

Chapter 3: Trigonometric Functions Exercise 3.4

NCERT · Class 11 · Maths

Questions

Question 1

$$\tan x = \sqrt{3}$$

Solution

It is given that

$$\tan x = \sqrt{3}$$

We know that

$$\tan \frac{\pi}{3} = \sqrt{3}$$

It can be written as

$$\tan \left(\frac{4\pi}{3} \right) = \tan \left(\pi + \frac{\pi}{3} \right)$$

So we get

$$= \tan \frac{\pi}{3} = \sqrt{3}$$

Hence, the principal solutions are $x = \pi/3$ and $4\pi/3$

$$\tan x = \tan \frac{\pi}{3}$$

We get

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Question 2

$$\sec x = 2$$

Solution

It is given that

$$\sec x = 2$$

We know that

$$\sec \frac{\pi}{3} = 2$$

It can be written as

$$\sec \frac{5\pi}{3} = \sec \left(2\pi - \frac{\pi}{3} \right)$$

So we get

$$= \sec \frac{\pi}{3} = 2$$

Hence, the principal solutions are $x = \pi/3$ and $5\pi/3$.

$$\sec x = \sec \frac{\pi}{3}$$

We know that $\sec x = 1/\cos x$

$$\cos x = \cos \frac{\pi}{3}$$

So we get

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Question 3

$$\cot x = -\sqrt{3}$$

Solution

It is given that

$$\cot x = -\sqrt{3}$$

We know that

$$\cot \frac{\pi}{6} = \sqrt{3}$$

It can be written as

$$\cot \left(\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

And

$$\cot \left(2\pi - \frac{\pi}{6} \right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

So we get

$$\cot \frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot \frac{11\pi}{6} = -\sqrt{3}$$

Hence, the principal solutions are $x = 5\pi/6$ and $11\pi/6$.

$$\cot x = \cot \frac{5\pi}{6}$$

We know that $\cot x = 1/\tan x$

$$\tan x = \tan \frac{5\pi}{6}$$

So we get

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Question 4

$$\operatorname{cosec} x = -2$$

Solution

It is given that

$$\operatorname{cosec} x = -2$$

We know that

$$\operatorname{cosec} \frac{\pi}{6} = 2$$

It can be written as

$$\operatorname{cosec} \left(\pi + \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

And

$$\operatorname{cosec} \left(2\pi - \frac{\pi}{6} \right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

So we get

$$\operatorname{cosec} \frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec} \frac{11\pi}{6} = -2$$

Hence, the principal solutions are $x = 7\pi/6$ and $11\pi/6$.

$$\operatorname{cosec} x = \operatorname{cosec} \frac{7\pi}{6}$$

We know that $\operatorname{cosec} x = 1/\sin x$

$$\sin x = \sin \frac{7\pi}{6}$$

So we get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Find the general solution for each of the following equations:

