

Chapter 15: Statistics Exercise 15.1

NCERT · Class 11 · Maths

Exercise 15.1

Question 1

Calculate the mean deviation about mean for the numbers

4, 7, 8, 9, 10, 12, 13, 17.

Solution

Ans.

We are provided the numbers

4, 7, 8, 9, 10, 12, 13, 17.

Therefore, mean of the numbers,

$$\begin{aligned}\bar{x} &= \frac{4 + 7 + 8 + 9 + 10 + 12 + 13 + 17}{8} \\ &= \frac{80}{8} \\ &= 10\end{aligned}$$

The deviations for the given numbers with the mean $\bar{x} = 10$,

that is $x_i - \bar{x}$, for $i = 1, 2, \dots, 8$ are

−6, −3, −2, −1, 0, 2, 3, 7.

Also, the absolute values of the obtained deviations, that is $|x_i - \bar{x}|$ are

6, 3, 2, 1, 0, 2, 3, 7.

Thus, the mean deviation about the mean $\bar{x}$ for the given numbers is

$$\begin{aligned} M.D. (\bar{x}) &= \frac{\sum_{i=1}^8 |x_i - \overline{line}x|}{8} \\ &= \frac{6 + 3 + 2 + 1 + 0 + 2 + 3 + 7}{8} \\ &= \frac{24}{8} \\ &= 3 \end{aligned}$$

Question 2

Calculate the mean deviation about the mean for the following numbers.

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution

Ans.

We are provided that numbers

38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

Therefore, the mean of the numbers,

$$\begin{aligned} \bar{x} &= \frac{38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44}{10} \\ &= \frac{500}{10} \\ &= 50 \end{aligned}$$

Now, the deviations for the given numbers with the mean $\bar{x} = 50$, that is

$x_i - \bar{x}$, for $i = 1, 2, 3, \dots, 10$ are

$-12, 20, -2, -10, -8, 5, 13, -4, 4, -6$.

So, the absolute values of the obtained deviations for the given numbers, that is $|x_i - \bar{x}|$ are

$12, 20, 2, 10, 8, 5, 13, 4, 4, 6$.

Thus, the mean deviation about the mean $\bar{x}$ for the given numbers is

$$\begin{aligned} M.D. (\bar{x}) &= \frac{\sum_{i=1}^{10} |x_i - \bar{x}|}{10} \\ &= \frac{12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6}{10} \\ &= \frac{84}{10} \\ &= 8.4 \end{aligned}$$

Question 3

Calculate the mean deviation about the median for the following numbers.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

Solution

Ans.

We are provided the numbers

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

There is total 12 numbers, and it is even.

Now, arranging the numbers in the ascending order, gives

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.

It is known that,

$$\text{Median, } M = \frac{\left(\frac{n}{2}\right)^{th} \text{ number} + \left(\frac{n}{2} + 1\right)^{th} \text{ number}}{2}, \text{ when } n \text{ is even.}$$

Therefore, the median

$$\begin{aligned}
 M &= \frac{\left(\frac{12}{2}\right)^{th} \text{ number} + \left(\frac{12}{2} + 1\right)^{th} \text{ number}}{2} \\
 &= \frac{6^{th} \text{ number} + 7^{th} \text{ number}}{2} \\
 &= \frac{13 + 14}{2} \\
 &= \frac{27}{2} \\
 &= 13.5
 \end{aligned}$$

Now, the deviations about the median M for the given numbers, that is $x_i - M$, for $i = 1, 2, 3, \dots, 12$ are

$$-3.5, -2.5, -2.5, -1.5, -0.5, -0.5, 0.5, 2.5, 3.5, 3.5, 3.5, 4.5$$

So, the absolute values of the resulted deviations, that is $|x_i - M|$, for $i = 1, 2, 3, \dots, 12$ are $3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5$.

Thus, the mean deviation about the median M for the given numbers,

$$\begin{aligned}
 M.D.(M) &= \frac{\sum_{i=1}^{12} |x_i - M|}{12} \\
 &= \frac{3.5 + 2.5 + 2.5 + 1.5 + 0.5 + 0.5 + 0.5 + 2.5 + 2.5 + 3.5 + 3.5 + 4.5}{12} \\
 &= \frac{28}{12} \\
 &= 2.33
 \end{aligned}$$

Question 4

Calculate the mean deviation about the median for the following numbers.

36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

Solution

Ans.

