

Chapter 15: Statistics Exercise 15.2

NCERT · Class 11 · Maths

Exercise 15.2

Question 1

Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12.

Solution

Ans: Given data: 6, 7, 10, 12, 13, 4, 8, 12.

Mean can be given as:

$$\bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8} = \frac{72}{8} = 9$$

The following table is obtained:

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	$6 - 9 = -3$	9
7	$7 - 9 = -2$	4
10	$10 - 9 = 1$	1
12	$12 - 9 = 3$	9
13	$13 - 9 = 4$	16
4	$4 - 9 = -5$	25

8	$8 - 9 = -1$	1
12	$12 - 9 = 3$	9
		74

Variance can be given as:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{8} \times 74$$

$$\sigma^2 = 9.25.$$

Therefore, mean is 9 and variance is 9.25.

Question 2

Find the mean and variance for the first n natural numbers.

Solution

Ans: The mean of first n natural numbers can be calculated as:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Variance,

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n \left(x_i - \left(\frac{n+1}{2} \right) \right)^2$$

As we know, $(a - b)^2 = a^2 - 2ab + b^2$. Hence:

$$\begin{aligned} &\Rightarrow \frac{1}{n} \sum_{i=1}^n x_i^2 - \frac{1}{n} \sum_{i=1}^n 2 \left(\frac{n+1}{2} \right) x_i + \frac{1}{n} \sum_{i=1}^n \left(\frac{n+1}{2} \right)^2 \\ &\Rightarrow \frac{1}{n} \frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{2} \right) \left[\frac{n(n+1)}{2} \right] + \frac{(n+1)^2}{4n} \times n \\ &\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4} \end{aligned}$$

Taking LCM, we get:

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

Taking $(n+1)$ as common, we get:

$$\begin{aligned} &\Rightarrow (n+1) \left[\frac{4n+2-3n-3}{12} \right] \\ &\Rightarrow \frac{(n+1)(n-1)}{12} \end{aligned}$$

Now, we know, $(a+b)(a-b) = a^2 - b^2$. Hence,

$$\Rightarrow \frac{n^2-1}{12}$$

Therefore, mean is $\frac{n+1}{2}$ and variance is $\frac{n^2-1}{12}$.

Question 3

Find the mean and variance for the first 10 multiples of 3.

Solution

Ans: First 10 multiples of 3 can be written as: 3, 9, 12, 15, 18, 21, 24, 27, 30. Hence, $n = 10$.

Mean,

$$\bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30}{10} = \frac{165}{10} = 16.5$$

The following table is obtained:

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	$3 - 16.5 = -13.5$	182.25
6	$6 - 16.5 = -10.5$	110.25
9	$9 - 16.5 = -7.5$	56.25
12	$12 - 16.5 = -4.5$	20.25
15	$15 - 16.5 = -1.5$	2.25
18	$18 - 16.5 = 1.5$	2.25
21	$21 - 16.5 = 4.5$	20.25
24	$24 - 16.5 = 7.5$	56.25
27	$27 - 16.5 = 10.5$	110.25
30	$30 - 16.5 = 13.5$	182.25
		742.5

Variance can be given as:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{10} \times 742.5$$

$$\sigma^2 = 74.25.$$

Therefore, mean is 16.5 and variance is 74.25.

Question 4

Find the mean and variance for the data:

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution

Ans: Calculating the given data we get the following table:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
6	2	12	$6 - 19 = -13$	169	338
10	4	40	$10 - 19 = -9$	81	324
14	7	98	$14 - 19 = -5$	25	175
18	12	216	$18 - 19 = -1$	1	12
24	8	192	$24 - 19 = 5$	25	200
28	4	112	$28 - 19 = 9$	81	324
30	3	90	$30 - 19 = 11$	121	363
	40	760			1736

Here,

$$N = 40$$

,

$$\sum_{i=1}^7 f_i x_i = 760$$

,

$$\sum_{i=1}^7 f_i (x_i - \bar{x})^2 = 1736$$

Mean,

$$\bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{N} = \frac{1}{40} \times 760 = 19$$

$$\text{Variance, } \sigma^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

Therefore, mean is 19 and variance is 43.4.