Question 5

$$\cos 4x = \cos 2x$$

Solution

It is given that

$$\cos 4x = \cos 2x$$

We can write it as

$$\cos 4x - \cos 2x = 0$$

Using the formula

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

We get

$$-2 \sin\left(\frac{4x+2x}{2}\right) \sin\left(\frac{4x-2x}{2}\right) = 0$$

By further simplification

$$\sin 3x \sin x = 0$$

We can write it as

$$\sin 3x = 0 \text{ or } \sin x = 0$$

By equating the values

$$3x = n\pi \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$$

We get

$$x = n\pi/3 \text{ or } x = n\pi, \text{ where } n \in \mathbb{Z}$$

Question 6

$$\cos 3x + \cos x - \cos 2x = 0$$

Solution

It is given that

$$\cos 3x + \cos x - \cos 2x = 0$$

We can write it as

$$2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$$

Using the formula

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

We get

$$2 \cos 2x \cos x - \cos 2x = 0$$

By further simplification

$$\cos 2x (2 \cos x - 1) = 0$$

We can write it as

$$\cos 2x = 0 \text{ or } 2 \cos x - 1 = 0$$

$$\cos 2x = 0 \text{ or } \cos x = 1/2$$

By equating the values

$$2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

We get

$$x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Question 7

$$\sin 2x + \cos x = 0$$

Solution

It is given that

$$\sin 2x + \cos x = 0$$

We can write it as

$$2 \sin x \cos x + \cos x = 0$$

$$\cos x (2 \sin x + 1) = 0$$

$$\cos x = 0 \text{ or } 2 \sin x + 1 = 0$$

$$\text{Let } \cos x = 0$$

$$\cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in \mathbb{Z}$$

$$2 \sin x + 1 = 0$$

So we get

$$\sin x = \frac{-1}{2} = -\sin \frac{\pi}{6}$$

We can write it as

$$\begin{aligned} &= \sin \left(\pi + \frac{\pi}{6} \right) = \sin \left(\pi + \frac{\pi}{6} \right) \\ &= \sin \frac{7\pi}{6} \end{aligned}$$

We get

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$(2n+1)\frac{\pi}{2} \text{ or } n\pi + (-1)^n \frac{7\pi}{6}, n \in \mathbb{Z}$$

Question 8

$$\sec^2 2x = 1 - \tan 2x$$

Solution

It is given that

$$\sec^2 2x = 1 - \tan 2x$$

We can write it as

$$1 + \tan^2 2x = 1 - \tan 2x$$

$$\tan^2 2x + \tan 2x = 0$$

Taking common terms

$$\tan 2x (\tan 2x + 1) = 0$$

Here

$$\tan 2x = 0 \text{ or } \tan 2x + 1 = 0$$

$$\text{If } \tan 2x = 0$$

$$\tan 2x = \tan 0$$

We get

$$2x = n\pi + 0, \text{ where } n \in \mathbb{Z}$$

$$x = n\pi/2, \text{ where } n \in \mathbb{Z}$$

$$\tan 2x + 1 = 0$$

We can write it as

$$\tan 2x = -1$$

So we get

$$= -\tan \frac{\pi}{4} = \tan \left(\pi - \frac{\pi}{4} \right)$$

$$= \tan \frac{3\pi}{4}$$

Here

$$2x = n\pi + 3\pi/4, \text{ where } n \in \mathbb{Z}$$

$$x = n\pi/2 + 3\pi/8, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is $n\pi/2$ or $n\pi/2 + 3\pi/8$, $n \in \mathbb{Z}$.

Question 9

$$\sin x + \sin 3x + \sin 5x = 0$$

Solution

It is given that

$$\sin x + \sin 3x + \sin 5x = 0$$

We can write it as

$$(\sin x + \sin 5x) + \sin 3x = 0$$

Using the formula

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\left[2 \sin \left(\frac{x+5x}{2} \right) \cos \left(\frac{x-5x}{2} \right) \right] + \sin 3x = 0$$

By further calculation

$$2 \sin 3x \cos (-2x) + \sin 3x = 0$$

It can be written as

$$2 \sin 3x \cos 2x + \sin 3x = 0$$

By taking out the common terms

$$\sin 3x (2 \cos 2x + 1) = 0$$

Here

$$\sin 3x = 0 \text{ or } 2 \cos 2x + 1 = 0$$

$$\text{If } \sin 3x = 0$$

$$3x = n\pi, \text{ where } n \in \mathbb{Z}$$

We get

$$x = n\pi/3, \text{ where } n \in \mathbb{Z}$$

$$\text{If } 2 \cos 2x + 1 = 0$$

$$\cos 2x = -1/2$$

By further simplification

$$= -\cos \pi/3$$

$$= \cos (\pi - \pi/3)$$

So we get

$$\cos 2x = \cos 2\pi/3$$

Here

$$2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Dividing by 2

$$x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Hence, the general solution is

$$\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$
