cos\left(\frac{3x}{2}\right) \left[2\sin\left\{\frac{\left(\frac{3x}{2}\right) + \left(\frac{x}{2}\right)}{2}\right\} \cos\left\{\frac{\left(\frac{3x}{2}\right) - \left(\frac{x}{2}\right)}{2}\right\} \right] \\
 &= 2\cos\left(\frac{3x}{2}\right) \cdot 2\sin x \cos\left(\frac{x}{2}\right)
 \end{aligned}$$

Therefore

$$\begin{aligned}
 \text{L.H.S} &= 4\sin x \cos\left(\frac{x}{2}\right) \cos\left(\frac{3x}{2}\right) \\
 &= \text{R.H.S}
 \end{aligned}$$

Question 8

Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\tan x = -\frac{4}{3}$, x lies in 2nd quadrant.

Solution

Ans: Here, x is in 2nd quadrant.

Therefore,

$$\begin{aligned}
 &\frac{\pi}{2} < x < \pi \\
 \text{i.e., } &\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}
 \end{aligned}$$

hence $\frac{x}{2}$ lies in 1st quadrant.

Therefore,

$$\sin\frac{x}{2}, \cos\frac{x}{2}$$

and $\tan\frac{x}{2}$ are positive.

$$\text{Given that } \tan x = -\frac{4}{3}$$

We know that, $\sec^2 x = 1 + \tan^2 x$

$$\sec^2 x = 1 + \tan^2 x$$

$$= 1 + \left(-\frac{4}{3}\right)^2$$

$$= \frac{25}{9}$$

As

x

is in 2nd quadrant, $\sec x$ is negative.

$$\text{Therefore, } \sec x = -\frac{5}{3}$$

$$\text{Then } \cos x = -\frac{3}{5}$$

$$\text{Now we know that, } 2\cos^2 \frac{x}{2} = \cos x + 1$$

$$\text{Computing we get, } 2\cos^2 \frac{x}{2} = \frac{2}{5}$$

Hence

$$\cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$

$$\text{Now we know that, } \sin^2 x + \cos^2 x = 1$$

$$\text{Therefore substituting } \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \text{ and computing,}$$

$$\sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

Hence,

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$= 2$$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$
are $\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, 2$.

Question 9

Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if $\cos x = -\frac{1}{3}$, x lies in 3rd quadrant.

Solution

Ans: Here, x is in 3rd quadrant.

Therefore,

$$\pi < x < \frac{3\pi}{2}$$

$$\text{i.e., } \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

hence $\frac{x}{2}$ lies in 2nd quadrant.

Therefore, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are negative and $\sin \frac{x}{2}$ is positive.

$$\text{Given that } \cos x = -\frac{1}{3}$$

$$\text{Now we know that, } 2\cos^2 \frac{x}{2} = \cos x + 1$$

$$\text{Computing we get, } 2\cos^2 \frac{x}{2} = \frac{2}{3}$$

$$\text{Hence } \cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$$

$$\text{Now we know that, } \sin^2 x + \cos^2 x = 1$$

Therefore substituting $\cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$ and computing ,

$$\sin \frac{x}{2} = \sqrt{\frac{2}{3}}$$

Hence,

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$= -\sqrt{2}$$

Thus, the respective values of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, $\tan \frac{x}{2}$

$$\text{are } \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -\sqrt{2}.$$

Question 10

Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if $\sin x = \frac{1}{4}$, x lies in 2nd quadrant.

Solution

Ans: Here, x lies in 2nd quadrant.

Therefore,

$$\frac{\pi}{2} < x < \pi$$

$$\text{i.e., } \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

hence $\frac{x}{2}$ lies in 1st quadrant.

Therefore, $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are positive.

$$\text{Given that } \sin x = \frac{1}{4}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore substituting $\sin x = \frac{1}{4}$ and computing ,

$$\cos x = -\frac{\sqrt{15}}{4}$$

since x lies in 2nd quadrant, $\cos x$ is negative.

Now we know that, $2\sin^2 \frac{x}{2} = 1 - \cos x$

Computing we get, $2\sin^2 \frac{x}{2} = 1 + \frac{\sqrt{15}}{4}$

$$\text{Hence } \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

Now we know that, $\sin^2 x + \cos^2 x = 1$

Therefore substituting $\sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}}$ and computing ,

$$\cos \frac{x}{2} = \sqrt{\frac{4 - \sqrt{15}}{8}}$$

Hence ,

$$\begin{aligned} \tan \frac{x}{2} &= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \\ &= \frac{\sqrt{4 + \sqrt{15}}}{\sqrt{4 - \sqrt{15}}} \\ &= 4 + \sqrt{15} \end{aligned}$$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$

are $\sqrt{\frac{4 + \sqrt{15}}{8}}, \sqrt{\frac{4 - \sqrt{15}}{8}}, 4 + \sqrt{15}$.