

Chapter 3: Trigonometric Functions Exercise 3.3

NCERT · Class 11 · Maths

Exercise 3.3

Question 1

Prove that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

Solution

Ans: Substituting the values of $\sin \frac{\pi}{6}, \cos \frac{\pi}{6}, \tan \frac{\pi}{4}$ on left hand side,

$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1$$

$$= -\frac{1}{2}$$

= R.H.S.

Hence proved.

Question 2

Prove that $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

Solution

Ans:

Substituting the values of $\sin \frac{\pi}{6}, \operatorname{cosec} \frac{7\pi}{6}, \cos \frac{\pi}{3}$ on left hand side,

$$\text{L.H.S.} = 2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3}$$

$$\begin{aligned}
&= 2 \left(\frac{1}{2} \right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6} \right) \left(\frac{1}{2} \right)^2 \\
&= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6} \right)^2 \left(\frac{1}{4} \right) \\
&= \frac{1}{2} + (-2)^2 \left(\frac{1}{4} \right)
\end{aligned}$$

Since cosec x repeat its value after an interval of 2π ,

$$\text{we have, } \operatorname{cosec} \frac{7\pi}{6} = -\operatorname{cosec} \frac{\pi}{6}$$

$$\begin{aligned}
\text{L.H.S} &= \frac{1}{2} + \frac{4}{4} \\
&= \frac{3}{2}
\end{aligned}$$

$$= \text{R.H.S.}$$

Hence proved.

Question 3

$$\text{Prove that } \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$$

Solution

Ans:

Substituting the values of $\cot \frac{\pi}{6}, \operatorname{cosec} \frac{5\pi}{6}, \tan \frac{\pi}{6}$ on left hand side,

$$\begin{aligned}
\text{L.H.S.} &= \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} \\
&= \left(\sqrt{3} \right)^2 + \operatorname{cosec} \left(\pi - \frac{\pi}{6} \right) + 3 \left(\frac{1}{\sqrt{3}} \right)^2 \\
&= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3}
\end{aligned}$$

Since cosec x repeat its value after an interval of 2π ,

we have, $\operatorname{cosec} \frac{5\pi}{6} = \operatorname{cosec} \frac{\pi}{6}$

$$\text{L.H.S} = 3 + 2 + 1$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

Question 4

Prove that $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$

Solution

Ans:

Substituting the values of $\sin \frac{3\pi}{4}, \cos \frac{\pi}{4}, \sec \frac{\pi}{3}$ on left hand side,

$$\begin{aligned}\text{L.H.S.} &= 2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} \\ &= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2 \\ &= 2\left\{\sin \frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8\end{aligned}$$

Since $\sin x$ repeats its value after an interval of 2π ,

we have, $\sin \frac{3\pi}{4} = \sin \frac{\pi}{4}$

$$\text{L.H.S} = 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S.}$$

Hence proved.

Question 5

Find the value of :

(i) $\sin 75^\circ$ (ii) $\tan 15^\circ$ **Solution**

Ans: We have,

$$\sin 75^\circ = \sin (45^\circ + 30^\circ)$$

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

Since we know that, $\sin (x+y) = \sin x \cos y + \cos x \sin y$

Therefore we have,

$$\sin 75^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Ans: We have,

$$\tan 15^\circ = \tan (45^\circ - 30^\circ)$$

$$= \frac{(\tan 45^\circ - \tan 30^\circ)}{1 + (\tan 45^\circ \tan 30^\circ)}$$

$$\text{Since we know, } \tan (x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Therefore we have,

$$\tan 15^\circ = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \left(\frac{1}{\sqrt{3}} \right)}$$

$$\begin{aligned}
 &= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} \\
 &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \\
 &= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}
 \end{aligned}$$

Further computing we have,

$$\begin{aligned}
 \tan 15^\circ &= \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2} \\
 &= \frac{4-2\sqrt{3}}{3-1} \\
 &= 2-\sqrt{3}
 \end{aligned}$$

Question 6

Prove that $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Solution

Ans: We know that, $\cos(x+y)=\cos x\cos y-\sin x\sin y$

$$\begin{aligned}
 \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) &= \cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right] \\
 &= \cos\left[\frac{\pi}{2}-(x+y)\right] \\
 &= \sin(x+y)
 \end{aligned}$$

L.H.S = R.H.S.

Hence proved.

Question 7

Prove that $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Solution

Ans:

We know that, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

and $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

Using the above formula,

$$\text{L.H.S.} = \frac{\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}\right)}{\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}}$$

$$= \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)}$$

Substituting $\tan\frac{\pi}{4} = 1$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

= R.H.S.