We are provided the numbers

36, 72, 46, 42, 60, 45, 53, 46, 51, 49.

There is total 10 numbers, and it is even.

Now, arranging the numbers in the ascending order, gives

36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

It is known that,

$$\text{Median, } M = \frac{\left(\frac{n}{2}\right)^{th} \text{ number} + \left(\frac{n}{2} + 1\right)^{th} \text{ number}}{2}, \text{ when } n \text{ is even.}$$

Therefore, the median

$$\begin{aligned} M &= \frac{\left(\frac{10}{2}\right)^{th} \text{ number} + \left(\frac{10}{2} + 1\right)^{th} \text{ number}}{2} \\ &= \frac{5^{th} \text{ number} + 6^{th} \text{ number}}{2} \\ &= \frac{46 + 49}{2} \\ &= \frac{95}{2} \\ &= 47.5 \end{aligned}$$

Now, the deviations about the median M for the given numbers, that is $x_i - M$, for $i = 1, 2, 3, \dots, 10$ are

-11.5, -5.5, -2.5, -1.5, -1.5, 1.5, 3.5, 5.5, 12.5, 24.5

So, the absolute values of the resulted deviations, that is $|x_i - M|$, for $i = 1, 2, 3, \dots, 10$ are

11.5, 5.5, 2.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5.

Thus, the mean deviation about the median M for the given numbers,

$$\begin{aligned}
 M.D.(M) &= \frac{\sum_{i=1}^{10} |x_i - M|}{10} \\
 &= \frac{11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5}{10} \\
 &= \frac{70}{10} \\
 &= 7.
 \end{aligned}$$

Question 5

Use the following data to calculate the mean deviation about the mean.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution

Ans.

Consider the following table of data.

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18

25	5	125	11	55
Total	$\sum f_i = 25$	$\sum f_i x_i = 350$		$\sum f_i x_i - \bar{x} = 158$

Thus, from the above table we have,

$$N = \sum_{i=1}^5 f_i = 25,$$

$$\sum_{i=1}^5 f_i x_i = 350$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^5 f_i x_i$$

$$= \frac{1}{25} \times 350$$

$$= 14$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^5 f_i |x_i - \bar{x}|$$

$$= \frac{1}{25} \times 158$$

$$= 6.32$$

Question 6

Use the following data to calculate the mean deviation about the mean.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution

Ans.

Consider the following table of data.

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
Total	$\sum f_i = 80$	$\sum f_i x_i = 4000$		$\sum f_i x_i - \bar{x} = 1280$

Thus, from the above table we have,

$$N = \sum_{i=1}^5 f_i = 80,$$

$$\sum_{i=1}^5 f_i x_i = 4000$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^5 f_i x_i$$

$$= \frac{1}{80} \times 4000$$

$$= 50$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^5 f_i |x_i - \bar{x}|$$

$$= \frac{1}{80} \times 1280$$

$$= 16$$

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Question 7

Use the following data to calculate the mean deviation about the median.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution

Ans.

Notice that the given data is already in ascending order.

Consider the following table that shows the cumulative frequencies of the provided data.

x_i	f_i	$c.f.$

5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Thus, from the above table we have,

$N = 26$, that is an even number.

Therefore, the median can be obtained by calculating the mean of the 13^{th} and 14^{th} observations and both the observations belongs to the cumulative frequency 14.

Therefore,

Median,

$$M = \frac{13^{th} \text{ observation} + 14^{th} \text{ observation}}{2}$$

$$= \frac{7 + 7}{2}$$

$$= 7$$

Now, the following table shows the absolute values of the deviations about the median, that is $|x_i - M|$.

$ x_i - M $	2	0	2	3	5	8	Total
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f_i	8	6	2	2	2	6	$\sum f_i = 26$
$f_i x_i - M $	16	0	4	6	10	48	$\sum f_i x_i - M = 84$

Thus, from the above table, we have

$$\sum_{i=1}^6 f_i = 26 \text{ and } \sum_{i=1}^6 f_i |x_i - M| = 84.$$

Hence, the required mean deviation about the median,

$$M.D(M) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - M|$$

$$= \frac{1}{26} \times 84$$

$$= 3.23$$

Question 8

Use the following data to calculate the mean deviation about the median.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution

Ans.

Notice that the given data is already in ascending order.

Consider the following table that shows the cumulative frequencies of the provided data.

x_i	f_i	$c. f.$
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

Thus, from the above table we have,

$N = 29$, that is an odd number.

