

Chapter 2: Relations And Functions Miscellaneous Exercise

NCERT · Class 11 · Maths

Questions

Question 1

The relation f is defined by

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

The relation g is defined by

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

Show that f is a function and g is not a function.

Solution

The given relation f is defined as

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

It is seen that for $0 \leq x < 3$,

$$f(x) = x^2 \text{ and for } 3 < x \leq 10,$$

$$f(x) = 3x$$

Also, at $x = 3$

$$f(x) = 3^2 = 9 \text{ or } f(x) = 3 \times 3 = 9$$

i.e., at $x = 3$, $f(x) = 9$ [Single image]

Hence, for $0 \leq x \leq 10$, the images of $f(x)$ are unique.

Therefore, the given relation is a function.

Now,

In the given relation g is defined as

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

It is seen that, for $x = 2$

$$g(x) = 2^2 = 4 \text{ and } g(x) = 3 \times 2 = 6$$

Thus, element 2 of the domain of the relation g corresponds to two different images, i.e., 4 and 6.

Therefore, this relation is not a function.

Question 2

If $f(x) = x^2$, find

$$\frac{f(1.1) - f(1)}{(1.1 - 1)}$$

Solution

Given,

$$f(x) = x^2$$

Hence,

$$\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Question 3

Find the domain of the function

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

Solution

Given function,

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It is clearly seen that the function f is defined for all real numbers except at $x = 6$ and $x = 2$, as the denominator becomes zero otherwise.

Therefore, the domain of f is $\mathbb{R} - \{2, 6\}$.

Question 4

Find the domain and the range of the real function f defined by $f(x) = \sqrt{x - 1}$.

Solution

Given real function,

$$f(x) = \sqrt{x - 1}$$

Clearly, $\sqrt{x - 1}$ is defined for $(x - 1) \geq 0$.

So, the function $f(x) = \sqrt{x - 1}$ is defined for $x \geq 1$.

Thus, the domain of f is the set of all real numbers greater than or equal to 1.

Domain of $f = [1, \infty)$.

Now,

$$\text{As } x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{x - 1} \geq 0$$

Thus, the range of f is the set of all real numbers greater than or equal to 0.

Range of $f = [0, \infty)$.

Question 5

Find the domain and the range of the real function f defined by $f(x) = |x - 1|$.

Solution

Given, a real function,

$$f(x) = |x - 1|$$

Clearly, the function $|x - 1|$ is defined for all real numbers.

Hence,

Domain of $f = \mathbf{R}$

Also, for $x \in \mathbf{R}$, $|x - 1|$ assumes all real numbers.

Therefore, the range of f is the set of all non-negative real numbers.

Question 6

Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$ be a function from $\mathbf{R}$ into $\mathbf{R}$. Determine the range of f .

Solution

Given function,

$$f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$

Substituting values and determining the images, we have

$$= \left\{ (0, 0), \left(\pm 0.5, \frac{1}{5} \right), \left(\pm 1, \frac{1}{2} \right), \left(\pm 1.5, \frac{9}{13} \right), \left(\pm 2, \frac{4}{5} \right), \left(3, \frac{9}{10} \right), \left(4, \frac{16}{17} \right), \dots \right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

Or,

We know that, for $x \in \mathbf{R}$,

$$x^2 \geq 0$$

Then,

$$x^2 + 1 \geq x^2$$

$$1 \geq x^2 / (x^2 + 1)$$

Therefore, the range of $f = [0, 1)$

Question 7

Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and f/g .

Solution

Given, the functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$f(x) = x + 1, g(x) = 2x - 3$$

Now,

$$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$$

$$\text{Thus, } (f + g)(x) = 3x - 2$$

$$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$$

$$\text{Thus, } (f - g)(x) = -x + 4$$

$$f/g(x) = f(x)/g(x), g(x) \neq 0, x \in \mathbb{R}$$

$$f/g(x) = x + 1 / 2x - 3, 2x - 3 \neq 0$$

$$\text{Thus, } f/g(x) = x + 1 / 2x - 3, x \neq 3/2$$

Question 8

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $f(x) = ax + b$, for some integers a, b . Determine a, b .

Solution

$$\text{Given, } f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$$

$$\text{And the function defined as, } f(x) = ax + b$$

$$\text{For } (1, 1) \in f$$

$$\text{We have, } f(1) = 1$$

$$\text{So, } a \times 1 + b = 1$$

$$a + b = 1 \dots (i)$$

$$\text{And for } (0, -1) \in f$$

$$\text{We have } f(0) = -1$$

$$a \times 0 + b = -1$$

$$b = -1$$

On substituting $b = -1$ in (i), we get

$$a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$$

Therefore, the values of a and b are 2 and -1 , respectively.

Question 9

Let R be a relation from N to N defined by $R = \{(a, b): a, b \in N \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in N$

(ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Solution

Given relation $R = \{(a, b): a, b \in N \text{ and } a = b^2\}$

(i) It can be seen that $2 \in N$; however, $2 \neq 2^2 = 4$

Thus, the statement “ $(a, a) \in R$, for all $a \in N$ ” is not true.

(ii) Its clearly seen that $(9, 3) \in N$ because $9, 3 \in N$ and $9 = 3^2$

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin N$

Thus, the statement “ $(a, b) \in R$, implies $(b, a) \in R$ ” is not true.

(iii) It's clearly seen that $(16, 4) \in R, (4, 2) \in R$ because $16, 4, 2 \in N$ and $16 = 4^2$ and $4 = 2^2$

Now, $16 \neq 2^2 = 4$; therefore, $(16, 2) \notin N$

Thus, the statement “ $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$ ” is not true.

Question 10

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B .

Justify your answer in each case.

Solution

Given,

$$A = \{1, 2, 3, 4\} \text{ and } B = \{1, 5, 9, 11, 15, 16\}$$

So,

$$A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$$

Also, given that

$$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

It's clearly seen that f is a subset of $A \times B$.

Therefore, f is a relation from A to B .

(ii) As the same first element, i.e., 2 corresponds to two different images (9 and 11), relation f is not a function.

Question 11

Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z : justify your answer.

Solution

Given, the relation f is defined as

$$f = \{(ab, a + b) : a, b \in Z\}$$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B .

$$\text{As } 2, 6, -2, -6 \in Z, (2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$$

$$\text{i.e., } (12, 8), (12, -8) \in f$$

It's clearly seen that the same first element, 12, corresponds to two different images (8 and -8).

Therefore, the relation f is not a function.

Question 12

Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \rightarrow \mathbf{N}$ be defined by $f(n)$ = the highest prime factor of n . Find the range of f .

Solution

Given,

$$A = \{9, 10, 11, 12, 13\}$$

Now, $f: A \rightarrow \mathbf{N}$ is defined as

$f(n)$ = The highest prime factor of n

So,

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

Thus, it can be expressed as

$f(9)$ = The highest prime factor of 9 = 3

$f(10)$ = The highest prime factor of 10 = 5

$f(11)$ = The highest prime factor of 11 = 11

$f(12)$ = The highest prime factor of 12 = 3

$f(13)$ = The highest prime factor of 13 = 13

The range of f is the set of all $f(n)$, where $n \in A$.

Therefore,

Range of $f = \{3, 5, 11, 13\}$