Hence proved.

Question 8

Prove that
$$\frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

Solution

Ans: Observe that $\cos x$ repeats the same value after an interval 2π and $\sin x$ repeat the same value after an interval 2π .

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} \\ &= \frac{[-\cos x] [\cos x]}{(\sin x) (-\sin x)} \\ &= \frac{-\cos^2 x}{-\sin^2 x} \\ &= \cot^2 x \end{aligned}$$

Hence proved.

Question 9

Prove that,

$$\cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right] = 1$$

Solution

Ans:

We know that $\cot x$ repeats the same value after an interval 2π .

$$\text{L.H.S} = \cos\left(\frac{3\pi}{2}+x\right) \cos(2\pi+x) \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x) \right]$$

$$= \sin x \cos x [\tan x + \cot x]$$

$$\text{Substituting } \tan x = \frac{\sin x}{\cos x} \text{ and}$$

$$\cot x = \frac{\cos x}{\sin x},$$

$$\text{L.H.S.} = \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

Question 10

Prove that $\sin (n+1)x \sin (n+2)x + \cos (n+1)x \cos (n+2)x = \cos x$

Solution

Ans:

We know that, $\cos (x-y) = \cos x \cos y + \sin x \sin y$

$$\text{L.H.S.} = \sin (n+1)x \sin (n+2)x + \cos (n+1)x \cos (n+2)x$$

$$= \cos [(n+1)x - (n+2)x]$$

$$= \cos (-x)$$

$$= \cos x$$

$$= \text{R.H.S}$$

Question 11

Prove that $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$

Solution

Ans: We know that , $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$$\therefore \text{L.H.S.} = \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right) + \left(\frac{3\pi}{4}-x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4}+x\right) - \left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$= -2\sin\left(\frac{3\pi}{4}\right) \sin x$$

Since $\sin x$ repeats the same value after an interval 2π ,

$$\text{we have, } \sin\left(\frac{3\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right)$$

$$\text{Therefore, L.H.S.} = -2\sin\frac{\pi}{4} \sin x$$

$$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\sqrt{2}\sin x$$

$$= \text{R.H.S}$$

Hence proved.

Question 12

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution

Ans: We know that, $\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$

And $\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$

$$\therefore \text{L.H.S} = \sin^2 6x - \sin^2 4x$$

$$= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)$$

$$= \left[2 \sin \left(\frac{6x+4x}{2} \right) \cos \left(\frac{6x-4x}{2} \right) \right] \left[2 \cos \left(\frac{6x+4x}{2} \right) \sin \left(\frac{6x-4x}{2} \right) \right]$$

$$= (2 \sin 5x \cos x) (2 \cos 5x \sin x)$$

Now we know that, $\sin 2x = 2 \sin x \cos x$,

Therefore we have,

$$\text{L.H.S} = (2 \sin 5x \cos 5x) (2 \sin x \cos x)$$

$$= \sin 10x \sin 2x$$

$$= \text{R.H.S.}$$

Question 13

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution

Ans: We know that,

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\text{And } \cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\text{L.H.S.} = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[2\cos\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] \left[-2\sin\left(\frac{2x+6x}{2}\right) \sin\left(\frac{2x-6x}{2}\right) \right]$$

Further computing, we have,

$$\text{L.H.S} = [2\cos 4x \cos (-2x)] [-2\sin 4x \sin (-2x)]$$

$$= [2\cos 4x \cos 2x] [-2\sin 4x (-\sin 2x)]$$

$$= (2\sin 4x \cos 4x) (2\sin 2x \cos 2x)$$

Now we know that, $\sin 2x = 2\sin x \cos x$

Therefore we have,

$$\text{L.H.S} = \sin 8x \sin 4x$$

$$= \text{R.H.S}$$

Hence proved.

Question 14

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution

$$\text{Ans: We know that, } \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S} = \sin 2x + 2\sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2\sin 4x$$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] + 2\sin 4x$$

$$= 2\sin 4x \cos (-2x) + 2\sin 4x$$

Further computing,

$$\text{We have, L.H.S} = 2\sin 4x \cos 2x + 2\sin 4x$$

$$= 2\sin 4x (\cos 2x + 1)$$

Now we know that, $\cos 2x + 1 = 2\cos^2 x$

Therefore we have,

$$\text{L.H.S} = 2\sin 4x (2\cos^2 x)$$

$$= 4\cos^2 x \sin 4x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 15

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Solution

Ans: We know that, $\sin A + \sin B = 2\sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$