Therefore, the median will be the $\frac{29 + 1}{2} = \frac{30}{2} = 15^{th}$ observation.

Notice that, the 15^{th} observation belongs to the cumulative frequency 21. So, the median is the corresponding observation, that is 30.

Now, the following table shows the absolute values of the deviations about the median, that is $|x_i - M|$.

$ x_i - M $	15	9	3	0	5	Total
f_i	3	5	6	7	8	$\sum f_i = 29$
$f_i x_i - M $	45	45	18	0	40	$\sum f_i x_i - M = 148$

Thus, from the above table, we have

$$\sum_{i=1}^5 f_i = 29 \text{ and } \sum_{i=1}^5 f_i |x_i - M| = 148.$$

Hence, the required mean deviation about the median,

$$\begin{aligned} M.D(M) &= \frac{1}{N} \sum_{i=1}^5 f_i |x_i - M| \\ &= \frac{1}{29} \times 148 \\ &= 5.1 \end{aligned}$$

Question 9

Use the following data to calculate the mean deviation about the mean.

Income/ day	Number of persons
0 – 100	4
100 – 200	8
200 – 300	9
300 – 400	10
400 – 500	7
500 – 600	5
600 – 700	4
700 – 800	3

Solution

Ans.

Consider the following table of data.

Income/day	Number of persons f_i	Mid-point x_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1168
700 – 800	3	750	2250	392	1176
Total	$\sum f_i = 50$		$\sum f_i x_i = 17900$		7896

Thus, from the above table we have,

$$N = \sum_{i=1}^8 f_i = 50$$

,

$$\sum_{i=1}^8 f_i x_i = 17900$$

Therefore, the mean,

$$\begin{aligned}\bar{x} &= \frac{1}{N} \sum_{i=1}^8 f_i x_i \\ &= \frac{1}{50} \times 17900\end{aligned}$$

$$= 358$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^8 f_i |x_i - \bar{x}|$$

$$= \frac{1}{50} \times 7896$$

$$= 157.92$$

Question 10

Use the following data to calculate the mean deviation about the mean.

Height (cm)	Number of men
95 – 105	9
105 – 115	13
115 – 125	26
125 – 135	30
135 – 145	12
145 – 155	10

Solution

Ans.

Consider the following table of data.

Height (cm)	Number of men f_i	Mid-point x_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8
125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 – 155	10	150	1500	24.7	247
Total	$\sum f_i = 100$		$\sum f_i x_i = 12530$		1128.8

Thus, from the above table we have,

$$N = \sum_{i=1}^6 f_i = 100$$

,

$$\sum_{i=1}^6 f_i x_i = 12530$$

Therefore, the mean,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^6 f_i x_i$$

$$= \frac{1}{100} \times 12530$$

$$= 125.3$$

Hence, the mean deviation about the mean is given by

$$M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - \bar{x}|$$

$$= \frac{1}{100} \times 1128.8$$

$$= 11.28$$

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Question 11

Use the following data including ages of 100 men to find the mean deviation about the median.

Age	Number of men
16 – 20	5
21 – 25	6
26 – 30	12
31 – 35	14

36 – 40	26
41 – 45	12
46 – 50	16
51 – 55	9

Solution

Ans.

Observe that the provided data is not continuous. So, to convert these discrete data to continuous data, subtract 0.5 from the lower limit and add 0.5 to the upper limit of the ages.

So, consider the following table of data.

Age	Number of men f_i	Cumulative frequency (c.f.)	Mid-point x_i	$ x_i - Med. $	$f_i x_i - Med. $
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90
25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	26	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5 – 50.5	16	91	48	10	160
50.5 – 55.5	9	100	53	15	135

Total	100				735
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Therefore, here $N = 100$ and it is an even number.

So, $\frac{N}{2} = \frac{100}{2} = 50$. That is, the cumulative frequency greater than 50 is 63. Thus, the median class = 35.5 – 40.5.

Now, we know that,

$$\text{Median, } M = l + \frac{\frac{N}{2} - C}{f} \times h$$

We have $l = 35.5$, $C = 37$, $f = 26$, $h = 5$. Also, $N = 100$.

Therefore,

$$M = 35.5 + \frac{50 - 37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38.$$

Hence, the mean deviation about the median for the given data is

$$M.D.(M) = \frac{1}{N} \sum_{i=1}^8 f_i |x_i - M|$$

$$= \frac{1}{100} \times 735$$

$$= 7.35$$