Question 5

Find the mean and variance for the data:

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution

Ans: Calculating the given data we get the following table:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	$92 - 100 = -8$	64	192
93	2	186	$93 - 100 = -7$	49	98
97	3	291	$97 - 100 = -3$	9	27
98	2	196	$98 - 100 = -2$	4	8
102	6	612	$102 - 100 = 2$	4	24
104	3	312	$104 - 100 = 4$	16	48
109	3	327	$109 - 100 = 9$	81	243
	22	2200			640

Here,

$$N = 22$$

,

$$\sum_{i=1}^7 f_i x_i = 2200$$

,

$$\sum_{i=1}^7 f_i (x_i - \bar{x})^2 = 640$$

Mean,

$$\bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{N} = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance, } \sigma^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

Therefore, mean is 100 and variance is 29.09.

Question 6

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution

Ans: Suppose mean $A = 54$ and here, $h = 1$.

Calculating the given data, we get the following table:

x_i	f_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40

67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{\sum_{i=1}^9 N} \times h$$

$$= 64 + \frac{0}{100} \times 1$$

$$= 64 + 0$$

$$= 64$$

$$\begin{aligned} \text{Variance, } \sigma^2 &= \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right] \\ &= \frac{(1)^2}{(100)^2} [100(286) - (0)^2] \\ &= 2.86 \end{aligned}$$

Hence, standard deviation, $\sigma = \sqrt{2.86} = 1.69$

Therefore, mean is 64 and standard deviation is 1.69.

Question 7

Find the mean and variance for the following frequency distribution.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution

Ans: Suppose mean $A = 105$ and here, $h = 30$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	-3	9	-6	18
30-60	3	45	-2	4	-6	12
60-90	5	75	-1	1	-5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^7 f_i y_i}{N} \times h$$

$$= 105 + \frac{2}{30} \times 30$$

$$= 105 + 2$$

$$= 107$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^7 f_i y_i^2 - \left(\sum_{i=1}^7 f_i y_i \right)^2 \right]$$

$$\begin{aligned}
 &= \frac{(30)^2}{(30)^2} [30(76) - (2)^2] \\
 &= 2280 - 4 \\
 &= 2276
 \end{aligned}$$

Therefore, mean is 107 and variance is 2276.

Question 8

Find the mean and variance for the following frequency distribution.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution

Ans: Suppose mean $A = 25$ and here, $h = 10$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	-2	4	-10	20
10-20	8	15	-1	1	-8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^5 f_i y_i}{\sum_{i=1}^5 N} \times h$$

$$= 25 + \frac{10}{50} \times 10$$

$$= 25 + 2$$

$$= 27$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right]$$

$$= \frac{(10)^2}{(50)^2} [50(68) - (10)^2]$$

$$= \frac{1}{25} [50 \times 68 - (10)^2]$$

$$= \frac{3300}{25}$$

$$= 132$$

Therefore, mean is 27 and variance is 132.

Question 9

Find the mean, variance and standard deviation using short-cut method.

Height in cm	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	2	1	12	29	25	12	10	4	5

Solution

Ans: Suppose mean $A = 92.5$ and here, $h = 5$.

Calculating the given data, we get the following table:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	- 4	16	-12	48
75-80	4	77.5	- 3	9	- 12	36
80-85	7	82.5	- 2	4	- 14	28
85-90	7	87.5	- 1	1	- 7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{\sum_{i=1}^9 f_i} \times h$$

$$= 92.5 + \frac{6}{60} \times 5$$

$$= 92.5 + 0.5$$

$$= 93$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right]$$

$$\begin{aligned}
 &= \frac{(5)^2}{(60)^2} [60(254) - (6)^2] \\
 &= \frac{25}{3600} (15204) \\
 &= 105.58
 \end{aligned}$$

Hence, standard deviation, $\sigma = \sqrt{105.58} = 10.27$

Therefore, mean is 93, variance is 105.58 and standard deviation is 10.27.

Question 10

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

(Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 – 48.5, 48.5 – 52.5 and then proceed.)

Solution

Ans: Suppose mean $A = 42.5$ and here, $h = 4$.

Calculating the given data, we get the following table:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	y_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22

48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^5 f_i y_i}{\sum_{i=1}^5 N} \times h$$

$$= 42.5 + \frac{25}{100} \times 4$$

$$= 42.5 + 1$$

$$= 43.5$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right]$$

$$= \frac{(4)^2}{(100)^2} \left[100(19) - (25)^2 \right]$$

$$= \frac{16}{10000} (19275)$$

$$= 30.84$$

Hence, standard deviation, $\sigma = \sqrt{30.84} = 5.55$

Therefore, mean is 43.5, variance is 30.84 and standard deviation is 5.55.