$$\text{L.H.S.} = \cot 4x (\sin 5x + \sin 3x)$$

$$= \frac{\cot 4x}{\sin 4x} \left[2\sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right) \right]$$

$$= \left(\frac{\cos 4x}{\sin 4x} \right) [2\sin 4x \cos x]$$

$$= 2\cos 4x \cos x$$

Now also, we know that, $\sin A - \sin B = 2\cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$

$$\text{R.H.S} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cos x}{\sin x} \left[2\cos \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right) \right]$$

$$= \frac{\cos x}{\sin x} [2\cos 4x \sin x]$$

$$= 2\cos 4x \cos x$$

Therefore, we can conclude that,

L.H.S = R.H.S

Hence proved.

Question 16

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Solution

Ans: We know that,

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\text{And } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\begin{aligned}\text{L.H.S} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\ &= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}\end{aligned}$$

(Following the formula)

$$\begin{aligned}&= \frac{-2\sin 7x \cdot \sin 2x}{2\cos 10x \cdot \sin 7x} \\ &= -\frac{\sin 2x}{\cos 10x}\end{aligned}$$

R.H.S.

Hence proved.

Question 17

Prove that: $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Solution

Ans: We know that

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right),$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\text{Now, L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right)}{2 \cos \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right)}$$

(Using the formula)

$$= \frac{2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right)}{2 \cos \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right)}$$

$$= \frac{2 \sin 4x \cos x}{2 \cos 4x \cos x}$$

Further computing we have,

$$\text{L.H.S.} = \tan 4x$$

$$= \text{R.H.S.}$$

Question 18

Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

Solution

Ans: We know that,

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right),$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{2 \cos \left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right)}{2 \cos \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right)}$$

$$= \frac{\sin \left(\frac{x-y}{2} \right)}{\cos \left(\frac{x-y}{2} \right)}$$

$$= \tan \left(\frac{x-y}{2} \right)$$

Therefore L.H.S = R.H.S

Hence proved.

Question 19

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Solution

Ans: We know that

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right),$$

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\text{Now, L.H.S} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

(Using the formula)

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

Therefore, L.H.S = R.H.S.

Hence proved.

Question 20

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Solution

Ans: We know that,

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\text{And } \cos^2 A - \sin^2 A = \cos 2A$$

$$\text{L.H.S} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

Therefore , we have,

$$\text{L.H.S} = 2\sin x$$

= R.H.S.

Hence proved.

Question 21

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Solution

Ans: We know that,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{And, } \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{Now, L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

(Using the Formulas)

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

Further computing, we obtain,

$$\text{L.H.S} = \frac{\cos 3x (2\cos x + 1)}{\sin 3x (2\cos x + 1)}$$

$$= \cot 3x$$

= R.H.S.

Hence proved.

Question 22

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Solution

Ans:

We know that, $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

Now, L.H.S. = $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - (\cot 2x + \cot x) (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x)$$

Further computing we obtain,

$$\text{L.H.S.} = \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

Question 23

Prove that $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Solution

Ans:

We know that $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$$\text{L.H.S.} = \tan 4x$$

$$= \tan 2(2x)$$

$$= \frac{2 \tan 2x}{1 - \tan^2(2x)}$$

(Using the formula)

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

Further computing, we obtain,

$$\text{L.H.S} = \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

$$= \text{R.H.S.}$$

Hence proved.

Question 24

Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Solution

Ans:

We know that, $\cos 2x = 1 - 2 \sin^2 x$

And $\sin 2x = 2\sin x \cos x$

$$\text{L.H.S.} = \cos 4x$$

$$= \cos 2(2x)$$

$$= 1 - 2\sin^2 2x$$

$$= 1 - 2(2\sin x \cos x)^2$$

Further computing we get,

$$\text{L.H.S} = 1 - 8\sin^2 x \cos^2 x$$

$$= \text{R.H.S.}$$

Hence proved.

Question 25

Prove that $\cos 6x = 32x\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution

Ans:

We know that, $\cos 3A = 4\cos^3 A - 3\cos A$

and $\cos 2x = 1 - 2\sin^2 x$

$$\text{L.H.S} = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4\cos^3 2x - 3\cos 2x$$

$$= 4 \left[(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1) \right]$$

Further computing,

$$\text{L.H.S} = 4 \left[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x) \right] - 6\cos^2 x + 3$$

$$= 4 \left[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x) \right] - 6\cos^2 x + 3$$

$$=4 [8\cos^6 x - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3 = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$$

Therefore we have,

$$\text{L.H.S} = \text{R.H.S}$$

Hence proved.
